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# Endogenizing the Growth Effects of Human Capital, Technology, and Urbanization: A Derivation*

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**Abstract**
This paper seeks to derive an econometrically testable model which gauges the effects of human capital, technology, and urbanization on economic growth, with technological progress endogenized as a function of human capital and urbanization. This paper does not, however, empirically test the model(s) derived. It only seeks to build such a model which incorporates the impacts of human capital and technological progress, as two potential sources of growth emanating from non-tangible inputs, on sound theoretical basis.
**Keywords:** Modeling Human Capital and Technology in Economic Growth, Modeling Urbanization in Economic Growth

**The Paper**

Starting with a Solow-type Constant Returns to Scale (CRS) production function for the whole economy (1956), Mankiw, Romer, and Weil (1990) add human capital H to obtain:

(1) $Y_t = K_t^{\alpha} H_t^{\beta} (A_t L_t)^{1-\alpha-\beta}$, with $\alpha+\beta<1$,

where Y is output, K is capital, L is labor, A is the level of technology, and H is human capital, while the subscript t indicates that all of the above are being measured at a given time t.

In this construction, A and L are assumed to be growing exogenously at the rates g and n respectively according to the following equations:

(2) $A_t = A_0 e^{gt}$ and

(3) $L_t = L_0 e^{nt}$

As Mankiw, Romer, and Weil (1990) emphasize, the assumption that $\alpha+\beta<1$ is a crucial one because it implies that there are diminishing returns to all capital. "If $\alpha+\beta=1$, then there are constant returns to scale in reproducible factors. In this case, there is no steady state for this model" (p.12). If there's no steady state, there's no convergence. This means that the rich countries can go on getting richer without poor countries ever being able to catch up with them. Since the intention is to formulate and test a model with diminishing returns to all capital, we impose the condition above that $\alpha+\beta<1$.

**Two Modifications**

The model to be developed introduces two modifications to the model of Mankiw, Romer, and Weil (henceforth MRW) above.

These two modifications are:

1) the inclusion of an index of urban concentration into MRW's function to gauge the effects of agglomeration economies in economic growth. The rationale for this is discussed below.
2) partially endogenizing $A_t$, otherwise defined as the level of technology by some, and total factor productivity by others, by making it a function of urbanization and human capital. Making $A_t$ a function of human capital has been explored by Benhabib and Spiegel (1994) and making it explicitly a function of urbanization has been loosely treated in Jane Jacobs (1969) and James Rauch (1993).

The first modification transforms MRW's production function into:

(4) $Y_t = K_t^{\alpha} H_t^{\beta} U_t^{\gamma} (A_t L_t)^{1-\alpha-\beta-\gamma}$

where U is urban capital, measured by an index of urban concentration. Because of the role large cities play in economic growth we should perhaps speak of metropolitan concentration and metropolitan capital, or urban agglomeration and agglomeration capital. In what follows, "urban capital" will refer to urban agglomeration.

The second modification makes it possible to rewrite (4) as:

(6) $Y_t = K_t^{\alpha} H_t^{\beta} U_t^{\gamma} (A_t (U_t,H_t) L_t)^{1-\alpha-\beta-\gamma}$

To be consistent with the neoclassical assumption of diminishing returns to scale in all capital, we impose the overarching condition that $\alpha+\beta+\gamma<1$ on the new model which incorporates urban capital above.

**Theoretical Justification for Adding U to the Aggregate Production Function**

This is the age of non-tangible inputs. If capital K may be decomposed into human and physical components by virtue of a distinct contribution to productivity precipitated by investment in non-material means of production such as knowledge, skills, or experience, a case can be also made for the existence of another non-tangible input to be called urban capital $U_t$.

If urban economists are justified in indicating the presence of agglomeration economies, prompted by savings arising from rational location decisions (as separate from economies of scale, prompted by the sheer scale of production), then those agglomeration economies make a unique and independent contribution to output. Arising solely from the interaction of the location decisions of many firms and individuals, the congregation, or lack of, of these economic agents, and the patterns of their congregations is taken here to create a unique input: urban capital.

