

Kinematics of Lower Limb Segments during Cycling Session

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Abstract

This paper presents the kinematics of the leg-bicycle five-bar linkage system mechanism including foot segment. Kinematics is very important in the analysis of rigid bodies system whenever theoretical analysis is being sought especially in bicycle-leg linkage mechanism. A lot of experimental works has already been carried out on lower limbs segments biomechanics during cycling which involved the use of positions of the lower limb segments, but there has not been a complete theoretical analysis of the lower limb segments positions in the literature. Therefore, there is need for a complete kinematics of the bicycle-leg linkage mechanism which includes the position analysis of the lower limb segments. The position, velocity, and acceleration equations were derived and the profiles of the thigh, shank and foot segments were plotted against crank angles. It was found that the profiles obtained are reasonable and agrees with experiments.

Keywords: Cycling; Five-bar linkage; Kinematics Analysis; Lower Limb Segments.

1. Introduction

Cycling has been used for transportation and recreation throughout the world for over a century [1]. Many works have been carried out on cycling and cycling related subjects. The cycling exercise could be useful in the following ways: (1) as therapy for rehabilitation of accident patients, (2) for competition in sports (3) for recreation (4) for keeping fit (exercise) and (5) for electricity generation during exercise. Several works has been done in monitoring pedal forces and crank torques [2-6] and also design parameters for a bicycle-rider system which maximizes the power output from the muscles of human lower limb, and developmental progression that characterizes the interaction of muscular and non-muscular forces in tasks constrained by contact with the environment and biomechanics of the hip, knee and ankle during a progressive resistance cycling protocol in an effort to detect and measure the presence of muscle fatigue were investigated [6-10]. Four spaces Jacoby matrix was used to analyze a five-link mechanism and dynamic model of a four bar linkage with clearance between coupler and rocker were solved [11-12]. This paper presents a complete kinematics analysis of the leg-bicycle five-bar linkage mechanism thigh, shank and foot bone.

2. Equation Derivations

Figure 1 below shows the model of lower limb segments, bicycle frame and pedal crank. ***OABCD*** is the bicycle frame, l_1, l_2, and l_3 are respectively thigh, shank and foot segments and l_4 is the pedal crank arm.

2.1 Position Analysis

The loop equation of the model below in figure 1 can be written in as follows, if we assume the bicycle frame as a fixed link:

$$\begin{aligned} &l_1[\cos\theta_1 + j\sin\theta_1] + l_2[\cos\theta_2 - j\sin\theta_2] + l_3[\cos\theta_3 + j\sin\theta_3] - \\ &l_4[\cos\theta_4 - j\sin\theta_4] + l_5[\cos\pi/2 + j\sin\pi/2] + l_6[\cos\pi + j\sin\pi] = 0 \end{aligned} \tag{1}$$

The vector loop equation 1 can be written for the real and imaginary axes as

$$l_1\cos\theta_1 + l_2\cos\theta_2 + l_3\cos\theta_3 - l_4\cos\theta_4 + l_5\cos\pi/2 + l_6\cos\pi = 0 \tag{2a}$$

$$l_1\sin\theta_1 - l_2\sin\theta_2 + l_3\sin\theta_3 + l_4\sin\theta_4 + l_5\sin\pi/2 + l_6\sin\pi = 0 \tag{2b}$$

