ABSTRACT: Consciousness could be thought of as the problem to which propositions belong and concomitantly correspond as they indicate particular responses, signify instances of general solutions, with its essential configurations, rational representations confessional extrinsiness, interfacial interference, syncopated justices, heterogeneous variations testimonies, apodeictic knowledge of ideological turgidification, suspectful reality, sleaty sciolisms, tierated vaticinations, anchorite aperitif anomalous alienisms and manifest subjective acts of resolution. Consciousness in its organization of singular points, series and displacements, is doubly generative; it not only engenders the logical propositions with its determinate dimensions but also its correlates. The equivocality, ambiguity, in the synchronicity of the problem and proposition both in the sets and subsets of the ontological premises and logical boundaries, “error in perception” arises in the field of consciousness. Far from indicating the subjective and provisional state of empirical knowledge consciousness refers to an ideational objectivity or to a structure constitutive of space and time, the knowledge and the known, the proposition and its correlates. The question of “question” in consciousness does not bear any resemblance to the proposition which subsumes it, but rather it determines its own conditionality and representationality and assigns them to its constituents in various permutations and combinations, that are done with corporate signification, personalized manifestation, individual denotation and organizational individuation. Consciousness is only the shadow of the problem projected or rather constructed based on empirical propositions. It is the same ‘illusion’ which does not allow it to be reduced to any empirical thesis or antithesis for that matter. Retroactive movement of consciousness based on morphemes, semitones and relational openness leads to disintegration of external relations and dysfunctional fissures in the personality domains of resolvability are relativistic in the self determination of the consciousness problem. Consciousness makes signification as the condition of truth and proposition as the conditional truth; it is necessary that we should not vie the condition as the one who is conditioned, lest the biases of internationality and subject object conflict arise. Witness consciousness is the best answer to the problems that we face in science. Static genesis sets right the “aham brahasmi” (I AM Brahman) and “from Brahman we came” problem. Consciousness thus is neutral but never the double of the propositions which express it. “Events” have critical points like say liquids have, or water has. in all its pristine glory and primordial mortification consciousness is just “knowledge”, expressed in bytes, visual field capacity is also expressed. We make an explicit assumption that the storage is measured based on the number of bytes and that ASCII is used. Further assumption in gratification deprivation is that gratification increases in arithmetic progression, and deprivation in geometric progression. More you think, more you get angry. The still more you think you go mad. Repetitive actions and thoughts which are themselves actions are assumed to be recorded by a hypothetical “neuron DNA”. We thus record everything in the general ledger of the universe. And lo! The grand design simulated by someone, with people like us with Tamás, rajas (dynamism) and sattva (the transcendental form of Tamás and rajas) react. The height is the murder, mayhem calypso and cataclysm. the depth is “non reaction ability”. With this we state that this universe is a grand design simulated and we are really playing our roles to fit in a virtual drama.

INTRODUCTION:
We take in to considerations following parameters:

(1) Consciousness(just the amount of bytes recorded and visual representations measured by Information field capacity)
(2) Perception (What we see-It is said by many people like Kant and Indian Brihadyaranyaka Sutra that what you see is not what you see; what you do not see is not what you do not see; what you see is what you do not see and what you do not see is what you see –Here we assume that perception is what we see. And note in the Model we are making a case for the “augmented reality” or “dissipated reality” if the observer has “consciousness”, by which we mean what exactly is happening. If two crime syndicates are fighting each other, you may only see a terrible traffic and do not see anything else!)

(3) Gratification (we assume that it increases, the balance increases by arithmetic progression .The more you think, the same sentences form again and can be measured by ASCII numbers…Too much needless to say leads to paranoid schizophrenia. All actions are performed by people to achieve gratification or deprivation, that includes sadists and masochists)

(4) Deprivation(Balance here increases by GP ;again ASCII is used)

(5) Space

(6) Time

(7) Vacuum Energy

(8) Quantum Field

(9) Quantum Gravity

(10) Environmental Coherence

(11) Mass

(12) Energy

CONSCIOUSNESS AND PERCEPTION MODULE NUMBERED ONE
NOTATION:

\[ G_{13} \text{ : CATEGORY ONE OF PERCEPTION} \]
\[ G_{14} \text{ : CATEGORY TWO OF PERCEPTION} \]
\[ G_{15} \text{ : CATEGORY THREE OF PERCEPTION} \]
\[ T_{13} \text{ : CATEGORY ONE OF THE CONSCIOUSNESS} \]
\[ T_{14} \text{ : CATEGORY TWO OF THE CONSCIOUSNESS} \]
\[ T_{15} \text{ : CATEGORY THREE OF THE CONSCIOUSNESS} \]

SPACE AND TIME MODULE NUMBERED TWO:

\[ G_{16} \text{ : CATEGORY ONE OF TIME} \]
\[ G_{17} \text{ : CATEGORY TWO OF TIME} \]
\[ G_{18} \text{ : CATEGORY THREE OF TIME} \]
\[ T_{16} \text{ : CATEGORY ONE OF SPACE} \]
\[ T_{17} \text{ : CATEGORY TWO OF SPACE} \]
\[ T_{18} \text{ : CATEGORY THREE OF SPACE} \]

GRATIFICATION AND DEPRIVATION (MOSTLY UNCONSERVATIVE HOLISTICALLY AND INDIVIDUALLY! WORLD IS AN EXAMPLE) MODULE NUMBERED THREE:

\[ G_{20} \text{ : CATEGORY ONE OF DEPRIVATION} \]
\[ G_{21} \text{ : CATEGORY TWO OF DEPRIVATION} \]
\[ G_{22} \text{ : CATEGORY THREE OF DEPRIVATION} \]
\[ T_{20} \text{ : CATEGORY ONE OF GRATIFICATION} \]
\[ T_{21} \text{ : CATEGORY TWO OF GRATIFICATION} \]
\[ T_{22} \text{ : CATEGORY THREE OF GRATIFICATION} \]

MASS AND ENERGY: MODULE NUMBERED FOUR:

\[ G_{24} \text{ : CATEGORY ONE OF MATTER} \]
\[ G_{25} : \text{CATEGORY TWO OF MATTER} \]
\[ G_{26} : \text{CATEGORY THREE OF MATTER} \]
\[ T_{24} : \text{CATEGORY ONE OF ENERGY} \]
\[ T_{25} : \text{CATEGORY TWO OF ENERGY} \]
\[ T_{26} : \text{CATEGORY THREE OF ENERGY} \]

VACUUM ENERGY AND QUANTUM FIELD: MODULE NUMBERED FIVE:

\[ G_{28} : \text{CATEGORY ONE OF QUANTUM FIELD} \]
\[ G_{29} : \text{CATEGORY TWO OF QUANTUM FIELD} \]
\[ G_{30} : \text{CATEGORY THREE OF QUANTUM FIELD} \]
\[ T_{28} : \text{CATEGORY ONE OF VACUUM ENERGY} \]
\[ T_{29} : \text{CATEGORY TWO OF VACUUM ENERGY} \]
\[ T_{30} : \text{CATEGORY THREE OF VACUUM ENERGY} \]

ENVIRONMENTAL COHERENCE AND QUANTUM GRAVITY: MODULE NUMBERED SIX:

\[ G_{32} : \text{CATEGORY ONE OF ENVIRONMENTAL COHERENCE} \]
\[ G_{33} : \text{CATEGORY TWO OF ENVIRONMENTAL COHERENCE} \]
\[ G_{34} : \text{CATEGORY THREE OF ENVIRONMENTAL COHERENCE} \]
\[ T_{32} : \text{CATEGORY ONE OF QUANTUM GRAVITY} \]
\[ T_{33} : \text{CATEGORY TWO OF QUANTUM GRAVITY} \]
\[ T_{34} : \text{CATEGORY THREE OF QUANTUM GRAVITY} \]

\[
\begin{align*}
(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)} & , (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}, (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}, \\
(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)} & , \\
(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)} & , \\
(b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, \\
(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)} & 
\end{align*}
\]

are Accentuation coefficients

\[
\begin{align*}
(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)} & , (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}, \\
(a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)} & , \\
(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)} & , \\
(b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)} & , \\
(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)} & 
\end{align*}
\]
are Dissipation coefficients

The differential system of this model is now (Module Numbered one)

**CONSCIOUSNESS AND PERCEPTION MODULE NUMBERED ONE**

\[
\frac{dg_{13}}{dt} = (a_{13})^{(1)}g_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)]g_{13} \\
\frac{dg_{14}}{dt} = (a_{14})^{(1)}g_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)]g_{14} \\
\frac{dg_{15}}{dt} = (a_{15})^{(1)}g_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)]g_{15} \\
\frac{dt_{13}}{dt} = (b_{13})^{(3)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)]T_{13} \\
\frac{dt_{14}}{dt} = (b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)]T_{14} \\
\frac{dt_{15}}{dt} = (b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)]T_{15} \\
+(a'_{13})^{(1)}(T_{14}, t) = \text{First augmentation factor} \\
-(b''_{13})^{(1)}(G, t) = \text{First detritions factor}
\]

The differential system of this model is now (Module number two)

**SPACE AND TIME MODULE NUMBERED TWO**

\[
\frac{dg_{16}}{dt} = (a_{16})^{(2)}g_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)]g_{16} \\
\frac{dg_{17}}{dt} = (a_{17})^{(2)}g_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)]g_{17} \\
\frac{dg_{18}}{dt} = (a_{18})^{(2)}g_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)]g_{18} \\
\frac{dt_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}, t)]T_{16} \\
\frac{dt_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}, t)]T_{17} \\
\frac{dt_{18}}{dt} = (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}, t)]T_{18} \\
+(a'_{16})^{(2)}(T_{17}, t) = \text{First augmentation factor} \\
-(b''_{16})^{(2)}(G_{19}, t) = \text{First detritions factor}
\]

The differential system of this model is now (Module number three)

**GRATIFICATION AND DEPRIVATION(MOSTLY UNCONSERVATIVE HOLISTICALLY AND INDIVIDUALLY! WORLD IS AN EXAMPLE) MODULE NUMBERED THREE**

\[
\frac{dg_{20}}{dt} = (a_{20})^{(3)}g_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)]g_{20} \\
\frac{dg_{21}}{dt} = (a_{21})^{(3)}g_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)]g_{21} \\
\frac{dg_{22}}{dt} = (a_{22})^{(3)}g_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)]g_{22}
\]

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The differential system of this model is now (Module numbered Four)

\[
\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b_{20}^{(3)} - (b_{20}^{(3)})(G_{23}, t)]T_{20}
\]

\[
\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b_{21}^{(3)} - (b_{21}^{(3)})(G_{23}, t)]T_{21}
\]

\[
\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b_{22}^{(3)} - (b_{22}^{(3)})(G_{23}, t)]T_{22}
\]

\[
+(a_{20}^{(3)})(G_{21}, t) = \text{First augmentation factor}
\]

\[-(b_{20}^{(3)})(G_{23}, t) = \text{First detritions factor}
\]

**MASS AND ENERGY: MODULE NUMBERED FOUR:**

The differential system of this model is now (Module numbered Four)

\[
\frac{dg_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a_{24}^{(4)} + (a_{24}^{(4)})(G_{25}, t)]G_{24}
\]

\[
\frac{dg_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a_{25}^{(4)} + (a_{25}^{(4)})(G_{25}, t)]G_{25}
\]

\[
\frac{dg_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a_{26}^{(4)} + (a_{26}^{(4)})(G_{25}, t)]G_{26}
\]

\[
\frac{dt_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b_{24}^{(4)} - (b_{24}^{(4)})(G_{27}, t)]T_{24}
\]

\[
\frac{dt_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b_{25}^{(4)} - (b_{25}^{(4)})(G_{27}, t)]T_{25}
\]

\[
\frac{dt_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b_{26}^{(4)} - (b_{26}^{(4)})(G_{27}, t)]T_{26}
\]

\[
+(a_{25}^{(4)})(G_{27}, t) = \text{First augmentation factor}
\]

\[-(b_{25}^{(4)})(G_{27}, t) = \text{First detritions factor}
\]

The differential system of this model is now (Module number five)

**VACUUM ENERGY AND QUANTUM FIELD: MODULE NUMBERED FIVE**

\[
\frac{dg_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a_{28}^{(5)} + (a_{28}^{(5)})(G_{29}, t)]G_{28}
\]

\[
\frac{dg_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a_{29}^{(5)} + (a_{29}^{(5)})(G_{29}, t)]G_{29}
\]

\[
\frac{dg_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a_{30}^{(5)} + (a_{30}^{(5)})(G_{29}, t)]G_{30}
\]

\[
\frac{dt_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b_{28}^{(5)} - (b_{28}^{(5)})(G_{31}, t)]T_{28}
\]

\[
\frac{dt_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b_{29}^{(5)} - (b_{29}^{(5)})(G_{31}, t)]T_{29}
\]

\[
\frac{dt_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b_{30}^{(5)} - (b_{30}^{(5)})(G_{31}, t)]T_{30}
\]

\[
+(a_{28}^{(5)})(T_{29}, t) = \text{First augmentation factor}
\]
The differential system of this model is now (Module numbered Six)