Location in a metropolitan area for example, albeit expensive, occurs to take advantage of agglomeration economies. The most specialized inputs, which produce output with the lowest per capita demand, locate in the largest possible metropolitan areas, brain surgeons being an example. That allows agglomeration economies to fully materialize. Therefore, a less than optimal location decision would contribute negatively to profit just like a less than optimal allocation of labor, physical capital, or raw materials.

In that sense, location is not a geographical concept but an economic one. The same bridge or highway in the same exact place may have a much higher productivity, and therefore value, on the verge of the twenty-first century say than in Roman times. A higher urban concentration in absolute and relative terms around the facility in modern times may perhaps explain the difference.

"Moomaw (1988) concludes agglomeration economies induce firms to locate close to each other to minimize production and transportation costs. Manufacturing firms which locate in large cities minimize production cost more than firms in smaller cities, even if input prices are higher in large cities. Moomaw (1981) finds that the productivity advantages of larger cities are much larger for the non-manufacturing sector than the manufacturing sector" (Ibid).

Urban capital then is not just location in an abstract sense, but the configuration and degree of concentration of the aforementioned business and public services arising from individual location decisions. For example, infrastructure in the "wrong" place, as a result of some government plan or decree, does not yield as much urban capital as infrastructure in the "right" place resulting from the presumably rational location decisions of firms and individuals, even though the cost of constructing such infrastructure might be the same in both cases.

On the level of the economy as a whole, a rising urban concentration imparts worth and creates demand for business and public services. To the extent that those services tend to be more concentrated in larger than in small cities, agglomeration economies arise more in metropolitan areas and thus large metropolitan areas become our proxy for urban capital. To qualify as capital, though, urban concentration has to generate dynamic rather than merely static externalities, or agglomeration economies. Static externalities on the other hand may serve as shifters of the production function. Dynamic externalities cause the urban economy to grow over time, and thus propel the national economy forward.

Empirically, there has not been a lack of evidence on a significant relationship between proxies for urbanization and economic growth (Moomaw and Shatter 1993). This study however takes the further step of explicitly incorporating urbanization or urban concentration as an input, and later as a shifter, in the growth equation. A general survey of the growth literature by Barro and Sala-I-Martin (1995), indicates that this contribution is original. What remains though is to work out the MRW model mathematically with U included and then to see how well the new specification fits the data or if it contributes to the relevant questions posed by the growth literature.

**Endogenizing Technology**

Alternatively urban capital could be viewed not as a separate input but merely as a shifter that affects the economy's production function through its impact on technology. In this case, urban capital plays the role of enhancers of total factor productivity through their effect on the level of technology in a country. Total factor productivity here should not be defined in the narrow sense of production technology only, but in the general sense of a country's institutions and infrastructure.

Thus, the second modification to MRW's production function is a specification of variables that affect the level of technology. MRW assumed that technological progress will change at an exogenous rate g as in equation (2) above. This rate was taken as uniform across all countries in the sample.

This specification includes technological progress as an exogenous and uniform rate of change, but it also allows urban and human capital to affect the level of technology at time t.

The level of accumulated human and urban capital in a given country will thus contribute to a higher level of technology, if we assume technological innovation to be a positive externality generated at least partially by 1) a generally higher level of knowledge and skills, and 2) the more intense competition and interaction of firms and employees from diverse industries in the same urban place or locale (Rauch 1993).

Thus the second modification implies that $A_t$ is now a function of human and urban capital accumulation as in:

(5) $A_t=A_0e^{gt}A(H,U)$, where $A(H,U)= H^{c1}U^{c2}e^{c3HU}$ ➔
(5a) $A_t=A_0e^{gt} H^{c1}U^{c2}e^{c3HU}$ ➔ $\ln A_t = \ln(A_0e^{gt+c3HU} H_t^{c1}U_t^{c2})$ ➔
(5b) $\ln A_t = \ln A_0 + gt + c_1 \ln H_t + c_2 \ln U_t + c_3 H_tU_t$

where the interaction term HU implies that the preponderance of urbanization and human capital generates a higher level of technology than the sum of the parts. For the economy as a whole, this is the Rauch effect.