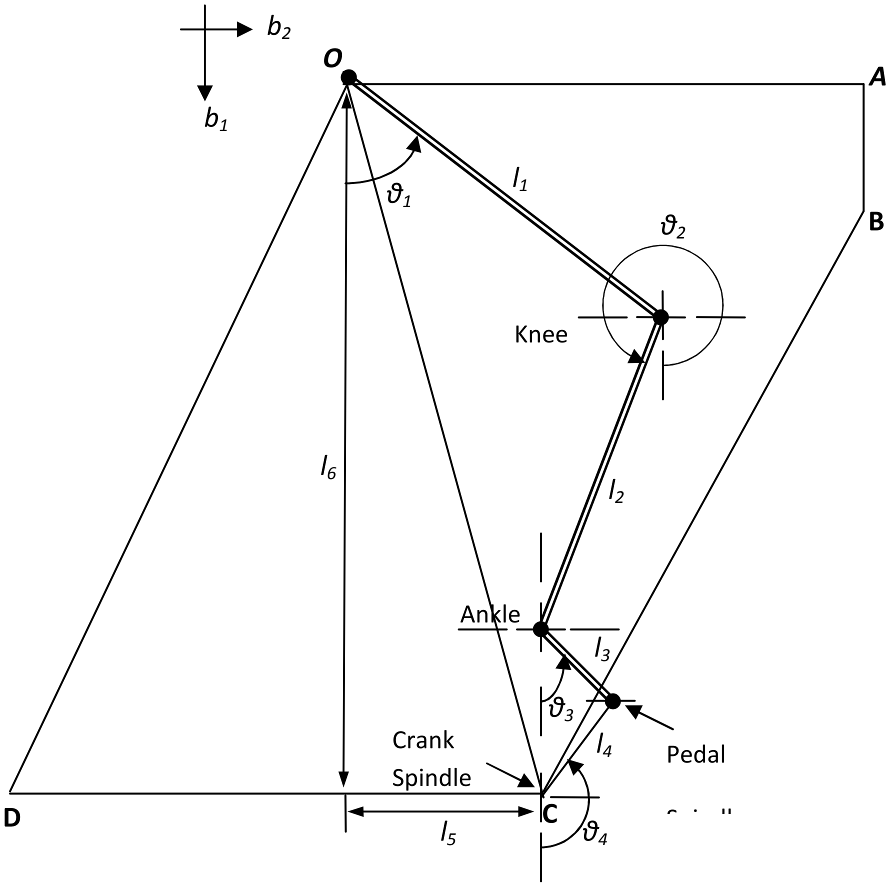

Figure 1: A model of a lower limb segments, pedal crank and bicycle frame

2.2 Derivation of Thigh and Knee Angles

Fig 2: shows the resolutions of the leg segments in the real and imaginary axis as well as the position analysis of the leg segments and the space analysis of the thigh and knee angles.

From triangle ABF in fig 2, it can be found that

$$\tan\theta_2 = \frac{l_4 Sin\theta_4 - l_3 Sin\theta_3 - l_1 Sin\theta_1 + l_5}{l_4 Cos\theta_4 - l_3 Cos\theta_3 - l_1 Cos\theta_1 + l_6} \tag{3}$$

$$\theta_2 = \tan^{-1}\left[\frac{l_4 Sin\theta_4 - l_3 Sin\theta_3 - l_1 Sin\theta_1 + l_5}{l_4 Cos\theta_4 - l_3 Cos\theta_3 - l_1 Cos\theta_1 + l_6}\right] \tag{4}$$

The expression for the thigh angle θ_1 can also be gotten from triangle OAH in fig 2 as shown below:

Using Pythagoras' theorem,

$$l_2^2 = [(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3) - l_1 Cos\theta_1]^2 + [(l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3) - l_1 Sin\theta_1]^2 \tag{5}$$

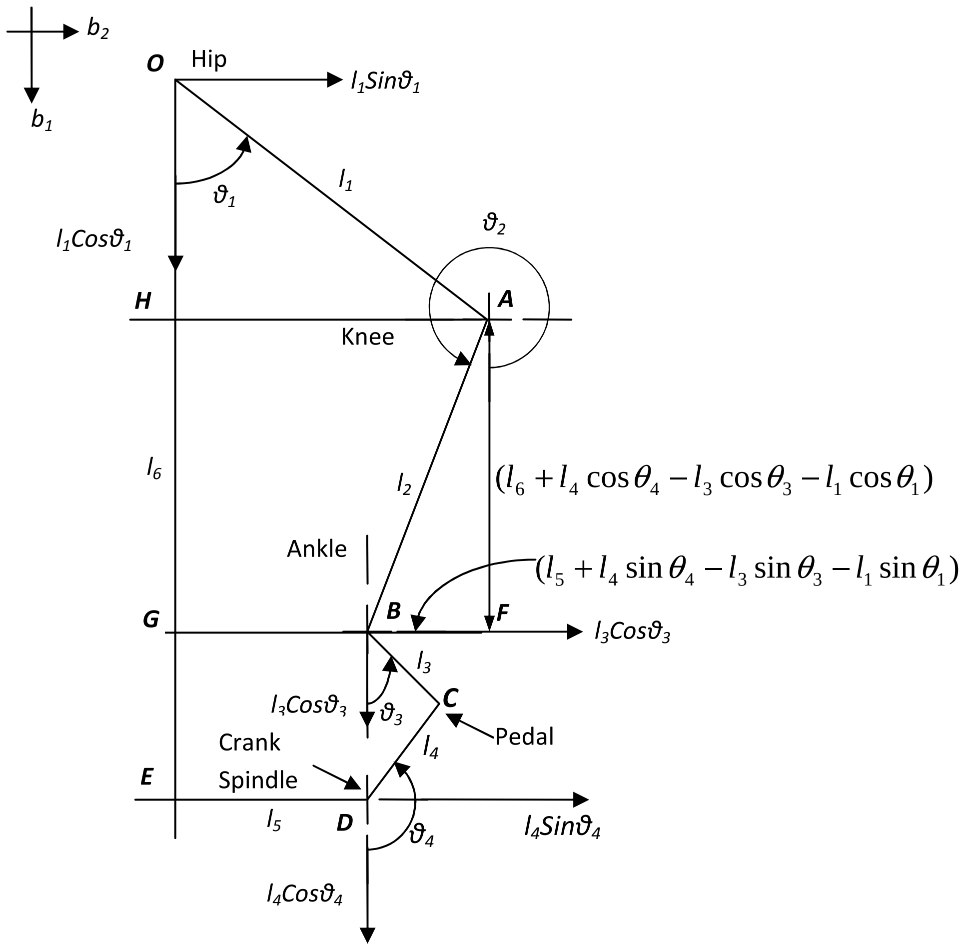

Figure 2: Space Analysis of thigh, θ_1 and knee, θ_2 angles

$$l_2^2 = \left\{ \begin{matrix} (l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)^2 + (l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)^2 + l_1^2 \\ -2[(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)(l_1 Cos\theta_1)] - 2[(l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)(l_1 Sin\theta_1)] \end{matrix} \right\} \tag{6}$$

$$\begin{aligned} &\{2[(l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)(l_1 Sin\theta_1)] + 2[(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)(l_1 Cos\theta_1)]\} \\ &= \left\{(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)^2 + (l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)^2 + l_1^2 - l_2^2\right\} \end{aligned} \tag{7}$$

