

Theoretical Analysis and Practical Prediction of Moment of Resistance of Timber Beams Strengthened By External CFRP Composites Using Universal Design Model

Orumu S.T.[1], Ephraim M.E. [2]

[1]Department of Civil Engineering and Hydrology
Niger Delta University Wilberforce Island, Nigeria
Tel: +234 8037112365, Email:solorumu@yahoo.com

[2]Department of Civil Engineering
Rivers State University of Science and Technology Port Harcourt, Nigeria
Tel: +234 8034043544, Email:solorumu@yahoo.com
The research was sponsored by ABSOVI @ COY NIG.

Abstract

The recently developed Universal design model is extended to Timber reinforced with CFRP in this paper. The required area of reinforcement CFRP in the Timber for any given Moment can be easily found using the model. Where the area of reinforcing CFRP is fixed, then the Moment of resistance can be estimated. The research reveals that for a fixed area of reinforcing CFRP an upper bound and lower bound moment of resistance matching the maximum moment that can be carried in compression by the weaker material (Timber) and the maximum moment that can be carried in tension by the stronger material (CFRP) respectively, when they act in composite. The paper reveals that the stronger material will assist the weaker material to the extent where there will be equilibrium of Moment of resistance, at which point the CFRP reinforced Timber section will fail. Existing Experimental results were confirmed. The only requirements in the method are the yield stresses of CFRP and Timber, and the section modulus of the cross section chosen. Though a rectangular section is discussed in this work, the model can be used for any cross section and shape with striking results.

Keywords: working stress; max bending moment; section moduli, Moment of resistance, timber, carbon fiber-reinforced polymer (CFRP).

1.0 Introduction

The paper is focused on external reinforcement based on CFRP composites for strengthening timber beams to increase their bending moment resistance.

The usage of FRP composite external reinforcement is very often applied for strengthening concrete structures, because of suitable FRP material parameters in comparison of concrete material properties, that concrete structures strengthened by FRP are widely used in practice, mainly in the case of reconstructions caused by the load increasing necessity. Also in the field of timber structures the methods of strengthening by external bonded reinforcement based on FRP using glass or carbon fibres are applied frequently in the case of structural members (composed of glued lamella wood), to increase the load-carrying capacity. However, in the case of timber structural members composed of grown wood the strengthening utilizing FRP is not applied so widely in the practice.

1.1 The basic information and assumption

Strengthening flexural members using reinforcement based on fibre-reinforced polymers is given by advanced material properties of used fibres. CFRP composites usually use carbon fibres with very high tensile strength and high modulus of elasticity. In the case of timber beams strengthened by CFRP external reinforcement, the Young's modulus of elasticity of CFRP is very high compared to Young's modulus of timber, as well as the strength. On that account for the calculation of the timber beams resistance the partial plastic approach neglecting the tensile part of timber beams has been applied by several authors. For the calculation of assumed resistances basic principles mentioned above in accordance with [2], [3], [4], have been used. Practically the bending moment resistance has been calculated using [5], [6], [7]. Orumu [1] in introducing the universal design model for flexure, showed that the Youngs Modulli for the materials acting in composite are not necessary parameters for design. Results of application verified the claim of [1]. According to [1] if a material section is to be reinforced, then the area of reinforcement required is directly proportional to the Area of the reinforcing material assuming it is to be used alone without the chief material.

2.0 Review of Universal Design Model and Adaptation to CFRP reinforced Timber
Materials and method

There are basically three requirements to have a design done. These are the

a) Stress resultants (moments, shear etc) needed for design and is established from analytical tools of structural analysis (calculations).
b) The section modulus (geometric property of section) required to find the maximum fibre stresses at the tension and compression zones of the section chosen. These maximum stresses are used to determine the area of reinforcement required to carter for the out of balance stresses (permissible strength less the working stress of the material)
c) The material properties obtained from laboratory experiments

2.1 Theoretical formulation

The governing equation for choice of material or of section of a pre-selected material in flexure is given by

$$\sigma = \frac{M}{Z} \qquad \text{eq.1}$$

Where

M is the maximum bending moment of the idealized structure
Z is the section modulus of the cross-section
σ is the permissible working stress of the material in flexure

If the section is not reinforced, then the area of material is given as

$$A_{material} = A_{actual} \qquad \text{eq.2}$$

Where

A_{actual} is the actual area of the cross-section

If the section is to be reinforced, then the area of reinforcement required is directly proportional to the Area of the reinforcing material assuming it is to be used alone without the chief material i.e

$$A_{rf} \, \alpha \, A_r \qquad \text{eq.3}$$

Where

A_{rf} is the total area of the reinforcement required
and A_r is the equivalent area of the cross-section if the reinforcing material where used alone.

