# Students' learning difficulties in secondary mathematics classroom in Bungoma County and pedagogical remedies by the teachers to help students overcome these difficulties. 

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#### Abstract

The study was designed to identify students' learning difficulties in secondary mathematics classroom, to investigate the possible causes of the errors and pedagogical remedies by the teachers to help students overcome their difficulties that hinder in-depth learning in secondary mathematics.

Descriptive survey, design was adopted for this study. Based on the input of two hundred and forty form four students and twelve secondary school mathematics teachers, an attempt was made to develop an overview of difficult curriculum topics in secondary mathematics. Teachers' knowledge on students' errors was investigated together with strategies for remedial teaching. Data was collected using a test that was conducted personally by the researchers. There were ten conceptual areas determined and included in the test viz: statistics, probability, vectors, longitudes and latitudes, linear programming, loci, circle, trigonometry, matrices and three dimensional geometry. Multiitems were developed in each of the areas with different difficulties in order to have an idea of which of the selected ten areas was posing threat of errors and misconceptions amongst the sample students. Semi structured interviews were also carried out among 25 selected students to identify the errors they had made in the process of solving mathematics. Data were analyzed using SPSS. Itemwise was conducted to identify errors students committed. Providing reflective accounts from teachers' experience and presenting illustrative examples from their classrooms, the study provides abroad picture of the context in which students learn mathematics. The study identifies the factors that hinder students from gaining in-depth learning of mathematics concepts. The major reasons given for errors made were lack of understanding, procedures being forgotten, carelessness, negligence in transcribing information from the question and guesswork. The findings showed that item 4 (probability), item 3 (latitude and longitude), item 6 (linear programming), item 8 (lines planes in 3 dimensions) and loci was the most difficult item in mathematics. Content analysis showed that 148 (61.7\%) students had conceptual errors, 48 ( $20 \%$ ) made careless errors and 15 ( $6.2 \%$ ) made value errors. The study builds on new conceptions about the professional identity and the critical relevance of teachers' pedagogical content knowledge (Shulman, 1987). This study has implications for the student learning process and understanding of mathematical concepts since mathematics is widely used in daily life. The errors made by students in solving mathematics problems provide useful input for mathematics teachers to design teaching and learning activities that help students to be more competent in mathematics.


Key words: Secondary mathematics, mathematical concepts, learning difficulties, pedagogical remedies, Errors.

## INTRODUCTION

Mathematics is regarded as one of the important subjects in the secondary schools curriculum. It is indeed the backbone of most applied subjects, that is, it is a pre-requisite for studying science and technical subjects. Development in the commerce and industry also rely heavily on mathematical knowledge. Mathematics is important in the day to day human activities such as farming, politics and military operations (cockcraft, 1982, Mutungu and Breakel, 1992). Competency in mathematics is the key towards the goal of establishing a scientific and technologically oriented society. Hence any citizen of any country should attain a reasonable level of mathematical literacy if adequate levels of scientific and technological development are to be achieved.

Buxton (1982) asserts that:
"Mathematics is the gate and key of science". Eshiwani (1984) seem to agree with Buxton (1982) when he remarks that a minimum level of science and technology is crucial in the present world of scientific and
technological revolution. It is therefore important that all pupils should acquire basic mathematical skills while a large number of them need affirm grounding in mathematics before they can effectively train to be technicians, accounts clerks, professional scientists or successful businessmen. All training institution whether nursing or others, require a credit, pass or higher in mathematics (Kitagn, 1998). It is perhaps of its importance that the Kenya Government made mathematics a compulsory subject both at primary and secondary school level soon after independence in 1963 (K.I.E, 1979). One of the objectives secondary school mathematics in Kenya is to produce competent graduates who will apply the mathematical skills acquired to develop their mental faculties and thus improve their society. The Kenya certificate of secondary education (KCSE) mathematical syllabus asserts that:

Secondary mathematics aims at producing a person who will be numerate orderly, logical, accurate and precise in thought. The person should be competent in appraising and utilizing mathematics skills in playing a positive role in the development of a modern society (KIE, 2002:3)

The mathematics curriculum in Kenya is based on activities that define the critical abilities of scientifically literate students. These are:

Application of mathematical knowledge and thinking mathematically. The doing and reflecting on mathematical knowledge are performance indicators that are connected with knowledge and skills students should acquire in the subject.

