Regression Equations for Predicting the Corrosion of Steel

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Abstract
Regression analysis was applied on the results of corrosion experiments performed on two grades of steel exposed to the laboratory atmosphere and 0.1M solutions of sodium chloride, ammonium hydroxide and hydrochloric acid. The regression equations showed acceptable coefficients of correlation between experimental and calculated extents of corrosion; indicating that the derived equations are applicable within the range of experimental values utilized.
Keywords: Regression analysis, Mild steel, Medium carbon steel, Laboratory atmosphere, 0.1M solutions

1. Introduction
Steel is the most commonly utilized industrial metal. Thus, the study of its degradation due to corrosion is of utmost relevance. In this respect, numerous studies on corrosion patterns and trends with time have been carried out; and in particular, the application of statistical methods for predicting corrosion behaviour is of great interest to researchers (Kaminski, 2014; Kienzler et al, 1991; Oshionwu et al, 2015; Suleiman et al, 2013). As a further contribution, the present study undertakes regression analyses of the corrosion – time data obtained from earlier reported sets of experiments (Sodiki, 2002) to arrive at equations which could be more readily utilized in predicting corrosion extents than corrosion-time graphs, within the limits of experimental values employed.

2. Generation of Data
Utilizing the method of measurement of weight change with time, the extents of corrosion of machined cylindrical specimens of two grades of steel exposed to the laboratory atmosphere and 0.1M solutions of sodium chloride, ammonium hydroxide and hydrochloric acid were obtained. The selected 0.1M solutions respectively represent salt, basic and acid environments that may be encountered by the test metals in actual service. The chemical compositions of the selected two grades of steel (as obtained from the stockist) are shown in Table 1.

Table 1: Chemical Composition of Test Metals

Test Metal	Main Element	Composition of Other Elements (Wt. %)	
Mild steel	Iron	Carbon	0.150
		Sulphur	0.023
		Phosphorus	0.030
		Manganese	0.500
		Silicon	0.250
Medium carbon steel	Iron	Carbon	0.350
		Sulphur	0.020
		Phosphorus	0.035
		Manganese	0.600
		Silicon	0.170

The experimental procedures involved, namely preparation of exposure environments and test specimens, corrosion measurements, and production of corrosion – time graphs had been reported in earlier publications (Sodiki, 2002; Sodiki, 2015). In order to achieve the desired control in experimentation, factors which normally influence the extent of corrosion over time such as the specimen's dimensions (i.e. length and diameter), surface roughness index and temperature were fixed. This was achieved by making the specimens as identical as possible and by placing them as close as possible on the same laboratory bench.

The corrosion-time data generated from the earlier study (Sodiki, 2002) are presented in Tables 2 to 5 for the mild steel specimens and Tables 6 to 9 for the medium carbon steel specimens; while the graphs derived from those data are shown in Figure 1 for mild steel and Figure 2 for medium carbon steel.

Table 2: Atmospheric Exposure of Mild Steel (Surface Finish Value 0.95 μm)

Exposure Time (h)	Weight Increase (10^{-3}mg/mm^2)
77	3.7
97	2.9
119	4.3
170	4.7
240	6.0
318	8.0
341	7.8

Table 3: Exposure of Mild Steel in 0.1M Sodium Chloride (Surface Finish Value 1.09 μm)

Exposure Time (h)	Weight Loss (10^{-3}mg/mm^2)
26	4.5
97	4.5
143	7.6
170	8.3
234	10.5
264	11.3
283	12.0

Table 4: Exposure of Mild Steel in 0.1M Ammonium Hydroxide (Surface Finish Value 1.21 μm)

Exposure Time (h)	Weight Loss (10^{-3}mg/mm^2)
25	0.9
76	6.2
125	10.1
165	13.7
195	15.6
290	23.9
313	26.5

Table 5: Exposure of Mild Steel in 0.1M Hydrochloric Acid (Surface Finish Value 1.17 μm)

Exposure Time (h)	Weight Loss (10^{-3}mg/mm^2)
17	22.8
45	52.0
93	57.2
112	56.0
137	55.4
165	56.6
184	57.6

Table 6: Atmospheric Exposure of Medium Carbon Steel (Surface Finish Value 1.01 μm)

Exposure Time (h)	Weight Increase (10^{-3}mg/mm^2)
44	0.318
110	0.636
163	0.636
188	0.509
256	0.573
286	0.527
318	0.730

