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INTRODUCTION
Grossman’s human capital model is a model that explains the fundamental elements of health economist program for which every health economist student should be able to understand its theoretical implications and application to the real-world situation. Students find it very difficult to understand the mathematical manipulations of the model. This review is to break the model into its simplest form. The review of this model will also bring to the notice of the policy makers on the relevance of the Grossman human capital model to the society and the world at large. The main purpose of the review is to educate the public on the knowledge of health as a capital good and also as consumption good with specific reference from Grossman (2000) human capital model. The model assumed health as capital and consumption good. He argues that health is a capital good because it depreciates with age; it is also used for investment and for further production. Again, Grossman see health as consumption good because individual consumers derive a direct utility or satisfaction from health. Thus the fact that one is healthy makes him/her happy. Also it is consumption good because health is produced by an individual consumer and cannot be sold to anyone since it is already imbedded in the consumer.

Household Utility Function
According to Grossman, the household/individual length of life depends on their health stock (i.e. the quantity of health capital). Therefore the optimal quantity of health stock can be obtained by maximizing the individual utility function subject to their resource constraint. Grossman assumed that, the individual utility function (U) depends on the total health stock accumulated (H) and the consumption of other health related goods (Z) over time (t). Therefore the individual utility function over time becomes;

$$U_t = U(\Phi H_t, Z_t)$$

From equation (1), $$H_t$$ represents stock of health accumulated over time, $$\Phi$$ represents the service one derive from health or the satisfaction one obtained from health stock, $$PH$$ represents the total health stock or the total consumption of health services or the total satisfaction derived from health stock accumulated and $$Z_t$$ represents consumption of other goods or commodities such as exercise, good diet e.t.c.

Analysis on the Health stock over time
$$H_t$$ = Health stock over time where ‘t’ is the time element or period and ‘H’ is the health stock.
$$H_0$$ = Initial health stock or the Nascent or early health stock when a child is born before investing
$$H_1$$ = Health stock at period one
$$H_{min}$$ = The minimum health stock which cannot sustain life but rather calls for death of a person
$$H_t \leq H_{min}$$ = This represent a death condition, which therefore implies that, an individual will die when the health stock over time ‘t’ in him/her is less than or equal to the minimum health stock imbedded in the individual.
$$H_t > H_{min}$$ = This represent an individual staying alive condition, which therefore implies that, an individual will be alive or live, when the health stock over time ‘t’ in him/her is greater than the minimum health stock imbedded in the individual.

Grossman assumed also that, the individual is a rational being and always want to invest into their health stock. He further assumed that, the individual can only invest into their health stock based on their resources even though the individual will be always and willing to invest into their health stock to maximize their length of life. Grossman further assumed that, the individual can only invest into their health stock through the production and consumption of $$I_t$$ and $$Z_t$$ in order to add to their health stock already imbedded in them. Grossman assumed the production of $$I_t$$ through the purchase of healthcare (i.e. medication) to improve on their health stock or in order to invest into their health stock. As shown below;

$$I_t = I_t(M_t, TH_t, E)$$

From equation (2), Grossman defined $$I_t$$ as the health stock investment which depends on health care, time spend to access health care and education. Where Grossman defined $$M_t$$ as health care (medication) or vector input for the production of $$I_t$$, $$TH_t$$ be equal to time spend on medication or to access health care and ‘E’ equal to educational level of the individual, where education improves the efficiency in the production of $$I_t$$.

Grossman further assumed the production of $$Z_t$$ through the purchase of other goods (i.e. exercising, Visiting the gym center e.t.c) to improve on their health stock or in order to invest into their health stock. As shown below;
From equation (3), Grossman defined $Z_t$ as the production of other related health improving goods which depends on the consumption of other related health improves good (i.e. regular exercising, good diet, gym visitations e.t.c), time spend exercising and education. Where Grossman defined $X_t$ as the consumption of other related health improves commodity (such as regular exercising, good diet, gym visitations e.t.c), $T_t$ to be equal to time spend exercising to improve on their health status and ‘E’ equal to educational level of the individual, where education improves the efficiency in the production of $Z_t$.