ENVIRONMENTAL COHERENCE AND QUANTUM GRAVITY: MODULE NUMBERED SIX

\[ \frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{32} - \left[ (a'_{32})^{(6)} + (a''_{32})^{(6)} (T_{33}, t) \right] G_{32} \]

\[ \frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \left[ (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}, t) \right] G_{33} \]

\[ \frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \left[ (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}, t) \right] G_{34} \]

\[ \frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \left[ (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}, t) \right] T_{32} \]

\[ \frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \left[ (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}, t) \right] T_{33} \]

\[ \frac{dT_{34}}{dt} = (b_{34})^{(6)} T_{33} - \left[ (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}, t) \right] T_{34} \]

\[ + (a''_{35})^{(6)} (T_{33}, t) = \text{First augmentation factor} \]

\[ - (b''_{35})^{(6)} (G_{35}, t) = \text{First detritions factor} \]

HOLISTIC CONCATENATE SYSTEMAL EQUATIONS HENCEFORTH REFERRED TO AS “GLOBAL EQUATIONS”

CONSCIOUSNESS AND PERCEPTION MODULE NUMBERED ONE

SPACE AND TIME MODULE NUMBERED TWO

GRATIFICATION AND DEPRIVATION (MOSTLY UNCONSERVATIVE HOLISTICALLY AND INDIVIDUALLY! WORLD IS AN EXAMPLE) MODULE NUMBERED THREE

VACUUM ENERGY AND QUANTUM FIELD: MODULE NUMBERED FIVE

MASS AND ENERGY: MODULE NUMBERED FOUR

ENVIRONMENTAL COHERENCE AND QUANTUM GRAVITY: MODULE NUMBERED SIX
\[
\frac{dg_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix}
(a_i^{(1)}(T_{14}, t) + (a_{15})^{(1)}(T_{14}, t) & \ldots & (a_{18})^{(2, 2)}(T_{17}, t) + (a_{22})^{(3, 3)}(T_{21}, t) \\
+ (a_{20})^{(4, 4, 4)}(T_{25}, t) & \ldots & + (a_{20})^{(4, 4, 4)}(T_{25}, t) + (a_{20})^{(5, 5, 5, 5)}(T_{29}, t) + (a_{20})^{(6, 6, 6, 6)}(T_{33}, t)
\end{bmatrix} G_{15}
\]

Where \((a_{15})^{(1)}(T_{14}, t)\), \((a_{15})^{(1)}(T_{14}, t)\), and \((a_{15})^{(1)}(T_{14}, t)\) are first augmentation coefficients for category 1, 2 and 3.

\[
\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix}
(b_{14}^{(1)}(G, t) - (b_{14}^{(2, 2)}(G, t) & \ldots & (b_{14}^{(3, 3)}(G, t) \\
(b_{20}^{(4, 4, 4)}(G, t) - (b_{20}^{(5, 5, 5, 5)}(G, t) & \ldots & (b_{20}^{(6, 6, 6, 6)}(G, t)
\end{bmatrix} T_{13}
\]

\[
\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix}
(b_{14}^{(1)}(G, t) - (b_{14}^{(2, 2)}(G, t) & \ldots & (b_{14}^{(3, 3)}(G, t) \\
(b_{20}^{(4, 4, 4)}(G, t) - (b_{20}^{(5, 5, 5, 5)}(G, t) & \ldots & (b_{20}^{(6, 6, 6, 6)}(G, t)
\end{bmatrix} T_{14}
\]

\[
\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix}
(b_{15}^{(1)}(G, t) - (b_{15}^{(2, 2)}(G, t) & \ldots & (b_{15}^{(3, 3)}(G, t) \\
(b_{20}^{(4, 4, 4)}(G, t) - (b_{20}^{(5, 5, 5, 5)}(G, t) & \ldots & (b_{20}^{(6, 6, 6, 6)}(G, t)
\end{bmatrix} T_{15}
\]

Where \((b_{15}^{(2, 2)}(G, t)\), \((b_{15}^{(2, 2)}(G, t)\), and \((b_{15}^{(2, 2)}(G, t)\) are first detrition coefficients for category 1, 2 and 3.

\[
\frac{dg_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix}
(a_{16}^{(2)}(T_{17}, t) + (a_{16}^{(2)}(T_{17}, t) & \ldots & (a_{16}^{(2, 2)}(T_{17}, t) + (a_{16}^{(2, 2)}(T_{17}, t) \\
+ (a_{20}^{(4, 4, 4)}(T_{25}, t) & \ldots & + (a_{20}^{(5, 5, 5, 5)}(T_{29}, t) + (a_{20}^{(6, 6, 6, 6)}(T_{33}, t)
\end{bmatrix} G_{16}
\]

\[
\frac{dg_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix}
(a_{17}^{(2)}(T_{17}, t) + (a_{17}^{(2)}(T_{17}, t) & \ldots & (a_{17}^{(2, 2)}(T_{17}, t) + (a_{17}^{(2, 2)}(T_{17}, t) \\
+ (a_{20}^{(4, 4, 4)}(T_{25}, t) & \ldots & + (a_{20}^{(5, 5, 5, 5)}(T_{29}, t) + (a_{20}^{(6, 6, 6, 6)}(T_{33}, t)
\end{bmatrix} G_{17}
\]

\[
\frac{dg_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix}
(a_{18}^{(2)}(T_{17}, t) + (a_{18}^{(2)}(T_{17}, t) & \ldots & (a_{18}^{(2, 2)}(T_{17}, t) + (a_{18}^{(2, 2)}(T_{17}, t) \\
+ (a_{20}^{(4, 4, 4)}(T_{25}, t) & \ldots & + (a_{20}^{(5, 5, 5, 5)}(T_{29}, t) + (a_{20}^{(6, 6, 6, 6)}(T_{33}, t)
\end{bmatrix} G_{18}
\]
Where

\[ \frac{dG_1}{dt} = (a_{10})^{(3)}G_1 - \left[ (a_{20})^{(3)} + (a_{20})^{(2)}(T_{21}, t) + (a_{10})^{(2)}(T_{17}, t) + (a_{01})^{(2)}(T_{12}, t) \right] G_1 \]

\[ \frac{dG_2}{dt} = (a_{21})^{(3)}G_2 - \left[ (a_{21})^{(3)} + (a_{21})^{(2)}(T_{21}, t) + (a_{11})^{(2)}(T_{17}, t) + (a_{02})^{(2)}(T_{12}, t) \right] G_2 \]

\[ \frac{dG_3}{dt} = (a_{22})^{(3)}G_3 - \left[ (a_{22})^{(3)} + (a_{22})^{(2)}(T_{21}, t) + (a_{12})^{(2)}(T_{17}, t) + (a_{03})^{(2)}(T_{12}, t) \right] G_3 \]

\[ \frac{dG_4}{dt} = (a_{23})^{(3)}G_4 - \left[ (a_{23})^{(3)} + (a_{23})^{(2)}(T_{21}, t) + (a_{13})^{(2)}(T_{17}, t) + (a_{04})^{(2)}(T_{12}, t) \right] G_4 \]

\[ \frac{dG_5}{dt} = (a_{24})^{(3)}G_5 - \left[ (a_{24})^{(3)} + (a_{24})^{(2)}(T_{21}, t) + (a_{14})^{(2)}(T_{17}, t) + (a_{05})^{(2)}(T_{12}, t) \right] G_5 \]

\[ \frac{dG_6}{dt} = (a_{25})^{(3)}G_6 - \left[ (a_{25})^{(3)} + (a_{25})^{(2)}(T_{21}, t) + (a_{15})^{(2)}(T_{17}, t) + (a_{06})^{(2)}(T_{12}, t) \right] G_6 \]

\[ \frac{dG_7}{dt} = (a_{26})^{(3)}G_7 - \left[ (a_{26})^{(3)} + (a_{26})^{(2)}(T_{21}, t) + (a_{16})^{(2)}(T_{17}, t) + (a_{07})^{(2)}(T_{12}, t) \right] G_7 \]

\[ \frac{dG_8}{dt} = (a_{27})^{(3)}G_8 - \left[ (a_{27})^{(3)} + (a_{27})^{(2)}(T_{21}, t) + (a_{17})^{(2)}(T_{17}, t) + (a_{08})^{(2)}(T_{12}, t) \right] G_8 \]

\[ \frac{dG_9}{dt} = (a_{28})^{(3)}G_9 - \left[ (a_{28})^{(3)} + (a_{28})^{(2)}(T_{21}, t) + (a_{18})^{(2)}(T_{17}, t) + (a_{09})^{(2)}(T_{12}, t) \right] G_9 \]

\[ \frac{dG_{10}}{dt} = (a_{29})^{(3)}G_{10} - \left[ (a_{29})^{(3)} + (a_{29})^{(2)}(T_{21}, t) + (a_{19})^{(2)}(T_{17}, t) + (a_{010})^{(2)}(T_{12}, t) \right] G_{10} \]

\[ \frac{dG_{11}}{dt} = (a_{31})^{(3)}G_{11} - \left[ (a_{31})^{(3)} + (a_{31})^{(2)}(T_{21}, t) + (a_{21})^{(2)}(T_{17}, t) + (a_{11})^{(2)}(T_{12}, t) \right] G_{11} \]

\[ \frac{dG_{12}}{dt} = (a_{32})^{(3)}G_{12} - \left[ (a_{32})^{(3)} + (a_{32})^{(2)}(T_{21}, t) + (a_{22})^{(2)}(T_{17}, t) + (a_{12})^{(2)}(T_{12}, t) \right] G_{12} \]

\[ \frac{dG_{13}}{dt} = (a_{33})^{(3)}G_{13} - \left[ (a_{33})^{(3)} + (a_{33})^{(2)}(T_{21}, t) + (a_{23})^{(2)}(T_{17}, t) + (a_{13})^{(2)}(T_{12}, t) \right] G_{13} \]

\[ \frac{dG_{14}}{dt} = (a_{34})^{(3)}G_{14} - \left[ (a_{34})^{(3)} + (a_{34})^{(2)}(T_{21}, t) + (a_{24})^{(2)}(T_{17}, t) + (a_{14})^{(2)}(T_{12}, t) \right] G_{14} \]

\[ \frac{dG_{15}}{dt} = (a_{35})^{(3)}G_{15} - \left[ (a_{35})^{(3)} + (a_{35})^{(2)}(T_{21}, t) + (a_{25})^{(2)}(T_{17}, t) + (a_{15})^{(2)}(T_{12}, t) \right] G_{15} \]

\[ \frac{dG_{16}}{dt} = (a_{36})^{(3)}G_{16} - \left[ (a_{36})^{(3)} + (a_{36})^{(2)}(T_{21}, t) + (a_{26})^{(2)}(T_{17}, t) + (a_{16})^{(2)}(T_{12}, t) \right] G_{16} \]

\[ \frac{dG_{17}}{dt} = (a_{37})^{(3)}G_{17} - \left[ (a_{37})^{(3)} + (a_{37})^{(2)}(T_{21}, t) + (a_{27})^{(2)}(T_{17}, t) + (a_{17})^{(2)}(T_{12}, t) \right] G_{17} \]

\[ \frac{dG_{18}}{dt} = (a_{38})^{(3)}G_{18} - \left[ (a_{38})^{(3)} + (a_{38})^{(2)}(T_{21}, t) + (a_{28})^{(2)}(T_{17}, t) + (a_{18})^{(2)}(T_{12}, t) \right] G_{18} \]

\[ \frac{dG_{19}}{dt} = (a_{39})^{(3)}G_{19} - \left[ (a_{39})^{(3)} + (a_{39})^{(2)}(T_{21}, t) + (a_{29})^{(2)}(T_{17}, t) + (a_{19})^{(2)}(T_{12}, t) \right] G_{19} \]

\[ \frac{dG_{20}}{dt} = (a_{40})^{(3)}G_{20} - \left[ (a_{40})^{(3)} + (a_{40})^{(2)}(T_{21}, t) + (a_{30})^{(2)}(T_{17}, t) + (a_{20})^{(2)}(T_{12}, t) \right] G_{20} \]

\[ \frac{dG_{21}}{dt} = (a_{41})^{(3)}G_{21} - \left[ (a_{41})^{(3)} + (a_{41})^{(2)}(T_{21}, t) + (a_{31})^{(2)}(T_{17}, t) + (a_{21})^{(2)}(T_{12}, t) \right] G_{21} \]

\[ \frac{dG_{22}}{dt} = (a_{42})^{(3)}G_{22} - \left[ (a_{42})^{(3)} + (a_{42})^{(2)}(T_{21}, t) + (a_{32})^{(2)}(T_{17}, t) + (a_{22})^{(2)}(T_{12}, t) \right] G_{22} \]
\[ \frac{d^2T_{20}}{dt} = (b_{20})^3T_{21} - \begin{bmatrix}
(b_{21})^3 & -b_{21}^0 & 0 \\
-b_{21}^0 & (b_{22})^3 & 0 \\
0 & 0 & (b_{23})^3
\end{bmatrix} \begin{bmatrix}
G_{23}(t) \\
G_{31}(t) \\
G_{32}(t)
\end{bmatrix}
\]

\[ \frac{d^2T_{21}}{dt} = (b_{21})^3T_{22} - \begin{bmatrix}
(b_{22})^3 & -b_{22}^0 & 0 \\
-b_{22}^0 & (b_{23})^3 & 0 \\
0 & 0 & (b_{24})^3
\end{bmatrix} \begin{bmatrix}
G_{23}(t) \\
G_{31}(t) \\
G_{32}(t)
\end{bmatrix}
\]

