An Application of Linear Programming for Efficient Resource Allocation

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Abstract

Every business organisation exits primarily to make profit while ensuring its continued existence. In a bid to maximise profit with the limited available resources, it becomes imperative therefore that efficient allocation be made of these resources. This paper ex-rays the concept of linear programming as an optimisation technique for maximising profit with the available resources. It goes further to address a problem encountered by a business (Eat Plaza)who due to an increment in price of raw material wants to strim- line its production. The linear programming model is used to analyse this problem and an optimum solution is reached as well as relevant recommendations to the management.

Keywords: linear programming model, limited resources, optimum, profit

1. Introduction

Every business enterprise strives to profitably exist while ensuring growth and expansion of its business endeavours. In its bid to achieve the above objectives it becomes expedient that critical decisions be made as regards allocation of its scarce resources effectively and also efficiently. These decisions includes but are not limited to, competitive price, identification of target market, product standard, level of technology involvement, manpower and so on. Owing to the fact that such decisions can become the make or break of the business thus distinguishing between a mediocre and an outstanding business.

It is important to note however that economic problems arise when available resources in organisations are not properly allocated or utilised [Akiniyi, .J.A, 2008]. It is a well known that a Nation's economy is said to be buoyant when a lot of business within the country is thriving. This can be said as the reason why attempts are always being made by government to ensure measures are put in place to create an enabling external environment for businesses within their jurisdiction.

Every entrepreneur or business manager is faced with the challenge of decision making as regards efficient allocation of its available resources to various areas of need that pertains to the organisation. This above is in an attempt to minimize cost incurred while maximizing profit earned. Such resource allocation decisions can be facilitated and enhanced through the use of an operations research technique termed linear programming.

Linear programming is an optimisation instrument, which allows the rationalisation of many managerial and/or technological decisions. Linear programming is a technique designed to help operations managers plan and make decisions relative to the trade-offs necessary to allocate resources [Heizer, J andRender, B. 2004]. Linear programming is a method for the formalization and analysis of constrained optimization problems in which the objective function is a linear function and is to be maximized or minimized subject to a number of linear inequality constraints [Akiniyi, .J.A, 2008]

1.1Why Linear Programming Technique

[Agbadudu .B. A, 1996] Asserts that linear programming is a mathematical technique for finding the best uses of a firm's or organizational limited resources. He further explained that it is a mathematical programming method for resolving problems associated with resource allocation.

Linear programming is a mathematical problem when the objective function as well as all the constraints are expressed by linear affine functions [Brandimarte .P, 2011]

Linear programming was aptly defined by [Dwivedi .N. D, 2003] as a technique for finding an optimum solution to the problem of resource allocation in order to achieve certain ends under the prescribed conditions.

Linear programming is a mathematical technique designed to assist management in solving optimisation problem when a decision has to be reached as to achieving the desired objective subject to limitations imposed in terms of restricted resources available, and/or desired quantities and/or qualities of output [Chartered Institute of management Accounting, 2009]. The author also asserts that allocation of scarce and limited resources amongst competing needs is a daily occurrence. There exist real life situations where linear programming comes handy in resource allocation which are but not limited to the following:

     a)   It enables development of a production schedule and inventory policy that will satisfy sales in future periods and at the same time minimize total production and inventory costs.

     b)   Developing an investment portfolio from a variety of stock and bond options in a bid to maximise return on investment.

     c)   Choosing amongst alternative advertising media such as radio, television, newspaper etc with the aim of maximising advertising effectiveness.

d) It is a useful tool for solving transportation problems, assignment problems with the aim of minimizing costs.etc.

1.1.1 Basic Assumptions of Linear Programming Model

. Components of a linear programming model have been identified by [Taha, A. H, 2011], [Bronson, .R, Naadimuthu, .G, 1997], [Stevenson .J.W, 2005], [Krajewski .J. L, Ritzman .P.L, Malhotra .k. M, 2007], [Dwivedi .N. D, 2008], [Hirschey .M, 2003] and [Budnick .S. F, Mcleavey .D, Mojena .R, 1988] as follows:

  i. Proportionality: The level of activity is proportional to the contribution as well as consumption of resources.
  ii. Additivity: Activities contribution and consumption are additive.
  iii. Non-negativity: The values of the activities cannot be negative.
  iv. Linearity: There exist linear relationships between the output of each product and the total quantity of each resource consumed.
  v. Single or one Objective function: There can be only one objective in a particular problem, either to maximise profit or to minimize cost not both.
  vi. Certainty/ Deterministic: This presupposes that all values and quantities are known.
  vii. Fixed external factors: This implies that the external environment is assumed not to vary.

