Estimation of Garch Models for Nigerian Exchange Rates Under Non-Gaussian Innovations

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Abstract

Financial series often displays evidence of leptokurticity and in that case, the empirical distribution often fails normality. GARCH models were initially based on normality assumption but estimated model based on this assumption cannot capture all the degree of leptokurticity in the return series. In this paper, we applied variants of GARCH models under non-normal innovations-t-distribution and Generalized Error Distribution (GED) on selected Nigeria exchange rates. The Berndt, Hall, Hall, Hausman (BHHH) numerical derivatives applied in the estimation of models converged faster and the time varied significantly across models. Asymmetric GARCH model with t-distribution (GARCH-t) was selected in most of the cases whereas for Nigeria-US Dollar exchange rate, GARCH-GED was specified. Both distributions showed evidence of leptokurticity in Naira exchange rate return series. The result is of practical importance to practitioners.

Key Words: GARCH, Exchange rate, Model specification, Non-Gaussian distribution.

1. INTRODUCTION

The recent economic crises in the world have awakened economist and financial econometricians towards monitoring the financial assets such as stocks and exchange rates, which are characterized by different forms of volatility. Researches have been concentrated on the study and modelling of volatility.

Due to the fact that conditional distribution of the innovations of financial asset is normal, the unconditional distribution has fatter tails than the normal distribution, hence the usual time series models such as Vector Autoregressive (VAR) and Autoregressive Integrated Moving Average (ARIMA) that assume normality and homoscedasticity cannot be used to model volatility (Pinho and Santos, 2012). However, the magnitude of leptokurtosis introduced by the GARCH process does not always capture all the Leptokurtosis that is present in the high-frequency financial asset (Xekalaki and Degiannakis, 2010). Thus, there is a fair amount of evidence that the conditional distribution of \( \epsilon_t \) is non-normal as well.

The problem of non-normality of innovations of these financial assets has been considered lately. Bollerslev (1987) proposed the standardized student t distribution; which is symmetric around zero. Nelson (1991) introduced Generalized Error Distribution (GED) which accounts for fat tail, which is a symmetric distribution.

Few articles on Nigerian naira exchange rates employed interpreting and estimating properties of the series in different dimension. These few ones include Olowe (2009), Shittu (2009), Awogbemi and Alagbe (2011) and Ezike and Amah (2011). The analysis of the Naira exchange rate returns indicate that the empirical distribution of returns in the foreign exchange rate market is non-normal and this have very thick tails (Olowe, 2009). Shittu (2009) applied the Intervention Analysis Approach (IAA) on the exchange rate and the diagnostic tests were satisfied at both points of the intervention. Awogbemi and Alagbe (2011) examined the volatility in the Naira-US dollar and Naira-UK pound exchange rates using GARCH model and obtained estimates of volatility persistence. Their results further showed evidence of asymmetries in the residual series, and this is an indication for asymmetric volatility models. Ezike and Amah (2011) checked for possible long run relationship between...
exchange rates, demand and supply of foreign exchange rate in the Dutch Auction Market (DAS) and obtained a significant relationship in the variables. All these authors have considered monthly data in their investigation, and based on the frequency of the data applied, characteristics of the series were not well captured. Secondly, they consider one or two Naira exchange rates out of many. Thirdly, though they have considered different analysis approach but their models did not assume different distributional forms different from the normal distribution. Fourthly, exchange rates are volatility series and are asymmetric at time, and daily data need to be applied to really examine these properties.

In this work, we consider modelling some Nigeria exchange rate returns series with Generalized Autoregressive Conditionally Heteroscedastic (GARCH) models with normal and non-normal distribution innovations. The rest of the paper is structured as: Section 2 deals with the distributional assumptions of GARCH models and log likelihood functions; section 3 presents the data as well as the results of the model specification based on the model selection criteria and section 4 renders the conclusions remark.