Despite its importance, mathematics is viewed in Kenya and even in the developed world as a difficult subject to learn and thus most students leave school without grasping the fundamental arithmetic.

Quite a number of students who leave school after national examinations hardly transact the day to day business because they are incapable of calculating simple arithmetic.

The Kenya Institute of Education and the Kenya National Examination Council lay emphasis on specific concepts and skill that must be mastered by students at the secondary school level (KIE, 2002, KNEC 2000). This includes measurement, probability, calculus and scale drawing.

An analysis of the Kenya Certificate of Secondary Education Examination question papers and reports indicates that questions on probability, geometry, trigonometry, vectors, latitude and longitudes are being set every year, yet no remarkable improvement in students' performance has been realized (KNEC, 2006).

The analysis for the year (2000-2003) of Kenya Certificate of Secondary Education Examination results indicates that students have continued to maintain a low mean mark of less than $20 \%$ in mathematics.
A further analysis of KCSE mathematics performance from 2005 to 2008 reveals similar trends of results.
A wide range of factors are responsible for poor quality of mathematics education. The factors such as teachers' poor subject knowledge, teachers lacking in pedagogical competence and teachers perception about mathematical knowledge hinder students from developing mathematical understanding (Amirali \& Halal, 2010).
The purpose of this study was designed to identify students' learning difficulties in secondary mathematics classroom. The researcher wished to identify the kinds of errors and misconceptions students make at secondary level and investigate the possible causes for these errors and misconceptions and to suggest remedial measures for the problems faced by the students. The errors and misconceptions that students develop during previous classes or bring with them to the school from the community can create hindrances in the ongoing learning of mathematics/conceptions, consequently producing poor achievement in mathematics. Students bring with them a variety of conceptions, abilities, skills, knowledge, interest, attitudes, beliefs, aspirations, expectations, habits and preferences, which may not be in harmony with the demands of mathematics. There is, therefore, a need to identify not only the conceptual areas where most of the students make errors or construct wrong generalizations but also the reasons responsible for those and how to rectify or correct them. This need has been translated into this study.
During the $39^{\text {th }}$ meeting of international commission for the study and improvement of mathematics teaching (CIEAEM) in 1987 (in Sherbrooke, Canada) the key issue deliberated upon was the role played by errors in the learning of mathematics. According toMigon, J. (2007), in the CIEAEM (1987) it was states that "an error takes place when a person chooses the false as the truth". When the actual results is different from the erroneous
results, when the procedure adopted is different from the accepted procedures, erroneous conceptions can hinder problem solving and produce irrational results.

An error might be caused due to a misconception, or other factors which may include carelessness, problems in reading or interpreting a question.

A misconception is the result of lack of understanding or misapplication (Spooner, 2002). Misconceptions interfere with learning when students use them to interpret new experiences. Usually, students become emotionally and intellectually attached to their misconceptions, because they have actively constructed them. Knowing students misconceptions will help the teachers to teach better. Bloom (1984) in his taxonomy of educational objectives believed, that the unit tests most teachers used did little more than show for whom the initial instruction was or was not appropriate. He asserts that, checking on students' learning at the end of each unit are useful instructional techniques and if these checks on learning were accompanied by a feedback and corrective procedure, they could serve as valuable learning tools.

Instead of marking the end of the unit, Bloom recommended these test be used to diagnose individual learning difficulties (feedback) and to prescribe specific remediation procedures (corrections). This study was designed to identify learning difficulties in secondary mathematics classrooms.

Dumont (1994), distinguishes two types of learning problems: "a learning disability" and "a learning difficulty".
A learning disability is situated in the child's own cognitive development where as the cause of a learning difficulty is situated outside the child. As cited by (Carnine, Jitendra\&Silbert, 1977), "individuals who exhibit learning difficulties may not be intellectually impaired, rather, their learning problems may be the results of an inadequate design of instruction in curricular materials". To improve teaching and learning process in mathematics classrooms requires a better understanding of the real nature of the common difficulties that hinder conceptual learning, particularly at secondary level as well as the pedagogical remedies by the teachers, to help students overcome these difficulties. The concern is to see students learn more than memorization and recalling factual information provided in the textbook- a situation that is happening in many mathematics classrooms in Kenya. Teachers try to transmit knowledge to students that is prescribed in textbook, asses students' learning through getting them to define or apply rules in a prescribed way (Amirali\&Halai, 2010).