**Analysis on the Budget Constraint**

Grossman assumed that, the individual is choosing $I_t$ and $Z_t$ which is not sold in the real-world market but determines solely by the individual. He further assumed a shadow pricing for each of the two non-market products, where the price of $M_t$ is equal to $P_t$ and the price of $X_t$ equal to $Q_t$.

Therefore the total expenditure (T.E) the individual faced is equal to equation (4) given below;

$$T.E = P_t M_t + Q_t X_t$$

He assumed that, the individual spends all his/her total resource or wealth on their expenditure on health care and the consumption of other goods. Note the identity of the total expenditure equals the total resource spends on goods and services is an idea from the classical economist which includes the following monetarist Irvin Fisher, A.C. Pigou e.t.c. which revolves around the Walrasian general equilibrium.

Grossman further assumed that, the individual total wealth or resource is equal to the summation of the individual initial endowment ($A_0$) and the individual monthly earnings (salary). The individual monthly earnings were view to be equal to the number of hours worked ($T_w$) multiply by the wage rate ($W_t$). Therefore, the total monthly earnings becomes $W_t T_w$, then total resources (T.R) becomes;

$$T.R = A_0 + W_t T_w$$

Based on the classical general equilibrium, then equation (4) is equal to equation (5). This implies that, $T.E = T.R$, thus $P_t M_t + Q_t X_t = A_0 + W_t T_w$ .........................................................

He also assumed that, the individual total available time ($\Omega$) is made up of; time spent working ($T_w$), time spend on health care purchase (medication ($TH_t$)), time spend on the consumption of other health related goods ($T_t$) and time spend on sickness or illness ($T_l$).

Therefore, individual total available time ($\Omega$) = $TH_t + T_t + T_w + T_l$ .................................

From equation (7) make $T_w$ the subject to get equation (8) and then substitute equation (8) into equation (6) to arrive at equation (9) as shown below;

$$T_w = \Omega - (TH_t + T_t + T_l)$$

$$P_t M_t + Q_t X_t = A_0 + W_t [\Omega - (TH_t + T_t + T_l)]$$

From equation (9), the individual lifetime budget constraint can be expressed as below where ‘$r$’ represents the discount rate and (1+$r$) is the discount factor use to discount the future values of the constraints in equation (9) into the present values. As shown below;

$$\left(\frac{Pt M_t + Qt X_t}{(1+r)t}\right) = A_0 + \left[\frac{Wt [\Omega - (TH_t + T_t + T_l)]}{(1+r)t}\right]$$

Hence, introduce the summation sign to sum all the future discounted values to the present discounted values as shown below;

$$\sum\left(\frac{Pt M_t + Qt X_t}{(1+r)t}\right) = A_0 + \sum\left[\frac{Wt [\Omega - (TH_t + T_t + T_l)]}{(1+r)t}\right]$$

Then expand equation (11) and juggle to solve for the budget constraint to get equation (12) as shown below;

$$\sum\left(\frac{Pt M_t + Qt X_t}{(1+r)t}\right) = A_0 + \sum\left[\frac{Wt \Omega}{(1+r)t}\right] - \sum\left[\frac{Wt (TH_t + T_t + T_l)}{(1+r)t}\right]$$

$$\sum\left(\frac{Pt M_t + Qt X_t}{(1+r)t}\right) + \sum\left[\frac{Wt (TH_t + T_t + T_l)}{(1+r)t}\right] = A_0 + \sum\left[\frac{Wt \Omega}{(1+r)t}\right]$$

This therefore implies that;

$$\sum\left(\frac{Pt M_t + Qt X_t + Wt (TH_t + T_t + T_l)}{(1+r)t}\right) = A_0 + \sum\left[\frac{Wt \Omega}{(1+r)t}\right]$$

The equation (12) above shows the budget constraint for the individuals, where the left-hand-side of the budget constraint $\sum\left(\frac{Pt M_t + Qt X_t + Wt (TH_t + T_t + T_l)}{(1+r)t}\right)$ represents the present value of the individual life-time expenditure and the right-hand-side of the budget constraint $A_0 + \sum\left[\frac{Wt \Omega}{(1+r)t}\right]$ represents the individual full wealth or the present value of the individual life-time wealth (resources).
Interpretation of the Budget Constraint (BC)

The budget constraint implies that, if the individual spend all his/her time working, part of the resources is spent on the purchase of health care or medication ($M_t$) and on the consumption of other health improves related goods ($X_t$) and part is lost to illness.