2. GARCH MODELS AND DISTRIBUTIONAL ASSUMPTIONS

Following Bollerslev (1986), the Generalized Autoregressive Conditionally Heteroscedastic (GARCH) model of order (p, q) is given as:

$$\sigma_t^2 = w + \sum_{i=1}^{p} \alpha_i e_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \quad e_t = \sigma_t z_t$$

(1)

where $e_t$ are the returns series of the financial asset; $\sigma_t$ is the volatility at time $t$ and $z_t$ gives the assumed distribution. The parameters $w > 0$, $\alpha_i \ (i = 1, 2, \ldots, p)$, $\beta_j \ (i = 1, 2, \ldots, q)$ and for stationarity of the whole process, $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$.

The model in (1) is symmetric in the sense that the magnitude of the innovations (returns), $e_t$, is expected to predict the future volatility. The asymmetric specifications allow for the signs of the innovations (returns) to have impact on the volatility apart from the magnitude. The first asymmetric GARCH(p,q) model is Exponential GARCH (EGARCH)

$$\log \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \frac{e_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{q} \beta_j \log \sigma_{t-j}^2 + \sum_{i=1}^{q} \gamma_i \left( \frac{e_{t-i}}{\sigma_{t-i}} \right)$$

(2)

proposed in Nelson (1991). The GJR(p,q) model

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i e_{t-i}^2 + \sum_{i=1}^{q} \left[ \gamma_i d(e_{t-i} < 0) e_{t-i}^2 \right] + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

(3)

was proposed in Glosten et al.(1993). The APARCH(p,q) model of Ding et al.(1993) is given as,
\[
\sigma_i^2 = \omega + \sum_{j=1}^{p} \alpha_i (|\epsilon_{i-j}| - \gamma_i \epsilon_{i-j})^\delta + \sum_{j=1}^{q} \beta_i \log \sigma_{i-j}^2
\]  

(4)

where, in the three models, \( w, \alpha_i (i = 1, 2, \ldots, p), \beta_i (i = 1, 2, \ldots, q) \) are the parameters. The \( \gamma_i \) are the asymmetric parameter and \( \delta > 0 \) in APARCH model is the Box and Cox (1964) power transformation.

The GARCH \((p, q)\) model in (1-4) are specified with normal innovations \( z_t \) distributed as standardized normal,

\[
z_t \sim N(0,1)
\]  

(5)

and this suggests approaching the estimation of GARCH \((p,q)\) process via maximum likelihood estimation but in most cases, the distribution of the residuals (innovations) presents fatter tail than the normal distribution,

\[
f(z_t) = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{1}{2} z_t^2 \right)
\]  

(6)

Thus there is a fair amount of evidence that the conditional distribution of \( \varepsilon_t \) is a non-normal as well.

Bollerslev (1987) circumvented the problem of non-normality of innovations of GARCH \((p, q)\) model by proposing the standardized student t-distribution with \( v > 2 \) degree of freedom as,

\[
f(z_t, v) = \frac{\Gamma((v+1)/2)}{\Gamma(v/2)\sqrt{\pi(v-2)}} \left( 1 + \frac{z_t^2}{v-2} \right)^{-(v+1)/2}
\]  

(7)

where \( \Gamma(.) \) is the gamma function. This distribution is as well symmetric about zero and with \( v > 2 \). At \( v > 4 \), its kurtosis becomes \( 3(v-2)(v-4) \) which is larger than 3, the corresponding value for the normal distribution. As \( v \to \infty \), the distribution converges to the standard normal distribution in (3).

Nelson (1991) proposed another competing distribution with standardized student t-distribution. This is the Generalized Error Distribution (GED) with distribution function,

\[
f(z_t, v) = v \exp \left( \frac{-0.5|z_t|/\lambda^2}{\Gamma(v+1)/(\Gamma(v)\lambda)} \right)
\]  

(8)

with \( v > 0 \), where \( v \) is the tail fatness parameter and \( \lambda \equiv \sqrt{2^{-2/v}\Gamma(v^{-1})/\Gamma(3v^{-1})} \).