## STATEMENT OF THE PROBLEM

Mathematics, as one of the core curriculum subjects, is taught in all schools in Kenya from grade 1 to form four. The mathematics curriculum prescribed for secondary classes contains a wide spectrum of concepts that have to be learnt and mastered by the students. Mathematics curriculum contains specialized knowledge which needs certain attitudes, frame of mind (analytical and logical thinking) and demand efforts on the part of learner (Ellis, 2011). Unfortunately, in many high schools in Kenya teachers usually fail to instill and nurture these critical abilities in students. Teachers try to transmit the knowledge to students that is prescribed in textbook, assess students' learning through getting them to define or apply rules in a prescribed way. (Amirali\&Holai, 2010). Students memorize rules without understanding why they are doing any of it (Mohammad, 2002). Teaching and learning processes in mathematics classrooms needs to be improved and it requires a better understanding of the real nature of the common difficulties that hinder conceptual learning, especially at secondary school level, as well as the pedagogical remedies by the teachers, to help students overcome the difficulties. The concern primarily arises from a desire to see students learn more than memorization and recalling factual information provided in the textbook.

## THEORETICAL BACKGROUND

The study is based on Bloom'swork of mastery learning which was influenced by Harvard University Professor John B. Caroll entitled "A model for school learning". Bloom (1776) believed that the history of any leaner (cognitive entry behaviours and affective entry characteristics) interact with cues, participation, reinforcement, and feedback/correctives to determine the quality of instruction. According to Bloom (1976), the concepts and material to be learned is organized into small learning units where students' learning is checked at the end of each unit. He believed that checks (tests) on learning should be accompanied by a feedback and corrective
procedure. Bloom recommended the tests be used to diagnose individual learning difficulties (feedback) and to prescribe specific remediation procedures (correctives). Under feedback and corrective procedure, the teacher points out the error (feedback) when the student makes a mistake and then follows up with further explanation and clarification (corrective). Successful students follow up the mistakes they make on quizzes and tests, seeking further information and greater understanding so that their errors are not repeated.

## RESEARCH QUESTIONS

The following are the questions that guided the study:

1. What common conceptual learning difficulties do students face in their mathematics classroom?
2. What are the possible causes of these conceptual difficulties?
3. What pedagogical remedies and moves do the teachers use to help students overcome these difficulties?

## METHODOLOGY

The study intends to identify students' learning difficulties (errors and misconceptions) in secondary mathematics classroom, to investigate the possible causes of the errors and pedagogical remedies designed by the teachers to help students overcome their difficulties that hinder in depth learning of mathematics. A test consisting of 10 items including ten conceptual areas was used to collect data. After conduction test, the errors made by the sample students were identified. In addition to identification of the errors, semi-structured interviews were also carried out among 25 selected students to identify the errors they had made in the process of solving mathematics. The study sets to describe and explain how ten high school teachers identified in their day -to-day teaching activities, try to use pedagogical remedies to help their students overcome the difficulties that hinder in depth learning in secondary mathematics classrooms.

## SAMPLE AND SAMPLING

There are 254 secondary schools in Bungoma County with an enrolment of approximately 15000 form four students attending school.

The sample constituted form four students $(\mathrm{N}=240)$ from ten secondary schools and ten teachers teaching mathematics in form four. Teachers on average have 16.6 years of experience in education. On average they have 10.58 years of experience in the current class they teach.

## DATA COLLECTION INSTRUMENTS

Data was collected using a test and semi-structured interviews amongst the sample students to identify the errors they had made in the process of solving mathematics. Multi-items were constructed in relation to ten mathematics domains that reoccur in each of them. Specific questionnaires were presented to ten identified teachers of mathematics. The questionnaire required teachers to mark the test and identify errors and misconceptions made by sample students in the process of solving mathematics, investigate the possible causes of the errors and pedagogical remedies by the teachers to help students overcome their difficulties that hinder in depth learning of mathematics.

