Reactive Power Flow Calculation Under Line Outage Condition using Differential Evolution (DE)

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Abstract
This paper presents an optimization based technique for calculation of reactive power flow under line outage condition. Traditional sensitivity based method provides an approximate solution due to presence of non-linearity. This deficiency has been overcome by formulating line outage as non-linear constrained optimization problem for a bounded network. A population based search algorithm namely Differential Evolution (DE) has been applied for optimization. The methodology is tested on IEEE 14 bus system and simulated results are compared with the results of AC power flow which shows that the proposed method improves the accuracy in reactive power flow and bus voltage values under line outage condition.
Keywords- Differential Evolution, Reactive Power flow, Bus Voltage, Line outage.

1.INTRODUCTION Modern power system is a very large complex network operating most of the time in stressed condition due to increased load demand. Isolation of one of the transmission line from network due to fault or any other abnormality results several line overloads in other branches and system voltage deviation. In order to alleviate these stresses it is required to develope fast and accurate methods to predict the post outage status of the system. However accuracy and speed of the solution are conflicting requirements.One of the most powerful tools for calculating post outage quantities is distribution factors based on sensitivity analysis. Line outage distribution facors and DC power flow methods [1] are effectively used for Megawatt flows. These methods provide Fairly fast and reliable results for MW flows but fail to address voltage security analysis.
Several methods based on power flow [2-5] have been developed to handle reactive power flows and voltage magnitude problems. Ilic and Phadke [5] devloped a method for calculating reactive power flows in contingency analysis but it requires nine different factors which are prone to computational errors. Lee and Chen [6] proposed voltage distribution factors which are based on FDLF and network sensitivities. The method suffers accuracy as the assumption of P-δ and Q-V coupling is not generally applicable during stressed system condition. Methods proposed by Preston et al [7] for calculating line currents under muliple outage condition suffers from high computational errors due to linearised network equations. Singh and Shrivastava [8] developed a new set of distribution factors based on decoupled power flow Jacobian matrix. Power system behaviour is highly non-linear under stressed condition and any attempt to develop the methods for reactive power flow using linearised model to enhance computational speed may result serious errors. Minor deviation in calculating voltage magnitude may result in high errors in reactive power flows. Therefore an improved model is required for calculating post outaged voltage magnitudes.
An improved model is presented in ths paper for voltage and reactive power calculation under line outage condition without increasing any significant computational time. A bounded network comprising the outaged branch and the neighbouring branches has been considered. Line outage is formulated as a non linear constrained optimization problem for this network. Voltage magnitude at different nodes of the bounded network is obtained using DE to minimize the mismatch between the actual and calculated reactive powers resulting from linearized network relatonship. The proposed method makes use of base case variables and the linearized MW power flows for a restricted set of network variables. Therefore it does not result any significant increase in computational time. However, the optimization cycle provides a non linear feedback for reactive power mismatch, which in turn minimizes the load bus voltage magnitude error due to linearised network constraints. The resulting bus voltage magnitudes and reactive power flows are much better than the one obtained by the traditonal approaches. The accuracy of the proposed algorithm has been verified on IEEE-14 bus test system .The results are compared with those of the actual power flows to examine the strength of the method.
2. Expression for Reactive Power flow through a transmission line Consider a transmission line connected between buses k and m in the system as shown in Fig.1

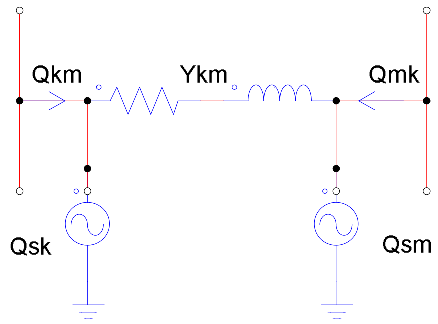

Fig. 1: Transmission line with external sources at both ends

Reactive power in the line can be expressed as follows

$Q_{km} = -V_k^2 b_{km} + V_k V_m [b_{km} \cos\delta_{mk} + g_{km} \sin\delta_{mk}]$ (1)

$Q_{mk} = -Q_{km}$

Where, V_k, V_m = bus voltage magnitude at kth & mth bus respectively

δ_k, δ_{jm} = bus voltage phase angles of kth & nth bus

$\delta_{mk} = \delta_m - \delta_k$

g_{km}, b_{km} are the conductance & susceptance of line connected between kth & mth

but

$y_{ij} = g_{km} + jb_{km}$ line admittance.

Generally it is preferred to simulate branch outage using a pair of fictitious sources to preserve the network topology. Fig 1 shows a simulated line outage. An exact outage simulation by fictitious sources creates the same effect of the line outage without changing the reactive power balance of the network i.e.