Table 7: Exposure of Medium Carbon Steel in 0.1 M Sodium Chloride (Surface Finish Value 1.17 μm)

Exposure Time (h)	Weight Loss (10^{-3}mg/mm^2)
23	5.1
94	9.8
118	13.6
169	16.2
187	18.3
233	22.7
281	26.3

Table 8: Exposure of Medium Steel in 0.1M Ammonium Hydroxide (Surface Finish Value 1.0 μm)

Exposure Time (h)	Weight Loss (10^{-3}mg/mm^2)
16	1.1
45	4.1
111	8.8
139	12.5
163	15.8
188	17.0
209	18.9
260	22.0

Table 9: Exposure of Medium Carbon Steel in 0.1 M Hydrochloric Acid (Surface Finish Value 1.10 μm)

Exposure Time (h)	Weight Loss (10^{-3}mg/mm^2)
2	28.4
22	45.6
48	50.6
74	51.4
145	59.7
192	64.3
236	67.7

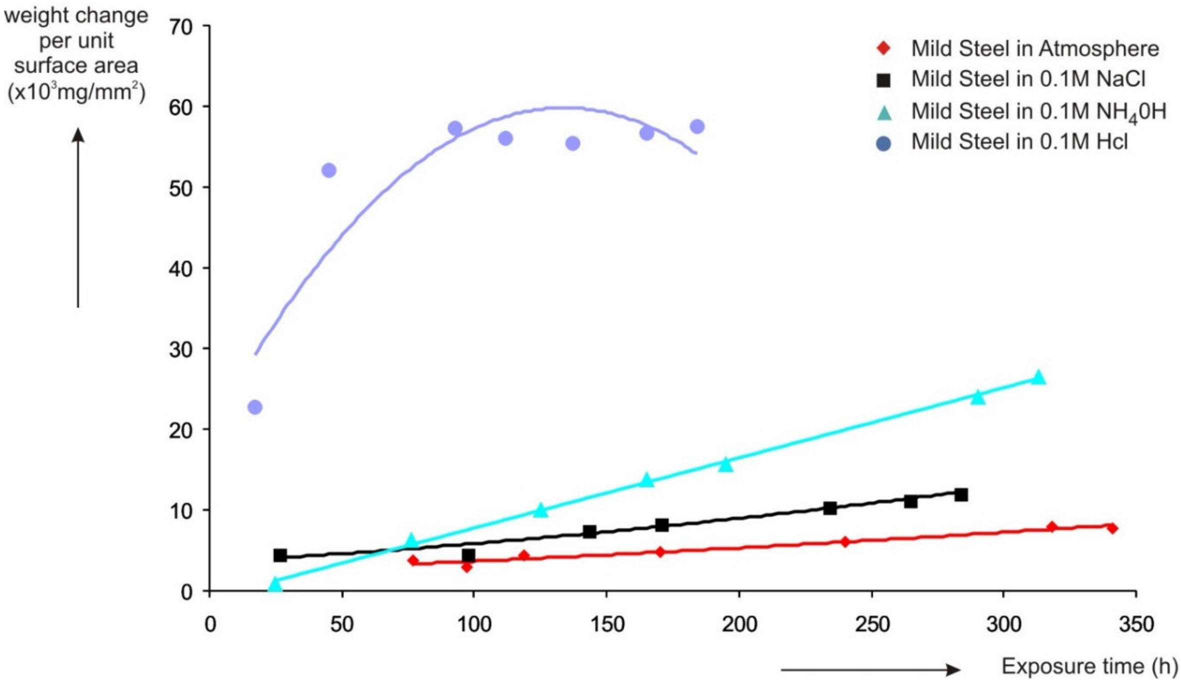

Figure 1: Corrosion Time Graphs of Mild Steel in Test Environments

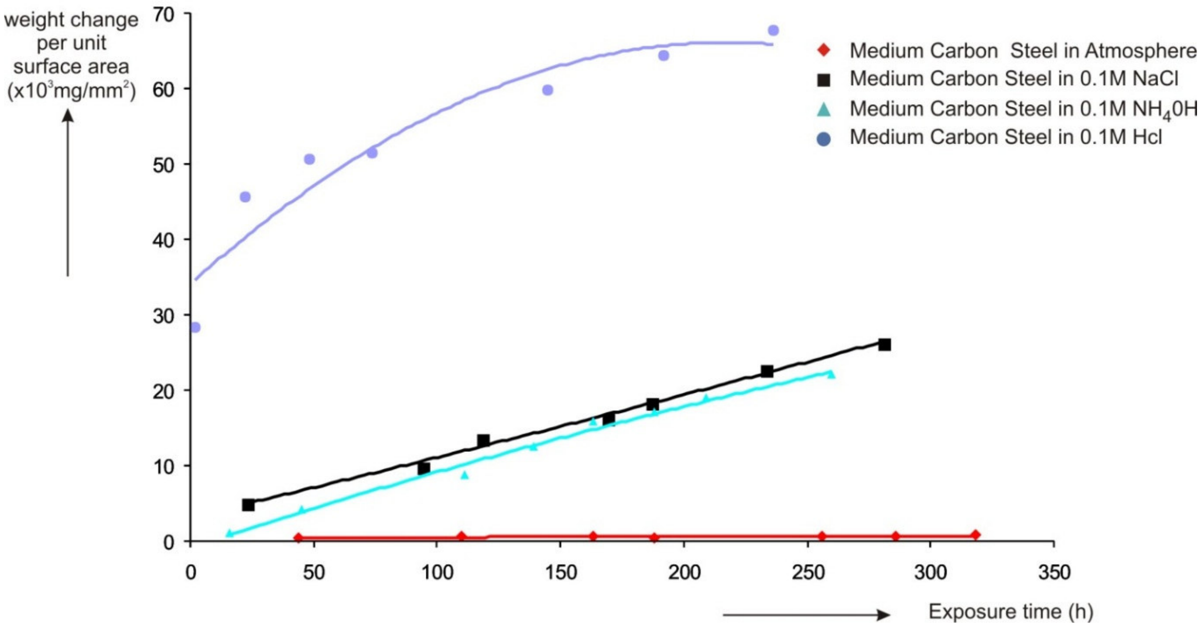

Figure 2: Corrosion-Time Graphs of Medium Carbon Steel in Test Environments

3. Regression Analysis of Corrosion Trends

In this study the statistical tool of regression analysis is employed to obtain equations which represent the change in extent of corrosion, expressed as weight change of the test specimen with time of exposure. Thus, weight change denoted as y is regressed on time, denoted as t.

As the graphs of Figures 1 and 2 generally show second order variations of extent of corrosion with time of exposure, an appropriate variation equation is

$$y = a_0 + a_1 t + a_2 t^2 \qquad \text{- - - - (1)}$$

