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To Develop Linear Programming (LP) Model to Determine the Best Combination or Mix of Products to Produce to Reach the Maximum Profit

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Abstract
Lending is a life wire or prominent business activity for banks. Loan portfolio therefore form a substantial amount of the assets of banks because it is the predominate source of interest income. The study was carried out to establish the impact of Linear programming model on the financial performance of banks, focusing on the Rural Bank X Lamashegu branch so that it can allocate funds to prospective loan seekers in order to maximize profits. To achieve this goal, a secondary data from the annual reports and financial statements were extracted for this study. Based on this data, LP model was formulated. A computerized software application called QM windows solver based on Revised Simplex Algorithm was used to solve the problem. The results from the model showed that Rural Bank X will be making annual profit of GH¢ 189144.14 from the amount GH ¢825812.77 invested on loan portfolio as against GH¢ 96879.00 in 2013 if the bank had the LP model in place. These product mix (ie. Midrofinance, Over draft and Commercial loans) were recommended for investment. The study also identified ineffective loan monitoring and poor credit vetting as the major factors accounting for some of the loan not performing in the loan portfolio, especially the Personal and Agricultural and Agro processing loans.
Keywords: Loan portfolio, Linear programming, Simplex method, Bad debt and Over draft

1. Introduction

Banks can no longer manage loan books in isolation. The disease called “Bad debt” in the banking industry which caused about ninety percent (90%) of the inefficiencies of the Commercial banks in Ghana from making maximum profit of the capital they invested in the loan portfolio cannot be over emphasis. This can be related to what the Executive Secretary of the Ghana Bankers Association Mr D. K. Mensah said in 2014. He complained that the high rate of non-performing loans on the books of the Commercial banks is sparking worries among bankers and industry watchers. This has compelled Commercial banks to review their internal controls, credit appraisal and risk management policies to reduce the soaring rate of loan default or bad debts.

Lending is the principal business activity for most commercial banks. The loan portfolio is typically the largest asset and the predominate source of revenue. As such, it is one of the greatest sources of risk to a bank’s safety and soundness. Whether due to lax credit standards, poor portfolio risk management, or weakness in the economy, loan portfolio problems have historically been the major cause of bank losses and failures. (Comtroller, 1998).

It is an indisputable fact that most commercial banks operating in Ghana are faced with the complex problem of how to manage their loans portfolio in such a manner that the goals of the bank are best achieved.

Feldman and Valdez-Flores (2010), stated that, financial institutions aim at giving loan packages to their customers in order to maximize profit. Loan portfolio management (LPM) is a key function for banks with large, multifaceted portfolios of credit, often including illiquid loans. Historically, its role has been to understand the institution’s aggregate credit risk, improve returns on those risks, sometimes by trading loans in the secondary market, and hedging and identifying and managing concentrations of risk. In contrast to traditional origination and credit risk-management functions that look only at individual deals or borrowers, LPM looks across the entire credit book.

Portfolio management is the art and science of making decisions about investment mix and policy, matching investments to objectives, asset allocation for individuals and institutions, and balancing risk against performance. Portfolio management is all about determining strengths, weaknesses, opportunities and threats in the choice of debt vs. equity, domestic vs. international, growth vs. safety, and many other trade-offs encountered in the attempt to maximize return at a given appetite for risk. (https://www.investopedia.com/terms/p/portfoliomangement.asp#ixzz4YY4XxL).

The risk of a trading partner not fulfilling his or her obligation as per the contract on due date or anytime thereafter can seriously jeopardize the smooth running bank’s loan portfolio management. On the other hand, a bank with high credit risk has bankruptcy risk that puts the depositors and stakeholders in jeopardy. In a bid to survive and maintain adequate profit level in this highly competitive environment, banks have tended to take excessive risks. But then the increasing tendency for greater risk taking has resulted in insolvency and failure of a large number of the banks. However, the higher the volume of loans income and hence the profit potentials for the commercial banks.

Commercial Lending is a key business activity in the financial services sector. The loan portfolio is one of the largest assets and a chief source of revenue for banks, but is also a great source of risk to a bank's safety and soundness. Whether due to lenient credit standards, poor portfolio risk management, or weaknesses in the economy, loan portfolio problems have historically been among the major cause of financial institutions' losses and failures.