$Q_{sk} = Q^T_{km}$ (2)

$Q_{sm} = -Q^T_{km}$ (3)

Q^T_{km} is the post outage reactive power flow which is very close to pre outaged flow.

3. Problem Formulation: For a bounded network line outage simulation can be carried out as a constrained optimization problem. This sub network comprises the outaged branch and the neighbouring branches. An exact simulation satisfies both the reactive power flow equation and fictitious source constraints (2) and (3).

An incremental relationship between reactive power and load bus voltage magnitudes mismatches as follows:

$\Delta Q = J\Delta V$ (4)

Where,

ΔQ= Reactive power mismatch error

ΔV= Change in load bus voltage magnitude

Load bus voltage magnitudes calculated from above do not satisfy (2) and (3) because of linearization errors. Therefore, part of the reactive power generation circulates through the system. The proposed formulation aims the minimization of difference between the reactive power flow in the line in pre and post outaged conditions. This can be stated as:

Min $F = Q_{sk} - Q_{km}^T$ (5)

Subject to

$\Delta Q = B\,\Delta Vb$

For all load buses

Where B = Bus susceptance matrix

The proposed DE optimization technique provides a feedback by which the load bus voltage magnitudes are revised to minimize the effect of fictitious sources on the network reactive power distribution. The time required is also not considerable as it is confined in the bounded network and makes the use of base case variables.

DC power flow equation provides bus voltage phase angles for each outage. Phase angles are assumed constant as it has very weak coupling with reactive power flow. DE is preferred for optimization as discussed below.

4. Overview of Differential Evolution Differential evolution (DE) developed by Storn and Price [10] is a very simple population based, stochastic function minimiser and has been found very powerful to solve various nature of engineering problems [11,12]. DE attacks the optimization problem by sampling the objective function at multiple randomly chosen initial points. Preset parameter bounds define the region from which 'M' vectors in this initial population are chosen. DE generates new solution points in 'D' dimensional space that are perturbations of existing points. It perturbs vectors with the scaled difference of two randomly selected population vectors. To produce a mutated vector, DE adds the scaled, random vector difference to a third selected population vector (called as base vector). Further DE also employs a uniform cross over to produce trial vector from target vector and mutated vector. The three fundamental steps are explained below.

Step-(a) Initialization: A initial population of size 'M' is generated as follows

$$S^{(0)} = [X_1^{(0)}, X_2^{(0)}, ...X_M^{(0)}] \tag{6}$$

$$X_i^{(0)} = [x_{i1}^{(0)}, x_{i2}^{(0)}, ...x_{iD}^{(0)}]^T \tag{7}$$

$x^{(0)}_{ij}$ i.e. j[th] parameter of X_i vector is obtained from uniform distribution as follows

$$x_{ij}^{(0)} = x_{j,\min} + (x_{j,\max} - x_{j,\min}) rand_j \tag{8}$$

$x_{j,min}$ and $x_{j,max}$ are lower and upper bounds on variable x_j. $rand_j$ is a random digit in the range [0,1].

Step-(b) Mutation: DE mutates and recombines the population to produce a population of 'M' trial vectors. Differential mutation adds a scaled, randomly sampled, vector difference to a third vector as follows

$$\underline{V}_i^{(k)} = X_{base}^{(k)} + \sigma(X_p^{(k)} - X_q^{(k)}) \tag{9}$$

σ is known as scale factor usually lies in the range [0, 1].

$X_p^{(k)}$ and $X_q^{(k)}$ are two randomly selected vectors ($p \neq q$).

$X_{base}^{(k)}$ is known as base vector.

$\underline{V}_i^{(k)}$ is a mutant vector.

The base vector index '*base*' may be determined in variety of ways. This may be a randomly chosen vector ($base \neq p \neq q$).

Step-(b) Crossover: DE employs a uniform crossover strategy. Crossover generates trial vectors $t_i^{(k)}$ as follows.

$$t_{i,j}^{(k)} = \begin{cases} v_{i,j}^{(k)} & \text{if } (rand_j \le C_r \text{ or } j = j_{rand}) \\ x_{i,j}^{(k)} & \text{otherwise} \end{cases} \tag{10}$$

C_r is crossover probability lies in the range [0, 1]. C_r is user defined value which controls the number of parameter values which are copied from the mutant. If the random number $rand_j$ is less than or equal to C_r, the trial parameter is adopted from the mutant $\underline{V}_i^{(k)}$. Further, the trial parameter with randomly chosen index, j_{rand} is taken from the mutant to ensure that trial vector does not duplicate target vector $X_i^{(k)}$. Otherwise the parameter is adopted from the target vector $X_i^{(k)}$.