If we let

$$A = (l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)^2 + (l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)^2 + l_1^2 - l_2^2 \tag{8}$$

$$B = 2(l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3) l_1 \tag{9}$$

$$C = 2(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3) l_1 \tag{10}$$

Then, equation (7) becomes

$$CSin\theta + BCos\theta = A \tag{11}$$

Using trigonometry identity and let $\tan\theta / 2 = t$

$$Sin\frac{\theta}{2} = \frac{t}{\sqrt{1+t^2}} \tag{12}$$

$$Cos\frac{\theta}{2} = \frac{1}{\sqrt{1+t^2}} \tag{13}$$

$$Sin\theta = 2Sin\frac{\theta}{2}Cos\frac{\theta}{2} = 2\cdot\frac{t}{\sqrt{1+t^2}}\cdot\frac{1}{\sqrt{1+t^2}} = \frac{2t}{1+t^2} \tag{14}$$

$$Cos\theta = 2Cos^2\frac{\theta}{2} - 1 = 2\left[\frac{1}{\sqrt{1+t^2}}\right]^2 - 1 = \frac{2}{1+t^2} - 1 = \frac{2-1-t^2}{1+t^2} = \frac{1-t^2}{1+t^2} \tag{15}$$

Recall equation (11)

$$CSin\theta + BCos\theta = A \tag{11}$$

Substituting equations (14) and (15) into equation (11), we have

$$C\left[\frac{2t}{1+t^2}\right] + B\left[\frac{1-t^2}{1+t^2}\right] = A \tag{12}$$

Multiplying through by $1+t^2$, we have

$$C(2t) + B(1-t^2) = A(1+t^2) \tag{13}$$

$$2Ct + B - Bt^2 = A + At^2 \tag{14}$$

$$A + At^2 - 2Ct - B + Bt^2 = 0 \tag{15}$$

$$(A+B)t^2 - 2Ct + (A-B) = 0 \tag{16}$$

Recall equations (8), (9) and (10)

$$A = (l_6 + l_4Cos\theta_4 - l_3Cos\theta_3)^2 + (l_5 + l_4Sin\theta_4 - l_3Sin\theta_3)^2 + l_1^2 - l_2^2 \tag{8}$$

$$B = 2(l_5 + l_4Sin\theta_4 - l_3Sin\theta_3)l_1 \tag{9}$$

$$C = 2(l_6 + l_4Cos\theta_4 - l_3Cos\theta_3)l_1 \tag{10}$$

Then equation (16) becomes

$$\left\{\begin{array}{l} \left\{\begin{array}{l}[(l_6 + l_4Cos\theta_4 - l_3Cos\theta_3)^2 + (l_5 + l_4Sin\theta_4 - l_3Sin\theta_3)^2 \\ + l_1^2 - l_2^2] + [2(l_5 + l_4Sin\theta_4 - l_3Sin\theta_3)l_1]\end{array}\right\} t^2 \\ -2[2(l_6 + l_4Cos\theta_4 - l_3Cos\theta_3)l_1]t \\ + \left\{\begin{array}{l}[(l_6 + l_4Cos\theta_4 - l_3Cos\theta_3)^2 + (l_5 + l_4Sin\theta_4 - l_3Sin\theta_3)^2 \\ + l_1^2 - l_2^2] - [2(l_5 + l_4Sin\theta_4 - l_3Sin\theta_3)l_1]\end{array}\right\} \end{array}\right\} = 0 \tag{17}$$

Equation (17) is a quadratic equation of the form $ax^2 + bx + c = 0$ where

$$a = \begin{Bmatrix} [(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)^2 + (l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)^2 + l_1^2 - l_2^2] \\ + [2(l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)l_1] \end{Bmatrix} \tag{18}$$

$$b = -2[2(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)l_1], \tag{19}$$

and

$$c = \begin{Bmatrix} [(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)^2 + (l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)^2 + l_1^2 - l_2^2] \\ - [2(l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)l_1] \end{Bmatrix} \tag{20}$$

The solution of equation (17) is then,

$$t = \left\{ \begin{matrix} 2\left(2(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)l_1\right) \\ \pm \left\{ \begin{matrix} \left(-2[2(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)l_1]\right)^2 \\ -4\begin{pmatrix} [(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)^2 \\ + (l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)^2 + l_1^2 - l_2^2] \\ + [2(l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)l_1] \end{pmatrix} \begin{pmatrix} [(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)^2 \\ + (l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)^2 \\ + l_1^2 - l_2^2] - \\ [2(l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)l_1] \end{pmatrix} \end{matrix} \right\}^{0.5} \\ \left\{ 2\begin{pmatrix} [(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)^2 \\ + (l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)^2 + l_1^2 - l_2^2] \\ + [2(l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)l_1] \end{pmatrix} \right\}^{-1} \end{matrix} \right\} \tag{21}$$