The formulation above keeps the growth rate of technology g exogenous, but makes the growth rate of technology function shift by a constant fraction of the interactive term HU. As long as human and urban capital are NOT assumed to be functions of time however, the slope of the growth rate of technology function remains exogenous. But the level of technology is now dependent on the infusion of human and urban capital, i.e., education as well as business and public services. In other words, the growth rate is still g, but the growth rate function of technology shifts up or down in proportion to the level of human and urban capital available.

Unlike the first modification which envisages urbanization as capital, the second modification contributes to total factor productivity by shifting the production function itself up or down depending on whether that country has more or less human and urban capital. This is tantamount to changing the intercept but not the slope of the growth function. Both possibilities will be explored theoretically in the context of the model developed here.

For example, take equation (7) below, after adding an intercept term to equation (6):
(7) $Y_t = A_0 K_t^{\alpha} (A_t (H_t,U_t)L_t)^{1-\alpha-\beta}$, where $A_0$ is an intercept term that denotes initial conditions.
Thus, equation (4) can be rewritten as
(7a) $Y_t = A_0 K_t^{\alpha} (A_t (H_t,U_t))^{1-\alpha-\beta} (L_t)^{1-\alpha-\beta}$ ➔ $Y_t = A_0 K_t^{\alpha} A_t^{1-\alpha-\beta} (L_t)^{1-\alpha-\beta}$ ➔
(7b) $Y_t = A_0 .A_t^{1-\alpha-\beta} .K_t^{\alpha} .L_t^{1-\alpha-\beta}$

which simply implies the same old production function with a new higher intercept term, $A_0 .A_t^{1-\alpha-\beta}$, by a proportion equivalent to the output elasticity with respect to labor times the coefficients of the relationship between A and H and U. If we make the new intercept, $A_0 .A_t^{1-\alpha-\beta}$, equal to $A_1$, then

$\ln A_1 = \ln A_0 + 1-\alpha-\beta \ln A_t$. This implies that a change of one percent in whatever affects $A_t$ will affect the intercept by $1-\alpha-\beta$, or the elasticity of output with respect to labor, times the coefficients of the function $A_t (H_t,U_t)$.

Note that this result is robust to any returns to scale. For example, even if we had a production function where $Y_t = A_0 K_t^{\alpha} (A_t (H_t,U_t)L_t)^{\eta}$, where $\alpha+\eta=?$, then we would still have $Y_t = A_0 K_t^{\alpha} (A_t (H_t,U_t))^{\eta}(L_t)^{\eta}$, and a coefficient for the proxy of the variable(s) that affect $A_t$ that is necessarily equal to the output elasticity with respect to labor multiplied by the coefficients of the relationship $A=f(H_t,U_t)$.

**Developing the Model**

Recall that with the two modifications combined, i.e., with (5) substituted back into (4), the general specification becomes (assuming constant returns to scale):
(6) $Y_t = K_t^{\alpha} H_t^{\beta} U_t^{\gamma} (A_t (H_t ,U_t)L_t)^{1-\alpha-\beta-\gamma}$

which is of course the same as equation (4) above, except that the formulation in equation (6), i.e., $Y_t = K_t^{\alpha} H_t^{\beta} U_t^{\gamma} (A_t (H_t ,U_t)L_t)^{1-\alpha-\beta-\gamma}$ is meant to emphasize the inclusion of human and urban capital in the production function both as possible inputs and as shifters of the technology function $A_t$.

Following Lucas (1988,1990), we assume that anything that enhances the productivity of the average worker affects A and relates to total factor productivity; otherwise it's an input. Thus a worker's decision to move to a metropolitan area or to earn a degree in anticipation of increasing his or her income plays out through its effect on that worker's marginal productivity through A.

Nevertheless, the total effect of these individual decisions is more than the sum of the parts. An increase in the metropolitan percentage of the population or the quality of the people one works with generates externalities reflected in the coefficients $\gamma$ and $\beta$ respectively. Then we can speak in terms of urban and human capital as inputs.

Subsequently equations (4) or (6) above may help us determine in what way and how much H and U contribute to output if any.

**Definitions**

Let AL be the effective units of labor,
then k = K/AL : Physical capital per effective unit of labor,
h = H/AL : Human capital per effective unit of labor,
u = U/AL : Urban capital per effective unit of labor,
y = Y/AL : Output per effective unit of labor.