Step-(d) Selection: Objective function is evaluated for target vector and trial vector, trial vector is selected if it provides better value of the function than target vector as follows

$$X_i^{(k+1)} = \begin{cases} t_i^{(k)} & \text{if } f(t_i^{(k)}) \le f(X_i^{(k)}) \\ X_i^{(k)} & \text{otherwise} \end{cases} \quad (11)$$

The process of mutation, crossover and selection is executed for all target vector index, *i,* and a new population is created till the optimal solution is obtained. The procedure is terminated if a maximum number of generations (k_{max}) have been executed or no improvement in objective function is noticed in a pre-specified generations. Various benchmark versions of DE that differ in the new generation methods largely are available [12]. In this the DE/best/1/bin has been selected. The first term after DE i.e. 'best' specifies the way base vector is chosen. In this selected scheme the base vector is the current best so far vector. '1' after best denotes that one vector difference contributes to the differential. Last term 'bin' denotes binomial distribution that result because of uniform crossover. Number of parameters donated by mutant vector closely follows binomial distribution. It is to be noted that best, target and difference vector indices are all different.

5. Results and Discussion The methodology presented in this paper is tested on IEEE 14 bus system [9]. Reactive power flows and voltage magnitudes for the given system are calculated both with AC power flow and with the proposed method under line outage condition. Simulation is carried out on MATLAB applying DE optimization technique. One of the line which is heavily loaded is selected for outage Errors in bus voltage magnitudes and reactive power flow are calculated as the difference of their values obtained using proposed method with that of AC power flow. Line connected between bus no 7 and 9 has been considered for outage. Pre outage reactive power flow is recorded 0.867pu. Table-1 and 2 depict the post outage voltage magnitude respectively as obtained with the proposed methodology Same table also contain the calculated values under contingency using AC power flow for comparison. Respective Errors are shown in the last column of table 1 and 2. The maximum % error in voltage magnitude is 0.4 which is less than 1pu which is reported. Error obtained in reactive power flow is quite high but still they are less than those reported in the literature. The error is bit dominant due to small size of the system. Algorithm has also been tested under different loading conditions and error in voltage margin and reactive power found remain low as compared to other existing methodology.

Table-1: Voltage magnitudes under line outage condition for IEEE-14 bus system.

BUS NO.	PRE OUTAGE VOLTAGE MAGNITUDE	AS OBTAINED USING PROPOSED METHOD	ERROR
1	1.060	1.060	0
2	1.045	1.045	0
3	1.010	1.010	0
4	1.015	1.015	0
5	1.016	1.016	0
6	1.070	1.070	0
7	1.066	1.068	0.002
8	1.090	1.090	0
9	0.988	0.991	0.011
10	0.994	0.997	0.003
11	1.027	1.030	0.003
12	1.050	1.054	0.003
13	1.040	1.043	0.003
14	0.992	0.896	0.006

Table-2 Reactive power flow under line outage condition for IEEE-14 bus system

LINE NO.	REACTIVE POWER FLOW USING AC LOAD FLOW	REACTIVE POWER FLOW USING PROPOSED METHOD	ERROR
1-2	-20.3	-20.1	0.2
1-5	5.4	4.9	0.5
2-3	3.6	3.5	0.1
2-4	0.2	0.15	0.05
2-5	2.8	1.8	1.0
3-4	5.3	5.0	0.3
4-5	12.0	10.6	1.4
4-7	-14.1	-15.0	0.9
4-9	13.2	12.3	1.07
5-6	12.8	13.5	0.9
6-11	14.6	13.1	1.1
6-12	3.7	3.3	1.1
6-13	13.0	11.8	1.1
7-9	**OUTAGED**	**87.2 SIMULATED**	
9-10	-5.5	-4.6	1.19
9-14	-2.6	-1.8	1.4
10-11	-11.3	-10.3	1.09
12-13	1.9	1.58	1.20
13-14	8.3	7.5	1.13
7-8	-14.5	-13.8	1.05

6. Conclusion An improved model has been presented in this paper for calculation of reactive power and voltage magnitudes. Considering the contingent line as part of bounded network it is then formulated as non-linear constrained optimization problem Voltage magnitude and reactive power flow in this bounded region are determined by DE in such a way to minimize the reactive power mismatch errors. Base case variables and linearised MW flows for bounded network have been used in the proposed method. This avoids excessive time of computation. Results are shown for IEEE 14 bus system. Clearly indicate that the bus voltage magnitude and reactive power flows are much better than those calculated by the traditional approaches. Accuracy of the method are found better for large size system compared to small size system.

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