\[ \frac{d^2T_{22}}{dt} = (b_{22})^3T_{21} - \begin{bmatrix}
(b_{23})^3 & -b_{23}^0 & 0 \\
-b_{23}^0 & (b_{24})^3 & 0 \\
0 & 0 & (b_{25})^3
\end{bmatrix} \begin{bmatrix}
G_{23}(t) \\
G_{31}(t) \\
G_{32}(t)
\end{bmatrix}
\]
Where \( \{a_{24}^{(s)}(T_{25}, t)\}, \{a_{25}^{(s)}(T_{25}, t)\}, \{a_{26}^{(s)}(T_{25}, t)\} \) are first augmentation coefficients for category 1, 2 and 3

\[ + (a_{24}^{(s)})^{(5,5)}(T_{29}, t), + (a_{25}^{(s)})^{(5,5)}(T_{29}, t), + (a_{26}^{(s)})^{(5,5)}(T_{29}, t) \]

are second augmentation coefficient for category 1, 2 and 3

\[ + (a_{24}^{(s)})^{(6,6)}(T_{35}, t), + (a_{25}^{(s)})^{(6,6)}(T_{35}, t), + (a_{26}^{(s)})^{(6,6)}(T_{35}, t) \]

are third augmentation coefficient for category 1, 2 and 3

\[ + (a_{14}^{(s)})^{(1,1,1)}(T_{14}, t), + (a_{14}^{(s)})^{(1,1,1)}(T_{14}, t), + (a_{14}^{(s)})^{(1,1,1)}(T_{14}, t) \]

are fourth augmentation coefficients for category 1, 2 and 3

\[ + (a_{14}^{(s)})^{(2,2,2)}(T_{19}, t), + (a_{14}^{(s)})^{(2,2,2)}(T_{19}, t), + (a_{14}^{(s)})^{(2,2,2)}(T_{19}, t) \]

are fifth augmentation coefficients for category 1, 2 and 3

\[ + (a_{21}^{(s)})^{(3,3,3)}(T_{21}, t), + (a_{21}^{(s)})^{(3,3,3)}(T_{21}, t), + (a_{21}^{(s)})^{(3,3,3)}(T_{21}, t) \]

are sixth augmentation coefficients for category 1, 2 and 3

\[
\frac{dT_{24}}{dt} = (b_{24}^{(s)})^{(4)}(T_{25}) - \begin{pmatrix}
(b_{24}^{(s)})^{(4)}(G_{27}, t) & -(b_{24}^{(s)})^{(5,5)}(G_{31}, t) & -(b_{24}^{(s)})^{(6,6)}(G_{35}, t) \\
-(b_{14}^{(s)})^{(1,1,1)}(G_{19}, t) & -(b_{20}^{(s)})^{(3,3,3)}(G_{23}, t) \\
-(b_{14}^{(s)})^{(2,2,2)}(G_{19}, t) & -(b_{20}^{(s)})^{(3,3,3)}(G_{23}, t) \\
\end{pmatrix} T_{24}
\]

\[
\frac{dT_{25}}{dt} = (b_{25}^{(s)})^{(4)}(T_{24}) - \begin{pmatrix}
(b_{25}^{(s)})^{(4)}(G_{27}, t) & -(b_{25}^{(s)})^{(5,5)}(G_{31}, t) & -(b_{25}^{(s)})^{(6,6)}(G_{35}, t) \\
-(b_{14}^{(s)})^{(1,1,1)}(G_{19}, t) & -(b_{20}^{(s)})^{(3,3,3)}(G_{23}, t) \\
-(b_{14}^{(s)})^{(2,2,2)}(G_{19}, t) & -(b_{20}^{(s)})^{(3,3,3)}(G_{23}, t) \\
\end{pmatrix} T_{25}
\]

\[
\frac{dT_{26}}{dt} = (b_{26}^{(s)})^{(4)}(T_{25}) - \begin{pmatrix}
(b_{26}^{(s)})^{(4)}(G_{27}, t) & -(b_{26}^{(s)})^{(5,5)}(G_{31}, t) & -(b_{26}^{(s)})^{(6,6)}(G_{35}, t) \\
-(b_{14}^{(s)})^{(1,1,1)}(G_{19}, t) & -(b_{20}^{(s)})^{(3,3,3)}(G_{23}, t) \\
-(b_{14}^{(s)})^{(2,2,2)}(G_{19}, t) & -(b_{20}^{(s)})^{(3,3,3)}(G_{23}, t) \\
\end{pmatrix} T_{26}
\]

Where \(- (b_{24}^{(s)})^{(4)}(G_{27}, t), -(b_{25}^{(s)})^{(4)}(G_{27}, t), -(b_{26}^{(s)})^{(4)}(G_{27}, t)\) are first detritus coefficients for category 1, 2 and 3

\[- (b_{24}^{(s)})^{(5,5)}(G_{27}, t), -(b_{25}^{(s)})^{(5,5)}(G_{27}, t), -(b_{26}^{(s)})^{(5,5)}(G_{27}, t)\]

are second detritus coefficients for category 1, 2 and 3

\[- (b_{24}^{(s)})^{(6,6)}(G_{27}, t), -(b_{25}^{(s)})^{(6,6)}(G_{27}, t), -(b_{26}^{(s)})^{(6,6)}(G_{27}, t)\]

are third detritus coefficients for category 1, 2 and 3

\[- (b_{14}^{(s)})^{(1,1,1)}(G_{19}, t), -(b_{14}^{(s)})^{(1,1,1)}(G_{19}, t), -(b_{14}^{(s)})^{(1,1,1)}(G_{19}, t)\]

are fourth detritus coefficients for category 1, 2 and 3

\[- (b_{14}^{(s)})^{(2,2,2)}(G_{19}, t), -(b_{14}^{(s)})^{(2,2,2)}(G_{19}, t), -(b_{14}^{(s)})^{(2,2,2)}(G_{19}, t)\]

are fifth detritus coefficients for category 1, 2 and 3

\[- (b_{20}^{(s)})^{(3,3,3)}(G_{23}, t), -(b_{21}^{(s)})^{(3,3,3)}(G_{23}, t), -(b_{22}^{(s)})^{(3,3,3)}(G_{23}, t)\]

are sixth detritus coefficients for category 1, 2 and 3

\[
\frac{dG_{28}}{dt} = (a_{28}^{(s)})^{(5)} G_{29} - \begin{pmatrix}
(a_{28}^{(s)})^{(5)}(T_{29}, t) & +(a_{28}^{(s)})^{(4,4)}(T_{25}, t) & +(a_{28}^{(s)})^{(6,6)}(T_{33}, t) \\
+(a_{18}^{(s)})^{(1,1,1,1,1)}(T_{14}, t) & +(a_{20}^{(s)})^{(3,3,3,3,3)}(T_{21}, t) \\
\end{pmatrix} G_{28}
\]

\[
\frac{dG_{29}}{dt} = (a_{29}^{(s)})^{(5)} G_{28} - \begin{pmatrix}
(a_{29}^{(s)})^{(5)}(T_{29}, t) & +(a_{29}^{(s)})^{(4,4)}(T_{25}, t) & +(a_{29}^{(s)})^{(6,6,6)}(T_{33}, t) \\
+(a_{18}^{(s)})^{(1,1,1,1,1)}(T_{14}, t) & +(a_{20}^{(s)})^{(3,3,3,3,3)}(T_{21}, t) \\
\end{pmatrix} G_{29}
\]

\[
\frac{dG_{30}}{dt} = (a_{30}^{(s)})^{(5)} G_{29} - \begin{pmatrix}
(a_{30}^{(s)})^{(5)}(T_{29}, t) & +(a_{30}^{(s)})^{(4,4)}(T_{25}, t) & +(a_{30}^{(s)})^{(6,6,6)}(T_{33}, t) \\
+(a_{18}^{(s)})^{(1,1,1,1,1)}(T_{14}, t) & +(a_{20}^{(s)})^{(3,3,3,3,3)}(T_{21}, t) \\
\end{pmatrix} G_{30}
\]

Where \( (a_{28}^{(s)})^{(5)}(T_{29}, t), (a_{29}^{(s)})^{(5)}(T_{29}, t), (a_{30}^{(s)})^{(5)}(T_{29}, t) \) are first augmentation coefficients for category 1, 2 and 3

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And \( + (a_{24}^{(4,4,4)}(T_{25}, t)) + (a_{24}^{(4,4,1)}(T_{25}, t)) + (a_{24}^{(4,4,3)}(T_{25}, t)) \) are second augmentation coefficients for category 1, 2 and 3.

\( + (a_{20}^{(6,6,6)}(T_{33}, t)) + (a_{20}^{(6,6,1)}(T_{33}, t)) + (a_{20}^{(6,6,3)}(T_{33}, t)) \) are third augmentation coefficients for category 1, 2 and 3.

\( + (a_{16}^{(1,1,1,1)}(T_{16}, t)) + (a_{16}^{(1,1,1,1)}(T_{16}, t)) + (a_{16}^{(1,1,1,1)}(T_{16}, t)) \) are fourth augmentation coefficients for category 1, 2, and 3.

\( + (a_{12}^{(2,2,2,2)}(T_{12}, t)) + (a_{12}^{(2,2,2,2)}(T_{12}, t)) + (a_{12}^{(2,2,2,2)}(T_{12}, t)) \) are fifth augmentation coefficients for category 1, 2, and 3.

\( + (a_{8}^{(3,3,3,3)}(T_{21}, t)) + (a_{8}^{(3,3,3,3)}(T_{21}, t)) + (a_{8}^{(3,3,3,3)}(T_{21}, t)) \) are sixth augmentation coefficients for category 1, 2, and 3.

\[
\frac{d}{dt} \left[ T_{28} \right] = \left( b_{28}^{(5)}(5) \right) T_{28} - \frac{\left( b_{28}^{(3)}(5) (G_{31}, t) \right)}{T_{28}} - \frac{\left( b_{28}^{(4,4)}(G_{27}, t) \right)}{T_{28}} - \frac{\left( b_{28}^{(6,6,6)}(G_{35}, t) \right)}{T_{28}} - \frac{\left( b_{31}^{(1,1,1,1)}(G, t) \right)}{T_{28}} - \frac{\left( b_{31}^{(2,2,2,2)}(G_{19}, t) \right)}{T_{28}} - \frac{\left( b_{31}^{(3,3,3,3)}(G_{23}, t) \right)}{T_{28}}
\]

where \( (b_{28}^{(5)}(5)(G_{31}, t)) \), \( (b_{28}^{(3)}(5)(G_{31}, t)) \), \( (b_{28}^{(4,4)}(G_{27}, t)) \) are first detrition coefficients for category 1, 2 and 3.

\( (b_{28}^{(6,6,6)}(G_{35}, t)) \), \( (b_{28}^{(6,6,1)}(G_{35}, t)) \), \( (b_{28}^{(6,6,3)}(G_{35}, t)) \) are second detritus coefficients for category 1, 2 and 3.

\( (b_{28}^{(1,1,1,1)}(G, t)) \), \( (b_{28}^{(1,1,1,1)}(G, t)) \), \( (b_{28}^{(1,1,1,1)}(G, t)) \) are third detritus coefficients for category 1, 2, and 3.

\( (b_{28}^{(2,2,2,2)}(G_{19}, t)) \), \( (b_{28}^{(2,2,2,2)}(G_{19}, t)) \), \( (b_{28}^{(2,2,2,2)}(G_{19}, t)) \) are fourth detritus coefficients for category 1, 2, and 3.

\( (b_{28}^{(3,3,3,3)}(G_{23}, t)) \), \( (b_{28}^{(3,3,3,3)}(G_{23}, t)) \), \( (b_{28}^{(3,3,3,3)}(G_{23}, t)) \) are fifth detritus coefficients for category 1, 2, and 3.

\( (b_{28}^{(4,4,4)}(G_{27}, t)) \), \( (b_{28}^{(4,4,4)}(G_{27}, t)) \), \( (b_{28}^{(4,4,4)}(G_{27}, t)) \) are sixth detritus coefficients for category 1, 2, and 3.

\[
\frac{d}{dt} \left[ G_{32} \right] = \left( a_{32}^{(6)} \right) G_{33} - \frac{\left( a_{32}^{(3)}(6)(T_{33}, t) + (a_{32}^{(6)}(6)(T_{33}, t) + (a_{32}^{(5,5,5)}(T_{25}, t)) + (a_{32}^{(4,4,4)}(T_{25}, t)) \right)}{G_{32}} - \frac{\left( a_{32}^{(1,1,1,1,1)}(T_{14}, t) + (a_{32}^{(2,2,2,2,2)}(T_{17}, t) + (a_{32}^{(3,3,3,3,3)}(T_{21}, t) \right)}{G_{32}}
\]

\[
\frac{d}{dt} \left[ G_{33} \right] = \left( a_{33}^{(6)} \right) G_{32} - \frac{\left( a_{33}^{(3)}(6)(T_{33}, t) + (a_{33}^{(6)}(6)(T_{33}, t) + (a_{33}^{(5,5,5)}(T_{25}, t)) + (a_{33}^{(4,4,4)}(T_{25}, t)) \right)}{G_{33}} - \frac{\left( a_{33}^{(1,1,1,1,1)}(T_{14}, t) + (a_{33}^{(2,2,2,2,2)}(T_{17}, t) + (a_{33}^{(3,3,3,3,3)}(T_{21}, t) \right)}{G_{33}}
\]

\[
\frac{d}{dt} \left[ G_{34} \right] = \left( a_{34}^{(6)} \right) G_{33} - \frac{\left( a_{34}^{(3)}(6)(T_{33}, t) + (a_{34}^{(6)}(6)(T_{33}, t) + (a_{34}^{(5,5,5)}(T_{25}, t)) + (a_{34}^{(4,4,4)}(T_{25}, t)) \right)}{G_{34}} - \frac{\left( a_{34}^{(1,1,1,1,1)}(T_{14}, t) + (a_{34}^{(2,2,2,2,2)}(T_{17}, t) + (a_{34}^{(3,3,3,3,3)}(T_{21}, t) \right)}{G_{34}}
\]

\( + (a_{32}^{(4,4,4)}(T_{25}, t)) + (a_{32}^{(4,4,4)}(T_{25}, t)) + (a_{32}^{(4,4,4)}(T_{25}, t)) \) are first augmentation coefficients for category 1, 2 and 3.