Now according to Grossman the individual faces two problems. First the individual faces the problem of utility maximization subject to their wealth constraint by choosing the equilibrium (optimal) levels of $I_t$ and $Z_t$. Secondly, the individual faces the problem of cost minimization in order to produce more of $I_t$ and $Z_t$.

The Grossman Utility Maximization Problem

The equilibrium level of $H_t$ (health stock) can be obtained by maximizing equation (1) subject to equation (2), (3) and (12). Thus;

Maximize $U_t = U(\Phi H_t, Z_t)$ where $t = (0, 1, 2 \ldots n)$ .................................................... (1)

Subject to

$I_t = I_t(M_t, TH_t, E)$ ............................................................... (2)

$Z_t = Z_t(X_t, T_t, E)$ ............................................................... (3)

$$\sum_{i=1}^{n} \left( \frac{W_t}{(1+r)^t} \right) = A_0 + \sum_{i=1}^{n} \left( \frac{W_t}{(1+r)^t} \right)$$ .................................................... (12)

Where $H_{t+1} - H_t = I_t - \delta H_t$, therefore $I_t = H_{t+1} - H_t + \delta H_t$ represents the gross investment for the optimization.

Here the maximization is done by both substitution and the lagrangean set-up (i.e. the introduction of $\lambda$) and it was done electronically but not mechanical manipulation as shown below;

First Order conditions for the optimization analysis

$$\frac{\pi_{t-1}}{(1+r)^{t-1}} = \frac{G_t W_t}{(1+r)^t} + \frac{G_{t+1} W_{t+1}}{(1+r)^{t+1}} + 
\ldots + (1-\delta_t) \ldots$$

$$\left( 1 - \delta_{n-1} \right) W_n G_n + \frac{U_{H_t} G_t}{\lambda} + (1-\delta_t) \ldots \ldots \ldots (1-\delta_{n-1})$$ .............. (*)

The equation (*) represents the Grossman geometric progression optimization first order condition. Which was properly arranged in equation (13) in simple terms as stated below.

The equilibrium Conditions after the Utility optimization is given as below to determine the optimal health stock of the individual;

$$\frac{\pi_{t-1}}{(1+r)^{t-1}} = G_t \left( \frac{W_t}{(1+r)^t} + \frac{U_{H_t}}{\lambda} \right)$$ .................................................... (13)

Where $\pi_t$ represents the marginal cost of gross Investment of health in period $t$, $G_t$ represents the marginal product of health capital in period $t$ ($G_t = \frac{\partial H_t}{\partial T_t} = -\frac{\partial T_t}{\partial H_t}$. $U_{H_t}$ represents the marginal utility of healthy time, $\left( \frac{W_t}{(1+r)^t} \right)$ represents the discounted monetary value of a unit increase in total amount of time available for market and non-market activities, $U_{Ht}$ represents the discounted monetary value of increase in utility due to a unit increase in healthy time, $\left( \frac{W_t}{(1+r)^t} + \frac{U_{H_t}}{\lambda} \right)$ represents the discounted monetary value to consumer of output produced by health capital. In the equation (13) above, the left-hand-side $\frac{\pi_{t-1}}{(1+r)^{t-1}}$ represents the present value of the marginal cost of gross investment in period $t-1$ (previous period) where as the right-hand-side $G_t \left( \frac{W_t}{(1+r)^t} + \frac{U_{H_t}}{\lambda} \right)$ represents the present value of the marginal benefit of gross investment in period $t$ (current period).

Undiscounted Equilibrium Condition

To get the undiscounted equilibrium condition, one has to recall equation (13) the discounted equilibrium condition and multiply through by $(1+r)^t$ in order to leave the values in future values (undiscounted) as shown below;
Then expand equation (14) to get equation (15);

\[
\frac{\Pi_{t-1}}{(1+r)^{t-1}} x (1+r)^t = G_t \left( W_t + \frac{U_h}{\delta} (1+r)^t \right)
\]

By assumption

\[
\frac{1}{(1+r)^{t-1}} = (r-\frac{\delta}{(t-1)+\delta})
\]