where the variation parameters a_0, a_1 and a_2, once obtained, provide explicit relations for the tested grades of steel and test environments, within the limits of utilized test conditions.

The variation parameters would be obtained by the analytical solution of the set of simultaneous equations (Lipson and Sheth, 1973)

$$\sum y = na_0 + a_1 \sum t + a_2 \sum t^2 \quad ----(2)$$

$$\sum yt = a_0 \sum t + a_1 \sum t^2 + a_2 \sum t^3 \quad ----(3)$$

$$\sum yt^2 = a_0 \sum t^2 + a_1 \sum t^3 + a_2 \sum t^4 \quad ----(4)$$

where n = number of data points

In order to aid the solution of Equations 2 to 4 for each laboratory test, Tables 10 to 17 are set up to facilitate the computations of the variables and terms which appear therein. Thus, substituting values from Table 10 into Equations 2 to 4, for instance, yields the simultaneous equations

$$37.4 = 7a_0 + 1362a_1 + 333404a_2 \quad -----(5)$$

$$8520.7 = 1362a_0 + 333404a_1 + 93600618a_2 \quad -----(6)$$

$$2307529.5 = 333404a_0 + 93600618a_1 + 28224520583a_2 \quad -----(7)$$

Solving for a_0, a_1 and a_2 yields the regression equation for the variation of extent of corrosion with time for the atmospheric exposure of mild steel as

$$y = 2.270 + 0.0124t + 1.366 \times 10^{-5} t^2 \quad ----(8)$$

The other regression equations are obtained in like manner and listed in Table 18.