1.1 Background of the Study

Rural Bank X Lamashegu branch loan portfolio recorded marginal profits with some running at a loss. Loan is granted in a form of contract between the creditor and the debtor. Due to poor allocation and loans disbursement, the bank is not able to optimize profits from the loans it disbursed out to its customers. Funds that could have been used to offer social services like provision of good drinking water, scholarship to brilliant but needy students, building of schools, hospitals etc in the community in which they operate runs into what is known as "Bad Debts". As a result, the Rural Bank X is seriously looking for a device or a mathematical model that will enable the bank to solve the problem of loan disbursement efficiently for both long term and short term basis in order to maximize profit. Hence the need for this study.
(https://www.casewareanalytics.com/solutions/business-solutions/loan-portfolio-monitoring)

1.2 Credit Facilities of Rural Bank X Limited

Table 1. The bank's credit facilities:

Type of Loans	Interest Rate	Probability of Bad Debt
Personal loan	27%	15%
Micro Finance	30%	4%
Overdraft	15%	1%
Commercial loans	28%	1%
Agricultural Loan	26%	7.5%

Source: Rural Bank X Limited (2014)

In the table, the first column shows the type of loans the banks provides to its customers, the second column shows the interest rate the bank charges on each loan and the third column shows the anticipated probability of default payment of each loan. This study covered the five composite variables with the aim of establishing their effect on profitability on Rural Bank X. For policy reasons, there are limits on how Rural Bank X allocate funds within its catchment area. The competition with other banking and financial institutions in the Municipality requires that the bank disburse forty percent (40%) of their funds as loans and sixty percent (60%) for their cash reserves. The following is how the bank disburses the forty percent (40%) of its funds for the year, 2014 (i.e. GH ¢825812.77). The policy of the bank requires that the bank allocate at least 20% of the total funds to Personal and Microfinance loans. To assist Agric and Agro processing in the region, must be at least 30% of the total funds, Commercial loans must be at 50% of funds allocated to Personal loans, Commercial loans and Agricultural loans.

The bank also has a stated policy specifying that the overall ratio for bad debts on all loans may not exceed 0.04.

1.3 Objective:

Developing linear programming (LP) model to determine the best mixture of credit products that is most likely to provide a combination of risk and expected return that is optimal for the loan portfolio performance of Rural Bank X Lamashegu branch.

1.4 Research Question:

The study aims at using the following research questions to act as guide to develop an effective and prudent LP model:

i) How should funds of Rural Bank X be allocated to these types of loans to maximize the net rate of return?

ii) Which credit facilities of Rural Bank X provide a best combination of risk and expected return that is optimal for the loan portfolio performance?

2. Linear programming for Bank Portfolio Management

The classical approach to portfolio optimization is based on these two conflicting optimization criteria: minimizing the risk of a loan portfolio and maximizing expected return of the loan portfolio. Return and risk are these items are in conflict of each other and finding a best loan mixture that give the highest return and lowest risk is not impossible. So Markowitz (1994) described a new method which avoids actual computation of the covariance matrix and Konno (1990) solved the risk quadratic function by reducing it to a linear programming and proposed a piecewise approach.

A linear programming model helps the banking industry to maximize the profit by using the available resources or to minimize the cost of expenses. Linear programming is a mathematical technique that deals with the optimization (maximizing or minimizing) of a linear function known as objective function subject to a set of linear equations or inequalities known as constraints. It is a mathematical technique which involves the allocation of scarce resources in a scientific manner, on the basis of a given criterion of optimality. The technique used here is linear programming because the decision variables in any given situation generate straight line when graphed. It is also programming because it involves the movement from one feasible solution to another until the best possible solution is attained.

According to International Association of Credit Portfolio Managers (IACPM) (2005), as risk management techniques in the equity, bond, derivatives, and loan markets converge, credit portfolio management demands the skill sets of both generalists and specialists. Generalist skills typically are needed in the analysis, underwriting, and monitoring functions. Specialist skills, such as advanced mathematics and statistical modeling backgrounds, are necessary in monitoring and optimizing the portfolio.