\( + (a_{33}^{(5,5,5)}(T_{25}, t)) + (a_{33}^{(5,5,5)}(T_{25}, t)) + (a_{33}^{(5,5,5)}(T_{25}, t)) \) are second augmentation coefficients for category 1, 2 and 3.

\( + (a_{34}^{(4,4,4)}(T_{25}, t)) + (a_{34}^{(4,4,4)}(T_{25}, t)) + (a_{34}^{(4,4,4)}(T_{25}, t)) \) are third augmentation coefficients for category 1, 2 and 3.
$$dT_{32} \over dt = (b_{32})^{(6)}T_{32} - \begin{bmatrix} (b'_{12})^{(6)} & -(b''_{12})^{(6)}(G_{32}, t) & -(b''_{29})^{(5,5,5)}(G_{31}, t) & -(b''_{28})^{(4,4,4)}(G_{27}, t) \\ -(b'_{13})^{(1,1,1,1,1)}(G, t) & -(b'_{14})^{(2,2,2,2,2)}(G_{19}, t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23}, t) \end{bmatrix} T_{32}$$

$$dT_{33} \over dt = (b_{33})^{(6)}T_{33} - \begin{bmatrix} (b'_{13})^{(6)} & -(b''_{13})^{(6)}(G_{32}, t) & -(b''_{29})^{(5,5,5)}(G_{31}, t) & -(b''_{28})^{(4,4,4)}(G_{27}, t) \\ -(b'_{14})^{(1,1,1,1,1)}(G, t) & -(b'_{15})^{(2,2,2,2,2)}(G_{19}, t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23}, t) \end{bmatrix} T_{33}$$

$$dT_{34} \over dt = (b_{34})^{(6)}T_{34} - \begin{bmatrix} (b'_{14})^{(6)} & -(b''_{14})^{(6)}(G_{32}, t) & -(b''_{29})^{(5,5,5)}(G_{31}, t) & -(b''_{28})^{(4,4,4)}(G_{27}, t) \\ -(b'_{15})^{(1,1,1,1,1)}(G, t) & -(b'_{16})^{(2,2,2,2,2)}(G_{19}, t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23}, t) \end{bmatrix} T_{34}$$

Where we suppose

(A) \( (a_i)^{(1)}, (a_j)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b_j)^{(1)}, (b''_i)^{(1)} > 0, i, j = 13,14,15 \)

(B) The functions \( (a''_i)^{(1)}, (b''_i)^{(1)} \) are positive continuous increasing and bounded.

**Definition of** \( (p_j)^{(1)}, (r_j)^{(1)} \):

\( (a_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (A_{13})^{(1)}(T_{14}, t) \)

\( (b_i)^{(1)}(G, t) \leq (r_i)^{(1)} \leq (B_{13})^{(1)}(G, t) \)

\( \lim_{T_{32} \to 0} (a''_i)^{(1)}(T_{14}, t) = (p_i)^{(1)} \)

\( \lim_{G \to 0} (b''_i)^{(1)}(G, t) = (r_i)^{(1)} \)

**Definition of** \( (A_{13})^{(1)}, (B_{13})^{(1)} \):

Where \( (A_{13})^{(1)}, (B_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)} \) are positive constants and \( i = 13,14,15 \)
They satisfy Lipschitz condition:

\[
|a'''(1)(T_{14}, t) - a'''(1)(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T'_{14}|e^{-\hat{\alpha}_{13}^{(1)}t}
\]

\[
|b'''(1)(G', t) - b'''(1)(G, t)| \leq (\hat{k}_{13}^{(2)})||G - G'||e^{-\hat{\alpha}_{13}^{(2)}t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions

\[
(a''(1)(T_{14}, t) \text{ and } a''(1)(T_{14}, t)) \text{ and } (T_{14}^{'} t) \text{ and } (T_{14}^{'} t)
\]

are points belonging to the interval

\[
[(\hat{k}_{13}^{(1)}, (M_{13})^{(1)})]
\]

It is to be noted that \((a''(1)||T_{14}, t||\) is uniformly continuous. In the eventuality of the fact, that if \((M_{13})^{(1)} = 1\) then the function \((a''(1)||T_{14}, t||\), the first augmentation coefficient \(WOULD\) be absolutely continuous.

**Definition of \((\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}\):**

\[\frac{(a_i^{(1)})}{(\hat{M}_{13})^{(1)}} < 1\]

**Definition of \((\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}\):**

\[
\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i^{(1)} + (A_{13})^{(1)} + (\hat{P}^{(1)}_{13} (\hat{k}_{13})^{(1)})] < 1
\]

\[
\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i^{(1)} + (B_{13})^{(1)} + (\hat{Q}^{(1)}_{13} (\hat{k}_{13})^{(1)})] < 1
\]

Where we suppose

\[(a_{i}^{(2)}, (a_{i}^{(2)}, (a_{i}^{(2)}), (b_{i}^{(2)}), (b_{i}^{(2)}), (b_{i}^{(2)})) > 0, \ i, j = 16, 17, 18\]

\[The \ functions \ (a_i^{(2)}), (b_i^{(2)}) \ are \ positive \ continuous \ increasing \ and \ bounded.\]

**Definition of \((p_{i})^{(2)}, (r_{i})^{(2)}\):**

\[\frac{(a_i^{(2)(T_{17}, t))}}{(p_i)^{(2)}} \leq (A_{16})^{(2)}\]

\[\frac{(b_i^{(2)(G_{19}, t))}}{(r_i)^{(2)}} \leq (B_{16})^{(2)}\]

\[\lim_{t \to \infty}(a_i^{(2)(T_{17}, t))} = (p_i)^{(2)}\]

\[\lim_{t \to \infty}(b_i^{(2)(G_{19}, t))} = (r_i)^{(2)}\]

**Definition of \((A_{16})^{(2)}, (B_{16})^{(2)}\):**

Where \((A_{16})^{(2)}, (B_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}\) are positive constants and \(i = 16, 17, 18\)

They satisfy Lipschitz condition:

\[
|a''(2)(T_{17}, t) - (a''(2)(T_{17}, t))| \leq (\hat{k}_{16})^{(2)}|T_{17} - T'_{17}|e^{-\hat{\alpha}_{16}^{(2)}t}
\]

\[
|b''(2)((G_{19}), t) - (b''(2)((G_{19}), t))| \leq (\hat{k}_{16}^{(2)})||G_{19} - (G_{19})||e^{-\hat{\alpha}_{16}^{(2)}t}
\]
With the Lipschitz condition, we place a restriction on the behavior of functions \((a''_i(t), T_{i2}, t)\) and \((a''_i(t), T_{i2}, t)\). \((T'_{i2}, t)\) and \((T_{i2}, t)\) are points belonging to the interval \([\hat{k}_{i16}^{(2)}, \hat{M}_{i16}^{(2)}]\). It is to be noted that \((a''_i(t), T_{i2}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{i16}^{(2)}) = 1\) then the function \((a''_i(t), T_{i2}, t)\), the SECOND augmentation coefficient would be absolutely continuous.

**Definition of \((\hat{M}_{i16}^{(2)}), (\hat{k}_{i16}^{(2)})\):**

\[
\begin{align*}
(\hat{M}_{i16}^{(2)}, (\hat{k}_{i16}^{(2)}), \text{ are positive constants} \\
\frac{(a_{i1})^{(2)}}{(\hat{M}_{i16}^{(2)})}, \frac{(b_{i1})^{(2)}}{(\hat{M}_{i16}^{(2)})} < 1
\end{align*}
\]

**Definition of \((\hat{p}_{i13}^{(2)}), (\hat{q}_{i13}^{(2)})\):**

There exists two constants \((\hat{p}_{i16}^{(2)}), (\hat{q}_{i16}^{(2)})\) which together with \((\hat{M}_{i16}^{(2)}), (\hat{k}_{i16}^{(2)}), (\hat{A}_{i16}^{(2)}), \text{and} (\hat{B}_{i16}^{(2)})\) and the constants

\[
\begin{align*}
(a_i^{(2)}), (a''_i^{(2)}), (b_i^{(2)}), (b''_i^{(2)}), (p_i^{(2)}), (r_i^{(2)}), i = 16, 17,18,
\end{align*}
\]

satisfy the inequalities

\[
\begin{align*}
\frac{1}{(\hat{M}_{i16}^{(2)})^2}[ (a_i^{(2)}), (a''_i^{(2)}), (\hat{A}_{i16}^{(2)}), (\hat{B}_{i16}^{(2)}), (\hat{k}_{i16}^{(2)}) ] < 1 \\
\frac{1}{(\hat{M}_{i16}^{(2)})^2}[ (b_i^{(2)}), (b''_i^{(2)}), (\hat{B}_{i16}^{(2)}), (\hat{Q}_{i16}^{(2)}), (\hat{k}_{i16}^{(2)}) ] < 1
\end{align*}
\]

Where we suppose

\[
\begin{align*}
(a_i^{(3)}), (a''_i^{(3)}), (b_i^{(3)}), (b''_i^{(3)}), (p_i^{(3)}), (r_i^{(3)}), i = 20, 21,22
\end{align*}
\]

The functions \((a''_i^{(3)}), (b''_i^{(3)})\) are positive continuous increasing and bounded.

**Definition of \((p_i^{(3)}), (r_i^{(3)})\):**

\[
\begin{align*}
(a_i^{(3)}), (T_{i21}, t) \leq (p_i^{(3)}) \leq (\hat{A}_{i20}^{(3)}) & \\
(b_i^{(3)}), (G_{i23}, t) \leq (r_i^{(3)}) \leq (\hat{B}_{i20}^{(3)})
\end{align*}
\]

\[
\begin{align*}
\lim_{T_{i21} \to +\infty}(a''_i^{(3)}), (T_{i21}, t) = (p_i^{(3)})
\end{align*}
\]

\[
\begin{align*}
\lim_{G_{i23} \to +\infty}(b''_i^{(3)}), (G_{i23}, t) = (r_i^{(3)})
\end{align*}
\]

**Definition of \((\hat{A}_{i20}^{(3)}), (\hat{B}_{i20}^{(3)}):**

Where \((\hat{A}_{i20}^{(3)}), (\hat{B}_{i20}^{(3)}), (p_i^{(3)}), (r_i^{(3)})\) are positive constants and \(i = 20, 21,22\)

They satisfy Lipschitz condition:

\[
\begin{align*}
\left|(a''_i^{(3)}), (T_{i21}, t) - (a''_i^{(3)}), (T_{i21}, t)\right| \leq (\hat{k}_{i20}^{(3)})|T_{i21} - T_{i2}'|e^{-(\hat{A}_{i20}^{(3)})t}
\end{align*}
\]

\[
\begin{align*}
\left|(b''_i^{(3)}), (G_{i23}, t) - (b''_i^{(3)}), (G_{i23}, t)\right| < (\hat{k}_{i20}^{(3)})|G_{i23} - G_{i23}'|e^{-(\hat{B}_{i20}^{(3)})t}
\end{align*}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a''_i^{(3)}), (T_{i21}, t)\) and \((a''_i^{(3)}), (T_{i21}, t)\). \((T_{i21}, t)\) and \((T_{i21}, t)\) are points belonging to the interval \([\hat{k}_{i20}^{(3)}, (\hat{M}_{i20}^{(3)})]\). It is to be noted that \((a''_i^{(3)}), (T_{i21}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{i20}^{(3)}) = 1\) then the function \((a''_i^{(3)}), (T_{i21}, t)\), the THIRD augmentation coefficient, would be absolutely continuous.