Recall $t = \tan\frac{\theta_1}{2}$

$$\theta_1 = 2\tan^{-1}\left\{ \begin{matrix} \left\{ \begin{matrix} 2\left(2(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)l_1\right) \\ \pm \left\{ \begin{matrix} \left(-2[2(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)l_1]\right)^2 \\ -4\begin{pmatrix} [(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)^2 \\ + (l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)^2 + l_1^2 - l_2^2] \\ + [2(l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)l_1] \end{pmatrix} \begin{pmatrix} [(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)^2 \\ + (l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)^2 \\ + l_1^2 - l_2^2] - \\ [2(l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)l_1] \end{pmatrix} \end{matrix} \right\}^{0.5} \end{matrix} \right\} \\ \left\{ 2\begin{pmatrix} [(l_6 + l_4 Cos\theta_4 - l_3 Cos\theta_3)^2 \\ + (l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)^2 + l_1^2 - l_2^2] \\ + [2(l_5 + l_4 Sin\theta_4 - l_3 Sin\theta_3)l_1] \end{pmatrix} \right\}^{-1} \end{matrix} \right\}$$

(23)

2.3 Derivation of the Foot Angle

The foot angle is θ_3 in figure 2 above, looking at triangle BCD, if a perpendicular is drawn from point C to line BD, and called *h*, as shown in figure 3 below. BC is assumed to be the foot segment while CD is the crank arm,

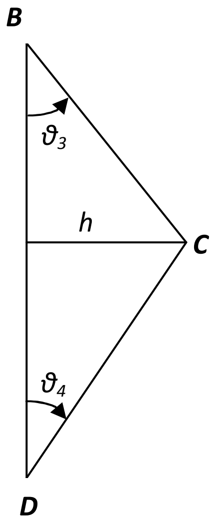

Figure 3: Analysis of foot angle

$$h = l_3 \sin\theta_3 = -l_4 \sin\theta_4 \tag{24}$$

$$l_3 \sin\theta_3 = -l_4 \sin\theta_4 \tag{25}$$

$$\theta_3 = \sin^{-1}\left(\frac{-l_4 \sin\theta_4}{l_3}\right) \tag{26}$$

2.4 Velocity Analysis

The velocity derivatives of the thigh and shank segments can be gotten by taking the

first time-derivatives of (1) above we obtain the angular velocities

$$\frac{d}{dt}\left\{\begin{matrix} l_1[\cos\theta_1 + j\sin\theta_1] + l_2[\cos\theta_2 - j\sin\theta_2] \\ + l_3[\cos\theta_3 + j\sin\theta_3] - l_4[\cos\theta_4 - j\sin\theta_4] \\ + l_5[\cos\pi/2 + j\sin\pi/2] + l_6[\cos\pi + j\sin\pi] \end{matrix}\right\} = 0 \tag{27}$$

Equation (27) is

$$\left\{\begin{matrix} l_1 \cdot \dot{\theta}\,[j\cos\theta_1 - \sin\theta_1] + l_2 \cdot \dot{\theta}_2[-j\cos\theta_2 - \sin\theta_2] \\ + l_3 \cdot \dot{\theta}_3[j\cos\theta_3 - \sin\theta_3] - l_4 \cdot \dot{\theta}_4[j\cos\theta_4 - \sin\theta_4] \end{matrix}\right\} = 0 \tag{28}$$

Separating equation (28) into real and imaginary parts and equating to zero

Real Part:

$$l_1[-\sin\theta_1]\dot{\theta}_1 + l_2[-\sin\theta_2]\dot{\theta}_2 + l_3[-\sin\theta_3]\dot{\theta}_3 - l_4[-\sin\theta_4]\dot{\theta}_4 = 0 \tag{29}$$

$$l_1[-\sin\theta_1]\dot{\theta}_1 + l_2[-\sin\theta_2]\dot{\theta}_2 + \{-l_3[\sin\theta_3]\dot{\theta}_3 + l_4[\sin\theta_4]\dot{\theta}_4\} = 0 \tag{30}$$

Imaginary part:

$$i\left[l_1[\cos\theta]\dot{\theta} + l_2[-\cos\theta_1]\dot{\theta}_1 + l_3[\cos\theta_2]\dot{\theta}_2 - l_4[-\cos\theta_3]\dot{\theta}_3\right] = 0 \tag{31}$$

Since $i \neq 0$, then

$$l_1[\cos\theta_1]\dot{\theta}_1 + l_2[-\cos\theta_2]\dot{\theta}_2 + l_3[\cos\theta_3]\dot{\theta}_3 - l_4[-\cos\theta_4]\dot{\theta}_4 = 0 \tag{32}$$

Therefore,

$$l_1[\cos\theta_1]\dot{\theta}_1 + l_2[-\cos\theta_2]\dot{\theta}_2 + \{l_4[\cos\theta_4]\dot{\theta}_4 + l_3[\cos\theta_3]\dot{\theta}_3\} = 0 \tag{33}$$