## DATA COLLECTION AND ANALYSIS

The following procedure was adopted:
A pre-test was administered to the sample students and then it was marked. The scores of the students provided initial information about the areas of mathematics where the students committed errors. The identification of
errors was done through the pre-test. The information regarding mean scores of the students per school and topic wise before item wise is given in Table 4.1

Table 1: Mean scores of the schools in 10 sub topics

| $\begin{aligned} & .0 .0 \\ & 2 \\ & 0 \\ & 0 \\ & 0 \\ & =1 \end{aligned}$ |  | $\begin{aligned} & n \\ & 0.0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | U U E E | 崖 |  | $\begin{aligned} & \text { 를 } \\ & \text { त్ర } \\ & \text { 을 } \end{aligned}$ |  |  | - |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| School A | 2.1700 | 1.6500 | 0.6500 | 0.6600 | 1.1300 | 0.9700 | 1.5800 | 2.1436 | 0.8321 | 1.2001 |
| School B | 1.6536 | 0.8302 | 0.6471 | 0.7413 | 0.5461 | 0.3117 | 0.1176 | 0.6547 | 0.3158 | 0.8421 |
| School C | 1.9475 | 1.4475 | 1.3125 | 0.9276 | 1.6000 | 0.6500 | 0.6500 | 2.1240 | 1.2450 | 0.9412 |
| School D | 2.8750 | 1.7250 | 0.6500 | 1.4840 | 0.4000 | 0.4000 | 0.7500 | 2.6431 | 0.3148 | 0.5300 |
| School E | 5.0432 | 1.0436 | 1.4347 | 1.3479 | 1.7481 | 1.3578 | 0.7291 | 1.0425 | 0.4281 | 1.1054 |
| School F | 1.5778 | 1.7368 | 1.6842 | 1.1489 | 0.7685 | 0.9548 | 0.9231 | 2.8840 | 1.1528 | 2.0412 |
| School G | 4.0400 | 2.0300 | 2.6530 | 3.1500 | 0.4800 | 2.8830 | 1.0214 | 2.0500 | 0.6832 | 1.8431 |
| School H | 1.0444 | 1.6000 | 1.8556 | 1.7000 | 0.6421 | 1.3000 | 1.2000 | 1.5789 | 1.0214 | 1.8120 |
| School I | 3.0000 | 1.4444 | 1.5349 | 1.1538 | 0.8956 | 1.8421 | 1.3135 | 2.5000 | 1.0000 | 1.2391 |
| School J | 5.1444 | 1.2556 | 1.0335 | 1.6000 | 0.7257 | 0.9538 | 0.6848 | 1.1200 | 0.8411 | 1.6400 |
| Overall measure | 2.85 | 1.48 | 1.3455 | 1.3914 | 0.8936 | 1.1623 | 0.8969 | 1.8741 | 0.7834 | 1.3194 |

The mean score is derived by adding the topic wise marks of the students of the particular school and dividing it by the number of the students in the test. The overall mean scores is obtained by adding the mean scores obtained by all the schools in a particular topic and dividing it by the number of schools.


Figure 1: mean score of students of form four.
The figure shows the overall mean scores of the sample schools attained by students of form four topic wise. It is evident from the figure that students obtained relatively higher mean score in the items that tested the concepts of statistics and circle.

However, students scored slightly above the mean of 1.00 in vectors, matrices, trigonometry, latitude and longitudes and probability indicating that these topics were still posing a threat to students in the sample schools. The topics that proved most difficult for students across the sample schools were three -dimensional geometry, latitude and longitude and loci where students' performance remained considerably low (below the mean of 1.00).

## Findings

The pretest consisted of 10 items that tested ten conceptual areas. The test was marked, marks awarded and analysis done on each student's processing of the problem in terms of accuracy, correct method followed and demonstrate understanding.

In line with the research questions, two major categories were used to process the data:
(1) Teachers' views about conceptual learning of subject matter and the issues underlying it, with particular focus on mathematical concepts prescribed in the curriculum at secondary level and
(2) The ways in which the teachers recognize the conceptual difficulties facing students in learning of the concepts, how they go about helping students overcome those difficulties. As a result of the analysis (Hiebert J. \& Carpenter, T.P. (1992), Yoe, K.K. (2010), a range of themes emerged relating to the contexts that reveal challenges to conceptual learning, the nature of these challenges and teachers' efforts to help students overcome them.
In the following, item-wise analysis of students process is discussed:
Question 1: This question was meant to test learners on construction. It required sample students to use ruler and compass only to construct a triangle ABC in which AB is 6 cm and BC 7 cm with angle $\mathrm{ABC} 75^{\circ}$. They were then asked to measure length AC and angle ACB. Sample students were also required to construct locus of Q such that Q is on one side of BC opposite A and angle CBQ as $30^{\circ}$. The response to this question showed that only 60 $(25 \%)$ students responded well while the rest $180(75 \%)$ did not respond to the item. This indicated that questions on construction are difficult for the sample students especially the construction of loci. The general problem encountered by the students was their inability to use ruler and compass only. Most of the students who did not respond to this question used protector to measure the angle.