**Definition of \((\hat{M}_{i20}^{(3)}), (\hat{k}_{i20}^{(3)})\):**


(K) \( (\bar{M}_{20})^{(3)}, (\bar{K}_{20})^{(3)} \), are positive constants
\[
\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}} < \frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} \leq 1
\]

There exists two constants \( \bar{P}_{20}^{(3)}, \bar{Q}_{20}^{(3)} \), which together with \( (\bar{M}_{20})^{(3)}, (\bar{K}_{20})^{(3)} \) and \( (\bar{M}_{20})^{(3)} \) and the constants \( (a_i)^{(3)}, (a_i^r)^{(3)}, (b_i)^{(3)}, (b_i^r)^{(3)}, (p_i)^{(3)}, (r_i)^{(3)} \), \( i = 20, 21, 22 \), satisfy the inequalities
\[
\frac{1}{(\bar{M}_{20})^{(3)}} [(a_i)^{(3)} + (a_i^r)^{(3)} + (\bar{A}_{20})^{(3)} + (\bar{P}_{20})^{(3)} (\bar{K}_{20})^{(3)}] < 1
\]
\[
\frac{1}{(\bar{M}_{20})^{(3)}} [(b_i)^{(3)} + (b_i^r)^{(3)} + (\bar{B}_{20})^{(3)} + (\bar{Q}_{20})^{(3)} (\bar{K}_{20})^{(3)}] < 1
\]

Where we suppose
\[
(a_i)^{(4)}, (a_i^r)^{(4)}, (a_i''^r)^{(4)}, (b_i)^{(4)}, (b_i^r)^{(4)}, (b_i''^r)^{(4)} > 0, \quad i, j = 24, 25, 26
\]

(M) The functions \( (a_i''^r)^{(4)}, (b_i''^r)^{(4)} \) are positive continuous increasing and bounded.

**Definition of** \( (p_i)^{(4)}, (r_i)^{(4)} \):
\[
(a_i''^r)^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\bar{A}_{24})^{(4)}
\]
\[
(b_i''^r)^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i^r)^{(4)} \leq (\bar{B}_{24})^{(4)}
\]

\[N\]
\[
\lim_{T_{25} \to 0} (a_i''^r)^{(4)}(T_{25}, t) = (p_i)^{(4)}
\]
\[
\lim_{G_{27} \to 0} (b_i''^r)^{(4)}((G_{27}), t) = (r_i)^{(4)}
\]

**Definition of** \( (\bar{A}_{24})^{(4)}, (\bar{B}_{24})^{(4)} \):
\[
(\bar{A}_{24})^{(4)}, (\bar{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}
\]

They satisfy Lipschitz condition:
\[
|(a_i''^r)^{(4)}(T_{25}, t) - (a_i''^r)^{(4)}(T_{25}, t)| \leq (\bar{K}_{24})^{(4)}|T_{25} - T_{25}'| e^{-(\bar{A}_{24})^{(4)}} t
\]
\[
|(b_i''^r)^{(4)}((G_{27}), t) - (b_i''^r)^{(4)}((G_{27}), t)| < (\bar{K}_{24})^{(4)} ||(G_{27}) - (G_{27})'|| e^{-(\bar{A}_{24})^{(4)} t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \( (a_i''^r)^{(4)}(T_{25}, t) \) and\( (a_i''^r)^{(4)}(T_{25}, t), (T_{25}, t) \) and \( (T_{25}, t) \) are points belonging to the interval \( [(\bar{K}_{24})^{(4)}, (\bar{M}_{24})^{(4)}] \). It is to be noted that \( (a_i''^r)^{(4)}(T_{25}, t) \) is uniformly continuous. In the eventuality of the fact, that if \( (\bar{M}_{24})^{(4)} = 4 \) then the function \( (a_i''^r)^{(4)}(T_{25}, t), \) the FOURTH augmentation coefficient WOULD be absolutely continuous.

**Definition of** \( (\bar{M}_{24})^{(4)}, (\bar{K}_{24})^{(4)} \):
\[
(\bar{M}_{24})^{(4)}, (\bar{K}_{24})^{(4)}, \text{ are positive constants}
\]
\[
\frac{(a_i)^{(4)}}{(\bar{M}_{24})^{(4)}} \leq \frac{(b_i)^{(4)}}{(\bar{M}_{24})^{(4)}} < 1
\]
Definition of \( \hat{P}_{24}^{(4)}, \hat{Q}_{24}^{(4)} \):

(Q) There exists two constants \( \hat{P}_{24}^{(4)} \) and \( \hat{Q}_{24}^{(4)} \) which together with \( \hat{M}_{24}^{(5)} \), \( \hat{k}_{24}^{(4)} \), \( \hat{A}_{24}^{(4)} \) and \( \hat{B}_{24}^{(4)} \) and the constants \( a_{ij}^{(4)}, a_{ij}^{(5)}, b_{ij}^{(5)}, b_{ij}^{(5)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24, 25, 26 \), satisfy the inequalities

\[
\frac{1}{(\hat{M}_{24}^{(5)})^2} \left[ (a_{ij}^{(4)})^2 + (a_{ij}^{(5)})^2 + (\hat{A}_{24}^{(4)})^2 + (\hat{P}_{24}^{(4)})^2 (\hat{k}_{24}^{(4)})^2 \right] < 1
\]

\[
\frac{1}{(\hat{M}_{24}^{(5)})^2} \left[ (b_{ij}^{(4)})^2 + (b_{ij}^{(5)})^2 + (\hat{B}_{24}^{(4)})^2 + (\hat{Q}_{24}^{(4)})^2 (\hat{k}_{24}^{(4)})^2 \right] < 1
\]

Where we suppose

\( a_{ij}^{(5)}, a_{ij}^{(5)}, a_{ij}^{(5)}, b_{ij}^{(5)}, b_{ij}^{(5)}, b_{ij}^{(5)}, \) are positive constants \( i, j = 28, 29, 30 \)

(S) The functions \( (a_{ij}^{(5)}), (b_{ij}^{(5)}) \) are positive continuous increasing and bounded.

**Definition of** \( (p_i)^{(5)}, (r_i)^{(5)} \):

\[
(a_{ij}^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28}^{(5)})
\]

\[
(b_{ij}^{(5)}(G_{31}, t) \leq (r_i)^{(5)} \leq (\hat{B}_{28}^{(5)})
\]

\( (T) \)

\[
\lim_{T_{29} \to 0}(a_{ij}^{(5)})(T_{29}, t) = (p_i)^{(5)}
\]

\[
\lim_{G_{31} \to 0}(b_{ij}^{(5)})(G_{31}, t) = (r_i)^{(5)}
\]

Definition of \( \hat{A}_{28}^{(5)}, \hat{B}_{28}^{(5)} \):

Where \( \hat{A}_{28}^{(5)}, \hat{B}_{28}^{(5)}, (p_i)^{(5)}, (r_i)^{(5)} \) are positive constants and \( t = 28, 29, 30 \)

They satisfy Lipschitz condition:

\[
|(a_{ij}^{(5)}(T_{29}, t) - (a_{ij}^{(5)}(T_{29}, t)) \leq (\hat{k}_{28}^{(5)})|T_{29} - T_{29}^*|e^{-(\hat{A}_{28}^{(5)})t}
\]

\[
|(b_{ij}^{(5)}(G_{31}, t) - (b_{ij}^{(5)}(G_{31}, t)) \leq (\hat{k}_{28}^{(5)})|G_{31} - G_{31}^*|e^{-(\hat{B}_{28}^{(5)})t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \( (a_{ij}^{(5)}(T_{29}, T)), (b_{ij}^{(5)}(G_{31}, T)) \) and \( b_{ij}^{(5)}(T_{29}, T), (G_{31}, T) \) and \( (T_{29}, T) \) and \( (G_{31}, T) \) are points belonging to the interval \( [\hat{k}_{28}^{(5)}, \hat{M}_{28}^{(5)}] \). It is to be noted that \( (a_{ij}^{(5)}(T_{29}, T) \) is uniformly continuous. In the eventuality of the fact, that if \( \hat{M}_{28}^{(5)} = 5 \) then the function \( (a_{ij}^{(5)}(T_{29}, T) \), the FIFTH augmentation coefficient attributable would be absolutely continuous.

**Definition of** \( \hat{M}_{28}^{(5)}, \hat{k}_{28}^{(5)} \):

\( \hat{M}_{28}^{(5)}, \hat{k}_{28}^{(5)} \) are positive constants

\[
\frac{1}{(\hat{M}_{28}^{(5)})^2} \left[ (a_{ij}^{(5)})^2 + (b_{ij}^{(5)})^2 \right] < 1
\]

**Definition of** \( \hat{P}_{28}^{(5)}, \hat{Q}_{28}^{(5)} \):

There exists two constants \( \hat{P}_{28}^{(5)} \) and \( \hat{Q}_{28}^{(5)} \) which together with \( \hat{M}_{28}^{(5)}, \hat{k}_{28}^{(5)}, \hat{A}_{28}^{(5)} \) and \( \hat{B}_{28}^{(5)} \) and the constants \( a_{ij}^{(5)}, a_{ij}^{(5)}, b_{ij}^{(5)}, b_{ij}^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28, 29, 30 \), satisfy the inequalities
\[
\frac{1}{(M_{28})^2} \left[ (a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{K}_{28})^{(5)} \right] < 1
\]

\[
\frac{1}{(M_{28})^2} \left[ (b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{K}_{28})^{(5)} \right] < 1
\]

Where we suppose

\[(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32, 33, 34\]

(W) The functions \((a_i'')^{(6)}, (b_i'')^{(6)}\) are positive continuous increasing and bounded.

**Definition of \((p_i)^{(6)}, (r_i)^{(6)}\):**

\[(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}\]

\[(b_i'')^{(6)}((G_{33}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}\]

\[(X) \quad \lim_{T_{2} \to 0} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}\]

\[\lim_{G \to 0} (b_i'')^{(6)} ((G_{33}), t) = (r_i)^{(6)}\]

**Definition of \((\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}\):**

Where \([\hat{A}_{32}]^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}\] are positive constants and \(i = 32, 33, 34\)

They satisfy Lipschitz condition:

\[| (a_i'')^{(6)}(T_{33}, t) - (a_i'')^{(6)}(T_{33}, t') | \leq (\hat{A}_{32})^{(6)} | T_{33} - T_{33}' | e^{- (\hat{A}_{32})^{(6)} t} \]

\[| (b_i'')^{(6)}((G_{33}), t) - (b_i'')^{(6)}((G_{33}), t') | \leq (\hat{B}_{32})^{(6)} | (G_{33}) - (G_{33})' | e^{- (\hat{B}_{32})^{(6)} t} \]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i'')^{(6)}(T_{33}, t)\) and \((a_i')^{(6)}(T_{33}, t)\). \((T_{33}, t)\) and \((T_{33}, t)\) are points belonging to the interval \([\hat{A}_{32}]^{(6)}, (\hat{B}_{32})^{(6)}\]. It is to be noted that \((a_i')^{(6)}(T_{33}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((M_{32})^{(6)} = 6\) then the function \((a_i'')^{(6)}(T_{33}, t)\), the SIXTH augmentation coefficient would be absolutely continuous.

**Definition of \((M_{32})^{(6)}, (\hat{A}_{32})^{(6)}\):**

\[(M_{32})^{(6)}, (\hat{A}_{32})^{(6)}\] are positive constants

\[\frac{(a_i)^{(6)}}{(M_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{A}_{32})^{(6)}} < 1\]

**Definition of \((\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}\):**

There exist two constants \((\hat{P}_{32})^{(6)}\) and \((\hat{Q}_{32})^{(6)}\) which together with \((\hat{M}_{32})^{(6)}, (\hat{A}_{32})^{(6)}\) and \((\hat{B}_{32})^{(6)}\) and the constants \((a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}\), \(i = 32, 33, 34\), satisfy the inequalities

\[\frac{1}{(M_{32})^{(6)}} \left[ (a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{K}_{32})^{(6)} \right] < 1\]

\[\frac{1}{(M_{32})^{(6)}} \left[ (b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{K}_{32})^{(6)} \right] < 1\]
**Theorem 1:** if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

Definition of \( G_i(0), T_i(0) \):

\[
G_i(t) \leq \left( \hat{p}_{13} \right)^{(1)} e^{(\hat{M}_{13})^{(1)} t} , \quad G_i(0) = G^0_i > 0
\]

\[
T_i(t) \leq \left( \hat{Q}_{13} \right)^{(1)} e^{(\hat{M}_{13})^{(1)} t} , \quad T_i(0) = T^0_i > 0
\]

Definition of \( G_i(0), T_i(0) \):

\[
G_i(t) \leq \left( \hat{p}_{16} \right)^{(2)} e^{(\hat{M}_{16})^{(2)} t} , \quad G_i(0) = G^0_i > 0
\]

\[
T_i(t) \leq \left( \hat{Q}_{16} \right)^{(2)} e^{(\hat{M}_{16})^{(2)} t} , \quad T_i(0) = T^0_i > 0
\]

Definition of \( G_i(0), T_i(0) \):

\[
G_i(t) \leq \left( \hat{p}_{20} \right)^{(3)} e^{(\hat{M}_{20})^{(3)} t} , \quad G_i(0) = G^0_i > 0
\]

\[
T_i(t) \leq \left( \hat{Q}_{20} \right)^{(3)} e^{(\hat{M}_{20})^{(3)} t} , \quad T_i(0) = T^0_i > 0
\]

Definition of \( G_i(0), T_i(0) \):

\[
G_i(t) \leq \left( \hat{p}_{24} \right)^{(4)} e^{(\hat{M}_{24})^{(4)} t} , \quad G_i(0) = G^0_i > 0
\]

\[
T_i(t) \leq \left( \hat{Q}_{24} \right)^{(4)} e^{(\hat{M}_{24})^{(4)} t} , \quad T_i(0) = T^0_i > 0
\]