Equations (31) and (33) can be set as simultaneous equations, and then solved for

$\dot{\theta}_1, and\dot{\theta}_2$ using second order determinant and simplify,

$$\dot{\theta}_1 = \frac{l_4 l_2 \dot{\theta}_4 \sin(\theta_2 + \theta_4) + l_3 l_2 \dot{\theta}_3 \sin(\theta_2 - \theta_3)}{l_1 l_2 \sin(\theta_1 + \theta_2)} \tag{34}$$

$$\dot{\theta}_2 = \frac{l_1 l_4 \dot{\theta}_4 \sin(\theta_1 + \theta_4) + l_1 l_3 \dot{\theta}_3 \sin(\theta_1 - \theta_3)}{l_1 l_2 \sin(\theta_1 + \theta_2)} \tag{35}$$

The velocity of the foot angle is gotten by taken the time derivatives of the foot angle thus

$$\frac{d}{dt}(\theta_3) = \frac{d}{dt}\left\{\sin^{-1}\left(\frac{-l_4\sin\theta_4}{l_3}\right)\right\} \tag{36}$$

$$\dot{\theta}_3 = \frac{-l_4\dot{\theta}_4\cos\theta_4}{l_3\sqrt{\left(1-\left(\frac{l_4}{l_3}\sin\theta_4\right)^2\right)}} \tag{37}$$

2.5. Acceleration Analysis

Taking the second time-derivates of loop equation (1), we have:

$$\frac{d^2}{dt^2}\begin{Bmatrix} l_1[\cos\theta_1 + j\sin\theta_1] + l_2[\cos\theta_2 - j\sin\theta_2] + l_3[\cos\theta_3 + j\sin\theta_3] \\ -l_4[\cos\theta_4 - j\sin\theta_4] + l_5[\cos\pi/2 + j\sin\pi/2] + l_6[\cos\pi + j\sin\pi] \end{Bmatrix} \tag{38}$$

$$= \begin{Bmatrix} l_1[-(\ddot{\theta}_1\sin\theta_1 + \dot{\theta}_1^2\cos\theta_1) + j(\ddot{\theta}_1\cos\theta_1 - \dot{\theta}_1^2\sin\theta_1)] \\ -l_2[(\ddot{\theta}_2\sin\theta_2 + \dot{\theta}_2^2\cos\theta_2) + j(-\dot{\theta}_2^2\sin\theta_2 + \ddot{\theta}_2\cos\theta_2)] \\ +l_3[-(\ddot{\theta}_3\sin\theta_3 + \dot{\theta}_3^2\cos\theta_3) + j(-\dot{\theta}_3^2\sin\theta_3 + \ddot{\theta}_3\cos\theta_3)] \\ +l_4[(\ddot{\theta}_4\sin\theta_4 + \dot{\theta}_4^2\cos\theta_4) - j(-\dot{\theta}_4^2\sin\theta_4 + \ddot{\theta}_4\cos\theta_4)] \end{Bmatrix} \tag{39}$$

Separating equation (39) into real and imaginary parts, we have

Real part:

$$\begin{Bmatrix} l_1(-\sin\theta_1)\ddot{\theta}_1 - l_1(\dot{\theta}_1^2\cos\theta_1) + l_2(-\sin\theta_2)\ddot{\theta}_2 - l_2\dot{\theta}_2^2\cos\theta_2] \\ + l_3(-\sin\theta_3)\ddot{\theta}_3 - l_3\dot{\theta}_3^2\cos\theta_3 + l_4\ddot{\theta}_4\sin\theta_4 + l_4\dot{\theta}_4^2\cos\theta_4)] \end{Bmatrix} = 0 \tag{40}$$

$$l_1(-\sin\theta_1)\ddot{\theta}_1 + l_2(-\sin\theta_2)\ddot{\theta}_2 + \begin{Bmatrix} -l_1\dot{\theta}_1^2\cos\theta_1 - l_2(\dot{\theta}_2^2\cos\theta_2) - l_3\ddot{\theta}_3\sin\theta_3 \\ -l_3\dot{\theta}_3^2\cos\theta_3 + l_4\ddot{\theta}_4\sin\theta_4 + l_4\dot{\theta}_4^2\cos\theta_4 \end{Bmatrix} = 0 \tag{41}$$

Imaginary part:

$$i\begin{Bmatrix} l_1[(\ddot{\theta}_1\cos\theta_1 - \dot{\theta}_1^2\sin\theta_1)] - l_2[(-\dot{\theta}_2^2\sin\theta_2 + \ddot{\theta}_2\cos\theta_2)] \\ + l_3[(-\dot{\theta}_3^2\sin\theta_3 + \ddot{\theta}_3\cos\theta_3)] - l_4[(-\dot{\theta}_4^2\sin\theta_4 + \ddot{\theta}_4\cos\theta_4)] \end{Bmatrix} = 0 \tag{42}$$