1.1.2 Basic Components of a Linear Programming model

A linear programming usually is expressed in inequalities, below are the various component that make up an LP model, [Dwivedi .N. D, (2003)], [Krajewski .J. L,Ritzman .P.L, Malhotra .k. M, 2007], [Dwivedi, N.D, 2008], [Hirschey .M, (2003) and [Stevenson, J.W, 2009]:

  a) Decision variables: e.g x_1, x_2
  b) Objective function: e.g Min C = $12x_1 + 10x_2$
  c) Constraints/limitations: e.g available labour hours, machine hours
  d) Non – negativity constraint.

1.1.3 Mathematical Statement of the Linear Programming Model

Definition of symbols:

x_j = $_j$th decision variable.

c_j = Coefficient on jth decision variable in the objective function.

a_{ij} = Coefficient in the ith constraint for the jth variable.

  b_i = Right-hand-side constant for the ith constraint.

  n = Number of decision variables.

  m = Number of structural constraints.

Based on the above definitions, the linear programming model can thus be stated as:

Optimize (maximise or minimise)

$$z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

Subject to structural constraints

$$A = \begin{bmatrix} a_{11}\,x_1 + a_{12}x_2 + \ldots + a_{1n}\,x_n(\geq)(\leq)b_1 \\ a_{21}\,x_1 + a_{22}x_2 + \ldots + a_{2n}x_n(\geq)(\leq)\,b_{2n} \\ \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n(\geq)(\leq)\,b_m \end{bmatrix}$$

$x_1, x_2, \ldots, x_n \geq 0$

The primary objective of this paper is to clearly show how a business organisation can efficiently allocate its limited resources amongst competing products or needs with the sole aim of concentrating its efforts on the highest yielding resource in terms of its contribution to profit generated. Having established a foundation on what linear programming is as well as state its general form, it becomes important to show its practical application in a business situation.

A fast food by name EAT PLAZA has a caterer that bakes meat pie, sausage and chicken pie. The owner wishes to know which of the above snacks to discontinue sales as a result of an increment in the price of flour and margarine. Expressed in table 1is the contributions made by each of the snacks and the quantity of materials required to make them.

STEP 1: Convert the above problem to a linear programming problem standard form.

$$\text{Max } \pi = 200x_1 + 250x_2 + 350x_3 \quad \text{objective function}$$

Subject to
$$500x_1 + 250x_2 + 500x_3 \leq 5000 \quad \text{flour constraint}$$
$$250x_1 + 100x_2 + 200x_3 \leq 3000 \quad \text{margarine constraint}$$
$$100x_1 + 150x_2 + 200x_3 \leq 8000 \quad \text{eggs constraint}$$
$$x_1, x_2, x_3 \geq 0 \quad \text{non-negativity constraint}$$

When the variables (x_i) are more than two the SIMPLEX method becomes most appropriate in obtaining the optimal solution, hence we shall be using the SIMPLEX method to solve this problem.

STEP 2: Introduce slack variables in the constraints [Azimi, .M, et al, 2013]
$$500x_1 + 250x_2 + 500x_3 + s_1 + 0s_2 + 0s_3 = 5000$$
$$250x_1 + 100x_2 + 200x_3 + 0s_1 + s_2 + 0s_3 = 3000$$
$$100x_1 + 150x_2 + 200x_3 + 0s_1 + 0s_{2\,+}s_3 = 8000$$
$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

STEP 3: Introduce the slack variable in the objective function [Fagoyinbo, I. S,et al,2011]
$$\pi = 200x_1 + 250x_2 + 350x_3 + 0s_1 + 0s_2 + 0s_3$$
$$\pi - 200x_1 - 250x_2 - 350x_3 + 0s_1 + 0s_2 + 0s_3 = 0 \text{ (the } s_i \text{ symbols is not affected because 0}$$
beingpositive or negative has no impact on its value)

STEP 4: Draw the initial tableau

STEP 5: To identify the pivot row, divide the entire pivot column by the entire 'b' column (the right end column) and select the smallest value. The intersection between the pivot column and the pivot row is known as the pivot element (it is the foundation on which the next tableau is introduced).

The Gauss- Jordan row operation shall be applied in solving this linear programming problem (LPP)

Where

R = Row

$\hat{R}$ = New Row

R1(b) ÷ 500(x_3 row 1) = 10   * pivot row

R2 (b) ÷ 200(x_3 row 2) = 15

R3 (b) ÷ 200(x_3 row 3) = 40

STEP 6: Draw a second tableau, introduce the pivot row into the slack variable (SV) where the pivot element was derived (x_3) and divide each number in the entire row by the pivot element.