At \( v = 2 \), \( z_t \) becomes standardized normal and so the distribution reduces to normal distribution in (6). At \( v < 2 \), the GED distribution of \( z_t \) has thicker tails than the normal distribution; it has double exponential or Laplace
distribution at $v = 1$, while at $v > 2$, the distribution has thinner tails than the normal distribution. As $v \to \infty$, the distribution becomes Uniform on the interval $(-\sqrt{3}, \sqrt{3})$.

Note that in the EGARCH(p,q) model, the component $E \frac{\varepsilon_{t-i}}{\sigma_{t-i}} = \sqrt{\frac{2}{\pi}}$ under normally distributed innovations. For the Student $t$ distribution and Generalized Error Distribution (GED),

\[ E \frac{\varepsilon_{t-i}}{\sigma_{t-i}} = \Gamma\left(\frac{v+1}{2}\right) \frac{\sqrt{v-2}}{\sqrt{\pi}}(v-1) \Gamma\left(\frac{v}{2}\right) \text{ and } E \frac{\varepsilon_{t-i}}{\sigma_{t-i}} = \lambda 2^{-v-1} \Gamma\left(\frac{2v-1}{v}\right) \Gamma\left(\frac{v}{v-1}\right) \text{ respectively.} \]

### 2.1 KURTOSIS OF GARCH MODELS

For a GARCH(p,q) model, $\varepsilon_t = \sigma_t z_t$, where $E(z_t) = 0$, $\text{Var}(z_t) = 1$ and $E(z_t^4) = k_z + 3$ where $k_z$ is the excess kurtosis of the innovation $z_t$. Also, $E(\varepsilon_t) = 0$.

\[
\text{Var}(\varepsilon_t) = E(\sigma_t^2) = w(1-\alpha - \beta) \quad \text{in a GARCH(1,1) model},
\]

then follows to write $E(\varepsilon_t^4) = E(\sigma_t^4)E(z_t^4)$. Under the assumption of independency, then,

\[
E(\varepsilon_t^4) = (k_z + 3) \sigma_t^4.
\]

Squaring both sides of GARCH(1,1) model in (9) above leads to

\[
\sigma_t^4 = w^2 + \alpha^2 \varepsilon_{t-4}^4 + \beta^2 \sigma_{t-4}^4 + 2w\alpha \varepsilon_{t-4}^2 \sigma_{t-4}^2 + 2w\beta \varepsilon_{t-4} \sigma_{t-4}^2 + 2\alpha \beta \varepsilon_{t-4} \sigma_{t-4}^2 \varepsilon_{t-4}^2
\]

(10)

Taking the expectation of the resulting expression and using the assumptions stated above,

\[
E(\sigma_t^4) = \frac{w^2 (1 + \alpha + \beta)}{(1 - \alpha - \beta) \left[1 - \alpha \alpha_t (k_z + 2) - (\alpha + \beta)^2\right]}.
\]

(11)

The excess kurtosis of $\varepsilon_t$ is then given as,

\[
k_z = \frac{E(\varepsilon_t^4)}{\left[E(\varepsilon_t^2)^2\right]} - 3
\]

\[
= \frac{(k_z + 3) \left[1 - (\alpha + \beta)^2\right]}{1 - 2\alpha^2 - (\alpha + \beta)^2 - k_z \alpha^2} - 3
\]

(12)

When $z_t$ is normally distributed ($k_z = 0$),
\[ k_e = \frac{6\alpha_i^2}{1 - 2\alpha_i^2 - (\alpha_i + \beta_i)^2} \]  

(13)

When \( z_t \) is not normally distributed \((k_z \neq 0)\),

\[
k_e = \frac{k_z - k_z (\alpha_i + \beta_i)^2 + 6\alpha_i^2 + 3k_z \alpha_i^2}{1 - 2\alpha_i^2 - (\alpha_i + \beta_i)^2 - k_z \alpha_i^2}
\]  