**Rates of Growth**

MRW, Nazrul Islam (1995), and others assume subsequently that k grows as follows:

(8) $k^{\bullet}_t = S_K y_t - (n + g + \delta) k_t$

where $S_K$ is the fraction of output invested in building physical capital, assumed constant.
n is the rate of growth of labor,
g is the rate of growth of technology,
and δ is depreciation.

Thus the equation above implies that the change of the capital-labor ratio, $k^{\bullet}$, is a function of the difference between the fraction of output that is invested and the growth rates of other inputs (labor and technology) and depreciation.

Similarly MRW assumed h would grow as:

(9) $h^{\bullet}_t = S_H y_t - (n + g + \delta) h_t$

where $S_H$ is the fraction of output invested in building human capital, assumed constant, with the rest of the variables as previously defined.

Along the same lines, we assume that u will grow as follows:

(10) $u^{\bullet}_t = S_\mu y_t - (n + g + \delta) u_t$

where $S_\mu$ is the fraction of income invested in building urban capital or infrastructure, i.e., business and public services, also assumed constant, with the rest of the variables as previously defined .

And following Mankiw, Romer, and Weil, we assume that the same production function applies to all three kinds of capital and to consumption, i.e., we adopt the assumption that one unit of physical capital for example can be transformed costlessly into one unit of urban capital or into one unit of consumption. Furthermore, we assume that all three different kinds of capital depreciate at the same rate. Recognizing that these are constraining assumptions, we adopt them to simplify the analysis.

**The Modified Production Function**

Equation (4) $Y_t = K_t^{\alpha} H_t^{\beta} U_t^{\gamma} (A_t L_t)^{1-\alpha-\beta-\gamma}$, can now be rewritten as

Equation (6) $Y_t = K_t^{\alpha} H_t^{\beta} U_t^{\gamma} (A_t(H_t, U_t) L_t)^{1-\alpha-\beta-\gamma}$ as pointed out before.

Dividing both sides by AL, and momentarily leaving aside the subscript t merely for convenience➔

$Y/AL = y = (K/AL)^{\alpha} (H/AL)^{\beta} (U/AL)^{\gamma}$ ➔

(11) $y_t = k_t^{\alpha} h_t^{\beta} u_t^{\gamma}$

which states that output per effective unit of labor is a function of physical capital, human capital, and urban capital per unit of effective labor.

**The Steady-State Levels of Physical and Human Capital**

Following MRW, except for adding urbanization, in the steady state, all of the growth rates of k, h, and u are equal to zero by definition. So,

$k^{\bullet}_t = 0 = S_K y_t - (n + g + \delta) k_t$ ➔

(12) $S_K y_t = (n + g + \delta) k_t$ ,

$h^{\bullet}_t = 0 = S_H y_t - (n + g + \delta) h_t$ ➔

(13) $S_H y_t = (n + g + \delta) h_t$ ,

$u^{\bullet}_t = 0 = S_\mu y_t - (n + g + \delta) u_t$ ➔

(14) $S_\mu y_t = (n + g + \delta) u_t$ .

Then substituting (11) above into $y_t$ in each of equations (12), (13), and (14), we obtain the following terms for $k_t$ , $h_t$ , and $u_t$ :

(15) $k_t = [(S_K \; h_t^{\beta} \; u_t^{\gamma}) / (n + g + \delta)]^{1/1-\alpha}$

(16) $h_t = [(S_H \; k_t^{\alpha} \; u_t^{\gamma}) / (n + g + \delta)]^{1/1-\beta}$

(17) $u_t = [(S_\mu \; k_t^{\alpha} \; h_t^{\beta}) / (n + g + \delta)]^{1/1-\gamma}$

Then Substituting (16) into (15), we obtain:

(15a) $k_t^* = [(S_K^{1-\beta} S_H^{\beta} \; u_t^{\gamma}) / (n + g + \delta)]^{1/1-\alpha-\beta}$

where $k^*$ is the steady-state level of physical capital per unit of effective labor, with the level of urbanization included.

Then substituting $k^*$ from (15a) back into (16), we obtain:

(16a) $h_t^* = [(S_H^{1-\alpha} S_K^{\alpha} u_t^{\gamma}) / (n + g + \delta)]^{1/1-\beta-\alpha}$,

where $h_t^*$ is the steady level of human capital per unit of effective labor, with the level of urbanization included.