2.1 Linear programming Sensitivity Analysis

Finding the optimal solution to a linear programming model is important, but it is not the only information available. There is a tremendous amount of sensitivity information, or information about what happens when data values are changed Sensitivity analysis allows researchers to determine how 'sensitive' the optimal solution is to changes in the data values. This include analyzing changes in: the Objective Function Coefficient and the Right Hand Side value of a constraint. *(cbafaculty.org/3_DM/chapter%204.ppt).*

When you use a mathematical model to describe reality you must make approximations. The world is more complicated than the kinds of optimization problems that we are able to solve. Linearity assumptions usually are significant approximations. Another important approximation comes because you cannot be sure of the data that you put into the model. Your knowledge of the relevant technology may be imprecise, forcing you to approximate values in

A, b or c. Moreover, information may change. Sensitivity analysis is a systematic study of how sensitive solutions are to (small) changes in the data. The basic idea is to be able to give answers to questions of the form:

1. If the objective function changes, how does the solution change?
2. If resources available change, how does the solution change?
3. If a constraint is added to the problem, how does the solution change? (http://econweb.ucsd.edu/~jsobel/172aw02/notes7.pdf)

2.2 Risk in Portfolio Management

Risk measures are key building blocks to set up and perform effective credit portfolio management. With the ever-growing size and complexity of credit portfolios in most financial institutions, portfolio management can no longer be practiced on the basis of qualitative analysis only, but must be supported by sound quantitative measures.

The concept that return should increase if risk increases is fundamental to modern management and finance. This relationship is regularly observed in the financial markets, and important clarification of it has led to Nobel prizes. To maximize share price, the financial manager must learn to assess two key determinants: risk and return. Each financial decision presents certain risk and return characteristics, and the unique combination of these characteristics has an impact on share price. Risk can be viewed as it is related either to a single asset or to a portfolio—a collection, or group, of assets.

2.2.1 Risk Defined

In the most basic sense, risk is the chance of financial loss. Assets having greater chances of loss are viewed as more risky than those with lesser chances of loss. More formally, the term risk is used interchangeably with uncertainty to refer to the variability of returns associated with a given asset. A $1,000 government bond that guarantees its holder $100 interest after 30 days has no risk, because there is no variability associated with the return.

A $1,000 investment in a firm's common stock, which over the same period may earn anywhere from $0 to $200, is very risky because of the high variability of its return. The more nearly certain the return from an asset, the less variability and therefore the less risk.

2.2.2 Return Defined

The total gain or loss experienced on an investment over a given period of time; calculated by dividing the asset's cash distributions during the period, plus change in value, by its beginning-of-period investment value

3. Methodology

The data collected for this study consist of secondary data from the annual report, journals and dailies of Rural Bank X. In order for the bank to maximize profit from the loan it disbursed to its customers, the proposed

Mathematical linear programming model will be based strictly on the Bank's loan policy and its previous history on loan allocations.

The Mathematical model will be solved using the Revised Simplex Algorithm with the aid of QM windows software solver. According to Render et al (2009) QM windows solver is considered the best option for LP because the software offers a very convenient data entry and editing features which allows for a greater understanding of how to construct linear programming models.

The QM windows solver was considered due to the fact that it is a popular software used by Operational researchers. (Quantitative Analysis for Management, Tenth Edition, by Render, Stair, and Hanna Power Point slides created by Jeff Hey 2009 Prentice-Hall, Inc.).

The use of linear and other types of mathematical programming techniques has received coverage in the extensive banking literature,(Chambers et al,1961), as well as (Cohen et al 1967;1972), developed a series of sophisticated linear programming models for managing the balance sheet of larger banks, while,(Waterman et al, 1963) and (Fortson et al,1977) proposed less elegant formulations which were better suited for the small to medium-sized banks.

3.1 Sources and data collection

The data used in this study is secondary source extracted from the published annual reports and financial statements of the bank. This category of data was mainly in quantitative form. Saunders et al, (2007) quote Stewart and Kamins (1993) as stating that secondary data are likely to be of higher-quality than could be obtained by collecting empirical data.

Rural Bank X Lamshegu branch Tamale is in the process of formulating a loan policy involving a total of GH¢ 825812.77 for the year 2013. Being a full-service facility, the bank is obligated to grant loans to different customers. The table 1 provides the type of loans, the interest rate charged by the bank, and the probability of bad debt as estimated from past experience.

Table 1 portrays the credit facilities together with their interest rate and probability of bad debts Rural Bank X Lamashegu branch Tamale offers to its customers and clients. The loan with the highest interest rate is Microfinance (30%), followed by Commercial (28%), Personal (27%), Agricultural (26%) and Overdraft (15%) respectively.