The equation (16) represents the supply price of capital. Then substitute equation (16) into equation (15) to get equation (17) as giving below;

\[
\Pi_{t-1}(r-\frac{\delta}{t-1}) = G_t \left( W_t + \frac{U_h}{\delta} (1+r)^t \right)
\]

The equation (17) symbolizes the undiscounted equilibrium condition which determines the optimal stock of capital for time \( t \). From equation (17) the left-hand-side i.e. \( \Pi_{t-1}(r-\frac{\delta}{t-1}) \) represents the undiscounted value of the marginal cost of health capital at time \( t \), where \( \delta \) represents the percentage change in the marginal cost between period \( t \) and \( t-1 \). On the other hand, the right-hand-side of the equation i.e. \( G_t \left( W_t + \frac{U_h}{\delta} (1+r)^t \right) \) represents the undiscounted value of the marginal product for the optimal stock of health capital at time \( t \) (any age).

**Cost Minimization of Health Production under Grossman Model**

According to Grossman the individual can produce more of the \( I_t \) and \( Z_t \) by minimizing the cost of their vector inputs in the market. Theoretically, firms output can increase if and only if cost is been minimized. Therefore the individual producer of health also faces the problem of cost minimization by minimizing the expenditure (equation (12) subject to equation (2) and (3) as shown below;

\[
\min \sum (P_t M_t + Q_t X_t + W_t (TH_t + T_t + TH_{t-1})) = A_0 + \sum \alpha_t (1+r)^t
\]

Subject to

\[
I_t = I_t (M_t, TH_t, E)
\]
\[
Z_t = Z_t (X_t, T_t, E)
\]

Here the minimization was done by only substitution, but the optimization was done electronically but not mechanical manipulation as shown below;

**First order condition of the cost minimization problem**

\[
\frac{\partial \Pi_{t-1}}{\partial W_t} = \frac{\partial W_t}{\partial \Pi_{t-1}}
\]

The equation (18) represents the necessary condition of cost minimization for the production of health through investment for a given quantity of \( I_t \). The equation (18) implies that, the cost of producing a given \( I_t \) is minimized at a point where the increase in the \( I_t \) from spending an additional amount ($) on medical care \( (M_{t-1}) \) equals the increase in the total cost from spending an additional amount ($) on time \( (TH_{t-1}) \).

**Note:** The stock of health capital cannot be sold because it is imbedded in the investor, which cannot be negative as well but always positive \( (I_t > 0) \). It is always positive because there is a user cost of capital that in equilibrium must equal to the value of the marginal product of the health stock of capital imbedded in the individual.

**The Optimal Length of Life**

The optimal length of life is very crucial to the individual as well as the policy maker. It is crucial to the individual because the optimal length of life depends on the health stock available or imbedded and invested in the individual. Since the planning horizon towards health stock investment is exogenous; thus an individual is alive in today and die tomorrow. Due to the crucial nature of the optimal length of life, it made Grossman to investigate into by making some assumption to help reveal the information to the policy makers and the individual producer of health.
Assumptions to the Optimal Life of Life

Grossman assumes that the individual health stock of capital depreciate over time as people ages. He assumed that the individual optimal health stock is always positive ($I_t > 0$), except in the last years of life. He assumed also that, the undiscounted monetary value of output of health is produced by health capital in period $t$, i.e. $V_t = \left( W_t + \frac{U_{Ht}}{\lambda} (1 + r)^t \right)$. He further assumed that, the output of health produced by health capital is limited to the maximum number of hours in a year by 8760 hours. Since the output of health produced by health capital is limited to a maximum of 8760 hours per year, the model assumes that $\frac{\partial G_t}{\partial H_t} < 0$ i.e. marginal product of the stock of capital diminishes as the stock increases. As shown in figure 1 below;

Figure 1: The Optimal Length of Life

Where $h_t$ represents the benefit derived from health stock, $H_{\text{min}}$ represents the minimum health stock imbedded in the individual given the time for health production and $H_t$ represents the health stock at the current period ($t$).

Given the undiscounted maximization problem for the optimality condition to the individual producer of health since the planning horizon is exogenous (i.e. alive in period $n$ and dead in period $n+1$). The individual behaves as if the rate of depreciation on the stock of health is equal to 1 in period $n$, which implies that, investment in period $(n-1)$ yields returns for period $n$ only. Therefore, it is necessary for the individual to maximize their optimal length of life in order to always stay healthy in the current period and avoid dead in the next period.

