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Forecasting Demand for Petroleum Products in Ghana using Time Series Models

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Abstract

The objective of this study was to forecast and analyse the demand for petroleum products in Ghana using annual data from 2000-2010. It focused on studying the feasibility forecast using nested conditional mean (ARMA) and conditional variance (GARCH, GJR, EGARCH) family of models under such volatile market conditions. A regression based forecast filtering simulation was proposed and studied for any improvements in the forecast results.

Keywords: time Series models, regression model, forecast filtering, petroleum products, stationarity of time Series data.

1. Introduction

The demand for petroleum has increased in the last decade all over the world including the United States, Middle Eastern nations, and other Asian nations, which has contributed in the high prices. The demand for petroleum products in India has been increasing at a rate higher than the increase of domestic availability (Banapurmath, et. al., 2011). At the same time, there is continuous pressure on emission control through periodically tightened regulations particularly in metropolitan cities. Over the period 1980-2008, the price of crude oil had fluctuated significantly, with a mean, minimum and maximum values of \$ 32.31 (bbl), \$ 12.72 (bbl) and \$ 140 (bbl) respectively(WAMA(2008). The above statistics, in addition to a standard deviation of 17.08 over the sample period show that the prices of crude have always been characterized with severe instability. Monthly fluctuations have in fact been more severe than these annual trends, with the price of crude oil reaching \$140 (bbl) in July 2008. Such instability in the prices of crude oil is bound to cause macroeconomic distortions, especially in net-oil importing countries, like some ECOWAS countries (WAMA, 2008).

Ghana's demand for crude oil and refined petroleum products has also been growing over the past decade, and, the country's demand for oil increased dramatically surprising many energy analysts. This growth has been driven by socio-economic and technical factors that have influenced each category of final energy use. These changing petroleum requirements are closely related to its high rates of growth in economic output and personal incomes. The growth in incomes and the accompanying changes in petroleum demand are themselves driven by an ongoing population shift from rural to urban areas. That growing urban population is demanding new vehicles and new roads, raising the demand for energy in the transportation sector. The growth in output in the industrial sector is driving the high demand for petrochemical feed stocks, including naphtha-based petrochemicals, which are similar in composition to motor gasoline. Fluid catalytic cracking of heavy ends to high-value liquid fuels is a common unit operation in oil refineries (Khan, et al., 2011). In this process, the heavy feedstock that contains sulfur is cracked to light products

At the core of the development of every nation is petroleum. Currently, petroleum is among the most important natural resources. Every nation uses petroleum products such as gasoline, jet fuel, liquefied petroleum gas and diesel to run cars, trucks, aircraft and other vehicles. There is therefore the need to build stocks to meet the seasonal demands. In the long term, blending non-petroleum additives into petroleum products such as ethanol or other oxygenating agents into gasoline will be necessary to reduce crude oil demand. Efficient refining capacity is a requirement to meet the demand of the nations. The past few years have witnessed an increased impetus toward renewable energy to replace fossil fuels that has been driven both by environmental and national security concerns (Hensel, 2011).

Many researchers analyzing the demand for petroleum products have looked at the aggregate consumption of petroleum. Sa'ad(2009), used annual time series data for the period 1973 to 2007 in two econometric techniques namely the structural time series model (STSM) and unrestricted error correction model (UECM) to estimate demand for petroleum products. The results from both models revealed that the demand for petroleum products is price inelastic. The robust optimization methodology is applied to deal with uncertainties in the prices of saleable products, operating costs, product demand, and product yield in the context of refinery operational planning (Leiras, et al , 2010 and Munim, et al.,2010

MATERIALS AND METHOD

Conditional mean models were used to forecast mean while the conditional variance models were used for forecasting variance or volatility in the demand for oil. In this study, the nesting of these two models was used first to forecast the conditional mean and then the conditional variance was estimated to get the value of forecast demands for oil(Shrivastava, 2009). After analysis of data for quarter 1 (Q1r) and quarter 2 (Q2r), ARMA(2,2) models had been found most appropriate for forecasting mean, hence ARMA(2,2) and (GARCH(1,1)/GJR(1,1) /EGARCH(1,1)) were used for forecasting oil demand for 2012 and 2013.

The conditional mean and variance models have been viewed from a linear filtering perspective, and then the application of the iterated conditional expectations to the recursive equations was conducted, one forecast period at a time (Dhar, et. al. , 2009). For example for forecasting demand for the second quarter (Q2), demand data were taken of quarter 1 (Q1) and forecast using above defined method has been done. Using observed quarter 2(Q2) demand as dependent variable and (ARMA/GARCH) forecast as independent variable regression model was obtained. Calculation of quarter 3 demand (Q3r) using observed data of Q2 was obtained using nested models and the regression model obtained in the previous step was used to filter quarter 3 (Q3) demand forecast.

Conditional variance models (Shrivastava, 2009), unlike the traditional or extreme value estimators, incorporate time varying characteristics of second moment/volatility explicitly. The following models fall into the category of conditional volatility models(Hull, 2006):

i. ARCH (m) Model (Auto Regressive Conditional Heteroscedasticity)
ii. EWMA Model (Exponentially Weighted Moving Average Model)
iii. GARCH(a,b)Model(Generalized Autoregressive Conditional Heteroscedasticity).
iv. EGARCH Model.

The stationarity was tested using ADF test with and without drift and trend, the AR(p) was determined using PACF and MA(q) was determined using ACF. The number of lagged terms to be included in the model was identified based on the minimum value of AIC and SBC criteria. The ARMA model was tested for ARCH effects using the ARCH LM test and the measures of performance were calculated for the static and dynamic forecasts made for the out-sample period. The in-sample data constituting 80% was used for estimating the coefficients of the parameters and 20% the out-of- sample data was forecasted.