Since $i \neq 0$, then

$$\begin{Bmatrix} l_1(\cos\theta_1)\ddot{\theta}_1 - l_1\dot{\theta}_1^2\sin\theta_1 - l_2(\cos\theta_2)\ddot{\theta}_2 + l_2\dot{\theta}_2^2\sin\theta_2 \\ - l_3\dot{\theta}_3^2\sin\theta_3 + l_3\ddot{\theta}_3\cos\theta_3 + l_4\dot{\theta}_4^2\sin\theta_4 - l_4\ddot{\theta}_4\cos\theta_4 \end{Bmatrix} = 0 \tag{43}$$

$$l_1(\cos\theta_1)\ddot{\theta}_1 - l_2(\cos\theta_2)\ddot{\theta}_2 + \begin{Bmatrix} -l_1\dot{\theta}_1^2\sin\theta_1 + l_2\dot{\theta}_2^2\sin\theta_2 - l_3\dot{\theta}_3^2\sin\theta_3 \\ + l_3\ddot{\theta}_3\cos\theta_3 + l_4\dot{\theta}_4^2\sin\theta_4 - l_4\ddot{\theta}_4\cos\theta_4 \end{Bmatrix} = 0 \tag{44}$$

The angular accelerations of the thigh and shank can be determined from the simultaneous equation (32) and equation (33), using second order determinant and simplify,

$$\ddot{\theta}_1 = \frac{\begin{Bmatrix} 2l_2^2\dot{\theta}_2^2\cos^2\theta_2 + l_1l_2\dot{\theta}_1^2\cos(\theta_1 - \theta_2) + l_3l_2\dot{\theta}_3^2\cos(\theta_3 - \theta_2) - \\ l_4l_2\dot{\theta}_4^2\cos(\theta_2 - \theta_4) + l_3l_2\ddot{\theta}_3\sin(\theta_3 - \theta_2) + l_4l_2\ddot{\theta}_4\sin(\theta_2 - \theta_4) \end{Bmatrix}}{l_1l_2\sin(\theta_1 + \theta_2)} \tag{45}$$

$$\ddot{\theta}_2 = \frac{\begin{Bmatrix} -l_1^2\dot{\theta}_2^2 - l_1l_2\dot{\theta}_2^2\cos(\theta_1 + \theta_2) - l_3l_1\dot{\theta}_3^2\cos(\theta_1 - \theta_3) + \\ l_1l_4\dot{\theta}_4^2\cos(\theta_1 - \theta_4) + l_3l_1\ddot{\theta}_3\sin(\theta_1 - \theta_3) - l_4l_1\ddot{\theta}_4\sin(\theta_1 + \theta_4) \end{Bmatrix}}{l_1l_2\sin(\theta_1 + \theta_2)} \tag{46}$$

The acceleration of the foot is gotten by taking the time derivatives of the angular velocity expression equation 38 above, assuming the differentiation of a quotient of two functions:

$$\frac{d}{dt}(\dot{\theta}_3) = \frac{d}{dt}\left(\frac{-l_4\dot{\theta}_4\cos\theta_4}{l_3\sqrt{\left(1 - \left(\frac{l_4}{l_3}\sin\theta_4\right)^2\right)}}\right) \tag{47}$$

If $u = -l_4\dot{\theta}_4\cos\theta_4$, $\dot{u} = l_4\left[\dot{\theta}_4\sin\theta_4 - \ddot{\theta}_4\cos\theta_4\right]$

and $v = l_3\sqrt{\left(1-\left(\frac{l_4}{l_3}\sin\theta_4\right)^2\right)}$, $\dot{v} = l_4\dot{\theta}_4\cos\theta_4\left[1-\left(\frac{l_4}{l_3}\sin\theta_4\right)^2\right]^{-\frac{1}{2}}$

$$\frac{d}{dt}\left(\dot{\theta}_3\right) = \ddot{\theta}_3 = \frac{u}{v} = \frac{v\dot{u} - u\dot{v}}{v^2} \tag{48}$$

$$\ddot{\theta}_3 = \frac{\left\{l_3\sqrt{\left(1-\left(\frac{l_4}{l_3}\sin\theta_4\right)^2\right)}\left(l_4\left[\dot{\theta}_4\sin\theta_4 - \ddot{\theta}_4\cos\theta_4\right]\right) + \left(\left(l_4\dot{\theta}_4\cos\theta_4\right)^2\left[1-\left(\frac{l_4}{l_3}\sin\theta_4\right)^2\right]^{-\frac{1}{2}}\right)\right\}}{l_3^2\left(1-\left(\frac{l_4}{l_3}\sin\theta_4\right)^2\right)}$$

(50)

3. Results and Discussions

For the studies of these segments, the values in Table 1, below is used. The profile of crank angles and the leg segments angles is shown in fig 4, below. These plots were obtained from our derivations for thigh, shank and foot angles in equations 4, 23 and 26 respectively.