 

**Economic Growth with level of Urbanization Included**

At this stage we can develop one version of the economic growth equation that can be tested econometrically.

Starting out from equation (11) and still following MRW:

$y_t = k_t^{\alpha}\ h_t^{\beta}\ u_t^{\gamma} \rightarrow\ Y/AL = k_t^{\alpha}\ h_t^{\beta}\ u_t^{\gamma} \rightarrow$

(11a) $Y_t/L_t = k_t^{\alpha}\ h_t^{\beta}\ u_t^{\gamma}\ A_t$.

Substituting the steady-state levels of k and h, i.e., (15a) and (16a) respectively back into (11a), we obtain:

$Y_t/L_t = \{[(S_K^{1-\beta} S_H^{\beta} u_t^{\gamma})/(n+g+\delta)]^{\alpha/1-\alpha-\beta}\}.\ \{[(S_H^{1-\alpha} S_K^{\alpha} u_t^{\gamma})/(n+g+\delta)]^{\beta/1-\beta-\alpha}\}.\ u_t^{\gamma}.\ A_t$

$\rightarrow Y_t/L_t = S_K^{\alpha/1-\alpha-\beta}\ .\ S_H^{\beta/1-\alpha-\beta}\ .\ u_t^{\gamma\{\alpha+\beta\}/1-\alpha-\beta}\ .\ (n+g+\delta)^{-\{\alpha+\beta\}/1-\alpha-\beta}\ .\ u_t^{\gamma}.\ A_t \rightarrow$

(11b) $Y_t/L_t = S_K^{\alpha/1-\alpha-\beta}\ .\ S_H^{\beta/1-\alpha-\beta}\ .\ u_t^{\gamma/1-\alpha-\beta}\ .\ (n+g+\delta)^{-\{\alpha+\beta\}/1-\alpha-\beta}\ .\ A_t$

Remember from Definitions above that u = U/AL, which implies that:

$Y_t/L_t = S_K^{\alpha/1-\alpha-\beta}\ .\ S_H^{\beta/1-\alpha-\beta}\ .\ (U_t/A_tL_t)^{\gamma/1-\alpha-\beta}\ .\ (n+g+\delta)^{-\{\alpha+\beta\}/1-\alpha-\beta}\ .\ A_t \rightarrow$

$Y_t/L_t = S_K^{\alpha/1-\alpha-\beta}\ .\ S_H^{\beta/1-\alpha-\beta}\ .(U_t/L_t)^{\gamma/1-\alpha-\beta}\ .\ (n+g+\delta)^{-\{\alpha+\beta\}/1-\alpha-\beta}\ .\ A_t^{(1-\alpha-\beta)/\gamma}\ .\ A_t \rightarrow$

Equation (18):

$Y_t/L_t = S_K^{\alpha/1-\alpha-\beta}\ .\ S_H^{\beta/1-\alpha-\beta}\ .(U_t/L_t)^{\gamma/1-\alpha-\beta}\ .\ (n+g+\delta)^{-\{\alpha+\beta\}/1-\alpha-\beta}\ .\ A_t^{(1-\alpha-\beta+\gamma)/\gamma}$

Taking natural logarithms, equation (18) becomes

(18a) $Ln\ (Y_t/L_t) = (\alpha/1-\alpha-\beta)\ Ln\ S_{Kt} + (\beta/1-\alpha-\beta)\ Ln\ S_{Ht}$
$- [(\alpha+\beta)\ /\ (1-\alpha-\beta)]\ Ln\ (n_t+g+\delta) + \gamma/(1-\alpha-\beta)\ Ln(U_t/L_t)$
$+ (1-\alpha-\beta+\gamma)/\gamma\ \ Ln\ A_t$

which is the basic prototype for the alternative specifications of the model.

**Two Possibilities**

Now, with respect to technology, we can assume it completely exogenous and dependent only on time as in equation (2) above where $A_t = A_0 e^{gt}$. In that case,

(2a) $Ln\ A_t = Ln\ A_0 + gt$ .