The forecasted results from random walk model, ARMA, ARMA-GARCH, ARMA-EGARCH models using static and dynamic forecasting were compared based on the predictive power using the three forecasting accuracy measures: Root Mean Square Error, Mean Absolute Error and Thiel Inequality Coefficient. Theil's U statistic was rescaled and decomposed into 3 proportions of inequality – bias, variance and covariance – such that bias + variance + covariance = 1 and these measures were also calculated.

1.1. Autoregressive Moving Average(ARMA)Models

The Autoregressive Moving Average (ARMA) Models have been used by many researcher for forecasting(Shrivastava, et, al. , 2010; Abu and Behrooz, 2011) . Given a time series of data Z_t, the ARMA model is a tool for understanding and, perhaps, predicting future values in this series. The model consists of two parts, an autoregressive (AR) part and a moving average (MA) part. The model is usually then referred to as the ARMA (a, b) model where "a" is the order of the autoregressive part and "b" is the order of the moving average part. The notation ARMA (a, b) refers to the model with "a" autoregressive terms and "b" moving-average terms. This model contains the AR(a) and MA(b) models. A time series Z_t follows an ARMA (1, 1) model if it satisfies $Z_1 = k + \omega_t + \sum_{i-1}^{a} \beta_i Z_{t-i} + \sum_{i=1}^{b} \alpha_i \omega_{t-i}$, where { ω_t } is a white noise series. The above equation implies that the forecast value is depended on the past value and previous shocks.

The notation MA(b) which refers to the moving average model of order b is written as $Z_t = k + \omega_t + \sum_{i=1}^{b} \alpha_i \omega_{t-i}$ and the notation AR(a) which refers to the autoregressive model of order a, is written as $Z_1 = k + \omega_t + \sum_{i-1}^{a} \beta_i Z_{t-i}$ where the $\alpha_1,\dots,\alpha_b$ are the parameters of the model, μ is the expectation of Z_t (often assumed to equal to 0), and the $\omega_t,\dots,\omega_{t-b}$ are again, white noise error terms.

The Autoregressive Moving Average model(ARMA) is a method which can be used when the time series variable is related to past values of itself. By regressing Z_t on some combination of its past values, we are able to derive a forecasting equation. We expect the autoregressive technique to perform reasonably well for a time series that:

1. Is not extremely volatile and does not contain extreme amounts of random movement.
2. Requires "q" short-term or medium- term forecast, that is less than two years

The fact that the autoregressive procedure does not perform well on a time series is not a serious disadvantage. Practically all forecasting techniques perform poorly in this situation. Suppose we want to predict the values of Z_t using the previous observation, we use the expanded equation: $Z_t = b_0 + b_1 Z_{t-1} + b_2 Z_{t-2}$, where t takes the values = 3, 4, 5, …. The values b_0, b_1, and b_2 are the least squares regression estimates obtained from any multiple linear regression. There are two predictor variables here, the lagged variables, Z_{t-1} and Z_{t-2}. The above equation is a second order autoregressive equation because it uses the two lagged terms. In general, the ath– order autoregressive equation is given as : $Z_t = b_0 + b_1 Z_{t-1} + \hat{b}_2 Z_{t-2} + \ldots + b_a Z_{t-a}$, and $t = a+1, a+2, \ldots$

The assumption underlying the ARMA model is that the future value of a variable is a linear function of past observations and random errors. In this model it is possible to find an adequate description of data set. This method consists of four steps:

1. Model identification,
2. Parameter estimation,
3. Diagnostic checking and
4. Forecasting.

In the identification step, it can be seen that a model generated from an ARMA process may contain some autocorrelation properties, so there will be some potential models that can fit the data set but the best fitted model is selected according to AIC information criteria. Stationarity is a necessary condition in building an ARMA model used for forecasting, so data transformation is often required to make the time series to be stationary. In this study, the unit root test, known as the Dickey and Fuller test (Shrivastava, et, al., 2010; Gujarati, 2006; Abu and Behrooz, 2011), is used to test the stationarity of the time series.

Based on the result obtained, the data set is stationary at first difference even with the existence of structural break. Once a tentative model is obtained, estimation of the model parameters is applicable. The parameters are estimated such that an overall measure of errors is minimized. The third step is diagnostic checking for model adequacy. Autocorrelation and also serial correlation of the residuals are used to test the goodness of fit of the tentatively obtained model to the original data. When the final model is approved then it will be used for prediction of future values of the oil demand.

1.2. The ARCH/GARCH Models

The first model that provides a systematic framework for volatility modeling is the ARCH model of Engle (Gujarati, 2006). The basic idea of the ARCH model is that the shock α_t of an asset return is serially uncorrected but dependent; also the dependence of α_t can be described by a simple quadratic function of its lagged values. Specifically, an ARCH (m) model assumes that $a_t = \sigma_t \varepsilon_t$, $\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \ldots.. + \alpha_m a_{t-m}^2$ (Gujarati, 2006), where $\{\varepsilon_t\}$ is a sequence of independent and identically distributed (i.i.d.) random variables with mean zero and variance 1, $\alpha_0 > 0$ and $\alpha_i \geq 0$ for $i > 0$. The coefficient α_i must satisfy some regularity condition to ensure that the unconditional variance of a_t is finite. In practices, ε_t is often assumed to follow the standard normal or a standardized student t distribution or a generalized error distribution.