Definition of \( G_i(0), T_i(0) \):

\[
G_i(t) \leq \left( \hat{p}_{28} \right)^{(5)} e^{(\hat{M}_{28})^{(5)} t} , \quad G_i(0) = G^0_i > 0
\]

\[
T_i(t) \leq \left( \hat{Q}_{28} \right)^{(5)} e^{(\hat{M}_{28})^{(5)} t} , \quad T_i(0) = T^0_i > 0
\]

Definition of \( G_i(0), T_i(0) \):

\[
G_i(t) \leq \left( \hat{p}_{32} \right)^{(6)} e^{(\hat{M}_{32})^{(6)} t} , \quad G_i(0) = G^0_i > 0
\]

\[
T_i(t) \leq \left( \hat{Q}_{32} \right)^{(6)} e^{(\hat{M}_{32})^{(6)} t} , \quad T_i(0) = T^0_i > 0
\]

**Proof:** Consider operator \( \mathcal{A}^{(1)} \) defined on the space of sextuples of continuous functions \( G_i, T_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[
G_i(0) = G^0_i , \quad T_i(0) = T^0_i , \quad G^0_i \leq \left( \hat{p}_{13} \right)^{(1)} , \quad T^0_i \leq \left( \hat{Q}_{13} \right)^{(1)} ,
\]

\[
0 \leq G_i(t) - G^0_i \leq \left( \hat{p}_{13} \right)^{(1)} e^{(\hat{M}_{13})^{(1)} t}
\]
where

\[ 0 \leq T_i(t) - T_i^0 \leq (\bar{Q}_{13})^{(1)}e^{(\bar{R}_{13})^{(1)}t} \]

By

\[ \bar{g}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)}G_{14}(s_{13}) - \left( (a'_{13})^{(1)} + a''_{13}^{(1)}(T_{14}(s_{13}), s_{13}) \right) G_{13}(s_{13}) \right] ds_{13} \]

\[ \bar{g}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)}G_{13}(s_{13}) - \left( (a'_{14})^{(1)} + a''_{14}^{(1)}(T_{14}(s_{13}), s_{13}) \right) G_{14}(s_{13}) \right] ds_{13} \]

\[ \bar{g}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)}G_{14}(s_{13}) - \left( (a'_{15})^{(1)} + a''_{15}^{(1)}(T_{14}(s_{13}), s_{13}) \right) G_{15}(s_{13}) \right] ds_{13} \]

\[ \bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)}T_{14}(s_{13}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)}G(s_{13}), s_{13} \right) T_{13}(s_{13}) \right] ds_{13} \]

\[ \bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)}T_{13}(s_{13}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)}G(s_{13}), s_{13} \right) T_{14}(s_{13}) \right] ds_{13} \]

\[ \bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)}T_{14}(s_{13}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)}G(s_{13}), s_{13} \right) T_{15}(s_{13}) \right] ds_{13} \]

Where \( s_{13} \) is the integrand that is integrated over an interval \((0, t)\)

**Proof:**

Consider operator \( \mathcal{A}^{(2)} \) defined on the space of sextuples of continuous functions \( G_t, T_t : \mathbb{R}_+ \to \mathbb{R}_+ \) which satisfy

\[ G_t(0) = G_t^0, \quad T_t(0) = T_t^0, \quad G_t^0 \leq (\bar{P}_{16})^{(2)}, \quad T_t^0 \leq (\bar{Q}_{16})^{(2)} \]

\[ 0 \leq G_t(t) - G_t^0 \leq (\bar{P}_{16})^{(2)}e^{(\bar{R}_{16})^{(2)}t} \]

\[ 0 \leq T_t(t) - T_t^0 \leq (\bar{Q}_{16})^{(2)}e^{(\bar{R}_{16})^{(2)}t} \]

By

\[ \bar{g}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)}G_{17}(s_{16}) - \left( (a'_{16})^{(2)} + a''_{16}^{(2)}(T_{17}(s_{16}), s_{16}) \right) G_{16}(s_{16}) \right] ds_{16} \]

\[ \bar{g}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)}G_{16}(s_{16}) - \left( (a'_{17})^{(2)} + a''_{17}^{(2)}(T_{16}(s_{16}), s_{16}) \right) G_{17}(s_{16}) \right] ds_{16} \]

\[ \bar{g}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)}G_{17}(s_{16}) - \left( (a'_{18})^{(2)} + a''_{18}^{(2)}(T_{17}(s_{16}), s_{16}) \right) G_{18}(s_{16}) \right] ds_{16} \]

\[ \bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[ (b_{16})^{(2)}T_{17}(s_{16}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)}G(s_{16}), s_{16} \right) T_{16}(s_{16}) \right] ds_{16} \]

\[ \bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[ (b_{17})^{(2)}T_{16}(s_{16}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)}G(s_{16}), s_{16} \right) T_{17}(s_{16}) \right] ds_{16} \]

\[ \bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[ (b_{18})^{(2)}T_{17}(s_{16}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)}G(s_{16}), s_{16} \right) T_{18}(s_{16}) \right] ds_{16} \]

Where \( s_{16} \) is the integrand that is integrated over an interval \((0, t)\)

**Proof:**

Consider operator \( \mathcal{A}^{(3)} \) defined on the space of sextuples of continuous functions \( G_t, T_t : \mathbb{R}_+ \to \mathbb{R}_+ \) which satisfy

\[ 64 \]
\[ G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\tilde{P}_{20})^{(3)}, T_i^0 \leq (\tilde{Q}_{20})^{(3)}, \]

\[ 0 \leq G_i(t) - G_i^0 \leq (\tilde{P}_{20})^{(3)}e^{(\theta_{20})^{(3)}t} \]

\[ 0 \leq T_i(t) - T_i^0 \leq (\tilde{Q}_{20})^{(3)}e^{(\theta_{20})^{(3)}t} \]

By

\[ \tilde{G}_{20}(t) = G_{20}^0 + \int_0^t \left[ (a_{20}^{(3)}G_{21}(s_{20}^0) - (a_{20}^{(3)} + a_{20}^{(3)}(T_{21}(s_{20}^0), s_{20}^0)) G_{20}(s_{20}) \right] ds_{20} \]

\[ \tilde{G}_{21}(t) = G_{21}^0 + \int_0^t \left[ (a_{21}'^{(3)}G_{20}(s_{20}^0) - (a_{21}'^{(3)} + a_{21}'^{(3)}(T_{21}(s_{20}^0), s_{20}) G_{21}(s_{20}) \right] ds_{20} \]

\[ \tilde{G}_{22}(t) = G_{22}^0 + \int_0^t \left[ (a_{22}'^{(3)}G_{21}(s_{20}^0) - (a_{22}'^{(3)} + a_{22}'^{(3)}(T_{21}(s_{20}^0), s_{20}) G_{22}(s_{20}) \right] ds_{20} \]

\[ \tilde{T}_{20}(t) = T_{20}^0 + \int_0^t \left[ (b_{20}')^{(3)}T_{21}(s_{20}^0) - ((b_{20}')^{(3)}(G(s_{20}^0), s_{20}^0)) T_{20}(s_{20}) \right] ds_{20} \]

\[ \tilde{T}_{21}(t) = T_{21}^0 + \int_0^t \left[ (b_{21}')^{(3)}T_{20}(s_{20}^0) - ((b_{21}')^{(3)}(G(s_{20}^0), s_{20}) T_{21}(s_{20}) \right] ds_{20} \]

\[ \tilde{T}_{22}(t) = T_{22}^0 + \int_0^t \left[ (b_{22}')^{(3)}T_{21}(s_{20}^0) - ((b_{22}')^{(3)}(G(s_{20}^0), s_{20}) T_{22}(s_{20}) \right] ds_{20} \]

Where \( s_{20}(\cdot) \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( \mathcal{A}^{(4)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\tilde{P}_{24})^{(4)}, T_i^0 \leq (\tilde{Q}_{24})^{(4)}, \]

\[ 0 \leq G_i(t) - G_i^0 \leq (\tilde{P}_{24})^{(4)}e^{(\theta_{24})^{(4)}t} \]

\[ 0 \leq T_i(t) - T_i^0 \leq (\tilde{Q}_{24})^{(4)}e^{(\theta_{24})^{(4)}t} \]

By

\[ \tilde{G}_{24}(t) = G_{24}^0 + \int_0^t \left[ (a_{24}^{(4)}G_{25}(s_{24}^0) - (a_{24}^{(4)} + a_{24}^{(4)}(T_{25}(s_{24}^0), s_{24}^0)) G_{24}(s_{24}) \right] ds_{24} \]

\[ \tilde{G}_{25}(t) = G_{25}^0 + \int_0^t \left[ (a_{25}^{(4)}G_{24}(s_{24}^0) - (a_{25}^{(4)} + a_{25}^{(4)}(T_{25}(s_{24}^0), s_{24}^0)) G_{25}(s_{24}) \right] ds_{24} \]

\[ \tilde{G}_{26}(t) = G_{26}^0 + \int_0^t \left[ (a_{26}^{(4)}G_{25}(s_{24}^0) - (a_{26}^{(4)} + a_{26}^{(4)}(T_{25}(s_{24}^0), s_{24}^0)) G_{26}(s_{24}) \right] ds_{24} \]

\[ \tilde{T}_{24}(t) = T_{24}^0 + \int_0^t \left[ (b_{24}'^{(4)}T_{25}(s_{24}^0) - ((b_{24}')^{(4)}(G(s_{24}^0), s_{24}^0)) T_{24}(s_{24}) \right] ds_{24} \]

\[ \tilde{T}_{25}(t) = T_{25}^0 + \int_0^t \left[ (b_{25}'^{(4)}T_{24}(s_{24}^0) - ((b_{25}')^{(4)}(G(s_{24}^0), s_{24})) T_{25}(s_{24}) \right] ds_{24} \]

\[ \tilde{T}_{26}(t) = T_{26}^0 + \int_0^t \left[ (b_{26}'^{(4)}T_{25}(s_{24}^0) - ((b_{26}')^{(4)}(G(s_{24}^0), s_{24})) T_{26}(s_{24}) \right] ds_{24} \]

Where \( s_{24}(\cdot) \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( \mathcal{A}^{(5)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy
\[ G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\hat{P}_{28})^{(5)}, \quad T_i^0 \leq (\hat{Q}_{28})^{(5)} , \]

\[ 0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\theta_{28})^{(5)} t} \]

\[ 0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\theta_{28})^{(5)} t} \]

By

\[ \tilde{G}_{29}(t) = G_{32}^0 + \int_0^t \left[ (a_{29})^{(5)} G_{29}(s_{28}) - (a_{29}^0)^{(5)} + a_{29}^{(5)} (T_{29}(s_{28}), s_{28}) \right] dS_{28} \]

\[ \tilde{G}_{30}(t) = G_{33}^0 + \int_0^t \left[ (a_{30})^{(5)} G_{29}(s_{28}) - (a_{30}^0)^{(5)} + a_{30}^{(5)} (T_{29}(s_{28}), s_{28}) \right] dS_{28} \]

Where \( s_{28} \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( \mathcal{A}^{(6)} \) defined on the space of sextuples of continuous functions \( G_i, T_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\hat{P}_{32})^{(6)}, \quad T_i^0 \leq (\hat{Q}_{32})^{(6)} , \]

\[ 0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\theta_{32})^{(6)} t} \]

\[ 0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\theta_{32})^{(6)} t} \]

By

\[ \tilde{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{32}) - (a_{32}^0)^{(6)} + a_{32}^{(6)} (T_{33}(s_{32}), s_{32}) \right] dS_{32} \]

\[ \tilde{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{32}(s_{32}) - (a_{32}^0)^{(6)} + a_{32}^{(6)} (T_{32}(s_{32}), s_{32}) \right] dS_{32} \]

\[ \tilde{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{32}) - (a_{34}^0)^{(6)} + a_{34}^{(6)} (T_{33}(s_{32}), s_{32}) \right] dS_{32} \]

Where \( s_{32} \) is the integrand that is integrated over an interval \((0, t)\)
(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} \left( G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{Q}_{13})^{(1)} t} \right) \right] \ dS_{(13)} =$$

$$\left( 1 + (a_{13})^{(1)} t \right) G_{14}^0 + \frac{(a_{13})^{(1)} (P_{13})^{(1)}}{(Q_{13})^{(1)}} \left( e^{(Q_{13})^{(1)} t} - 1 \right)$$

From which it follows that

$$(G_{13}(t) - G_{13}^0) e^{- (Q_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(Q_{13})^{(1)}} \left( (P_{13})^{(1)} + G_{14}^0 e^{\frac{(P_{13})^{(1)} + G_{14}^0}{(Q_{13})^{(1)}} - (P_{13})^{(1)}} \right)$$

$(G_{13}^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

(b) The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} \left( G_{17}^0 + (\hat{P}_{16})^{(2)} e^{(\hat{Q}_{16})^{(2)} t} \right) \right] \ dS_{(16)} =$$

$$\left( 1 + (a_{16})^{(2)} t \right) G_{17}^0 + \frac{(a_{16})^{(2)} (P_{16})^{(2)}}{(Q_{16})^{(2)}} \left( e^{(Q_{16})^{(2)} t} - 1 \right)$$