Table 10: Variables and Terms for Regression Analysis of Extent of Corrosion on Time of Atmospheric Exposure of Mild Steel

Exposure Time t (h)	Weight increase y (10^{-3} mg/mm^2)	yt	t^2	yt^2	t^3	t^4
77	3.7	284.9	5929	21937.3	456533	35153041
97	2.9	281.3	9409	27286.1	912673	88529281
119	4.3	511.7	14161	60892.3	1685159	200533921
170	4.7	799.0	28900	135830.0	4913000	835210000
240	6.0	1440.0	57600	345600.0	13824000	3317760000
318	8.0	2544.0	101124	808992.0	32157432	10226063380
341	7.8	2659.8	116281	906991.8	39651821	13521270960
$\sum$=1362	$\sum$=37.4	$\sum$= 8520.7	$\sum$= 333404	$\sum$= 2307529.5	$\sum$= 3600618	$\sum$= 28224520583

Table 11: Variables and Terms for Regression Analysis of Extent of Corrosion on Time of Exposure of Mild Steel in 0.1M Sodium Chloride

Exposure Time t (h)	Weight loss y (10^{-3} mg/mm^2)	yt	t^2	yt^2	t^3	t^4
26	4.5	117.0	676	3042.0	17576	456976
97	4.5	436.5	9409	42340.5	912673	88529281
143	7.6	1086.8	20449	155412.4	2863288	418161601
170	8.3	1411.0	28900	239870.0	4913000	835210000
234	10.5	2457.0	54756	574938.0	12812904	2998219536
264	11.3	2983.2	69696	787564.8	18399744	4857532416
283	12.0	3396.0	80089	961068.0	226651887	6414247921
$\sum$=1217	$\sum$=58.7	$\sum$= 11887.5	$\sum$= 263975	$\sum$= 2764235.7	$\sum$= 62584372	$\sum$= 14777147731

Table 12: Variables and Terms for Regression of Extent of Corrosion on Time of Exposure of Mild Steel in 0.1M Ammonium Hydroxide

Exposure Time $t\,(h)$	Weight loss y (10^{-3} mg/mm^2)	yt	t^2	yt^2	t^3	t^4
25	0.9	22.5	625	562.50	15625	390625
76	6.2	471.2	5776	35811.20	438976	33362176
125	10.1	1262.5	15625	15781.25	1953125	244140625
165	13.7	2260.5	27225	372982.50	4492125	741200625
195	15.6	3042.0	38025	593190.00	7414875	1445900625
290	23.9	6931.0	84100	2009990.00	24389000	7072810000
313	26.5	8294.5	97969	2596178.50	30664297	9597924961
$\sum=1189$	$\sum=96.9$	$\sum=$ 22284.2	$\sum=$ 269345	$\sum=$ 5624495.95	$\sum=$ 69368023	$\sum=$ 19135729637

Table 13: Variables and Terms for Regression Analysis of Extent of Corrosion on Time of Exposure of Mild Steel in 0.1M Hydrochloric Acid

Exposure Time $t\,(h)$	Weight loss y (10^{-3} mg/mm^2)	yt	t^2	yt^2	t^3	t^4
17	22.8	387.6	289	6589.2	4913	83521
45	52.0	2340.0	2025	105300.0	91125	4100625
93	57.2	5319.6	8649	489002.8	804357	14805201
112	56.0	6272.0	12544	702464.0	1404928	157351936
137	55.4	7589.8	18769	1039802.6	2571353	352275361
165	56.6	9339.0	27225	1540935.0	4492125	741200625
184	57.6	10598.4	33856	1950105.6	6229504	1146228736
$\sum=753$	$\sum=357.6$	$\sum=$ 41846.4	$\sum=$ 103357	$\sum=$ 5834199.2	$\sum=$ 15598305	$\sum=$ 2416046005

Table 14: Variables and Terms for Regression Analysis of Extent of Corrosion on Time of Atmospheric Exposure of Medium Carbon Steel

Exposure Time $t\,(h)$	Weight increase y (10^{-3} mg/mm^2)	yt	t^2	yt^2	t^3	t^4
44	0.32	14.08	1936	619.52	85184	3748096
110	0.64	70.40	12100	7744.00	1331000	146410000
163	0.64	104.32	26569	17004.16	4330747	705911761
188	0.51	95.88	35344	18025.44	6644672	1249198336
256	0.57	145.92	65536	37355.52	16777216	4294967296
286	0.53	151.58	81796	43351.88	23393656	6690585616
318	0.73	232.14	101124	73820.52	32157432	10226063380
$\sum=1365$	$\sum=3.94$	$\sum=$ 814.32	$\sum=$ 324405	$\sum=$ 197921.04	$\sum=$ 84719907	$\sum=$ 23316884500