GARCH models are used as a successful treatment to the financial data which often demonstrate time-persistence, volatility clustering and deviation from the normal distribution. Among the earliest models is Engel (1982) linear ARCH model, which captures the time varying
feature of the conditional variance. Bollerslev (1986) develops Generalized ARCH (GARCH) model, allowing for persistency of the conditional variance and more efficient testing. Engle and

Bollerslev (1986) invent the Integrated GARCH (IGARCH) model that provides consistent estimation under the unit root condition. Engle, Lilien, and Robins (1987) design the ARCH-in-Mean (ARCH-M) model to allow for time varying conditional mean. Nelson's (1990a & b) Exponential GARCH (EGARCH) model allows asymmetric effects and negative coefficients in the conditional variance function.

The leveraged GARCH (LGARCH) model documented in Glosten, Jagannathan and Runkle (1993) takes into account the asymmetric effects of shocks from different directions. Since their introduction by Engle (1982), Autoregressive Conditional Heteroskedastic (ARCH) models and their extension by Bollerslev (1986) to generalised ARCH (GARCH) processes, GARCH models have been used widely by practitioners. At a first glance, their structure may seem simple, but their mathematical treatment has turned out to be quite complex. Although the ARCH is simple, it often requires many parameters to adequately describe the volatility process of an asset return some alternative models must be sought. Shrivastava, et al. (2010) and Hull(2006) proposed a useful extension known as the generalized ARCH (GARCH) model. An important feature of GARCH-type models is that the unconditional volatility σ depends on the entire sample, while the conditional volatilities σ_t are determined by model parameters and recent return observations.

Let $(\varepsilon t)t \in \mathbb{Z}$ be a sequence of independent and identically distributed (i.i.d.) random variables, and let $p \in \mathbb{N} = \{1, 2, 3, \ldots,)$ and $p \in \mathbb{N}_o = \mathbb{N} \cup \{0\}$. Further, let $\alpha_0 > 0$, $\alpha_1, \ldots, \alpha_{p-1} \geq 0$, $\alpha_p > 0$, $\beta_1, \ldots, \beta_{q-1} \geq 0$ and $\beta_q > 0$ be non-negative parameters. A GARCH(p, q) process $(X_t)t \in \mathbb{Z}$ with volatility process is $(\sigma_t)t \in \mathbb{Z}$ is then a solution to the equations:

$$X_t = \sigma_t \varepsilon_t, \quad t \in \mathbb{Z} \tag{1}$$

$$\sigma_t^2 = \alpha_t + \sum_{i=1}^{p} \alpha_i X_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-1}^2, \quad t \in \mathbb{Z} \tag{2}$$

where the process $(\sigma_t)t \in \mathbb{Z}$ is non-negative. The sequence $(\varepsilon t)t \in \mathbb{Z}$ is referred to as the driving noise sequence. GARCH (p, 0) processes are called ARCH (p) processes. The case of a GARCH (0, q) process is excluded since in that case, the volatility equation (2) decouples from the observed process and the driving noise sequence.

It is a desirable property that σ_t should depend only on the past innovations $(\varepsilon t - h)h \in \mathbb{N}$, that is, it is measurable with respect to σ algebra generated by $(\varepsilon t - h)h \in \mathbb{N}$. If this condition holds, we shall call the GARCH (p, q) process causal. Then (X_t) is measurable with respect to σ algebra $\sigma(\varepsilon t - h : h \in \mathbb{N}_0)$, generated by $(\varepsilon t - h)h \in \mathbb{N}_0$. Also, σ_t is independent of $(\varepsilon t + h)h \in \mathbb{N}_0$, and X_t is independent of $\sigma(\varepsilon t + h : h \in \mathbb{N})$, for fixed *t*. The requirement that all the coefficients $\alpha_1, \ldots, \alpha_p$ and $\beta_1, \ldots, \beta_q$ are non-negative ensures that σ^2 is non-negative, so that σ_t can indeed be defined as the square root of σ^2.

Equation(1) is the mean equation and is specified as an AR(p) process. Equation (2) is the conditional variance equation and it is specified as the GARCH(1, 1) process. Conditional variance models (Shrivastava, 2009), unlike the traditional or extreme value estimators, incorporate time varying characteristics of second moment/volatility explicitly. By successively substituting for the lagged conditional variance into equation(2), the following expression is obtained:

$$h_t = \frac{\alpha_0}{1-\beta} + \alpha_1 \sum_{i=1}^{\infty} \beta_{i-1} \varepsilon_{t-i}^2 \tag{3}$$

An ordinary sample variance would give each of the past squares an equal weight rather than declining weights. Thus the GARCH variance is like a sample variance but it emphasizes the most recent observations. Since h_t is the one period ahead forecast variance based on past data, it is called the conditional variance. The squared residual is given by:

$$v_t = \varepsilon_t^2 - h_t \tag{4}$$

Equation (4) is by definition unpredictable based on the past. Substituting equation (4) into equation(2) yields an alternative expression as follows:

$$\varepsilon_t^2 = \omega + (\alpha_1 + \beta)\varepsilon_{t-1}^2 + v_t - \beta v_{t-1} \tag{5}$$

From the structure of the model, it is seen that large past squared shocks $\{a_{t-i}^2\}_{i=1}^m$ imply a large conditional variance σ_t^2 for the innovation a_t. Consequently, a_t tends to assume a large value (in modulus). This means that, under the ARCH framework, large shock tend to be followed by another shock; because a large variance does not necessarily produce a large realization. It only says that the probability of obtaining a large variate is greater than that of a smaller variance. To understand the ARCH models, it pays to carefully study ARCH (I) model