3.2 Proposed Linear Programming Model

Decision variables usually represent items that can be adjusted or controlled. An objective function can be defined as a mathematical expression that combines the variables to express goal and the constraints are expressions that combine variables to express limits on the possible solutions.

3.2.1 Decision variables

The variables of the model are defined as follows;

X_1 = Personal loans
X_2 = Micro Finance
X_3 = Overdrafts
X_4 = Commercial loans
X_5 = Agricultural loans

3.2.2 Model Formulation

In general the linear programming model can be formulated as follows:

Profit on loan is given by;

$Z = \beta_1 (1- \rho_1)\, x_1 + \beta_2(1- \rho_2)x_2 + \beta_3 (1- \rho_3)x_3 + \beta_4 (1- \rho_4)\, x_4 + \ldots - (\rho_1 x_1 + \rho_2 x_2 + \rho_3 x_3 + \ldots)$, where $\pi_i > 0$

The above equation can be summarized as;

$$\text{Maximize} \quad Z = \sum_{i=1}^{n} \beta_i (1 - \rho_i) x_i - \sum_{i=1}^{n} \rho_i x_i, \qquad \text{where}$$

Z = is the optimal solution,
βi = the coefficients of objective function, (i.e. interest rate)
x_i = the various loan items of the bank
$(1 – \rho i)\, xi$ = the amount contributing to profit and
ρi = the probability of bad debt

$$\text{Subject to} \sum_{i=1}^{n} x_{ij} \le \alpha i A i$$

x_i , is an integer

Where i = 1,2,3,…,5 and Ai is the amount and αi is the percentage impose on the loan allocated to various loan items.

3.2.3 Objective function of the Proposed Model

The objective function of the proposed model is to maximize BRB net returns, Z which comprise the difference between the revenue from interest and lost funds due to bad debts for each amount of loan disburses are shown in Table 2

3.2.3 Table 2 Revenue from Interest and Lost Funds Due To Bad Debts

Type of Loans	Amount of bad debts (ρixi)	Amount contributing to profit (1- ρi)xi
x_1	0.15	0.85
x_2	0.04	0.96
x_3	0.01	0.99
x_4	0.01	0.99
x_5	0.075	0.925

Source: Authors construct with data from Rural Bank X, June, 2014

Maximize $Z = 0.27(0.85)x_1 + 0.3(0.96)x_2 + 0.15(0.99)x_3 + 0.28(0.99)x_4 + 0.26(0.925)x_5 - 0.15x_1 - 0.04x_2 - 0.01x_3 - 0.01x_4 - 0.075x_5$

The pattern is simple, the first five terms represent the income due annual growth on the portfolio investment that do not lose money, and the second five terms represent the capital losses on the investment that lose money or that goes to bad debts.

3.3 Basic Assumptions of the formulation of the above Proposed LP Model

A subtle assumption in the formulation of the above model is that all loans are distributed at approximately the same time. This assumption allows us to ignore the differences in the time values of the funds allocated to the different loans.

All variables are restricted to non-negative values (i.e., their numerical value will be ≥ 0). Also Non – integer values of decision variables are accepted. This is referred to as the assumption of divisibility (Amponsah, 2007).

3.3.1 Computational Procedures

This simplifies the proposed model to;

$Z = 0.0795x_1 + 0.248x_2 + 0.1385x_3 + 0.2672x_4 + 0.1655x_5$

The program has five constraints and they are as follows.

1) Limit on total funds available (x_1, x_2, x_3, x_4, and x_5)

The total funds available is GH¢ 825812.77

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 825812.77$$

2) Personal and Microfinance loans

$$x_1 + x_2 \geq 0.2 \times 825812.77$$
$$x_1 + x_2 \geq 165162.554$$

3) Limit on Agric and Agro processing

$$x_5 \geq 0.3 \times 825812.77$$
$$x_5 \geq 247743.831$$

4) Limit on Commercial loans compared to personal, commercial, and agric

$$x_3 \geq 0.5\,(x_1 + x_4 + x_5)$$
$$0.5x_1 + 0x_2 - x_3 + 0.5x_4 + 0.5x_5 \geq 0$$

5) Limit on expected capital losses (i.e. bad debts). Thus,

$$\frac{\text{Expected capital lose}}{\text{Totalcapitalinvested}}$$

$$\frac{0.15x_1 + 0.04x_2 + 0.01x_3 + 0.01x_4 + 0.075x_5}{x_1 + x_2 + x_3 + x_4 + x_5} \leq 0.04$$

$$\Rightarrow 0.11x_1 + 0x_2 - 0.03x_3 - 0.03x_4 - 0.035x_5 \leq 0$$

Non negativity: $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x+_4\, x \geq 0$, $x_5 \geq 0$