For the maximization of the optimal length of life, recall equation (17) for $t < n$

$$\Pi_{t-1} = \left( G_t \left( W_t + \frac{U_{Ht}}{\lambda} (1 + r)^t \right) \right) \delta_t$$

Where $V_t = \left( W_t + \frac{U_{Ht}}{\lambda} (1 + r)^t \right)$

Then substitute equation (19) into equation (17) to get equation (20) below;

$$\Pi_{t-1} \left( G_t V_t \right) = \delta_t$$

Therefore, the optimality condition for $n=1$, for $\delta_t - \delta_{t-1}$ then collapses to one (1) to get;

$$G_n V_n = \Pi_{n-1} (r + 1)$$

The individual is to maximize equation (21) by choosing period $\theta$ and $n$. Then the first order condition is given as below:

$$\Pi_0 = \frac{G_1 V_1}{(1+r)} + d_2 \frac{G_2 V_2}{(1+r)^2} + d_3 \frac{G_3 V_3}{(1+r)^3} + \ldots \ldots \ldots \ldots + d_n \frac{G_n V_n}{(1+r)^n}$$

Where $I_n = 0$ because the individual will not be alive in period $n+1$ to collect the returns, $d_t = \frac{\partial H_t}{\partial I_0}$, $d_1 = \frac{\partial H_1}{\partial I_0}$. 

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Therefore dead will occur in \( n+1 \), when \( H_{n+1} \leq H_{min} \); since \( I_n = 0 \). This implies that \( H_{n+1} = (1-\delta_n)H_n \). Therefore to die in \( n \) years implies that \( I_{n-1} = 0 \). The application of this concept is that the individual who dies today or in the current year did not invest into his/her health stock in the previous years or day. Their previous health stock approaches zero or falls below the minimum health stock.

Since the individual can choose the optimal length of life “why should a policy makers be interested in the individual optimal length of life? And what kind of policy implication can be drawn from this concept of optimal length of life?

Even though the individual is to choose their own optimal length of life, such information is important to the policy makers because the information deducted or obtain implies that, as people age, their demand for health care could increase. Thus as the number of the aged or the proportion of the aged in a given population increases, the policy makers have to ensure that, they subsist enough infrastructures for health care, to avoid over-crowding and hence under-staffing of health care facilities. The implication is that, failure to prepare for this could increase the cost of care, reduce the quality of care and hence reduce the consumption of health care and in the nut shell deteriorate the health of the entire population.

ANALYSES ON HEALTH AS PURE INVESTMENT MODEL BY GROSSMAN

If health is assume to be solely an investment good or commodity, and then from the first order condition the marginal utility of healthy time must be equal to zero (i.e. \( U_{h_t} < 0 \)). Recall the undiscounted equilibrium condition and impose the restriction;

\[
\pi_{t-1}(r-\bar{H}_{t-1})\delta_t = G_t\left(W_t + \frac{U_{h_t}}{\lambda} (1 + r)^t\right) 
\]

Therefore, if \( U_{h_t} < 0 \), then equation (17) becomes equation (23) below;

\[
\pi_{t-1}(r-\bar{H}_{t-1})\delta_t = G_t\left(W_t \right) 
\]

From equation (23) make the opportunity cost of health or the supply prices of capital the subject by dividing through by \( \pi_{t-1} \) as shown below;

\[
\frac{(r-\bar{H}_{t-1})\delta_t}{\pi_{t-1}} = \frac{G_tW_t}{\pi_{t-1}} 
\]

Which implies that,

\[
(r-\bar{H}_{t-1})\delta_t = \frac{G_tW_t}{\pi_{t-1}} 
\]

When health is assume to be purely an investment good, then the optimal health stock \( H_t \) is found or obtained by equating the marginal rate of returns on health \( \frac{G_tW_t}{\pi_{t-1}} \) to the opportunity cost of health or the supply price of capital \( (r-\bar{H}_{t-1})\delta_t \). Where \( \frac{G_tW_t}{\pi_{t-1}} = (r - \bar{H}_{t-1})\delta_t = \gamma_t \), then the last year or period of life becomes:

\[
\frac{G_tW_t}{\pi_{t-1}} = \gamma_t = r + 1 
\]