$a_t = \sigma_t \varepsilon_t$, $\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2$, where $\alpha_0 > 0$ and $\alpha_I \geq 0$. The unconditional mean of a_t remains zero because

$$E(a_t) = E\left[E(a_t / F_{t-1})\right] = E\left[\sigma_t E(\varepsilon_t)\right] = 0 \tag{6}$$

The conditional variance if a_t can be obtained as

$$\operatorname{var}(a_t) = E(a_t^2) = E\left[E(a_t^2 / F_{t-1})\right] = E(\alpha_0 + \alpha_1 a_{t-1}^2) = \alpha_0 + \alpha_1 E(a_{t-1}^2).$$

Because a_t is a stationary process with $E(a_{t-1}^2) = 0$ $\operatorname{var}(a_t) = \operatorname{var}(a_{t-1}) = E(a_{t-1}^2)$. Therefore, we have $\operatorname{var}(a_t) = \alpha_0 + \alpha_1 \operatorname{var}(a_t)$ and $\operatorname{var}(a_t) = \dfrac{\alpha_0}{1-\alpha_1}$. Since the variance of a_t must be positive, we require $0 \leq \alpha_1 \leq 1$. In some applications, we need higher order moments of a_t to exist and, hence, α_1 must also satisfy some additional constraints. For instance, to satisfy its tail behavior, we require that the fourth moment of a_t is finite. Under the normality assumption, we have

$$E(a_t^4 / F_{t-1}) = 3\left[E(a_t^2 / F_{t-1})\right]^2 = 3(\alpha_0 + \alpha_1 a_{t-1}^2)^2$$ (Brockwell and Davis, 1996).

Therefore , $E(a_t^4) = E\left[E(a_t^4 / F_{t-1})\right] = 3E(\alpha_0 + \alpha_1 a_{t-1}^2)^2 = 3E(\alpha_0^2 + 2\alpha_0\alpha_1 a_{t-1}^2 + \alpha_1^2 a_{t-1}^4)$

If a_t is fourth – order stationary with

$m_4 = E(a_t^4)$, then we have

$$m_4 = 3\left[\alpha_0^2 + 2\alpha_0\alpha_1 \operatorname{var}(a_t) + \alpha_1 m_4\right]$$

$$= 3\alpha_0^2\left(1 + 2\frac{\alpha_1}{1-\alpha_1}\right) + 3\alpha_1^2 m_4$$

Consequently

$$m_4 = \frac{3\alpha_0^2(1+\alpha_1)}{(1-\alpha_1)(1-3\alpha_1^2)}.$$

This result has two important implications: since the fourth moment of a_t is positive, we see that α_1 must also satisfy the condition $1 - 3\alpha_1^3 > 0;$ that is, $0 \leq \alpha_1^2 \leq 1/3$; and the unconditional Kurtosis of a_t is

$$\frac{E\left(a_t^4\right)}{\left[\operatorname{var}\left(a_t\right)\right]^2}=3\frac{\alpha_0^2\left(1+\alpha_1\right)}{\left(1-\alpha_1\right)\left(1-3\alpha_1^2\right)}\times\frac{\left(1-\alpha_1\right)^2}{\alpha_0^2}=3\frac{1-\alpha_1^2}{1-3\alpha_1^{2>3}} \tag{7}$$

Thus, the excess of a_t is positive and the tail distribution of a_t is heavier than that of a normal distribution. In other words, the shock a_t of a conditional Gaussian ARCH (I) model is more likely than Gaussian white noise series to produce "outcome". This is in agreement with the empirical finding that "outliers" appear more often in asset returns than that implied by an iid sequence of normal random variates. These properties continue to hold for general ARCH models, but the formula becomes more complicated for higher order ARCH models.

The condition $\alpha_i \geq 0$ in $\sigma_t^2=\alpha_0+\alpha_1 a_{t-1}^2+....+\alpha_m a_{t-m}^2$ can be related. It is a condition to ensure that the conditional variance σ_t^2 is positive for all t. The model has some weakness: it assume that positive and negative shocks have the same effects on volatility because it depends on the square of the previous shocks. In practices it is well known that price of a financial asset responds differently to positive and negative shocks.

The ARCH model is rather restrictive. For instance, α_1^2 of an ARCH (I) model must be in the interval $\left[0\,{}^1\!/\!_3\right]$ if the series has a finite fourth moment. The constraint becomes complicated for higher order ARCH models. In limits, the ability of ARCH models with Gaussian innovations is to capture excess kurtosis. The ARCH model does not provide any new insight for understanding the sources of variation of a financial time series. It merely provides a mechanical way to describe the behavior of the conditional variation. It gives no indications of what causes such behavior to occur. ARCH models are likely to over predict the volatility because they respond slowly to large isolated shocks to the return series(Brockwell and Davis, 1996).