This becomes

$$A_{rf} = C\, A_r \qquad \text{eq.4}$$

The constant of proportionality C has been found to be

$$C = \frac{Zr}{Zm} \qquad \text{eq.5}$$

Therefore the area of reinforcing material becomes

$$A_{rf} = \frac{Zr}{Zm} A_r \qquad \text{eq.6}$$

Where

Z_r is the section modulus of the cross-section if the reinforcing material where used alone.
Z_m is the section modulus of the actual cross-section.

For CFRP reinforced Timber the equation 6 becomes

$$A_{cfrp} = \frac{Zf}{Zt} A_f \qquad \text{eq.7}$$

To implement equation 7 above, find each of Z_f and A_f in terms of yield stress f_f and Moment M. Express Z_t only in geometric form

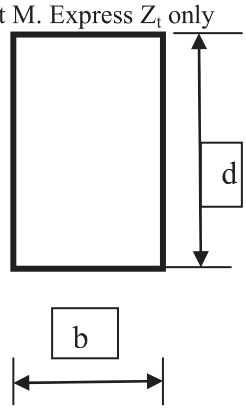
$$Z_f = \frac{M}{ff} \qquad \text{eq.8}$$

$$Z_t = \frac{bds^2}{6} \qquad \text{eq.9}$$

From where

$$d_f = \left(\frac{6M}{bff}\right)^{1/2} \qquad \text{eq.10}$$

$$Z_f = \frac{bd^2}{6} \qquad \text{eq.11}$$



For a common breadth b of the section, the total CFRP that should have been used for a CFRP section alone is

$$A_f = bd_f \qquad \text{eq.12}$$

Back substituting equations 8, 9, 10, 11 and 12 appropriately in equation 7, we have area of reinforcement A_{rf} as

$$A_{rf} = \frac{1}{d^2 b^{0.5}} \left(\frac{6M}{ff}\right)^{1.5} \quad \text{eq.13}$$

For fully plastic design eq.13 become

$$A_{rf} = \frac{1}{d^2 b^{0.5}} \left(\frac{4M}{ff}\right)^{1.5} \quad \text{eq.14}$$

And for elasto-plastic design (general equation)

$$A_{rf} = \frac{1}{d^2 b^{0.5}} \left(\frac{4M}{(1-\frac{x^2}{3d^2})ff}\right)^{1.5} \quad \text{eq.15}$$

If the strength of Timber in tension is considered then

$$A_{rs} = \frac{1}{d^2 b^{0.5}} \left(\frac{4(M-ft.Zt)}{(1-\frac{x^2}{3d^2})ff}\right)^{1.5} \quad \text{eq.16}$$

Where

$\frac{x}{d}$ is the ratio of elastic depth to total depth of the cross-section.

f_t is the compressive strength of the Timber used

2.1.1 Doubly reinforced Section

Available design codes show that when the moment of resistance M_r of the design cross-section is less than the working moment, the section should be doubly reinforced. The author can explain this as: If the working stress $\left(\frac{M}{Z}\right)$ of the designed section is greater than the compressive strength f_t of the Timber, the compression portion of the section needs to be reinforced for the excess stress i.e

$$\sigma = \frac{M}{Z} > f_t \quad \text{eq.38 17}$$

Reinforcement of the compression portion for a total stress of

$$\sigma - f_t \quad \text{eq.39 18}$$

is required from simple proportionality of moments and stresses.

The area of compression and tension reinforcement required is respectively

$$A_{comp} = As\left(1 - \frac{f\,cu}{\sigma}\right)^{1.5} \quad \text{eq.40 19}$$

$$A_{Ten} = A_s - A_{comp} \quad \text{eq.41 20}$$

2.1.2 Theoretical Doubly reinforced Section singly reinforced

When a supposedly doubly reinforced section is only singly reinforced at the tension zone, equilibrium of working stress and material strength of the section will come to play. In achieving this equilibrium eq,19 will equal zero, from where we have

$$\sigma = f_t$$

This is only possible when

$$f_{equil} = \{\sigma + f_t\}/2$$

When back substituted into eqn 18 the practical Moment of resistance can be assuredly predicted.