(14)

In the two cases discussed, the coefficient \( \alpha_i \) is important in determining the tail behaviour of \( \alpha_i \), since in both, once \( \alpha_i = 0, k_e = 0 \). Hence, \( k_e = k_z \) for the non-normally distributed case and it implies the similarity of the tail behaviours of both \( \varepsilon_t \) and the standardized \( z_t \). In the student \( t \)-distribution earlier discussed, \( E(\varepsilon_t^4) = k_z + 3 \) at degree of freedom \( \nu > 4 \) where \( k_z \), the excess kurtosis is set at \( k_z = 6/(\nu - 4) \).

2.2 LOG-LIKELIHOOD OF THE DISTRIBUTIONS AND ESTIMATION APPROACH

The log-likelihood functions of the standardized distributions discussed above are presented here.

For normally distributed innovations, \( z_t \) the log-likelihood function is

\[
L_i = \log \left( \frac{1}{\sqrt{2\pi}} \right) - \frac{\varepsilon_i^2}{2\sigma_i^2} - \frac{1}{2} \log \sigma_i^2
\]

(15)

where \( x_t \) denotes a \( k \times 1 \) vector of endogenous and exogenous explanatory variables in the information set \( I_{t-1} \) and \( N \) is the sample size of the time series. The full log-likelihood function is written as;

\[
L_i = \frac{-1}{2} \left[ N \log(2\pi) + \sum_{i=1}^{N} \frac{\varepsilon_i^2}{\sigma_i} + \sum_{i=1}^{N} \log \sigma_i^2 \right]
\]

(16)

where \( N \) is the sample size.

In a similar way, the log likelihood function for the standardized \( t \)-distribution is

\[
L_i = \frac{-1}{2} \left[ N \log \left( \frac{\pi^{(\nu-2)/2} \Gamma((\nu+1)/2)}{\Gamma((\nu+2)/2)} \right) \right] - \frac{1}{2} \sum_{i=1}^{N} \log \sigma_i^2 + \frac{\nu}{2} \sum_{i=1}^{N} \log \left( 1 + \frac{\varepsilon_i^2}{\sigma_i^2} \right)
\]

(17)

and that of GED is,
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\[
I_l = \frac{1}{2N} \left\{ N \log \left( \frac{\Gamma \left( \nu^2 \right)}{\Gamma \left( \nu^2/2 \right) \nu^2} \right) + \sum_{i=1}^{N} \log \sigma_i^2 + \left( \nu + 1 \right) \sum_{i=1}^{N} \left( \frac{\Gamma \left( 3\nu^2 \right) \epsilon_i^2}{\sigma_i^2 \Gamma \left( \nu^2 \right)} \right) \right\}
\]

(18)

These likelihood functions give in (7), (8) and (9) are then estimated using the numerical derivatives based on the fact that GARCH models lack closed form estimation. Berndt, Hall, Hall and Hausman (BHHH) algorithm of Berndt, et al (1974) is then used. This algorithm uses only first derivatives of the likelihood function and computes a set of parameter values as

\[
\psi_{(i+1)} = \psi_{(i)} - \left( \sum_{i=1}^{N} \frac{\partial^2 I_l}{\partial \psi_i \partial \psi_j} \right)^{-1} \frac{\partial I_l}{\partial \psi_i}
\]

(19)

where \( I_l \) is the likelihood function. The initial parameter set is given as \( \psi_{(0)} \) and the parameter set which maximize the likelihood function is denoted as \( \psi_{(i+1)} \).

The estimation of GARCH (p, q) model with t-distribution and GED follow Quasi Maximum Likelihood Estimation (QMLE) since normality assumption is violated in both cases.