1.3. The EGARCH Model

This model is used to allow for symmetric effects between positive and negative asset returns. An EGARCH (m, s) model can be written as (Dhar, et. Al. , 2009).

$$a_t=\sigma_t\varepsilon_t, \qquad In\left(\sigma_t^2\right)=\alpha_0\frac{1+\beta_1 B+...+\beta_{s-1}B^{s-1}}{1-\alpha_1 B....\alpha_m B^m}g\left(\varepsilon_{t-1}\right) \tag{8}$$

wher α_0 is a constant, *B* is the back-shift (or lag) operator such that $Bg\left(\varepsilon_t\right)=g\left(\varepsilon_{t-1}\right)$ and $1+\beta_{1B}+\beta_{s-B}+\cdots$ are polynomials with zeros outside the unit circle and have no common factors. By outside the unit circle, we mean that absolute values of the zeros are greater than 1. Here, it is understood that $\alpha_i=0$ for $i > m$ and $\beta_j=0$ for j > s. The latter constraint on α_i and β_i implies that the unconditional variance α_t is finite, whereas its conditional variance σ_t^2 evolves over time, and ε_t is often assumed to be a standard normal standardized student-t distribution or generalized error distribution:

$$\sigma_t^2=\alpha_0+\sum_{i=1}^{m}\alpha_i\alpha_{t-1}^2+\sum_{i=1}^{m}\beta_j\sigma_{t-j}^2 \tag{9}$$

reduces to a pure ARCH (m) model if $S=0$.

The α_i and β_i are referred to as ARCH and GARCH parameters respectively. The unconditional mean of $In(\sigma_t^2)$ is α_0. It uses logged conditional variance to relax the positiveness constraint of model coefficients. The use of $g(\varepsilon_t)$ enables the model to respond asymmetrically to the positive and negative lagged values of α_t. The model is nonlinear if $\theta \neq 0$. Since negative shocks tend to have larger impacts, we expect θ to be negative. For higher order EGARCH model, the nonlinearity becomes much more complicated. This model can be used to obtain multistep ahead volatility forecasts.

1.4. Fitting the Parameters of the Model

Once a model is selected and data are collected, it is the job of the researcher to estimate the parameters of the Model. These are values that best fit the historical data. It is hypothesized that the resulting model will provide a prediction of the future observation. It is also hypothesized that all values in a given sample are equal.

The time series model includes one or more parameters. We identify the estimated values with a hat. For instance, the estimated value of β is denoted $\hat{\beta}$. The procedure also provide estimates of the standard deviation of the noise, σ_ε.

1.5. Forecasting from The Model

The main purpose of modeling a time series is to make forecasts which are then used directly for making decisions. In this analysis, we let the current time be T, and assume that the demand for periods 1 through T are known. We now want to forecast the demand for the period (T+ς). The unknown demand is the random variable $X_{(T+\varsigma)}$, and its realization is $x_{(T+\varsigma)}$. Our forecast for the realization is $\bar{x}_{T+\varsigma}$.

1.6. Measuring the Accuracy of the Model

The forecast error is the difference between the realization and the forecast. Thus

$$e_\varsigma = x_{(T+\varsigma)} - \bar{x}_{T+\varsigma} \quad (10)$$

Assuming the model is correct, then we have

$$e_\varsigma = E[X_{T+\varsigma}] + \varepsilon_\varsigma - \bar{x}_{T+\varsigma} \quad (11)$$

We investigate the probability distribution of the error by computing its mean and variance. One desirable characteristics of the forecast $\bar{x}_{T+\varsigma}$ is that it is unbiased. For an unbiased estimate, the expected value of the forecast is the same as the expected value of the time series. Because ε_t is assumed to have a mean of zero, an unbiased forecast implies $E[\varepsilon_\varsigma]$. The fact that the noise in independent from one period to the next period means that the variance of the error is: $Var[\varepsilon_t] = Var\{E[X_{T+\varsigma}] - \bar{x}_{T+\varsigma}\} + Var[\varepsilon_{T+\varsigma}]$ and

$$\sigma_{\varepsilon^2}(\varsigma) = \sigma_{E^2}(\varsigma) + \sigma^2 \quad (12)$$

2.

3. DATA ANALYSIS AND RESULTS

3.1. Data and method of analysis

The data for the study was obtained from Tema Oil refinery. The AFC and the PACF of the time series are shown in Figure 1. The PACF shows a single spike at the first lag and the ACF shows a tapering pattern. The positive, geometrically decaying pattern of the AFC, coupled with single significant coefficient ϕ_{11} strongly suggest an AR(1){=ARMA(1,0)} process.

The time series plot(Figure 2) of the standardized residuals mostly indicates that there is no trend in the residuals, no outliers, and in general, no changing variance across time. The ACF of the residuals shows no significant autocorrelations, an indication of a good result. The Q-Q plot is a normal probability plot. It doesn't look too bad, so the assumption of normally distributed residuals looks okay. The bottom plot gives p-values for the Ljung-Box-Pierce statistics for each lag up to 20. These statistics consider the accumulated residual autocorrelation from lag 1 up to and including the lag on the horizontal axis. The dashed blue line is at 0.05. All p-values are above it indicating that this is a good result

The time series data ranged from January 2000 until December 2012. The coefficient of variation (V) was used to measure the index of instability of the time series data. The coefficient of variation (V) is defined as:

$$V = \frac{\sigma}{\bar{Y}} \quad (13)$$

where σ is the standard deviation and

$$\bar{Y} = \frac{1}{n}\sum_{t=1}^{n} Y_t \quad (14)$$

is the mean of petroleum prices changes.