See the graphical equilibrium condition for pure investment good in figure 3.2 below;
The $s$ represents supply curve, which shows the relationship between health stock ($H_t$) and the cost of capital $(r - \pi_{t-1} + \delta_t)$. The supply curve is horizontal because $\frac{\partial (r - \pi_{t-1} + \delta_t)}{\partial H_t} = 0$. The M.E.C represents the marginal efficiency curve which is negatively sloping because $G_t > 0$ and $\frac{\partial G_t}{\partial H_t} < 0$ or $\frac{\partial^2 h_t}{\partial H_t^2} < 0$. This explains the law of diminishing marginal returns, which implies that the marginal efficiency falls with the stock of health.

Depreciation Effects
The analysis is such that, whenever depreciation increases i.e. (from $\delta_0$ to $\delta_1$), the cost of investment increases, so the supply curve falls from $s_0$ to $s_1$. The high cost of investment reduces the return on investment, so demand also falls for the equilibrium health stock to fall from $H^*$ to $H^{**}$ as shown below;
Market and Non-Market Efficiency

People who face the same cost of capital will demand the same amount of health, if the return on capital is the same for all. Factors such as Wage rate ($W_t$) and Education ($E$) can shift the marginal efficiency curve (M.E.C) and then causes variation in the returns of health stock ($H_t$).

Effects of Change in Wages ($W_t$) on Health

Recall the value of marginal product of health stock ($W_tG_t$), thus an increase in wages lead to an increase in the marginal product of health capital (MP) and increases the benefit derived from health capital ($h_t$). Wage measures the workers efficiency, implying a positive correlation between wage rate ($W_t$) and the reduction in the time lost due to illness. Whenever wage rate increases, the individual reduces leisure until the value of marginal benefit from leisure equals the wage rate. In general, when wages increase, time spent on market and non-market goods fall (i.e. $T_t$, $TH_t$ and $T_l$ all falls).

Let $K$= fraction of total cost of $I_t$ accounted for by time, $(1-K)$ = fraction of total cost of $I_t$ accounted for by medical care. So the wage elasticity of capital is given as in equation (27);

$$E_{HW} = (1-K) \frac{\partial \ln H_t}{\partial \ln W_t}$$

(27)

The wage elasticity of capital ($E_{HW}$) is defined as the percentage change in demand for $H_t$ due to a percentage change in wage.

Effects of a Change in Wage Rate on Medical Care

There is a positive relationship between wage rate and medical care ($\frac{\partial M_t}{\partial W_t} > 0$). Thus when wage rate increases, the individual reduces time spent on market and non-market goods. However, if fixed proportions of $M_t$ and $TH_t$ are required for the production of $I_t$, then the impact of a change in $W_t$ on $M_t$ equals to the change in $W_t$ on $TH_t$. But because $TH_t$ can be substituted for $M_t$ as wage rate increases, $TH_t$ becomes pricey and the demand for $TH_t$ falls which leads to an increase in the demand for its substitute such as medical care ($M_t$) due to the elasticity of substitution ($\sigma_p$). In summary as wage rate increases people increase their purchases for more medical care and spend less time to improve on their health ($TH_t$). Also as wage rate increases people have the incentives to increase $I_t$ through medication since it brings greater earnings.

A positive wage elasticity of health ($E_{HW} > 0$)

It implies that an increase in wage improves the health above the minimum level. Thus people with high income, all things being equal are likely to have a longer life expectancy than those with low income.

How applicable is the above conclusion to the developing world where medication could be highly costly (regressive) in terms of cash price, transportation cost and waiting time?

Assuming full insurance

Given the assumption of full insurance, then the cost (price) of medical care is zero ($P_t = 0$), therefore care is rationed by waiting time and travel time. Suppose $q$ hours are needed to obtain a unit of $M_t$, then the price of $M_t=qW_t$. Suppose also that, three inputs are needed for the production of $I_t$ such as $M_t$, $TH_t$ and $X_t$. Then the Wage elasticity of Medical care becomes $E_{Mw} = (1-K) \frac{\partial M_t}{\partial W_t}$ where $\sigma_m$ is equal to the partial elasticity of substitution in production between $M_t$ and $X_t$.