The best model is determined for the series by employing necessary criterion. The commonly used criteria suggested in Harvey (1989) are the Akaike Information (AIC) and Schwarz Bayesian Information Criterion (SBIC) proposed by Akaike (1974) and Schwarz (1978). The AIC and BIC are defined by:

\[
AIC = -2I_l \left( \Theta \right) + 2k
\]

(20)

and

\[
SBIC = -2I_l \left( \Theta \right) + 2k \ln(N)
\]

(21)

where \( I_l \) is any of the likelihood function defined above. The \( \Theta \) is the parameter set in the AR-GARCH model, \( k \) is the number of parameters to be jointly estimated and \( N \) is the size of the time series.

3. DATA PRESENTATION, RESULTS AND DISCUSSION

The data considered in this work are the trading days Nigeria exchange rate with Euro, British pound, Japanese Yen and US dollars. These data span between 10/12/2001 to 14/12/2011. The data have been sourced from Central Bank of Nigeria website (www.cenbank.org).

The empirical analysis of these data is given in Yaya, Adepoju and Adeniyi (2012). Based on the results, the Naira-US Dollar exchange rate was the least volatile while Naira-British pound was the most volatile rate.

In the log return series, autocorrelation is only significant at first lag, therefore autoregressive model of order one [AR (1)] is the first estimated as the mean equation.
The best GARCH model is determined for each of the exchange rate returns based on the minimum AIC and SBIC, maximum log-likelihood estimates and normality of the GARCH residuals.

### Table 1: Nigeria Naira-Euro Exchange Rate

<table>
<thead>
<tr>
<th>Model</th>
<th>AIC</th>
<th>SBIC</th>
<th>Log-lik.</th>
<th>Skewness</th>
<th>Ex. Kurtosis</th>
<th>JB</th>
<th>Comp. Time (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)-GARCH(1,1)-Normal</td>
<td>-8.404</td>
<td>-8.392</td>
<td>10249.7</td>
<td>-3.436</td>
<td>80.779</td>
<td>6.68E5</td>
<td>1.373</td>
</tr>
<tr>
<td>AR(1)-GJR(1,1)-Normal</td>
<td>-8.388</td>
<td>-8.374</td>
<td>10231.3</td>
<td>-3.182</td>
<td>70.647</td>
<td>5.11E5</td>
<td>2.278</td>
</tr>
<tr>
<td>AR(1)-EGARCH(1,1)-Normal</td>