A completely stable data has V = 1, but unstable data are characterized by a V>1 (Telesca *et al*., 2008). Regression analysis was used to test whether trends and seasonal factors exist in the time series data. The existence of linear trend factors was tested through this regression equation:

$$Y = \beta_0 + \beta_1 T + \varepsilon \, , \; \varepsilon \sim WN\left(0,\sigma^2\right) \qquad (15)$$

Stationarity is tested using unit root test. The stochastic time series of interest is Z_t. Taking the first difference we have the following: $\Delta Z_t = A_1 + A_2 + A_3 Z_{t-1} + \mu$ where ΔZ_t is the first difference, t is the trend variable taking on the values from 1, 2, 3, …, n. and Z_{t-1} is the one period lagged value of the variable Z. The null hypothesis is that A_3, the coefficient of Z_{t-1} is zero. That is to say that the underlying time series is nonstationary. This is called the unit root hypothesis. We proceed to show that a_3, the estimated value of A_3 is zero. The unit root test is used since we have already assumed that the time series is nonstationary. The tau test whose critical values are tabulated by the creators on the basis on Monte Carlo simulation are used(Gujarati, 2006). The rule for testing the hypothesis is that if the computed t(tau) value of the estimated A_3 is greater (in absolute value) than the critical Dickey Fuller(DF) tau values, we reject the root hypothesis, that is, we conclude that the said time series is stationary. On the other hand, if the computed tau value is smaller (in absolute values) than the critical DF tau values, we do not reject the unit hypothesis. In that case, the time series is nonstationary.

Data in Table 1 describe the nested ARMA(2,2) and GARCH(1,1) models. The forecasts have closest mean with respect to observed mean while EGARCH(1,1) model has shown maximum correlation with the observed Q2 returns. In all these cases Q1 returns data is used to calculate GARCH family model's parameters.

3.2. Empirical Results

Eight model selection criteria as suggested by Ramanathan (2002) were used to choose the best forecasting models among ARIMA and GARCH models, while the best time series methods for forecasting demand for petroleum products was chosen based on the values of four criteria, namely RMSE, MAE, MAPE and U-statistics (Table 2). Finally, the selected model was used to perform short-term forecasting for the next twelve months for demand for petroleum products starting from January 2013 until December 2013.

The results showed that the coefficient of variation (V) of the time series data was 1.312 (V>1). Because the V value was closed to 1, it was concluded that the time series data was stable (Telesca *et al*., 2008). The results of the regression analysis had shown that positive linear trend factor existed in the time series data but seasonal factor was not. Referring to the Augmented Dickey-Fuller tests results, the time series data of the study was not stationary. But after the first order of differencing was carried out, the time series data became stationary.

The double exponential smoothing method was used as the regression result had shown that positive linear trend factor exists in the time series data. Double exponential smoothing models consisted with two parameters which were symbolized as α for the mean and β for the trend. The best model of the double exponential smoothing was selected based on the lowest value of MSE (Mean Square Error) from the combination of α and β with condition 0<α, β<1.

The result showed that the combination α = 0.9 and β = 0.1 was the best forecasting model of double exponential smoothing method (Table 3). The double exponential smoothing model was written in equation form, from Table 4, as $F_{T+k} = a + bh = 4764.2345 + h*(-32.3465)$

All models which fulfilled the criteria of p+q≤5 have been considered and compared in this study. There were twenty ARIMA (p, d, q) models which fulfilled the criteria(Table 5). The parameters of the models were estimated with the least square method. Parameters which were not significant at 5% confidence level were dropped from the model. Using the eight model selection criteria suggested by Ramanathan (2002), the ARIMA (3, 1, 2) model was selected as the best model among the other ARIMA models. However, the parameters of AR (1) and MA (1) were found not significant and thus dropped from the model.

Identification and estimation of GARCH (p, q) models in this study were done by following the four steps that were ARCH effect checking, estimation, model checking and forecasting. Four GARCH (p,q) models were selected and compared, namely GARCH (1, 1), GARCH (1, 2), GARCH (2, 1) and GARCH (2, 2). Using the eight model selection criteria suggested by Ramanathan (2002), the GARCH (1, 1) model was selected as the best model among the other three GARCH models(Table 6). ARCH effect which was tested by using a regression analysis existed in the ARIMA (3, 1, 2) model. What this meant was that the ARIMA (3, 1, 2) model could be mixed with the best GARCH model (i.e., GARCH(1, 1)). Four model selection criteria were used to select the best forecasting model from the four different types of time series methods. Based on the results of the ex-post forecasting (starting from January until December 2013), the ARIMA (3, 1, 2)/GARCH (1, 1) model was the best short-term forecasting model for the demand for petroleum products (Table 9).

A linear relationship between Q2r and GARCH family forecast for different combinations was also obtained. R-sq values in table of models gave the percentage of variations which the regression was able to explain. It was clear that relationship (7) best explains the variations in the actual returns and forecasted returns, while relationship (5) was the second best in explaining the variations while relationship (3) was the third best in Table 8.

To determine whether there is significant difference for the mean demand and the standard deviation values of the observed and predicted data for each month, a z-test (for means) and F-test (for standard deviations) were applied (Haan, 1977; Devore and Peck, 1993). Since monthly mean values from observed and predicted data is between z-critical table values (± 1.96 for 2 tailed at the 5% significance level), the data support the claim that there is no difference between the mean values of observed and predicted data. Similarly, monthly standard deviation values from observed and predicted data were smaller than F- critical table values at the 5% significance level. Furthermore, these results show that the predicted data preserve the basic statistical properties of the observed series.

The coefficient of correlation (R), which measures the strength of the association between 2 variables, and the significance level (R_{sig}) related to the R of regression shows that there is a statistically significant linear relationship between the observed and predicted data. On the other hand, the coefficient of determination (R-square), which is interpreted as the proportionate reduction of total variation associated with the use of the predictor variable (the observed data in this study), and adjusted R-square measure, which presents the sample response of the population for each regression, were close to one. In addition, the results (F-value and F_{Sig}) concerning tests applied for determining whether the estimated regression functions adequately fit the data emphasize that the association between the observed and predicted monthly data sequences is linear. Based on these results, it is concluded that the selected best ARIMA model for each station can make accurate estimates.