Or we can assume the level of technology is dependent on human and urban capital along with time as in (5a) $A_t = A_0 e^{gt} H_t^{a} U_t^{b} e^{cHU}$, and taking logarithms we obtain:

(5b) $lnA_t = lnA_0 + gt + c_1\ lnH_t + c_2\ lnU_t + c_3\ H_tU_t$

In what follows, we will explore the econometric specifications of both possibilities.

If we assume (2a), then

(18a) $Ln\ (Y_t/L_t) = (\alpha/1-\alpha-\beta)\ Ln\ S_K + (\beta/1-\alpha-\beta)\ Ln\ S_H$
$- [(\alpha+\beta)\ /\ (1-\alpha-\beta)]\ Ln\ (n+g+\delta) + \gamma/(1-\alpha-\beta)\ Ln\ (U_t/L_t)$
$+ (1-\alpha-\beta+\gamma)/\gamma\ \ Ln\ A_t$

becomes after substituting (2a) into (18a),

(18b) $Ln\ (Y_t/L_t) = (\alpha/1-\alpha-\beta)\ Ln\ S_K + (\beta/1-\alpha-\beta)\ Ln\ S_H$
$- [(\alpha+\beta)\ /\ (1-\alpha-\beta)]\ Ln\ (n+g+\delta) + \gamma/(1-\alpha-\beta)\ Ln\ (U_t/L_t)$
$+ (1-\alpha-\beta+\gamma)/\gamma\ \ Ln\ A_0\ +\ (1-\alpha-\beta+\gamma)/\gamma\ gt$

(Note that following the tradition in the growth literature, we drop the subscript t on the variables $S_K$, $S_H$, and n)

This will be the first equation to estimate, with $(1-\alpha-\beta+\gamma)/\gamma\ \ Ln\ A_0$ serving as the constant which when estimated under a fixed effects procedure can produce country effects obviously augmented by the output elasticity with respect to all of the three sorts of capital. $(U_t/L_t)$ is urban capital per capita.

However, if the level of technology is dependent on the level of human and urban capital in a country or region as in (5b) substituting it back into (18a) gives:

(18c) $Ln\ (Y_t/L_t) = (\alpha/1-\alpha-\beta)\ Ln\ S_K + (\beta/1-\alpha-\beta)\ Ln\ S_H$
$- [(\alpha+\beta)\ /\ (1-\alpha-\beta)]\ Ln\ (n+g+\delta) + \gamma/(1-\alpha-\beta)\ Ln\ (U_t/L_t)$
$+ (1-\alpha-\beta+\gamma)/\gamma\ \ Ln\ A_0\ +\ (1-\alpha-\beta+\gamma)/\gamma\ gt +\ (1-\alpha-\beta+\gamma)/\gamma\ c_1\ Ln\ H_t$
$+ (1-\alpha-\beta+\gamma)/\gamma\ c_2\ ln\ U_t +\ (1-\alpha-\beta+\gamma)/\gamma\ c_3\ H_t\ U_t$

Equation (18b) is different from (18a) in that levels of human and urban capital operate as shifters of the technology function and therefore of the whole production function. An interaction term $H_tU_t$ gauges the additional effect if any of the interaction of human and urban capital.

Equations (18a) and (18b) can be tested for a restricted version in which the sum of the first two coefficients minus the third should yield an estimate not significantly different from zero.

**Estimating Agglomeration Effects: Two More Specifications**

So far we've assumed urbanization a variable outside the system affecting the determination of the steady-states of physical and human capital, as in equations (15) and (16), but not in fact being affected by them. No steady state for the level of urban capital was determined or made use of. This was actually done to develop specifications (18a) and (18b) above where urbanization enters the picture as $U_t$.