Table 15: Variables and Terms for Regression Analysis of Extent of Corrosion on Time of Exposure of Medium Carbon Steel in 0.1M Sodium Chloride

Exposure Time $t(h)$	Weight loss y (10^{-3} mg/mm^2)	yt	t^2	yt^2	t^3	t^4
23	5.1	117.3	529	2697.9	12167	279841
94	9.8	921.2	8839	86622.2	830584	78074896
118	13.6	1604.8	13924	189366.4	1643032	193877776
169	16.2	2737.8	28561	462688.2	4826809	815730721
187	18.3	3422.1	34969	639932.7	6539203	1222830961
233	22.7	5289.1	54289	1232360.3	12649337	2947295521
281	26.3	7390.3	78981	2076674.3	22188041	6234839521
$\sum=1105$	$\sum=112.0$	$\sum=$ 21482.6	$\sum=$ 220072	$\sum=$ 4890342.0	$\sum=$ 48689173	$\sum=$ 11492929237

Table 16: Variables and Terms for Regression Analysis of Extent of Corrosion on Time of Exposure of Medium Carbon Steel in 0.1M Ammonium Hydroxide

Exposure Time $t(h)$	Weight loss y (10^{-3} mg/mm^2)	yt	t^2	yt^2	t^3	t^4
16	1.1	17.6	256	281.6	4096	65536
45	4.1	184.5	2025	8302.5	91125	4100625
111	8.8	976.8	12321	108424.8	1367631	151807041
139	12.5	1737.5	19321	241512.5	2685619	373301041
163	15.8	2575.4	26569	419790.2	4330747	705911761
188	17.0	3196.0	35344	600848.0	6644672	1249198336
209	18.9	3950.1	43681	825570.9	9129329	1908029761
260	22.0	5720.0	67600	1487200.0	17576000	4569760000
$\sum=1131$	$\sum=$ 100.2	$\sum=$ 18357.9	$\sum=$ 207117	$\sum=$ 3691930.5	$\sum=41829219$	$\sum=$ 8962174101

Table 17: Variables and Terms for Regression Analysis of Extent of Corrosion on Time of Exposure of Medium Carbon Steel in 0.1M Hydrochloric Acid

Exposure Time $t(h)$	Weight loss y (10^{-3} mg/mm^2)	yt	t^2	yt^2	t^3	t^4
2	28.4	56.8	4	113.6	8	16
22	45.6	1003.2	484	22070.4	10648	234256
49	50.6	2479.4	2401	121490.6	117649	5764801
74	51.4	3803.6	5476	281466.4	405224	29986576
145	59.7	8656.5	21025	1255192.5	3048625	442050625
192	64.3	12345.6	36864	2370355.2	7077888	1358954496
236	67.7	15977.2	55696	3770619.2	13144256	3102044416
$\sum=720$	$\sum=67.7$	$\sum=$ 44322.3	$\sum=$ 121950	$\sum=$ 7821307.9	$\sum=23804298$	$\sum=$ 4939035186

Table 18: Regression Equations and Correlation Coefficients for the Different Experiments

Corrosion Experiment	Regression Equation	Coefficient of Correlation
M.S.* in Laboratory Atmosphere	$y = 2.270 + 0.0124t + 1.366 \times 10^{-5}t^2$	0.962
M.S. in 0.1M NaCl	$y = 2.277 + 0.0416t - 2.98 \times 10^{-5} t^2$	0.948
M.S. in 0.1M NH_4OH	$y = -7.047 + 0.19t - 2.948 \times 10^{-4} t^2$	0.936
M.S. in 0.1M HCl	$y = 38.960 + 0.0293t + 6.079 \times 10^{-4}t^2$	0.997
M.C.S.* in Laboratory Atmosphere	$y = 0.297 + 2.403 \times 10^{-3}t - 4.376 \times 10^{-6} t^2$	0.401
M.C.S. in 0.1M NaCl	$y = 3.225 + 0.076t + 2.245 \times 10^{-5} t^2$	0.995
M.C.S. in 0.1M NH_4OH	$y = -0.736 + 0.105t - 6.085 \times 10^{-5} t^2$	0.994
M.C.S. in 0.1M HCl	$y = 33.970 + 0.295t – 6.793 \times 10^{-4} t^2$	0.929

M.S*: Mild Steel, M.C.S*: Medium Carbon Steel

Table 18 also shows the computed coefficients of correlation between measured and calculated extents of corrosion for the different experiments.