i.e

$$M_{practical} = Z\{\sigma + f_t\}/2$$

This implies that M when found after substitution of Area of reinforcement in equation x is substituted in equation 17 to obtain the working stress. Then the average of this working stress and material stress multiplied by the section modulus gives the practical Moment of resistance of the section

3.0 Application

Application 1

A typical problem from [7] will be handled here to illustrate the model adapted to CFRP reinforced Timber. In the work three samples each of 4m long timber beams of width 100mm and depth of 220mm, 200mm and 180mm respectively where reinforced with 50mm x 1.2mm CFRP at the bottom and another three samples each of 3m long timber beams of width 100mm and depth of 160mm, 140mm and 120mm respectively where reinforced with 50mm x 1.2mm CFRP at the bottom where tested to failure. The methods described in [6], [8] and [9] were used to predict the collapse load for the experimentation by [7]. The Timber was of class C24 of Strength 24N/mm^2 and CFRP with tensile strength of 3000N/mm^2.

3.1 Solution

The Universal model as describeb in [1] and adapted as described above shall be used to handle the problem above and compared with [7]

3.11 Elastic Solution

If the given data is imputed into eqn 13

$$A_f = \frac{1}{d^2 b^{0.5}}\left(\frac{6M}{ff}\right)^{1.5} \quad \text{eq.13}$$

$M=\{f_f[A_f d^2 b^{0.5}]^{1/1.5}\}/6$

for

A_f=60 mm^2

f_f=3000N/mm^2

for the first sample with depth of 220mm

$M=\{3000[60x220^2x100^{0.5}]^{1/1.5}\}/6$

M=47.239KNm

To check if compression steel is needed, we consider equations 17 and 18

$$\frac{M}{Z} = \frac{6x47.239x10^6}{100x220^2}$$

=58.561N/ mm^2 < 24N/mm^2

This shows that compression CFRP is required but not provided.

Therefore

$M_{practical}$=100x220^2(58.561+24)/2/6

=33.30KNm

3.1.2 Plastic Solution

If the given data is imputed into eqn 14

$$A_f = \frac{1}{d^2 b^{0.5}}\left(\frac{4M}{ff}\right)^{1.5} \quad \text{eq.14}$$

$M=\{f_f[A_f d^2 b^{0.5}]^{1/1.5}\}/4$

for

A_f=60 mm^2

f_f=3000N/mm^2

for the first sample with depth of 220mm

$M=\{3000[60x220^2x100^{0.5}]^{1/1.5}\}/4$

M=70.86KNm

To check if compression steel is needed, we consider equations 17 and 18

$$\frac{M}{Z} = \frac{4x70.86x10^6}{100x220^2}$$

=58.561N/ mm^2 < 24N/mm^2

This shows that compression CFRP is required but not provided.

Therefore

$M_{practical}$=100x220^2(58.561+24)/2/4

=49.95KNm

3.1.3 Elastic Solution without CFRP reinforcement

For the first sample with depth of 220mm

$$\frac{M}{Z} = 24$$

This shows that compression CFRP is required but not provided.

Therefore

$M_{practical}$=100x220^{2}x24/6

=19.36KNm

3.1.4 Plastic Solution without CFRP reinforcement

$M_{practical}$=100x220^{2}x24/4

=29.04KNm

Table: Showing theoretical and experimental results for various timber sections reinforced with CFRP in some cases and not reinforced in other cases.

depth	Elastic moment of resistance with CFRP (proposed)	Avg. Elastic moment of resistance with CFRP (quoted)	Elastic moment of resistance without CFRP (proposed)	Avg. Elastic moment of resistance without CFRP (quoted)	Elxperimental moment of resistance with CFRP exp predicted (proposed)	moment of resistance with CFRP exp test (quoted)	Experimental moment of resistance without CFRP (proposed)	Avg. Experimental moment of resistance without CFRP quoted	Plastic moment of resistance with CFRP (proposed)	Avg. plastic moment of resistance with CFRP (quoted)
220	47.24	45	19.36	21	33.3	33.71	29.04	33.56	49.95	70.86
200	41.60	41.3	16	17.5	28.80	28.69	24	23.14	43.20	62.40
180	36.15	32.4	12.96	14.3	24.55	21.16	19.44	19.44	36.83	54.22
160	30.9	26.57	10.24	11.2	20.57	21.01	15.36	15.61	30.85	46.34
140	25.86	21.1	7.84	8.7	16.85	16.91	11.76	16.64	25.27	38.79
120	21.05	16.2	5.76	6.6	13.41	12.74	8.64	8.6	20.11	31.58