By contrast, if the steady-states of k and h, $k^*$ and $h^*$ respectively are substituted into equation (17) $u_t = [(S_\mu\ k_t^\alpha\ h_t^\beta\ ) / (n + g + \delta)\ ]^{1/1-\gamma}$, then we obtain the steady-state value for urban capital per unit of effective labor,

(17a) $u_t^* = [(S_\mu^{1-\alpha-\beta}\ S_K^\alpha\ S_H^\beta\ ) / (n + g + \delta)\ ]^{1/1-\alpha-\beta-\gamma}$

Substituting (17a) into (11b):

$Y_t/L_t = S_K^{\alpha/1-\alpha-\beta}\ .\ S_H^{\beta/1-\alpha-\beta}\ .\ u_t^{\gamma\,/\,1-\alpha-\beta}\ .\ (n+g+\delta)^{-\{\alpha+\beta]\,/\,1-\alpha-\beta}\ .\ A_t$ ➔

$Y_t/L_t = S_K^{\alpha/1-\alpha-\beta}\ .S_H^{\beta/1-\alpha-\beta}.[(S_\mu^{1-\alpha-\beta}S_K^\alpha S_H^\beta)/(n+g+\delta)]^{\gamma\,/\,(1-\alpha-\beta)(1-\alpha-\beta-\gamma)}$
$.(n+g+\delta)^{-\{\alpha+\beta]\,/\,1-\alpha-\beta}\ .A_t$ ➔

(19) $Y_t/L_t = S_K^{\alpha/1-\alpha-\beta-\gamma}.S_H^{\beta/1-\alpha-\beta-\gamma}.S_\mu^{\gamma/(1-\alpha-\beta-\gamma)}\ .\ (n+g+\delta)^{-\{\alpha+\beta]\,/\,1-\alpha-\beta-\gamma}\ .\ A_t$ ➔

(19a) $\ln (Y_t/L_t) = \alpha/(1-\alpha-\beta-\gamma)\ \text{Ln}\ S_K + \beta/(1-\alpha-\beta-\gamma)\ \text{Ln}\ S_H + \gamma/(1-\alpha-\beta-\gamma)\ \text{Ln}\ S_\mu$
$-\ [(\alpha+\beta+\gamma) / (\ 1-\alpha-\beta-\gamma)]\ \text{Ln}\ (n+g+\delta) + \text{Ln}\ A_t$

Again if $\text{Ln}\ A_t = \text{Ln}\ A_0\ + gt$ ➔

(19b) $\ln (Y_t/L_t) = \alpha/(1-\alpha-\beta-\gamma)\ \text{Ln}\ S_K + \beta/(1-\alpha-\beta-\gamma)\ \text{Ln}\ S_H + \gamma/(1-\alpha-\beta-\gamma)\ \text{Ln}\ S_\mu$
$-\ [(\alpha+\beta+\gamma) / (\ 1-\alpha-\beta-\gamma)]\ \text{Ln}\ (n+g+\delta) + \text{Ln}\ A_0\ + gt$

Or else if $\ln A_t = \ln A_0 + gt + c_1\ \ln H_t + c_2 \ln U_t + c_3\ H_t\ U_t$ ➔

(19c) $\ln (Y_t/L_t) = \alpha/(1-\alpha-\beta-\gamma)\ \text{Ln}\ S_K + \beta/(1-\alpha-\beta-\gamma)\ \text{Ln}\ S_H + \gamma/(1-\alpha-\beta-\gamma)\ \text{Ln}\ S_\mu$
$-\ [(\alpha+\beta+\gamma) / (\ 1-\alpha-\beta-\gamma)]\ \text{Ln}\ (n+g+\delta)$
$+\ \text{Ln}\ A_0\ + gt +\ c_1\ \ln H_t + c_2 \ln U_t + c_3\ H_t\ U_t$

where (19b) and (19c) are the third and fourth equations to estimate. They differ from (18b) and (18c) not only in the interpretation of the coefficients, but also in the nature of the variables included. Instead of $U_t\ /L_t$, $S_\mu$ the share of output devoted to building urban capital is the major urban explanatory variable here. The level of urbanization, $U_t$, used in (18c) and (19c) along with the level of human capital, $H_t$, act only as shifters.

Furthermore, a restricted version of equations (19b) and (19c) can test whether the sum of the first three coefficients minus the fourth coefficient is equal to zero.

The country effects in equations (19) are much more straightforward to recover and interpret since they are not intermingled with the output elasticities with respect to capital, education, and urbanization as in equations (18b) and (18c).

**Restricted Versions**

But to recover estimates of the output elasticities with respect to all three kinds of capital, K, H, and U, we need to estimate restricted versions of equations (18b) and (18c), and (19b) and (19c).