CONCLUSION

Seven multiple regression relationships for different combination of nested ARMA / GARCH were used to filter their Q3 demand forecast. Filtered result analysis shows improvements in the correlation coefficient of the forecast demands and observed Q3 demands. Correlation coefficient is positive in some simulations, which were always negative with GARCH family model's forecast. Regression filtered results follow market trend better, while other descriptive parameters like variance, skewness and kurtosis become more comparable to actual Q3 demand. Therefore the proposed simulation framework under given observations to some extent has improved nested conditional mean and variance models forecast of Q3 forecast for petroleum products under such market conditions of 2013. However, it is not generally possible to get a definite relationship between observed and forecasted result.

This study also investigated four different types of univariate time series methods, namely exponential smoothing, ARIMA, GARCH and the mixed ARIMA/GARCH. The results showed that the mixed ARIMA/GARCH model outperformed the exponential smoothing, ARIMA and GARCH for forecasting the demand for petroleum products.

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Tables

Table 1: Descriptive Statistics of ARMA/GARCH forecast and Regression filtered forecast

Variables	Mean	StDev	Variance	Minimum	Maximum	Range	Skewness	Kurtosis
Q3r	-0.0049	0.0398	0.0016	-0.1160	0.0790	0.1950	-0.0600	-0.0500
Q3g2211f	-0.0067	0.0178	0.0003	-0.0056	0.0065	0.0121	1.0100	5.7100
Q3j2211f	0.0088	0.0707	0.0050	-0.0088	0.0110	0.0198	0.0400	-1.5100
Q3e2211f	-0.0072	0.0141	0.0002	0.0637	-0.0358	-0.0996	0.1111	-0.0767
Q3g2211rf	-0.0010	0.0283	0.0008	0.1173	-0.0698	-0.1872	0.0044	-0.8067
Q3j2211rf	0.0001	0.0566	0.0032	0.1709	-0.1038	-0.2748	0.0122	-1.5367
Q3e2211rf	-0.0112	0.1131	0.0128	0.2245	-0.1378	-0.3624	0.0005	-2.2667
Q3gjrf	0.0201	0.2262	0.0512	0.2781	-0.1718	-0.4500	0.0013	-2.9967
Q3jerf	-0.0313	0.4524	0.2047	0.3317	-0.2058	-0.5376	0.0134	-3.7267

Table 2: Criteria for Assessing Forecast Accuracy

Criteria	**Formula**	Double Exponential Smoothing	ARIMA(3, 1,)	GARCH(1, 1)	ARIMA(3, 1, 2) / GARCH(1, 1)
RSME	$\sqrt{\frac{ESS}{N}}$	414.4506	193 .3087	158.8801	155.5007
MAE	$\frac{1}{n}\sum_{t=1}^{n}\left\|Y_t - \widehat{Y}_t\right\|$	392.6509	1348.9835	122.8083	126.7645
MAPE	$\frac{1}{n}\sum_{t=1}^{n}\left\|\frac{Y_t - \widehat{Y}_t}{Y_t}\right\| \times 100\%$	6.8052	2.8309	2.4178	2.5528
U- Statistics	$\frac{RSME}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}\widehat{Y}^2 + \frac{1}{n}\sum_{t=1}^{n}Y^2}}$	0.0237	0.0192	0.0161	0.0157

Table 3: Error Sum of Square (ESS) according to α and β values

β	α								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.1	19345	81246	46341	31941	24941	21943	121426	31244	451427
0.2	18311	78242	47398	34742	23946	20846	110478	30142	410672
0.3	14567	61965	35943	33943	22647	20143	101495	30245	401421
0.4	13425	56349	26332	22946	21548	20959	100433	30456	400231
0.5	12833	45247	22344	21948	21041	22932	221409	32537	421502

Table 4: Output of the double exponential smoothing model

Parameters	**Values**
α	0.9800
β	0.1000
Sum of squared residuals	187364
Root mean squared error	234.3465
Mean	4764.2345
Trend	-32.3465

Table 5: Estimation of ARIMA (3, 1, 2)

Variables	Coefficient	Standard error	Z-statistic	p-value
Constant	19.2319	28.2345	0.6934	0.4562
AR(2)	-0.7843	0.0453	-18.4653	0.0001*
AR(3)	0.1287	0.0321	2.8675	0.0023*
MA(2)	0.9128	0.0234	20.3465	0.0002*
MA(3)	0.3465	0.0458	28.9874	0.0134*

*significant at 0.005 level

Table 6: Estimation of GARCH (1, 1)

	Mean Equation			
Variables	Coefficient	Standard error	Z-statistic	p-value
Constant	2.2319	18.2345	0.1934	0.8502
Conditional Variance Equation				
Constant	5452.87	3214.03	2.8642	0.0123*
ε_{k-1}^2	0.3128	0.1026	2.3465	0.0302*
σ_{k-1}^2	0.6487	0.0858	8.9814	0.0134*

*significant at 0.005 level

Table 7: Estimation of ARIMA (3, 1, 2)/GARCH (1, 1)

Variables	**Coefficient**	**Standard error**	**Z-statistic**	**p-value**
ARIMA(3, 1, 2)				
Constant	12.2337	18.2311	0.6341	0.5568
AR(2)	-0.7423	0.0403	-18.4639	0.0021*
AR(3)	0.2384	0.0125	3.4672	0.0053*
MA(2)	0.8023	0.0336	30.5462	0.0042*
MA(3)	0.2662	0.0151	27.3873	0.0104*
GARCH(1, 1)				
Constant	4405.3452	2345.0987	2.4986	0.8373
ε_{k-1}^2	0.3254	0.0975	2.4863	0.0034*
σ_{k-1}^2	0.7654	0.1203	7.9073	0.0002*