R1(b) ÷ 1/2(x_2 row 1) = 20   * pivot row

R2 (b) ÷ 0   (x_2 row 2) = ~ (infinity hence disregard)

R3 (b) ÷ 50 (x_2 row 3) = 120

R2 – 200 R̂ 1:	250	100	200	0	1	0	[3000]
-	200	100	200	200/500	0	0	[2000]
	50	0	0	-200/500	1	0	[1000]

R3 – 200R̂ 1:	100	150	200	0	0	1	[8000]
-	200	100	200	200/500	0	0	[2000]
	-100	50	0	-200/500	0	1	[6000]

R4 + 350 R̂1:-	200	-250	-350	0	0	0	[0]
+	350	175	350	350/500	0	0	[3500]
	150	-75	0	7/10	0	0	[3500]

STEP 7: Since there is still a negative value (which indicates we have not reached optimum level) in the 4th row, we proceed to draw the third tableau following the same procedure.

$R2 + 0\hat{R}1$:

50	0	0	-200/500	1	0	[1000]
0	0	0	0	0	0	[0]
50	0	0	-200/500	1	0	[1000]

$R3 - 50\hat{R}1$:

-100	50	0	-200/500	0	1	[6000]
100	50	100	1/5	0	0	[1000]
0	0	-100	-300/500	0	1	[5000]

$R4 + 75\hat{R}1$:

150	-75	0	7/10	0	0	[3500]
150	75	150	75/250	0	0	[1500]
300	0	150	1	0	0	[5000]

The optimality condition of no negative value in the last column (π) has been meet hence this last tableau becomes the optimal.

In using the Gaussian row operation, we observe that the values under the pivot element must be manipulated to result in zero (0) in the next tableau drawn. This also serves as a check to ensure that the analysis is in order. Having no negative value in the last column is a signal that the optimum solution has been reached, thus bring the iteration to a halt and then evaluation is carried out on the values of the final (optimum) tableau.

1.1.4 Recommendations

Management can therefore engage the following solutions as they see fit given the findings derived from the analyzing the situation using SIMPLEX method:

1. Arbitrarily discontinue production of meat pie or chicken pie or both (meat pie and chicken pie) to concentrate on sausages.
2. Engage in staff training and ensure practical demonstrations of how to bake meat pie and chicken pie.
3. Check through the production system for possible lapses and take corresponding action.
4. Cease ordering for more stock of flour and margarine and harness the available resources

1.1.5 Conclusion

It has been clearly expressed from the above empirical data that resource allocation problems can be optimally and efficiently evaluated to enhance management decision making. It would have been difficult or impossible to know what to do with such statistics or data without skill application of optimisation model (in this case linear programming). As rightly indicated by [Taha, A. H 2011] there are computer softwares that enhance speed and accuracy in computing linear programming models such as Microsoft excel spread sheet, Riddy Ricks etc. These procedures were carefully outlined or explained to aid better understanding of how the optimum solution is generated using linear programming model.

It is important to express that the linear programming model is not without its limitations. [17]Some of which includes inability to predict prices, difficulty in accounting for diminishing marginal returns, its lack of operator risk preference, poor handling of decreasing cost. The above limitations not withstanding do not detract from its importance and usefulness as an optimisation technique.

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Table1.A representation of the major ingredients involved in the baking process.

Ingredients	Meat pie	Sausage	Chicken pie	Available resources
Flour	500	250	500	5000
Margarine	250	100	200	3000
Eggs	100	150	200	8000
Contribution	₦200	₦250	₦350	

Table 2. The Initial Tableau

SV	X_1	X_2	X_3	S_1	S_2	S_3	b
S_1	500	250	500	1	0	0	5000
S_2	250	100	200	0	1	0	3000
S_3	100	150	200	0	0	1	8000
π	-200	-250	-350	0	0	0	0

♂ _____ pivot column

Table 3. Second Tableau

SV	X_1	X_2	X_3	S_1	S_2	S_3	b
X_3	1	½	1	1/500	0	0	10
S_2	50	0	0	-200/500	1	0	1000
S_3	-100	50	0	-200/500	0	1	6000
π	150	-75	0	7/10	0	0	3500

♂ _____ pivot column

Table 4. Third Tableau

SV	X_1	X_2	X_3	S_1	S_2	S_3	b
X_2	2	1	2	1/250	0	0	20
S_2	50	0	0	- 2/5	1	0	1000
S_3	-200	0	-100	-3/5	0	1	5000
π	300	0	150	1	0	0	5000

Table 5.The findings of the linear programming problem (LPP):

Decision variable	Optimum value	Recommendation
X_2	20	Produce 20 grams of sausage daily
S_2	------	There is surplus in margarine and egg resources of 1000& 5000 respectively which need to be harnessed in order to maximise profit for the business.
S_3	-------	
π	5000	Daily profit ₦5,000