Thus (18b) can be rewritten as:

(18d) $\text{Ln}\ (Y_t/L_t) = (\alpha/1-\alpha-\beta)\ [\text{Ln}\ S_K\ - \text{Ln}\ (n+g+\delta)] +$
$(\beta/1-\alpha-\beta)\ [\text{Ln}\ S_H - \text{Ln}\ (n+g+\delta)]$
$+\ \gamma/(1-\alpha-\beta)\ \text{Ln}\ (U_t\ /L_t)\ + (1-\alpha-\beta+\gamma)/\gamma\ \ \text{Ln}\ A_0\ +\ (1-\alpha-\beta+\gamma)/\gamma\ gt$

Similarly, we can rewrite (18c) as:

(18e) $\text{Ln}\ (Y_t/L_t) = (\alpha/1-\alpha-\beta)\ [\text{Ln}\ S_K\ - \text{Ln}\ (n+g+\delta)] +$
$(\beta/1-\alpha-\beta)\ [\text{Ln}\ S_H - \text{Ln}\ (n+g+\delta)]$
$+\ \gamma/(1-\alpha-\beta)\ \text{Ln}\ (U_t\ /L_t)\ + (1-\alpha-\beta+\gamma)/\gamma\ \ \text{Ln}\ A_0\ +\ (1-\alpha-\beta+\gamma)/\gamma\ gt$
$+(1-\alpha-\beta+\gamma)/\gamma\ c_1\ \text{Ln}\ H_t + (1-\alpha-\beta+\gamma)/\gamma\ c_2\ \ln U_t +\ (1-\alpha-\beta+\gamma)/\gamma\ c_3\ H_t\ U_t$

The same applies to (19b) and (19c) which we can rewrite respectively as:

(19d) $\ln (Y_t/L_t) = \alpha/(1-\alpha-\beta-\gamma)\ [\text{Ln}\ S_K - \text{Ln}\ (n+g+\delta)]$
$+\beta/(1-\alpha-\beta-\gamma)\ [\text{Ln}\ S_H - \text{Ln}\ (n+g+\delta)]$
$+\ \gamma/(1-\alpha-\beta-\gamma)\ [\text{Ln}\ S_\mu - \text{Ln}\ (n+g+\delta)]$
$+\ \text{Ln}\ A_0 + gt_t$

(19e) $\ln (Y_t/L_t) = \alpha/(1-\alpha-\beta-\gamma)\ [\text{Ln}\ S_K - \text{Ln}\ (n+g+\delta)]$
$+\beta/(1-\alpha-\beta-\gamma)\ [\text{Ln}\ S_H - \text{Ln}\ (n+g+\delta)]$
$+\ \gamma/(1-\alpha-\beta-\gamma)\ [\text{Ln}\ S_\mu - \text{Ln}\ (n+g+\delta)]$
$+\ \text{Ln}\ A_0\ + gt_t\ + c_1 \ln H_t + c_2 \ln U_t + c_3\ H_t\ U_t$

A major difference between the pair (18d) and (18e) and the pair (19d) and (19e) is the fact that for the latter three restrictions are imposed whereas for the former there are only two restrictions. In either case, the estimated coefficients are set equal to their value in terms of α, β, and γ that is predicted by the equations (18d), (18e), (19d), or (19e). Then we solve for the specific numerical values of α, β, and γ in a system of two-equations two-unknowns in the case of two restrictions, and three-equations three-unknowns in the case of the three restrictions.

For example suppose in one of the equations (18d) or (18e) the estimates of the restricted coefficients $(\alpha/1-\alpha-\beta)$ and $(\beta/1-\alpha-\beta)$ was $(\alpha/1-\alpha-\beta) = (\beta/1-\alpha-\beta) = 1$. Then solving for α and β simultaneously we get α =

$\beta = 1/3$. And if in that same regression the value of $\gamma/(1-\alpha-\beta)$ was equal to ½, then using the values for $\alpha$ and $\beta$ obtained we can get a value of $\gamma$ equal to 1/6.

By the same token we can obtain values for $\alpha$, $\beta$, and $\gamma$ from the coefficients of the restricted regressions (19d) and (19e) by solving simultaneously for the three of them. Then we can consider if the output elasticities obtained are plausible.

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