In interview with the sample students, it was revealed that they were unable to drop a perpendicular from a given point to a stated line in the triangle, indicating that they did not understand what was required of them by not following instructions of the question that required them to use ruler and compass only. Infact most of the students from the sample schools used rulers to draw a perpendicular instead of constructing, it was disclosed during the interviews that the students had very little experience of using ruler, protractor, divider, compass etc. to measure line segments, angles and construct LOCI. Without doing with hands it seemed impossible a student can draw geometrical figures correctly

For teachers to overcome these difficulties encountered by the sample students, they unanimously agreed that during instructions on geometry, all students should have geometrical apparatus to practice with on construction. A variety of items testing concepts in geometry should be given to learners where they will get used to handling of geometrical apparatus during the construction.

Question 2: Question two tested concepts on vectors. It required sample students to determine the magnitude of a vector in three dimension and show collinearity and parallel vectors.

The response to this question showed that 80 students from the sample schools representing $33.3 \%$ responded well while the majority of the students 160 ( $66.7 \%$ ) did not understand the requirements of the item. They were unable to identify the magnitude symbol. They were also unable to find the column vector hence could not come up with the column vectors from the three given points. To overcome difficulties faced by the learners on vectors, teachers should provide a lot of practice on order of operations, recall of the magnitude symbol and encouraging students to engage in frequent consultations with their teachers. Frequent testing on the concepts was necessary if students are to grasp the concepts on vectors.

Question 3: This question tested the concepts in longitudes and latitudes. The diagram of the earth's surface was provided. The sample students were required to carry out a number of operations pertaining to the diagram: They were to locate the position of the points labelled in the diagram, calculate the distance between the given points in kilometres and nautical miles, calculate the shortest distance between two given points in kilometres and calculate local time at the town when the plane landed. This question again appeared difficult to most of the students in the sample schools. Only 35 students representing $14.6 \%$ answered it well but the rest 205 ( $85.4 \%$ ) students did not.

A number of errors committed by the sample students were identified during the marking of this question:
Sample students were unable to locate the position of the points asked to. Most of them started with the Eastings instead of Northings and the Eastings as it is required. In calculating the distance between the given two points on the earth's surface, the sample students could not identify which of the points is on the great circle and which one on the small circle. It is necessary for the learners to have this distinction of the points on the Earth's surface as this calls for different formulae when calculating distance in kilometres and in nautical miles. However in the case where the points are in the small circle, sample students found it difficult to calculate the distance between them. The error identified was the students were unable to use the correct formula for small circle which in this case $\left(\frac{\phi}{8 \pi} \boldsymbol{2 \pi R} \boldsymbol{\operatorname { C o s }} \boldsymbol{\alpha}\right)$ for calculating distance in kilometres where: $\boldsymbol{\infty}$ is the common latitude. In the last part of question three, students were given the time, the speed and the day the plane left a certain town. They were then required to calculate the local time when the plane landed in the next town using the shortest distance. However, in calculating the shortest distance, sample students did not realize that it is the distance along a great circle hence they were required to use formula for distance on great circle $\left(\frac{1}{9} \boldsymbol{x} \boldsymbol{2} \boldsymbol{\pi} \boldsymbol{R}\right)$ where $\boldsymbol{0}=(\mathbf{1 8 0} \mathbf{- 2} \mathbf{x}$ common latitude). It appears that questions on latitude and longitude are difficult to understand. It seems that the sample students did not see the difference between points in a small circle and those on a great circle which calls for different application of formulae. To correct the errors made in question three by the sample students, teachers emphasized on the use of illustrations on chalk board as well as the use of models representing the earth's surface. There is need for students to have a lot of practice for mastery of the concepts in longitude and latitude.