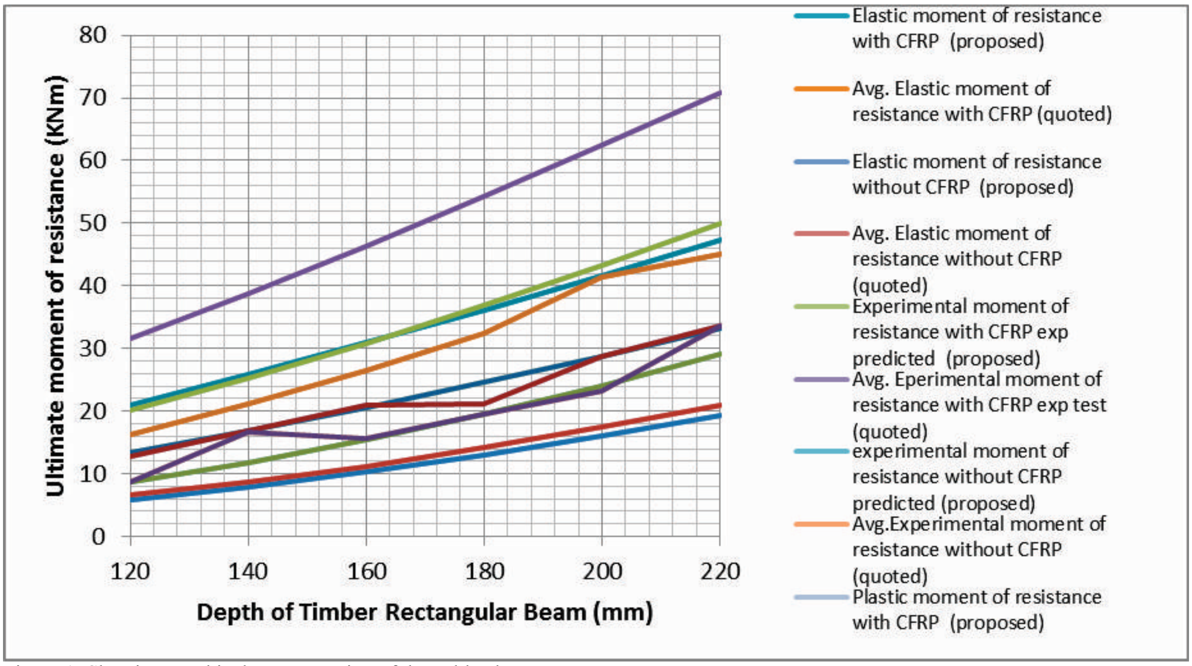


Figure 1: Showing graphical representation of the Table above

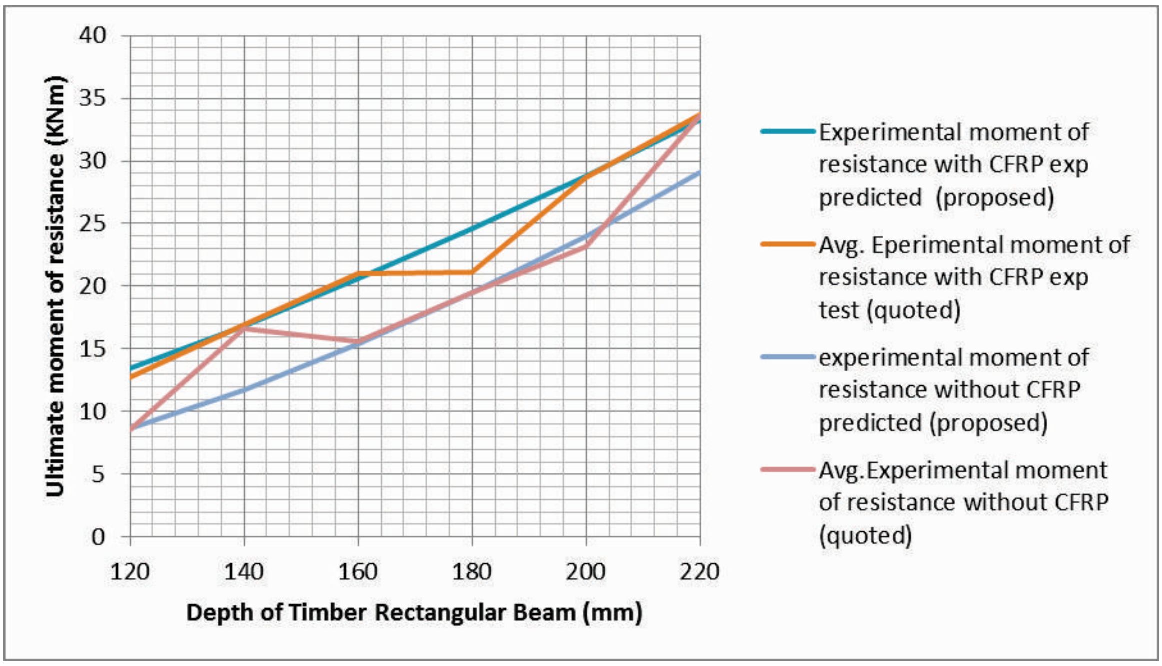


Figure 2: Showing Predicted and Tested Experimental Moment of Resistance for CFRP Reinforced and non Reinforced Timber beams of various depths

4.0 Discussion of Results:

Test results of the tests of CFRP reinforced timber beams have been evaluated using the recently published universal model for design in flexure for reinforced concrete. The theoretical Elastic and Plastic moment of resistance together with the Experimental Moment of resistance where computed with a lot of ease producing stricking results. This was done for both cases when the CFRP was used and not was not used. The results are compared with those of [7] who carried out extensive work on the application problems attempted using the universal design model. The table and Charts indicate that with CFRP five (5) of the six (6) different beams had their experimental moment of resistance correctly predicted. The Figures show that even the one (180mm deep beam) not predicted correctly may have had some issues within the limits of experimental errors. The results from the beams without CFRP predicted correctly four (4) of the six (6) different beams.

5.0 Conclusions

The bending moment resistances verified experimentally confirmed assumed (calculated) resistance for the proposed method. The check to confirm if the compression area of the beam needs to be reinforced is the reason why the predicted Experimental moment of resistance gave very reliable results. The effect of CFRP strengthening of the timber beams appears not significant if compared to the case where CFRP is not used, going by the results. This is not actually true. The need to reinforce the timber with a very little area of CFRP along the compression will bring about a very significant increase in the moment of resistance. The hitherto sought model to aid in the design of