The Condition for which the Wage elasticity of Medical care could be negative ($E_{Mw}<0$)

Assuming time is not require for the production of $M_t$ and $X_t$ is also assumed to be close substitutes in the production ($\sigma_m > 0$). Then it is possible for the consumption of medical care to fall when wage rate increases. Thus when wage rate increases, any production that requires time will be expensive relative to the other. All other things being equal, base on the law of demand, the lower the price, the higher the demand and the higher the price, the lower the demand, so that more of $X_t$ will be demanded since time is not require for it production and declares its cheaper nature which calls for more demand of $X_t$ and less of $M_t$.

In an extreme cases where there is no insurance: Medical care could be highly costly because health care would be rationed in terms of the cash price, transportation time and waiting time. Which will increase the price
of medical care extremely, so when wages increases the medical care will no longer be a normal good to the individual since the demand for it requires some amount of time elements in its production of a given $I_t$.

**Effects of Education on Health Production**

The effects of education on health production focus on years of formal schooling. The assumption of $I_t$ and $Z_t$ production are linearly homogenous in $M_t$ and $X_t$ respectively. Therefore education affects $I_t$ and $Z_t$ through $M_t$ and $X_t$ respectively. So whenever educational attainment increases the marginal product of $M_t$ also increases which in effects increase the $I_t$ (i.e. $\frac{\partial I_t}{\partial M_t} > 0$). There is also the neutrality assumption factor where $\frac{\partial I_t}{\partial E} = 0$. Here there is no incentive to substitute $M_t$ for $TH_t$ as education increases. But since marginal product of $M_t$ and $TH_t$ increases with an increase in education, less of both inputs are required to produce given levels of $I_t$. Hence assuming prices do not change but the marginal cost and the average cost of $I_t$ falls. Then the effects of a change in education on the health production can be illustrated in a diagram as shown in Figure 4.

When there is an improvement in the educational level attainment, it shift the marginal efficiency curve to the right from $M.E.C_1$ to $M.E.C_2$ and further leads to an increase in the optimal health stock from $H^*$ to $H^{**}$.

**Figure 4: Education Effects on the Equilibrium Condition of Investment Model**

Let $\pi = -pH$ equals the percentage change in $\pi$ due to per unit change in education and $pH$ represents education elasticity of $f$ (i.e. percentage increase in $I_t$ as a result of a unit increase in education). Let the percentage increase in $H$ due to a unit increase in education be equal to $H = -pH$. A person with a greater number of years of education receives more health, holding medical care ($M_t$) and $TH_t$ constant. Thus both highly educated and less educated may consume the same amount of $M_t$ and $TH_t$ but the highly educated gets more health.

Again, let $H = -pH$ (ε - 1) for $M_t$ to be equal to the percentage change in $M$ due to per unit change in education. This implies that, the more educated would demand more health but less medical care ($M$) if the elasticity of marginal efficiency curve is inelastic (ε <1).

**ANALYSES ON HEALTH AS PURE CONSUMPTION MODEL BY GROSSMAN**

Assuming ($\pi = 0$) and the cost of health capital is also assumed to be large relative to the monetary rate of returns, then the optimality condition can be approximated as;

$$\lambda = \frac{\pi(r+\delta)}{(1+r)^{\gamma}}$$  \hspace{1cm} (28)

The equilibrium condition can be interpreted as the equilibrium point between the monetary equivalent of...
marginal utility of health capital \( \left( \frac{U_h C_L}{\lambda} \right) \) and the discounted user cost of health capital \( \left( \frac{\pi (r+\delta)}{(1+r)^t} \right) \).

Assumption

\[ \text{MRS}_{H_t, H_{t+1}} = \text{depends only on } H_t \text{ and } H_{t+1} \]

\[ \text{MRS}_{H_t, Z_t} = \text{depends only on } H_t \text{ and } Z_t \]

\[ (1+r) = 1 + \text{rate of time preference for the present} \]

The relevant elasticity here is the elasticity of substitution in consumption between \( H_t \) and \( H_{t+1} \) which is replace by \( \varepsilon \).