Table 1**:** Parameters used for simulation

S/N	DESCRIPTION	SYMBOL	VALUES USED
1	Length of thigh bone	l_1	0.396m
2	Length of shank bone	l_2	0.435m
3	Length of foot bone	l_3	0.213m
4	Crank arm length	l_4	0.170m
5	Hip to crank axis (horizontal)	l_5	0.212m
6	Hip to crank axis (vertical)	l_6	0.673m
7	Angle between the vertical and thigh bone	θ_1	87^0
8	Angle between the vertical and the shank bone	θ_2	157^0
9	Angle between the vertical axis and the foot bone	θ_3	39^0
10	Angle of the crank arm	θ_4	45^0

Figs. 4 -6 are the profile of the segments angles versus the crank angles: The foot angle profile in Fig. 4 is sinusoidal which agrees with experimental observation of Hull and Jorge [6]. The profile of the thigh and shank angles versus crank angles also agrees with standard kinematics of four bar linkage mechanism [13], if the foot angle is fixed. The velocity profile of foot segment versus crank angles in figure 5 is cosine shape which agrees in theory with differentiation of the foot segment angle (a sine curve). The profile of the angular velocity of the shank and thigh segments peak at around 45^0 and also at 315^0 with lesser peaks at 135^0. The acceleration profiles in Fig. 6 reveal that the magnitude of the shank segment profile peaks at 45^0 and again at 330^0 but with lesser magnitude. It can be inferred from the graphs that the cycling activities will be felt more in the shank segment of the lower limb at around 45^0 and 315^0 which suggests likely stress and strain in the region of the knee since it is the knee angle that is associated with the shank segment in the analysis.

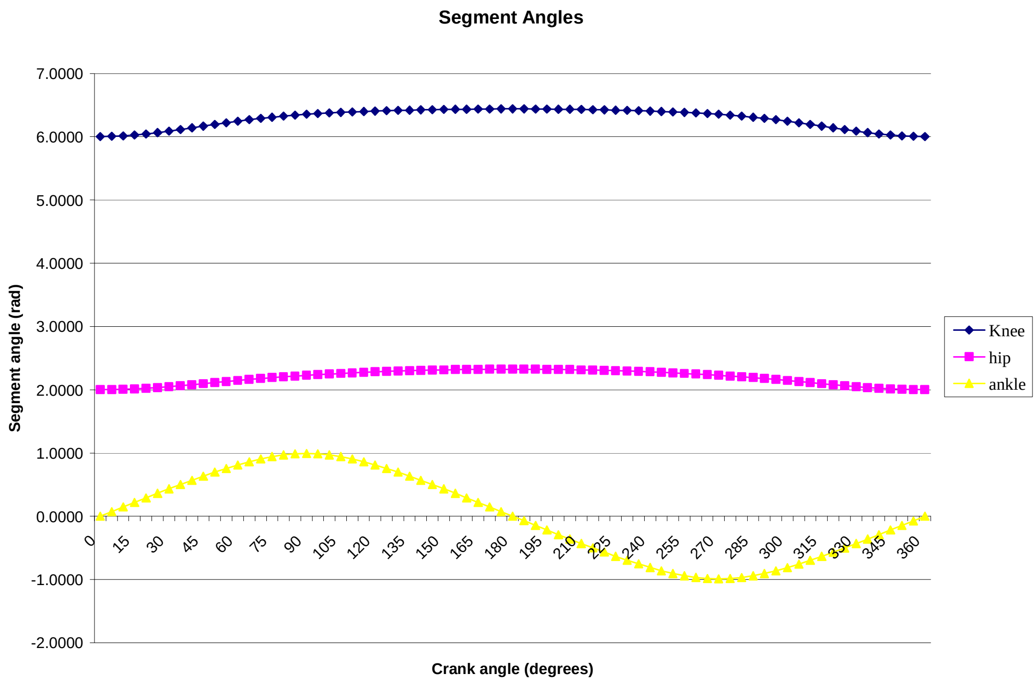

Figure 4: The profile of segments angles and crank angles

The profile of velocity and crank angles for thigh, shank and foot are shown in Fig. 5 and it was obtained from equations 34, 35 and 37 respectively.