3.4 The Proposed Model

In summary, the mathematical model of the Rural Bank X loan portfolio problem is:

Maximize $Z = 0.0795x_1 + 0.248x_2 + 0.1385x_3 + 0.2672x_4 + 0.1655x_5$

Subject to;

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 825812.77$$
$$x_1 + x_2 + 0x_3 + 0x_4 + 0x_5 \geq 165162.554$$
$$0x_1 + 0x_2 + 0x_3 + 0x_4 + x_5 \geq 247743.831$$

$0.5x_1 + 0x_2 + 0x_3 - x_4 + 0.5x_5 \geq 0$
$0.11x_1 + 0x_2 - 0.03x_3 + 0.035x_4 - 0.03x_5 \leq 0$
$x_i \geq 0, i = 1, 2, 3 \ldots 5$

3.4.1 Solution of the Proposed LP model

We used the LP software (QM windows) to solve the linear systems as shown below;
Writing the linear system in matrix form, we have,
Z = [0.0795 0.248 0.1385 0.2672 0.1655]

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0.5 & 0 & -1 & 0.5 & 0.5 \\ 0.11 & 0 & -0.03 & -0.03 & 0.035 \end{bmatrix} \begin{bmatrix} 825812.8 \\ 165162.5 \\ 247743.8 \\ 0 \\ 0 \end{bmatrix}$$

3.4.2 Table 3 Optimal Solution and Dual Value

Decision Variable	Solution Value	Objective Coefficient	Contribution	Dual Value
x_1	0	0.0795	0	0.2243
x_2	165162.5	0.248	40960.3	0.0237
x_3	220216.8	0.1385	30500.0268	0
x_4	440433.5	0.2672	117683.8312	0.0858
x_5	0	0.1655	0	0

Source: Authors construct, June, 2014

The results were found after four (4) iterations. See appendix A for details on iteration. Column 2 of table .3 represents the optimal solution each decision variable (i.e. loan) has obtained, thus x_1 = 0.0, x_2 = 165162.5, x_3 = 220216.8, x_4 = 440433.5 and x_5 = 0.0. The results of multiplying the values of column 2 with that of column 3 is given in column 4 which represent the contribution of each decision variables to the objective function of the proposed model. The sum of the values in column 4 is 189144.158 (i.e. GH¢ 189144.158) which is the optimal return of Rural Bank X after the loans disbursement.

The total contribution of each decision variable to the objective function is equal to the multiplication of its final solution and the objective function coefficient.

Thus 0.0795(0) + 0.248(165162.5) + 0.1385(220216.8) + 0.2672(440433.5) + 0.1655(0) = 189144.158

To achieve optimal solution the bank in question is to disburse GH¢165162.5, GH ¢ 220212.8 and GH ¢ 440433.5 to Microfinance, Overdraft and Commercial loans respectively.

Thus the product mix the bank should consider is Microfinance, Overdraft and Commercial loans. The product mix from table 3 yielded a maximum profit of GH ¢ 189144.158.

The Dual values are often called Shadow price and are the amount by which the value of the objective function would change (i.e. increase or decrease) with a one-unit change in the RHS value of a constraint. It is the value (i.e. in theory) that would be added to the optimal solution of GH ¢ 189144.158 if an additional units of constrains x_2 and x_4 is added to the loan investment. In this problem, a shadow (dual) price of GH ¢ 0.0237 for Microfinance and GH ¢ 0.0858 for Commercial loans indicates that their allocation of the funds were used up and, in theory, if additional loans on Microfinance and Commercial is added, the maximum profit of GH ¢ 189144.158 would increase by GH ¢ 0.0237 and GH ¢ 0.0858 respectively.

Refer to the following appendices A, B, C, and D for tables showing the Iterations, Linear Programming results, Range of results and Solution list as displayed by the software QM windows solver.

A change in the value of an objective function coefficient can cause a change in the optimal solution of a problem. This is shown in table 4.3.2.