<td>-8.358</td>
<td>-8.342</td>
<td>10196.1</td>
<td>-0.064</td>
<td>9.4796</td>
<td>9130.2</td>
<td>20.374</td>
</tr>
<tr>
<td>AR(1)-APARCH(1,1)-Normal</td>
<td>-8.426</td>
<td>-8.410</td>
<td>10278.8</td>
<td>-2.562</td>
<td>58.094</td>
<td>3.46E5</td>
<td>8.673</td>
</tr>
<tr>
<td>AR(1)-GARCH(1,1)-t</td>
<td>-8.722</td>
<td>-8.707</td>
<td>10637.8</td>
<td>-3.271</td>
<td>75.319</td>
<td>5.81E5</td>
<td>3.51</td>
</tr>
<tr>
<td>AR(1)-GJR(1,1)-t</td>
<td>-8.724</td>
<td>-8.707</td>
<td>10641.7</td>
<td>-3.280</td>
<td>77.109</td>
<td>6.08E5</td>
<td>3.9</td>
</tr>
<tr>
<td>AR(1)-EGARCH(1,1)-t</td>
<td>-8.687</td>
<td>-8.668</td>
<td>10597.1</td>
<td>-3.538</td>
<td>84.356</td>
<td>7.28E5</td>
<td>63.585</td>
</tr>
<tr>
<td>AR(1)-APARCH(1,1)-t</td>
<td>-8.740</td>
<td>-8.721</td>
<td>10662.6</td>
<td>-0.866</td>
<td>126.87</td>
<td>1.64E6</td>
<td>9.22</td>
</tr>
<tr>
<td>AR(1)-GARCH(1,1)-t</td>
<td>-8.701</td>
<td>-8.689</td>
<td>10611.8</td>
<td>-0.35</td>
<td>7.379</td>
<td>5583.8</td>
<td>2.465</td>
</tr>
<tr>
<td>AR(1)-GJR(1,1)-t</td>
<td>-8.695</td>
<td>-8.680</td>
<td>10604.6</td>
<td>-0.399</td>
<td>7.052</td>
<td>5116.6</td>
<td>2.839</td>
</tr>
<tr>
<td>AR(1)-EGARCH(1,1)-t</td>
<td>-8.590</td>
<td>-8.574</td>
<td>10478.6</td>
<td>-0.062</td>
<td>10.805</td>
<td>11860</td>
<td>25.178</td>
</tr>
<tr>
<td>AR(1)-APARCH(1,1)-t</td>
<td>-8.701</td>
<td>-8.689</td>
<td>10612.6</td>
<td>-3.373</td>
<td>79.567</td>
<td>6.48E5</td>
<td>2.995</td>
</tr>
<tr>
<td>AR(1)-GARCH(1,1)-GED</td>
<td>-8.676</td>
<td>-8.660</td>
<td>10583.6</td>
<td>-2.089</td>
<td>47.190</td>
<td>2.28E5</td>
<td>4.134</td>
</tr>
<tr>
<td>AR(1)-GJR(1,1)-GED</td>
<td>-8.583</td>
<td>-8.564</td>
<td>10470.8</td>
<td>-4.107</td>
<td>97.602</td>
<td>9.75E5</td>
<td>21.559</td>
</tr>
<tr>
<td>AR(1)-EGARCH(1,1)-GED</td>
<td>-8.675</td>
<td>-8.656</td>
<td>10583.0</td>
<td>N A N</td>
<td>N A N</td>
<td>N A N</td>
<td>N A N</td>
</tr>
<tr>
<td>AR(1)-APARCH(1,1)-GED</td>
<td>-8.695</td>
<td>-8.678</td>
<td>10606.8</td>
<td>N A N</td>
<td>N A N</td>
<td>N A N</td>
<td>N A N</td>
</tr>
</tbody>
</table>