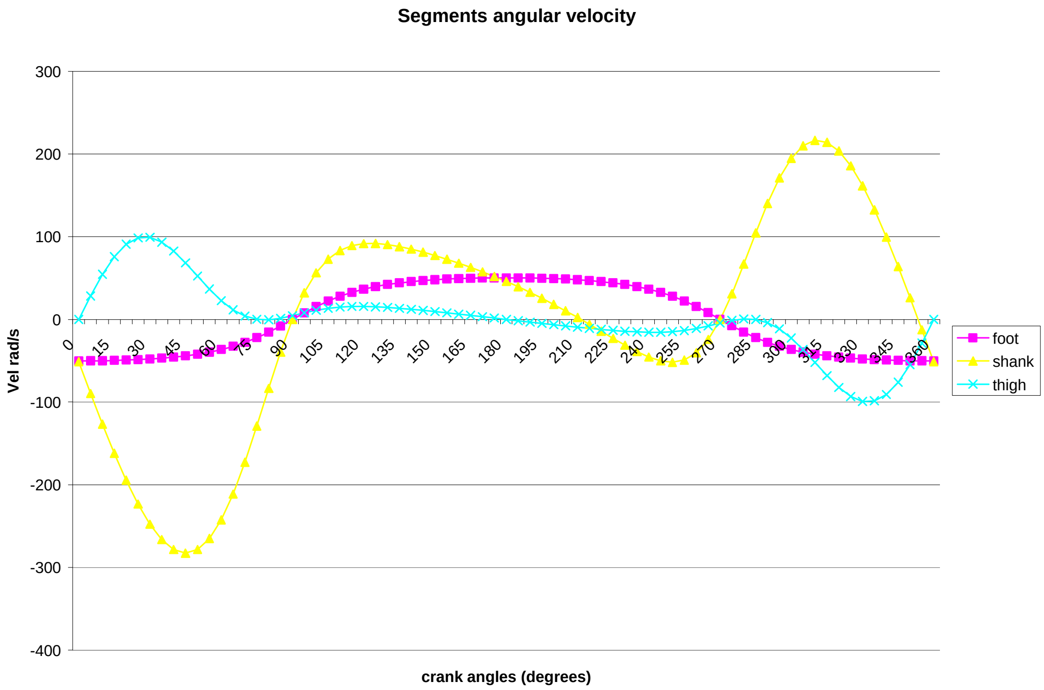

Figure 5: The profile of segments angular velocity and crank angles

The profile of angular acceleration and crank angles for thigh, shank and foot are shown in Fig. 6 as obtained from equations 45, 46 and 50 respectively.

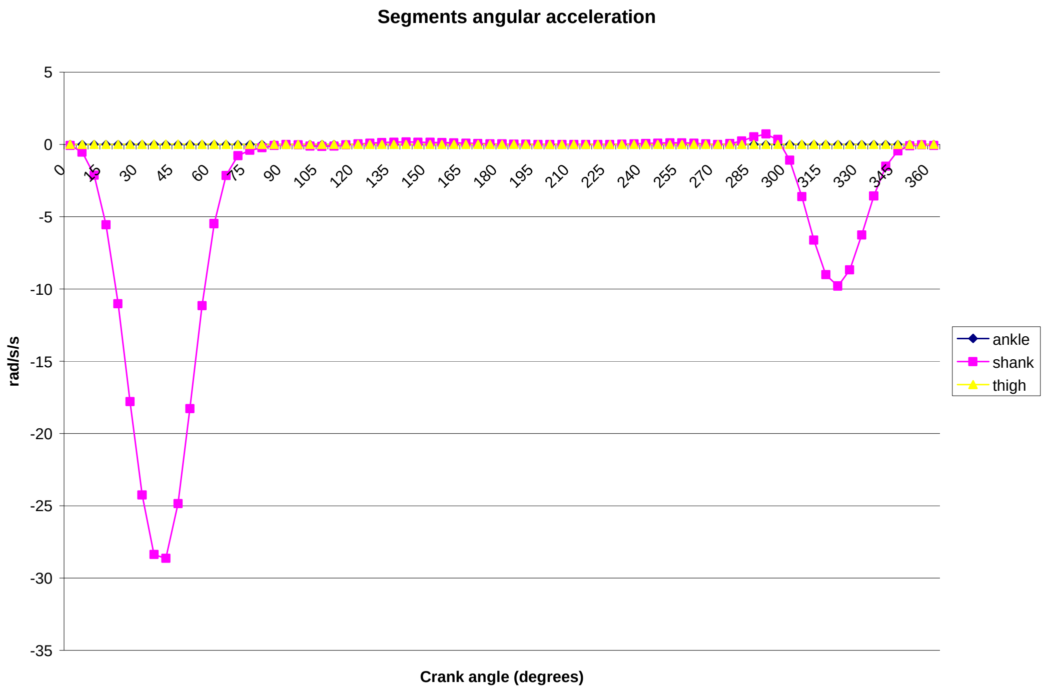

Figure 6: The profile of segments angular acceleration and crank angles

4. Conclusion

Kinematics analysis of the lower limb segments during cycling session has been presented. The position, velocity and acceleration analysis of the lower limb segments during cycling which will afford theoretical validation of experiments that can be carried out in this field of study. The equations for the position, velocity and acceleration of the foot, shank and thigh segments were derived. The profiles of position, velocity and acceleration and crank angles plotted. The plots agree with experiment results.

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Nomenclature

$b_1,\ b_2$: real and imaginary axes respectively ;

$l_1,\ l_2,\ l_3$: respectively lengths of thigh, shank and ankle to pedal spindle;

l_4 : crank arm length – from pedal to crank spindle;

$l_5,\ l_6$: respectively horizontal and vertical distances of the seat from the crank arm spindle;

$\theta_1,\ \dot{\theta}_1,\ \ddot{\theta}_1$: positional angle, angular velocity and angular acceleration for thigh with the vertical respectively;

$\theta_2,\ \dot{\theta}_2,\ \ddot{\theta}_2$: positional angle, angular velocity and angular acceleration for thigh with the vertical respectively;

$\theta_3,\ \dot{\theta}_3,\ \ddot{\theta}_3$: positional angle, angular velocity and angular acceleration for thigh with the vertical respectively;

θ_4, : crank angle measured from the vertical.