Question 4: Question four tested the concepts in probability. The question was: "A basket contains 7 white balls and 5 yellow balls all identical in size and shape. Jack selects 3 balls at random without replacement." Students were then asked to use a tree diagram to represent the information given in the item. Further they were asked to calculate the probabilities of choosing two white balls and one yellow ball, two yellow balls and one white ball, three balls of the same colour and atleast one white ball.

The response to this question showed that 72 (30\%) students responded well and 168 (70\%) students were unable either to attempt the question or did not understand how to go about answering it. This again demonstrates the fact that students from the sample schools have problems with items testing probability concepts. However, some errors were revealed during the marking of this item: First, sample students were not able to represent the information using a tree diagram. The inability to represent information using tree diagram showed that they could not answer all the subsequent questions that pertained to the diagram. Along with the drawings of a tree diagram, students were unable to differentiate between atleast and atmost. It appears students are not very familiar with the language of probability.

To counteract these errors the study revealed that teachers of mathematics should lay emphasis on the concepts and skills to be attained by their learners. While teaching probability, teachers of mathematics should instill in their learners the ability to distinguish between the following concepts of probability: atleast, atmost, independent and dependent events, mutually exclusive events, with and without replacement and how they are applied in computation. The concepts of probability are fundamental to the study of mathematics, especially at the upper secondary and pre-university levels. The study of pre-university probability and representation of Data, for example, depends heavily on probability concepts. Hence it is important to detect if errors and misconceptions exists at the initial secondary school stage of their learning in probability.

Question 5: Question five tested some statistical concepts. Students were given a grouped distribution table which showed marks obtained by a class in a mathematics mock paper. They were then asked to calculate the median, the standard deviation and the pass mark given the number of students who passed the test.

The response to this question was fantastic. More than a half of the sample (150) students representing $62.5 \%$ answered the question correctly. Only $90(37.5 \%)$ students found it difficult. It appears sample students find the questions from statistics easy to understand. The problem identified in this item was in the calculation of the standard deviation. Some of the students could not link calculation of standard deviation with the variance. Calculating the pass mark also posed a threat to those students who tried to work out the problem.

Question 6: Question six tested students on inequalities and linear programing. Students were required to write down the inequalities from the set of information provided. They were also asked to find the maximum if all the conditions are fulfilled. The response to this question was not satisfactory as only 76 (31.7\%) students tried while the rest $164(68.3 \%)$ found the question quite difficult. It was revealed that students committed errors in writing inequalities. Yet this is the initial stage that is required for linear programming. Students confused the terms atleast meaning less than and atmost which they thought is greater than. Such misconception led to incorrect writing down of the inequalities hence failing to answer the questions correctly. The indication is that these students lack proper foundation. The evidence suggest that students' difficulties in grasping mathematical concept at secondary level are mainly rooted in the knowledge gap they bring with them. For students to be able to solve problems on linear programming, they should be familiar with linear inequalities in two variables as the starting point. Later on teachers will involve their learners in solving systems of linear inequalities graphically.

Question 7: Question seven tested the angle properties of a circle. It required sample students to relate and compute angle subtended by an arc at the centre and circumference. This question was well done on average as $135(56.3 \%)$ students seem to have understood while 105 ( $43.7 \%$ ) still had problem of understanding.

Question 8: Question eight tested the concepts in three dimensional geometry. It required sample students to identify and calculate the angle between a line and a line and the angle between a line and a plane. The response to this question was quite poor across the sample schools. Only 83 (34.6\%) students understood and followed the procedure while 157 (64.4\%) students did not.This is an indication of students' difficult in calculating problems on three dimensional geometry. To counteract these difficulties, teachers of mathematics need to use illustrations and geometrical models which students can see the actual line and planes required.

Question 9: Question nine tested concepts in matrices and transformation. Students were asked to relate, identify matrix and transformation. The response to this question showed $83(34.6 \%)$ students understood what was needed hence followed the correct process and 157 ( $65.4 \%$ ) did not hence found the question difficult. They were unable to equate the given coordinates with its transformation which was a straight forward procedure and relate it to identify matrix. Teachers of mathematics should revisit matrix multiplication and identity matrix.