Table 4 Range of Objective Function Coefficients and Reduced Cost

Variable	Value	Reduced Cost	Original Val	Lower Bound	Upper Bound
X1	0.	0.2114	0.0795	-Infinity	0.29
X2	165162.5	0	0.248	0.22	Infinity
X3	220216.8	0	0.1385	-0.53	0.21
X4	440433.5	0	0.2672	0.17	0.3
X5	0.	0.1017	0.1655	-Infinity	0.27

However, not every change in the value of an objective function coefficient will lead to a changed solution; generally there is a range of values for which the optimal values of the decision variables will not change. In table 4 problem, if the profit on x_2 (i.e. Microfinance) increased from GH¢ 0.22 per unit to say, GH¢ 0.28 per unit, the optimal solution would still be to disburse GH¢ 165162.5, GH¢ 220216.8 and GH¢ 440433.5 for x_3 and x_4 respectively. Similarly, if the profit per x_2 decreased from GH¢ 0.22 to, say, GH¢ 0.20, disbursing of GH¢ 165162.5, GH¢ 220216.8 and GH¢ 440433.5 for x_3 and x_4 would still be optimal.

The reduced cost of x_1 and x_5 being non-zero show how much their coefficients should be increased before they can contribute significantly to the optimal function value. But the reduced cost of x_2, x_3 and x_4 are zero each indicating their allocations had been fully utilized completely. (See appendix c).

Table 5 Slack Values

Slacks	Right Hand Side	Status	Slack Value
Slack1 (x_1)	825812.77	Non-Basic	0
Slack2 (x_2)	165162.554	Non-Basic	0
Slack3 (x_3)	247743.831	Basic	247743.831
Slack4 (x_4)	0	Non-Basic	0
Slack5 (x_5)	0	Non-Basic	19819.51

Table 5 shows the absence of zeros slack values for x_3 and x_5 for the constraint equation. This means the budgeted amount for these loans were not fully utilized. But the zeros slack values for x_1, x_2 and x_4 indicate that the amount invested in them was fully utilized to the maximum.

Table 6 Right Hand Side Range (Single Change)

Constraint	Current RHS	Minimum RHS	Maximum RHS	Dual Price
x_1	825812.77	165162.6	Infinity	0.2343
x_2	165162.554	0	825812.8	0.0537
x_3	247743.831	0	Infinity	0
x_4	0	-660,650.3	330,325.1	0.0853
x_5	0	-19,819.15	Infinity	0

Columns 3 and 4 in table 6 shows the ranges or limits through which the right hand side, thus column 2 terms of the constraint equation could be varied to effect change in the optimal values. This would mean change in an investment portfolio to change optimality.

Column 5 contains the dual or shadow prices of the five constraints indicating which constraints are binding and those which are not. If a constraint is nonbinding, its shadow price is zero, meaning that increasing or decreasing its RHS value (i.e. investments) by one unit will have no impact on the value of the objective function. But a constraint is binding if substituting the values of the decision variables of that solution into the left side of the constraint results in a value that is equal to the RHS value.

Base on these facts, the managers of Rural Bank X need not increase their loan investment in Overdraft (x_3) and Agricultural (x_5) loans.

The dual price column gives us the amount of optimal value change per unit of budgeted investment. Thus one unit addition to x_2 the objective function value would increase by GH¢ 0.0237 (the shadow price) to GH ¢ 189144.14, addition of one unit to x_4 would increase GH ¢ 189144.14 by GH ¢ 0.0858.

The table further indicated that for x_3 it units need not be increase for they will add nothing to the objective function values. So for the optimality of the investment to be maintained in Rural Bank X, the following conditions must be met:

- Investment of x_1 should be within GH ¢ 165162.6 and infinity
- Investment of x_2 should be within 0 and GH ¢825816.8
- Investment of x_3 should be within 0 and infinity
- Investment of x_4 should be within GH ¢ 660650.3 and GH ¢ 330325.1
- Investment of x_5 should be within GH ¢ 19819.15 and infinity

5. Conclusion

The product mix or loan combination the bank should consider is Microfinance, Overdraft and Commercial loans.

The other loans, thus Personal and Agricultural loans must critical be looked at by management. The LP model developed for the bank will enabled it to increase their revenue returns on loans investment (i.e. GH¢ 825812.8) to make annual profit of GH¢ 189144.14 on loans alone as against GH¢ 96879.1 in 2013 if they are to stick to the model. The outcome further showed that the performing loans in the Loan portfolio are Microfinance, Overdraft and Commercial loans and the non-performing ones are Personal and Agricultural loans.

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