From which it follows that

$$(G_{16}(t) - G_{16}^0) e^{- (Q_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(Q_{16})^{(2)}} \left( (P_{16})^{(2)} + G_{17}^0 e^{\frac{(P_{16})^{(2)} + G_{17}^0}{(Q_{16})^{(2)}} - (P_{16})^{(2)}} \right)$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

(a) The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} \left( G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{Q}_{20})^{(3)} t} \right) \right] \ dS_{(20)} =$$

$$\left( 1 + (a_{20})^{(3)} t \right) G_{21}^0 + \frac{(a_{20})^{(3)} (P_{20})^{(3)}}{(Q_{20})^{(3)}} \left( e^{(Q_{20})^{(3)} t} - 1 \right)$$

From which it follows that

$$(G_{20}(t) - G_{20}^0) e^{- (Q_{20})^{(3)} t} \leq \frac{(a_{20})^{(3)}}{(Q_{20})^{(3)}} \left( (P_{20})^{(3)} + G_{21}^0 e^{\frac{(P_{20})^{(3)} + G_{21}^0}{(Q_{20})^{(3)}} - (P_{20})^{(3)}} \right)$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

(b) The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{24}(t) \leq G_{24}^0 + \int_0^t \left[ (a_{24})^{(4)} \left( G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{Q}_{24})^{(4)} t} \right) \right] \ dS_{(24)} =$$

$$\left( 1 + (a_{24})^{(4)} t \right) G_{25}^0 + \frac{(a_{24})^{(4)} (P_{24})^{(4)}}{(Q_{24})^{(4)}} \left( e^{(Q_{24})^{(4)} t} - 1 \right)$$

From which it follows that
\[(G_{24}(t) - G_{24}^0)e^{-((\tilde{M}_{24}(6))t)} \leq \frac{a_{24}(4)}{(\tilde{M}_{24}(4))}
\left\{((\tilde{P}_{24}(4)) + G_{25}^0)e\left(-\frac{(P_{24}(4) + G_{25}^0)}{\tilde{G}_{25}}\right) + (\tilde{P}_{24}(4))\right\}\]

\[(G_{25}^0)\] is as defined in the statement of theorem 1

(c) The operator \(\mathcal{A}^{(5)}\) maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

\[G_{28}(t) \leq G_{28}^0 + \int_0^t \left[\frac{a_{28}(5)}{(\tilde{M}_{28}(5))(\tilde{P}_{28}(5))}e(\tilde{M}_{28}(5)(s-28))\right] dS_{(28)} =
\]

\[\left(1 + (a_{28}(5)t)G_{29}^0 + \frac{(a_{28}(5)(\tilde{P}_{28})^5)}{(\tilde{M}_{28}(5))}\left(e(\tilde{M}_{28}(5)t - 1)\right)\]

From which it follows that

\[\begin{align*}
(G_{28}(t) - G_{28}^0)e^{-((\tilde{M}_{28}(5))t)} & \leq \frac{a_{28}(5)}{(\tilde{M}_{28}(5))}\left\{((\tilde{P}_{28}(5)) + G_{29}^0)e\left(-\frac{(P_{28}(5) + G_{29}^0)}{\tilde{G}_{29}}\right) + (\tilde{P}_{28}(5))\right\} \\
\end{align*}\]

\[(G_{25}^0)\] is as defined in the statement of theorem 1

(d) The operator \(\mathcal{A}^{(6)}\) maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

\[G_{32}(t) \leq G_{32}^0 + \int_0^t \left[\frac{a_{32}(6)}{(\tilde{M}_{32}(6))(\tilde{P}_{32}(6))}e(\tilde{M}_{32}(6)(s-32))\right] dS_{(32)} =
\]

\[\left(1 + (a_{32}(6)t)G_{33}^0 + \frac{(a_{32}(6)(\tilde{P}_{32})^6)}{(\tilde{M}_{32}(6))}\left(e(\tilde{M}_{32}(6)t - 1)\right)\]

From which it follows that

\[\begin{align*}
(G_{32}(t) - G_{32}^0)e^{-((\tilde{M}_{32}(6))t)} & \leq \frac{a_{32}(6)}{(\tilde{M}_{32}(6))}\left\{((\tilde{P}_{32}(6)) + G_{33}^0)e\left(-\frac{(P_{32}(6) + G_{33}^0)}{\tilde{G}_{33}}\right) + (\tilde{P}_{32}(6))\right\} \\
\end{align*}\]

\[(G_{11}^0)\] is as defined in the statement of theorem 6

Analogous inequalities hold also for \(G_{25}, G_{26}, T_{24}, T_{25}, T_{26}\)

It is now sufficient to take \(\frac{(a_{14}(1)}{(M_{13}(1)}\), \(\frac{(b_{14}(1)}{(M_{13}(1)}\) < 1 and to choose \(\tilde{P}_{13}(1)\) and \(\tilde{Q}_{13}(1)\) large to have

\[\begin{align*}
\frac{(a_{14}(1)}{(M_{13}(1)}\left[\tilde{P}_{13}(1) + ((\tilde{P}_{13}(1) + G_0^0)e\left(-\frac{(P_{13}(1) + G_0^0)}{\tilde{T}_j}\right)\right] & \leq (\tilde{P}_{13}(1)) \\
\frac{(b_{14}(1)}{(M_{13}(1)}\left[((\tilde{Q}_{13}(1) + T_0)e\left(-\frac{(\tilde{Q}_{13}(1) + T_0]}{\tilde{T}_j}\right) + (\tilde{Q}_{13}(1)\right] & \leq (\tilde{Q}_{13}(1)) \\
\end{align*}\]
In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i,T_i$ satisfying GLOBAL EQUATIONS into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d\left((G^{(1)},T^{(1)}),(G^{(2)},T^{(2)})\right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}^+} |G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-\left\langle R_{13}\right\rangle t} \max_{t \in \mathbb{R}^+} |T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-\left\langle R_{13}\right\rangle t} \}$$

Indeed if we denote

**Definition of $\tilde{G}, \tilde{T}$**:

$$(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G,T)$$

It results

$$|\tilde{G}^{(1)}_{13} - \tilde{G}^{(2)}_{13}| \leq \int_{0}^{t} (a_{13}^{(1)}) |G^{(1)}_{14} - G^{(2)}_{14}| e^{-\left\langle R_{13}\right\rangle s_{13}} e^{\left\langle R_{13}\right\rangle s_{13}} ds_{13} +$$

$$\int_{0}^{t} (a_{13}^{(1)}) |G^{(1)}_{13} - G^{(2)}_{13}| e^{-\left\langle R_{13}\right\rangle s_{13}} e^{\left\langle R_{13}\right\rangle s_{13}} +$$

$$(a_{13}^{(1)})(T^{(1)}_{14},s_{13})|G^{(1)}_{13} - G^{(2)}_{13}| e^{-\left\langle R_{13}\right\rangle s_{13}} e^{\left\langle R_{13}\right\rangle s_{13}} +$$

$$G^{(2)}_{13}(a_{13}^{(1)}(T^{(1)}_{14},s_{13}) - (a_{13}^{(1)})(T^{(2)}_{14},s_{13})| e^{-\left\langle R_{13}\right\rangle s_{13}} e^{\left\langle R_{13}\right\rangle s_{13}} d s_{13}$$

Where $s_{13}$ represents integrand that is integrated over the interval $[0,t]$.

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-\left\langle R_{13}\right\rangle t} \leq$$

$$\frac{1}{\left\langle R_{13}\right\rangle} \left( (a_{13}^{(1)}) + (a_{13}^{(1)}) + (\bar{A}_{13})^{(1)} + (\bar{P}_{13})^{(1)}(\bar{K}_{13})^{(1)} \right) d\left((G^{(1)},T^{(1)}; G^{(2)},T^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows

**Remark 1**: The fact that we supposed $(a_{13}^{(1)})$ and $(b_{13}^{(1)})$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\bar{P}_{13})^{(1)} e^{\left\langle R_{13}\right\rangle t}$ and $(\bar{Q}_{13})^{(1)} e^{\left\langle R_{13}\right\rangle t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_{13}^{(1)})$ and $(b_{13}^{(1)})$, $i = 13,14,15$ depend only on $T_{14}$ and respectively on $G$ (and not on $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2**: There does not exist any $t$ where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_{i0} \left| e^{-\left\langle R_{13}\right\rangle t} \int_{0}^{t} (a_{13}^{(1)} - (a_{13}^{(1)}) \int_{0}^{s} (s_{13}) ds_{13}) \right| \geq 0$$

$$T_i(t) \geq T_{i0} e^{-\left\langle R_{13}\right\rangle t} > 0 \quad \text{for} \quad t > 0$$

**Definition of** $(\bar{M}_{13})^{(1)}$, $(\bar{M}_{13})^{(1)}$, and $(\bar{M}_{13})^{(1)}$

**Remark 3**: If $G_{13}$ is bounded, the same property have also $G_{14}$ and $G_{15}$. Indeed if
\[ G_{13} < (M_{13})^{(1)} \) it follows \[ \frac{dG_{14}}{dt} \leq (M_{13})^{(1)} + (a_{14})^{(1)}G_{14} \] and by integrating

\[ G_{14} \leq (M_{13})^{(1)} + 2(a_{14})^{(1)}(M_{13})^{(1)}f(a_{14})^{(1)} \]

In the same way, one can obtain

\[ G_{15} \leq (M_{13})^{(1)} + 2(a_{15})^{(1)}(M_{13})^{(1)}f(a_{15})^{(1)} \]

If \[ G_{14} \] or \[ G_{15} \] is bounded, the same property follows for \[ G_{13} \], \[ G_{15} \] and \[ G_{13}, G_{14} \] respectively.

**Remark 4:** If \[ G_{13} \] is bounded, from below, the same property holds for \[ G_{14} \] and \[ G_{15} \]. The proof is analogous with the preceding one. An analogous property is true if \[ G_{14} \] is bounded from below.

**Remark 5:** If \[ T_{13} \] is bounded from below and \[ \lim_{t \to \infty} ((b_{14}'(t)) (G(t), t)) = (b_{14}')^{(1)} \] then \[ T_{14} \to \infty. \]

**Definition of** \((m)^{(1)}\) and \(\varepsilon_1\):

Indeed let \( t_1 \) be so that for \( t > t_1 \)

\[ (b_{14})^{(1)} - (b_{14}'(t)) (G(t), t) < \varepsilon_1, T_{13} (t) > (m)^{(1)} \]

Then \[ \frac{df_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14} \] which leads to

\[ T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_{14}^{(1)} e^{-\varepsilon_1 t} \]

If we take \( t \) such that \( e^{-\varepsilon_1 t} = \frac{1}{2} \) it results

\[ T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_1} \]

By taking now \( \varepsilon_1 \) sufficiently small one sees that \( T_{14} \) is unbounded.

The same property holds for \( T_{15} \) if \[ \lim_{t \to \infty} ((b_{15}'(t)) (G(t), t)) = (b_{15}')^{(1)} \]

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take \[ \frac{(a_{16})^{(2)}}{(M_{16})^{(2)}}, \quad \frac{(b_{16})^{(2)}}{(M_{16})^{(2)}} < 1 \] and to choose

\((\hat{P}_{16})^{(2)}\) and \((\hat{Q}_{16})^{(2)}\) large to have

\[ \frac{(a_{16})^{(2)}}{(M_{16})^{(2)}} \left( (\hat{P}_{16})^{(2)} + (\hat{P}_{16})^{(2)} + G_j (\hat{P}_{16})^{(2)} e^{-\left( (\hat{Q}_{16})^{(2)} + \tau_j \right)} \right) \leq (\hat{P}_{16})^{(2)} \]

\[ \frac{(b_{16})^{(2)}}{(M_{16})^{(2)}} \left( (\hat{Q}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} e^{-\left( (\hat{Q}_{16})^{(2)} + \tau_j \right)} \right) \leq (\hat{Q}_{16})^{(2)} \]

In order that the operator \( \mathcal{A}^{(2)} \) transforms the space of sextuples of functions \( G_i, T_i \) satisfying

The operator \( \mathcal{A}^{(2)} \) is a contraction with respect to the metric

\[ d((G_{13})^{(1)}, (T_{13})^{(1)}), ((G_{13})^{(2)}, (T_{13})^{(2)})) \]

\[ \sup_{t \in \mathbb{R}^+} [G_i^{(1)}(t) - G_i^{(2)}(t)] e^{-(M_{16})^{(2)} t}, \max_{t \in \mathbb{R}^+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(M_{16})^{(2)} t} \]

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Indeed if we denote

**Definition of** $\hat{G}_{19}, \hat{T}_{19}$ : $(\hat{G}_{19}, \hat{T}_{19}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results

$$
\left| \hat{G}_{19}^{(1)} - \hat{G}_{19}^{(2)} \right| \leq \int_0^T (a_{16})^{(2)} \left[ (\hat{G}_{17}^{(1)} - \hat{G}_{17}^{(2)}) e^{-((\hat{R}_{16})^{(2)}\hat{s}_{16})} e^{((\hat{R}_{16})^{(2)}\hat{s}_{16})} ds_{16} + \\
\int_0^T \left[ (a_{16}^{(2)} - \hat{a}_{16}^{(2)}) \hat{G}_{16}^{(1)} - \hat{G}_{16}^{(2)} \right] e^{-((\hat{R}_{16})^{(2)}\hat{s}_{16})} e^{((\hat{R}_{16})^{(2)}\hat{s}_{16})} + \\
(a_{16}^{(2)} - \hat{a}_{16}^{(2)}) \left[ \left( T_{17}^{(1)} \hat{s}_{16} \right) - \hat{a}_{16}^{(2)} \left( T_{17}^{(2)} \hat{s}_{16} \right) \right] e^{-((\hat{R}_{16})^{(2)}\hat{s}_{16})} e^{((\hat{R}_{16})^{(2)}\hat{s}_{16})} ds_{16} \right)
$$

Where $s_{16}$ represents integrand that is integrated over the interval $[0, t]$.