The coefficient of correlation r is given as (Lipson and Sheth, 1973)

$$r = \sqrt{1 - \left(\frac{s_{y.x}}{s_y}\right)^2} \qquad \text{- - - - - (9)}$$

where
$$S_{y..x} = \sqrt{\sum_{i-1}^{n} \frac{(y_i - y_{ic})^2}{n-3}} \qquad \text{- - - - - (10)}$$

with y_i = actual experimental value of y of the data set

y_{ic} = value of y computed from the derived regression equation

n-3 = degree of freedom, as the number of regression parameters is three: a_0, a_1 and a_2

and
$$S_y = \sqrt{\sum_{i=1}^{n} \frac{(y_i - \bar{y})^2}{n-1}} \qquad \text{- - - - - (11)}$$

with $\bar{y}$ = sample mean

In order to facilitate the computation of the correlation coefficient which is useful in testing the acceptability of each regression equation, tables are compiled as input to Equations 9 to 11. For instance, Table 19 gives the input to Equations 9 to 11 for the case of atmospheric exposure of mild steel, where y_i is the actual value of y obtained from the experiment, $\bar{y}$ is the mean of the experimental y values, and y_{ic} is the calculated value of y obtained from Equation 8 written as

$$y_{ic} = 2.270 + 0.0124 t_i + 1.366 \times 10^{-5} t_i^2$$

Table 19: Statistical Variables for Calculating Correlation Coefficient for Case of Exposure of Mild Steel in Laboratory Atmosphere

i	t_i	y_i	$y_i - \bar{y}$	$(y_i - \bar{y})^2$	y_{ic}	$y_i - y_{ic}$	$(y_i - y_{ic})^2$
1	77	3.7	-1.64	2.690	3.275	0.425	0.181
2	97	2.9	-2.44	5.954	3.601	-0.701	0.491
3	119	4.3	-1.04	1.082	3.939	0.361	0.131
4	170	4.7	-0.64	0.410	4.773	-0.073	0.005
5	240	6.0	0.66	0.436	6.033	-0.033	0.001
6	318	8.0	2.66	7.076	7.467	0.533	0.284
7	341	7.8	2.46	6.052	8.087	-0.287	0.082
		$\sum = 37.4$ $\bar{y} = \frac{37.4}{7} = 5.34$		$\sum = 23.7$			$\sum = 1.175$

Substituting values from Table 19 into Equations 9 to 11 yields

$$S_{y.x} = \sqrt{\frac{1.175}{4}} = 0.542$$

$$S_y = \sqrt{\frac{23.7}{6}} = 1.987$$

$$r = \sqrt{1 - \left(\frac{0.542}{1.987}\right)^2} = 0.962$$

The coefficients of correlation (listed in Table 18) for the other experiments are obtained in like manner.

4. Discussion of Results

Except the case of exposure of medium carbon steel in 0.1M ammonium hydroxide (which has an exposure time count n and, hence, number of weight change values of 8) all other tests have time count n and number of weight change values of 7. From statistical tables (Lipson and Seth, 1973), r required for 99% confidence level is 0.917 for n = 7, while it is 0.874 for n =8. Since all values of r obtained in the tests exceed 0.917 (except for the case of exposure of medium carbon steel in the laboratory atmosphere with r = 0.401), there is 99% confidence that for all tests (except for exposure of medium carbon steel in the laboratory atmosphere) the time dependent variation of extent of corrosion can be estimated from the derived regression equations, within the limits of experimental values utilized.

Furthermore, statistical data (Soper, 2014) indicate that in the case of exposure of medium carbon steel in the laboratory atmosphere, the coefficient of 0.401 falls within the 90% confidence interval of $0.321 \leq r \leq 0.804$. The derived regression equation for this case may, therefore, also be applied with reasonable accuracy.

5. Conclusions

Within the limits of experimental values utilized, estimates of the extents of corrosion of mild steel and medium carbon steel specimens of the test compositions, and environments, can be made using the derived regression equations. Following similar procedures to those applied in this paper, for extended exposure durations and for other test materials and environments, relevant regression equations can be derived for predicting corrosion extents.

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