In Table 1, APARCH(1,1) model with t-distribution was specified as the best model for Naira-Euro exchange rate based on the minimum AIC and SBIC values. This GARCH residuals of this model also presents longest tail (kurtosis = 126.87) among the other models.

### Table 2: Nigeria Naira-British Pound Exchange Rate

<table>
<thead>
<tr>
<th>Model</th>
<th>AIC</th>
<th>SBIC</th>
<th>Log-lik.</th>
<th>Skewness</th>
<th>Ex. Kurtosis</th>
<th>JB</th>
<th>Comp. Time (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)-GARCH(1,1)-Normal</td>
<td>-8.701</td>
<td>-8.689</td>
<td>10611.8</td>
<td>-0.3595</td>
<td>7.379</td>
<td>5583.8</td>
<td>2.465</td>
</tr>
<tr>
<td>AR(1)-GJR(1,1)-Normal</td>
<td>-8.695</td>
<td>-8.680</td>
<td>10604.6</td>
<td>-0.3991</td>
<td>7.052</td>
<td>5116.6</td>
<td>2.839</td>
</tr>
<tr>
<td>AR(1)-EGARCH(1,1)-Normal</td>
<td>-8.590</td>
<td>-8.574</td>
<td>10478.6</td>
<td>-0.062</td>
<td>10.805</td>
<td>11860</td>
<td>25.178</td>
</tr>
<tr>
<td>AR(1)-APARCH(1,1)-Normal</td>
<td>-8.695</td>
<td>-8.678</td>
<td>10606.8</td>
<td>N A N</td>
<td>N A N</td>
<td>N A N</td>
<td>N A N</td>
</tr>
<tr>
<td>AR(1)-GARCH(1,1)-t</td>
<td>-8.892</td>
<td>-8.878</td>
<td>10845.8</td>
<td>-0.3762</td>
<td>7.7926</td>
<td>6226.2</td>
<td>2.325</td>
</tr>
<tr>
<td>AR(1)-GJR(1,1)-t</td>
<td>-8.892</td>
<td>-8.875</td>
<td>10845.9</td>
<td>-0.3781</td>
<td>7.8562</td>
<td>6.327.8</td>
<td>2.683</td>
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<tr>
<td>AR(1)-EGARCH(1,1)-t</td>
<td>-8.842</td>
<td>-8.828</td>
<td>10786.1</td>
<td>0.2486</td>
<td>15.200</td>
<td>23496.0</td>
<td>22.839</td>
</tr>
<tr>
<td>AR(1)-APARCH(1,1)-t</td>
<td>-8.898</td>
<td>-8.882</td>
<td>10854.9</td>
<td>-0.2562</td>
<td>8.7319</td>
<td>7772.0</td>
<td>9.001</td>
</tr>
<tr>
<td>AR(1)-GARCH(1,1)-GED</td>
<td>-8.857</td>
<td>-8.843</td>
<td>10802.6</td>
<td>-0.2584</td>
<td>7.9064</td>
<td>6377.2</td>
<td>2.434</td>
</tr>
<tr>
<td>AR(1)-GJR(1,1)-GED</td>
<td>-8.856</td>
<td>-8.840</td>
<td>10802.7</td>
<td>-0.2618</td>
<td>7.9360</td>
<td>6425.6</td>
<td>3.323</td>
</tr>
<tr>
<td>AR(1)-EGARCH(1,1)-GED</td>
<td>-8.769</td>
<td>-8.750</td>
<td>10697.8</td>
<td>0.20702</td>
<td>14.353</td>
<td>20944.0</td>
<td>16.068</td>
</tr>
<tr>
<td>AR(1)-APARCH(1,1)-GED</td>
<td>-8.837</td>
<td>-8.818</td>
<td>10780.6</td>
<td>N A N</td>
<td>N A N</td>
<td>N A N</td>
<td>5.226</td>
</tr>
</tbody>
</table>

In Table 2, the best model for Naira-British Pound exchange rate is APARCH(1,1) with t-distribution of residuals.
In Table 3, the best model for Naira-Japanese Yen exchange rate is also APARCH (1,1) model with t-distribution in the residuals. This model presents kurtosis estimate of 162.38, which is the highest among the models estimated.

In Table 4, estimation of models for Naira-US dollar exchange rate posed more serious convergence problem due to more zeros in the return series as a result of series stability (less volatility). As it is observed in the results that normality tests were not computed for models with the t-distributions. Based on the Information Criteria, the best model here is GARCH(1,1) with GED. The model also records highest tail measure.
4. CONCLUSION

In this paper, variants of GARCH models for both symmetric and asymmetric types were considered in modelling daily Nigeria naira exchange rate returns series under non-normal GARCH distributions. Four common naira exchange rates selected were Naira-Euro, Naira-British Pound, Naira-Japanese Yen And Naira US Dollars exchange rates. GARCH models were estimated under both normality and non-normality assumptions of GARCH models. The t-distribution and Generalized Error Distribution (GED) were considered in the non-normally distributed case.

The complex log-likelihood from the Quasi Maximum Likelihood Estimation (QMLE) was simplified using the Berndt, Hall, Hall, Hausman (BHHH) numerical derivative to optimize the estimates of the parameters of the models. Computational time varied from model to model, and divergence was hardly experienced except in the case of Naira-US dollars exchange rate series in which return series gave more zeros as a result of series stability for some time periods.

Asymmetric GARCH models with t-distribution were specified for the series, except for Naira-US Dollars exchange rates, where GARCH model with GED was specified as the optimal model. More zeros in the return series of Naira-US dollars exchange rates affected the tail measure the series. Estimates of kurtosis for GARCH residuals also showed evidence for specifying GARCH variants with t-distribution.

This work can be generalized by considering all the available Nigeria naira exchange rates to confirm if the return series will always show longer tail in most cases.

REFERENCES