reinforced Timber (flexural members) to reflect actual behavior of the materials is herein formulated. Practicing Engineers would want a model as flexible as this, using a hand calculator for design. Extension of the work to Glass fibre, Steel and other materials as reinforcing materials for timber is in sight.

References

Orumu, S.T. (2013) Universal Design Model for Reinforced Concrete in Flexure International Journal of Civil & Environmental Engineering IJCEE-IJENS Vol:13 No:04 pg 32-40.

L. Ascione, A. Barbieri, A. Benedetti et al., (2007). Guidelines for the Design and Construction of Externally Bonded FRP Systems for Strengthening Existing Structures, National research council: Rome.

J.M.C. Cadei, T.J. Stratford, L.C. Hollaway and W.G. Duckett, (2004).Strengthening metallic structures using externally-bonded fibre reinforced- polymers, CIRIA: London

J. R. Gilfillan, S. R. Gilbert, G. R. H. Patrick (2003). The use of FRP composites in enhancing the structural behaviour of timber beams, Journal of reinforced plastics and composites, Vol.22, No.15/2003, pp. 1 373-1 388, Sage publications.

M. Karmazínová, J. J. Melcher, J. Prokeš, (2011) Fibre-reinforced composites based on CFRP and GFRP used as the external bonded reinforcement for the strengthening steel and timber beams, In Proceedings of the 16th International Conference on Composite Structures "ICCS 16", FEUP: Porto..

EN 1995-1-1 Design of Timber Structures (2006). – Part 1-1: General Rules and Rules for Buildings, CEN Brussels,
Marcela Karmazínová (2012) Theoretical analysis and experimental verification of moment resistance of steel and timber beams strengthened by external CFRP composites International Journal of Mechanics issue 3, Volume 6
EN 1994'1'1 Design of Composite Steel and Concrete Structures (2006). – Part 1'1: General Rules and Rules for Buildings, CEN Brussels,
EN 1993'1'1 Design of Steel Structures (2006)– Part 1'1: General Rules and Rules for Buildings, CEN Brussels,