*significant at 0.005 level

Table 10: Regression Statistics with Q2r on LHS and Q2g2211f, Q2e2211f on RHS

Relationship	Regression order	K	Q2g2211f	Q2j2211f	Q2e2211f	p- value	R-Sq	R-Sq(adj.)
1	1	-0.001	0.07	0	0	0.0264	0%	0%
2	1	0.007	0	3.23	0	0.0262	1.50%	0%
3	1	0.010	0	0	3.01	0.0257	4.90%	3.40%
4	2	0.008	0.45	3.41	0	0.0263	1.60%	0%
5	2	0.003	0	-17.6	0	0.0251	10.60%	7.70%
6	2	0.010	0.34	3.04	2.06	0.0259	5.0%	1.80%
7	3	0.008	-0.89	-17.7	11.3	0.0349	11.20%	7.60%

Figure 1: AFC and PACF Plots

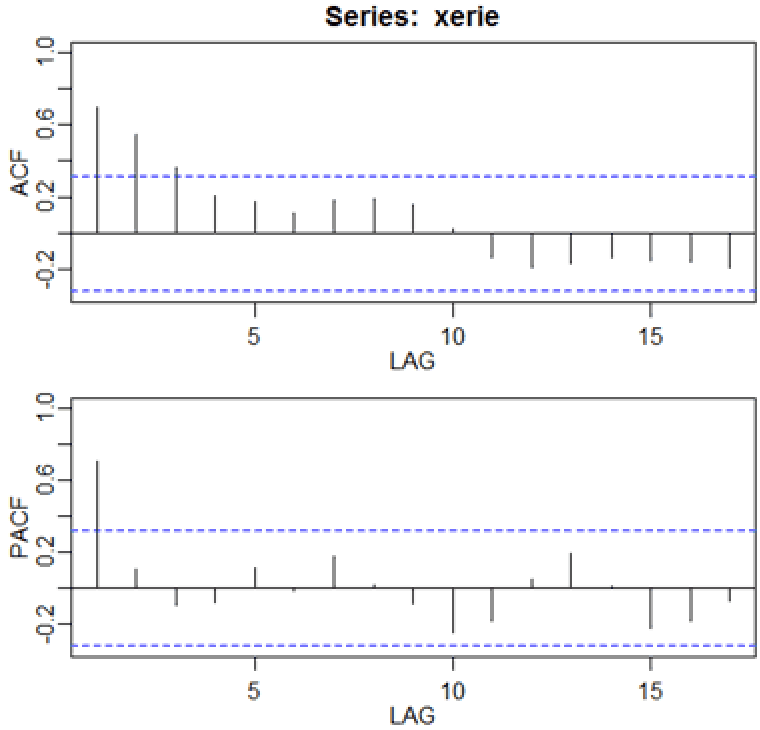
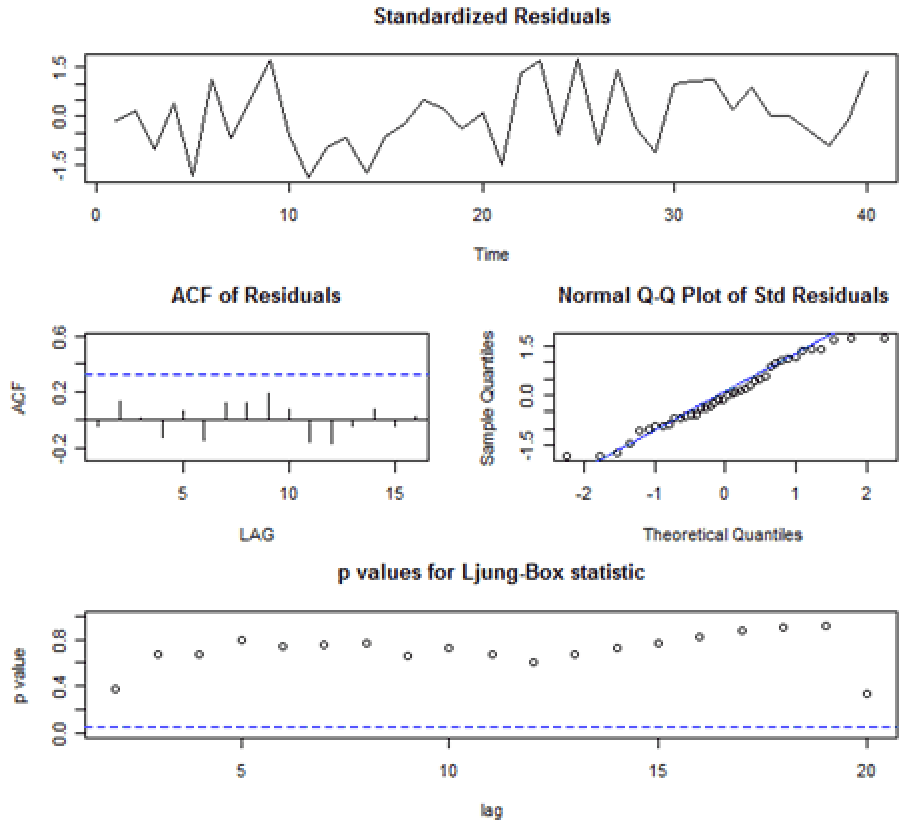


Figure 2: Time Series Plots of Residuals



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