Question 10: Questions ten tested concepts in Trigonometry. The first part of the questions however asked students to complete the table for $y=\frac{\vdots}{i}\left(u+20^{\circ}\right)$ and $y=\cos 2 u$. They were then asked to draw on the same set axis on the grid provided to draw the graphs from which they could solve the equation $\mathbf{y}$ $=\frac{1}{2} \sin \left(u+20^{\circ}\right)-\cos 2 u=\mathbf{0}$. The response to this question showed that $87(26.3 \%)$ students understood what they were asked to do but the rest 157 ( $65.4 \%$ ) could not. This indicated that trigonometry as a topic is still posing a threat to students.

From the marking, it was detected that students could not, from the onset try to complete the table. Those who were unable to complete the table skipped the subsequent questions pertaining to the table. However those who tried to draw the graphs were not able to solve the equation simply because they lacked the skill of finding where the two graph crossed as the required solution. Teachers of mathematics should subject their learners to plenty of practices exercise with graphs of trigonometric ratios.

Results:
The study was designed to identify students' learning difficulties in secondary mathematics classroom, to investigate the possible causes of the errors and pedagogical remedies by the teachers to help students overcome these difficulties. The results indicate that according to the teachers, the following curriculum topics pose consistently learning difficulties in the fourth form: probability, three dimension geometry, latitude and longitude, loci and liner programming. Next in the ranking are vectors, matrices and trigonometry. The findings indicated that $72 \%$ of the teachers used holistic marking while $28 \%$ used analytical making. The study revealed that the major difficulty seems to lie with the teachers' ability to make use of the knowledge they have on
student error, rather than being aware of the errors. The study showed that teachers need assistance in error identification and how the errors would be used in the learning process.

## Limitation of the study

The study was conducted in ten secondary schools in Bungoma County of western part of Kenya. It may not therefore be possible to generalize the results to a larger population. The research sample was- though representative -not chosen at random.

## Conclusion and implications

From an educational practice point of view, the present study points out that mathematics is experienced as a difficult subject during a student's entire secondary school career. Moreover, the study reveals that particular mathematics topics seem to be more difficult than others and that some curriculum topics are experienced to be difficult in all secondary school classes.

There are multiple reasons for conceptual difficulties students usually face in secondary mathematics classrooms. Some of the reasons include their not being conversant with the ways of learning concepts other than memorization because they might not have been exposed to such experiences before in the previous classes. Their approaches towards learning mathematical concepts seem to have been shaped by their previous classroom practices. Thus, the teachers' reflections together with the data generated by scoring of the test, imply that students have limited understanding of fundamental concepts as being the primary factor contributing to students' inability to gain command over subject matter knowledge at secondary level. The problems that constraint student understanding of subject matter knowledge in mathematics at secondary level is surrounded mainly by the poor prior knowledge background students bring with them. Lacking prior knowledge of concepts is a deficiency, which may not be addressed easily. Both teacher and student need to work hard during the critical stages of students learning primary concepts. The findings accords with the lessons reported by other studies that emphasize the link between students' prior knowledge and new knowledge (Gollub et al, 1993). Building in the overview of the difficulties experience $s$ in relation to mathematics curriculum topics, teachers can start to develop specific interventions to overcome the occurrences of mathematics learning difficulties. To sum up, students' mastery learning is more complex. It makes teaching harder because of the high demands it places on time, resources, energy, expertise, commitment and creative mental efforts on the part of both the teacher and the learners. The above conclusions have important implications for understanding of what actually exist in secondary mathematics classrooms.

In the context of teachers, it is relevant to consider what teachers need to know and be able to do in order to promote deeper understanding of the subject-matter knowledge in mathematics. This unavoidably places demand on teachers' knowledge of subject matter, pedagogical content knowledge, knowledge of the learner, knowledge of the curriculum and knowledge of test and evaluation, better understanding of new classroom management strategies, knowledge of resources management and rediness to accept and ability to cope with the diversified challenges associated with student learning. The high demands of conceptual learning require mathematics teachers to letting go of transmission-oriented practices, they need to carefully prepare lesson plans, blackboard work, home assignments and assessment tasks, in order to be able to think about and convey the subject matter in different ways. Schools should provide opportunities for teachers to enhance their content knowledge, deepen and widen their knowledge of innovative pedagogies and ongoing assessment techniques.

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## Suggestion for further research

Future research should focus on teachers' knowledge, teacher practices and student outcomes in order to develop a more profound picture of mathematics learning difficulties in secondary schools.

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