Rate of Depreciation

When time \((t)\) goes up or age increases, \( I_t \) increases and \( M_t \) and \( TH_t \) also increase if the elasticity of substitution between the current and the future health is less than one \((\varepsilon<1)\). The effects of depreciation on health as a consumption good could be expressed mathematically as: \( \frac{\partial H_t}{\partial \delta_t} < 0 \). Over the life cycle.

Wealth/Wages Effects on Health as a Pure Consumption Good

Since health enters the utility function directly, therefore an increase in wealth will lead to an increase in health \((H)\), if health is a normal good, assuming no change in wage.

Holding wealth constant, the wage elasticity of health is giving as below in equation (29);

\[ \frac{\delta H_t}{\delta W} = (1-\theta) (K - K_Z) \Omega_{HZ} \]

\[ \Omega_{HZ} \]

From equation (29), \( \theta \) represents the share of health in wealth, i.e. part of wealth due to health, \( K_Z \) represents the share of total cost of \( Z \) accounted for by time and \( \Omega_{HZ} \) represents the positive elasticity of substitution between \( H \) and \( Z \).

Analysis on \( \varepsilon_{\text{Hw}} \) as \( K < K_Z \) \( \Omega_{HZ} \) \( \frac{\delta H_t}{\partial \delta_t} < 0 \).

Effects of Education on Health as Pure Consumption Good

The human capital parameter in the consumption demand function is stated in equation (31) below;

\[ \hat{H} = \rho \eta_H^+ \]

\[ (\rho_H^Z \rho_Z^+ (1-\theta) \Omega_{HZ}) \]

\[ \Omega_{HZ} \]

Where \( \rho_Z \) represents a percentage increase in the health from the consumption of other goods \((Z)\) due to a unit increase in education \((E)\), \( \rho_H \) represents the percentage increase in health due to increase in education \((E)\), \( \theta \) represents part of health due to wealth, \( \eta_H \) represents the wealth elasticity of demand for health, \((1-\theta) \)

\( \Omega_{HZ} \) represents the price elasticity of demand for health and \( (\rho_H^Z \rho_Z^+ (1-\theta)) \)

\[ \Omega_{HZ} \]

represents the percentage increase in real wealth as education increases by assuming monetary wealth and wages to be held constant.

Again, from equation (31) the first term of the right-hand-side such that \( \rho \eta_H^+ \) represents the wealth effect and the second term of the right-hand-side such that \( (\rho_H^Z \rho_Z^+ (1-\theta) \Omega_{HZ}) \)

\[ \Omega_{HZ} \]

represents the substitution effects. The equation (31) as represents the Slutsky equation of education effects on health as pure consumption good which is made up of two effects the substitution effect and the income or output or wealth effect.

Analysis on the Effects of Education on Health

If the productivity effect for \( Z \) equals that of \( I_t \) then \( \rho_H = \rho_Z \) and \( \hat{H} \) shows wealth effect alone. Then education becomes a neutrality commodity.
If $\rho_H > \rho_Z$ then education is biased towards health. Thus its relative price falls, when wealth and substitution effects all move in the same direction. Therefore as education increases the health of the educated person also increases by the consumption of appropriate health care (medication).

If $\rho_H < \rho_Z$ then education is biased away from health. Thus its relative price increases when wealth and substitution effects all move in different direction. Therefore as education increases the health of the educated person also increases but through the consumption of other health related commodity ($Z$).

Human Capital Parameter in Consumption Demand Curve for Medical Care

$$\hat{M} = (\rho_H (\eta_H -1) + (\rho_H \rho_Z) ((1-\theta) \sigma_{HZ} -1))$$

If the shift in education is commodity neutrality, then medical care consumption and education are negatively correlated unless the wealth elasticity of demand for health is fairly elastic ($\eta_H \geq 1$)

Again, for education to favors $H, M$ and can still be negatively correlated unless wealth and price elasticity are both greater than one.

Differences between Consumption and Investment Models of Health

Wealth effects are not relevant in the investment model. This is because if the wealthy people faced lower interest rate in the investment model, then there could be a positive elasticity between wealth and health, but this did not exist in the pure investment model where the analysis assumed fixed wealth when education shifts.

The commodity neutrality is irrelevant in the investment model in assessing the impact of education on health (H) but highly relevant in the determination of the effects of education on health under the pure consumption model.

Reference