From the hypotheses it follows

$$
\frac{1}{(M_{16})^{(2)}} \left[ \left( a_{16}^{(2)} + (a_{16}^{(2)} - \hat{a}_{16}^{(2)}) \left( \hat{R}_{16}^{(2)} \hat{s}_{16} \right) \right) d \left( (G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)} \right) \right] 
$$

And analogous inequalities for $G_{19}$ and $T_{19}$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a_{16}^{(2)} - \hat{a}_{16}^{(2)})$ and $(b_{16}^{(2)} - \hat{b}_{16}^{(2)})$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{16}^{(2)} e^{(\hat{R}_{16}^{(2)} t)}$, and $(\hat{Q}_{16}^{(2)} e^{(\hat{R}_{16}^{(2)} t)})$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_{16}^{(2)} - \hat{a}_{16}^{(2)})$ and $(b_{16}^{(2)} - \hat{b}_{16}^{(2)})$, $i = 16, 17, 18$ depend only on $T_{17}$ and respectively on $(G_{19})$ (and not on $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_{i}(t) = 0$ and $T_{i}(t) = 0$.

From 19 to 24 it results

$$
G_{i}(t) \geq \left| e^{-\int_0^t \left( (a_{16}^{(2)} - \hat{a}_{16}^{(2)}) (T_{17}^{(1)} \hat{s}_{16} - T_{17}^{(2)} \hat{s}_{16}) ds_{16} \right) } \right| \geq 0 \\
T_{i}(t) \geq T_{i}^{0} e^{- \left( b_{i}^{(2)} t \right) } > 0 \quad \text{for } t > 0
$$

**Definition of** $(\overline{M}_{16}^{(2)})_1, (\overline{M}_{16}^{(2)})_2$ and $(\overline{M}_{16}^{(2)})_3$:

**Remark 3:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$$
G_{16} < (\overline{M}_{16}^{(2)}) \quad \text{it follows } \frac{dG_{17}}{dt} \leq (\overline{M}_{16}^{(2)})_1 - (a_{17}^{(2)}) G_{17} \quad \text{and by integrating}
$$

$$
G_{17} \leq (\overline{M}_{16}^{(2)})_2 = G_{17}^0 + 2(a_{17})^2 (\overline{M}_{16}^{(2)})_1/(a_{17}^{(2)})
$$

In the same way, one can obtain

$$
G_{18} \leq (\overline{M}_{16}^{(2)})_3 = G_{18}^0 + 2(a_{18})^2 (\overline{M}_{16}^{(2)})_2/(a_{18}^{(2)})
$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 4:** if $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below.
Remark 5: If $T_{16}$ is bounded from below and $\lim_{t \to \infty} (b_{17}''(t)) = (b_{17}'(t))$ then $T_{17} \to \infty$.

Definition of $(m)^{(2)}$ and $\varepsilon_2$:

Indeed let $t_2$ be so that for $t > t_2$

$$ (b_{17})^{(2)} - (b_{17}(G_{19})(t), t) < \varepsilon_2, T_{16} (t) > (m)^{(2)} $$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)} (m)^{(2)} - \varepsilon_2 T_{17}$ which leads to

$$ T_{17} \geq \left( \frac{(a_{17})^{(2)} (m)^{(2)}}{\varepsilon_2} \right) (1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t} $$

If we take $t$ such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$$ T_{17} \geq \left( \frac{(a_{17})^{(2)} (m)^{(2)}}{2} \right), \ t = \log \frac{2}{\varepsilon_2} $$

By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. The same property holds for $T_{18}$ if $\lim_{t \to \infty} (b_{18}''(t)) = (b_{18}'(t))$

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take $\frac{(a_{1})^{(3)}}{(M_{20})^{(3)}}, \frac{(b_{1})^{(3)}}{(M_{20})^{(3)}} < 1$ and to choose

$$(P_{20})^{(3)} \text{ and } (Q_{20})^{(3)} \text{ large to have}$$

$$ \frac{(a_{1})^{(3)}}{(M_{20})^{(3)}} \left[ (P_{20})^{(3)} + ((P_{20})^{(3)} + G_{10}^{0}) e^{-\frac{(P_{20})^{(3)} + G_{10}^{0}}{G_{10}^{0}\tau_{j}}} \right] \leq (P_{20})^{(3)} $$

$$ \frac{(b_{1})^{(3)}}{(M_{20})^{(3)}} \left[ (Q_{20})^{(3)} + \tau_{j}^{0} e^{-\frac{(Q_{20})^{(3)} + \tau_{j}^{0}}{\tau_{j}\tau_{j}}} + (Q_{20})^{(3)} \right] \leq (Q_{20})^{(3)} $$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_{1}, T_{1}$ into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric

$$ d\left((G_{23})^{(1)}(t), (T_{23})^{(1)}(t)), ((G_{23})^{(2)}(t), (T_{23})^{(2)}(t)) \right) = $$

$$ \sup_{t \in \mathbb{R}^{+}} \max_{i \in \mathbb{R}^{+}} \| G_{i}^{(1)}(t) - G_{i}^{(2)}(t) \| e^{-(M_{20})^{(3)}}, \max_{t \in \mathbb{R}^{+}} \| T_{i}^{(1)}(t) - T_{i}^{(2)}(t) \| e^{-(M_{20})^{(3)}} $$

Indeed if we denote

Definition of $G_{23}, \hat{T}_{23} : (\hat{G}_{23}, \hat{T}_{23}) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$

It results

$$ \left| \hat{G}_{20}^{(1)} - G_{20}^{(1)} \right| \leq \int_0^t (a_{20}^{(3)}(G_{21}^{(1)} - G_{21}^{(2)}) e^{-(M_{20})^{(3)} S_{20}(2)} e^{\hat{G}_{20}^{(3)} S_{20}(2)} ds_{20} $$

$$ \int_0^t (a_{20}^{(3)}(G_{20}^{(1)} - G_{20}^{(2)}) e^{-(M_{20})^{(3)} S_{20}(2)} e^{-(M_{20})^{(3)} S_{20}(2)} + $$

$$ (a_{20}^{(3)}(G_{21}^{(1)} - G_{21}^{(2)}) e^{-(M_{20})^{(3)} S_{20}(2)} e^{-(M_{20})^{(3)} S_{20}(2)} + $$

$$ G_{20}^{(2)}(a_{20}^{(3)}(G_{21}^{(1)} - G_{21}^{(2)}) e^{-(M_{20})^{(3)} S_{20}(2)} e^{-(M_{20})^{(3)} S_{20}(2)} ds_{20} $$
Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left| G^{(1)} - G^{(2)} \right| e^{-\left(\lambda_{20}\right)^{(3)} t} \leq \frac{1}{(\lambda_{20})^{(2)}} \left( (a_{20})^{(3)} + (a_{20})^{(3)} + (a_{20})^{(3)} + (\bar{P}_{20})^{(3)} (\bar{k}_{20})^{(3)} \right) d \left( (G_{23})^{(1)} (T_{23})^{(1)} ; (G_{23})^{(2)} (T_{23})^{(2)} \right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\bar{P}_{20})^{(3)} e^{(\lambda_{20})^{(3)} t}$ and $(\bar{Q}_{20})^{(3)} e^{(\lambda_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_{2i})^{(3)}$ and $(b''_{2i})^{(3)}$, $i = 20, 21, 22$ depend only on $T_{2i}$ and respectively on $(G_{2i})$ and not on $t$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i (t) = 0$ and $T_i (t) = 0$

From 19 to 24 it results

$$G_i (t) \geq G_i^0 e^{-\left(\lambda_{2i}^{(3)} t - (a_{2i}^{(3)} (\bar{P}_{2i} (s_{(20)}), s_{(20)}) d s_{(20)} \right) \geq 0}$$

$$T_i (t) \geq T_i^0 e^{-\left(\lambda_{2i}^{(3)} t \right) > 0 \text{ for } t > 0}$$

**Definition of** $((\bar{M}_{20})^{(3)})_1$, $((\bar{M}_{20})^{(3)})_2$ and $((\bar{M}_{20})^{(3)})_3$:

**Remark 3:** if $G_{20}$ is bounded, the same property have also $G_{21}$ and $G_{22}$ . indeed if $G_{20} < (\bar{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq ((\bar{M}_{20})^{(3)})_1 - (a_{21}^{(3)}) G_{21}$ and by integrating $G_{21} \leq ((\bar{M}_{20})^{(3)})_2 = G_{21}^0 + 2 (a_{21}^{(3)}) ((\bar{M}_{20})^{(3)})_1 / (a_{21}^{(3)})$

In the same way, one can obtain $G_{22} \leq ((\bar{M}_{20})^{(3)})_3 = G_{22}^0 + 2 (a_{22}^{(3)}) ((\bar{M}_{20})^{(3)})_2 / (a_{22}^{(3)})$

If $G_{21}$ or $G_{22}$ is bounded, the same property follows for $G_{20}$, $G_{22}$ and $G_{20}$, $G_{21}$ respectively.

**Remark 4:** If $G_{20}$ is bounded, from below, the same property holds for $G_{21}$ and $G_{22}$. The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below.

**Remark 5:** If $T_{20}$ is bounded from below and $\lim_{t \to \infty} ((b_{2i}^{(3)} ((G_{23}) (t), t)) = (b_{21}^{(3)} (t) )$ then $T_{21} \to \infty$.

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$:

Indeed let $t_3$ be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_{21})^{(3)} ((G_{23}) (t), t) < \varepsilon_3 T_{20} (t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)} (m)^{(3)} - \varepsilon_3 T_{21}$ which leads to

$$T_{21} \geq \left( (a_{21})^{(3)} (m)^{(3)} / \varepsilon_3 \right) (1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$$

If we take $t$ such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$$T_{21} \geq \left( (a_{21})^{(3)} (m)^{(3)} / 2 \right)$$

$t = \log \frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t \to 0^+} (b''_{22})^{(3)} \left( (G_{23})(t), t \right) = (b''_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions.

It is now sufficient to take $\left( \frac{a_0}{M_{24}} \right)^{(4)}, \left( \frac{b_0}{M_{24}} \right)^{(4)} < 1$ and to choose

$$( \hat{P}_{24} )^{(4)}$$ and $$( \hat{Q}_{24} )^{(4)}$$ large to have

$$\left( \frac{a_0}{M_{24}} \right)^{(4)} \left( ( \hat{P}_{24} )^{(4)} + \left( ( \hat{P}_{24} )^{(4)} + G_j^0 \right) e^{-\left( \frac{(P_{24})^{(4)} + \sigma_j^0}{\tau_j^0} \right)} \right) \leq ( \hat{P}_{24} )^{(4)}$$

and

$$\left( \frac{b_0}{M_{24}} \right)^{(4)} \left( ( \hat{Q}_{24} )^{(4)} + \tau_j^0 e^{-\left( \frac{(\hat{Q}_{24})^{(4)} + \tau_j^0}{\tau_j^0} \right)} + ( \hat{Q}_{24} )^{(4)} \right) \leq ( \hat{Q}_{24} )^{(4)}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G, T$ satisfying IN to itself.

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric

$$d \left( (G_{27})^{(1)}, (T_{27})^{(1)}, (G_{27})^{(2)}, (T_{27})^{(2)} \right) = \sup_i \max \left\{ \left| G_i^{(1)}(t) \right|, \left| G_i^{(2)}(t) \right| \right\}$$

Indeed if we denote

**Definition of** $(\overline{G_{27}}), (\overline{T_{27}}) : \left( (\overline{G_{27}}), (\overline{T_{27}}) \right) = \mathcal{A}^{(4)} ((G_{27}), (T_{27}))$

It results

$$\left| \overline{G}_{24}^{(1)} - G_i^{(2)} \right| \leq \int_0^t a_{24}^{(4)} \left| G_{25}^{(1)} - G_{25}^{(2)} \right| e^{-\left( \overline{G}_{24}^{(4)} \right)} s(24) \left( \overline{G}_{24}^{(4)} \right) S(24) dS(24) +$$

$$\int_0^t \left( a_{24}^{(4)} \right) \left| G_{24}^{(1)} - G_{24}^{(2)} \right| e^{-\left( \overline{G}_{24}^{(4)} \right)} s(24) \left( \overline{G}_{24}^{(4)} \right) S(24) +$$

$$(a_{24}^{(4)}) \left( (T_{25}^{(1)}), s(24) \right) \left| G_{24}^{(2)} - G_{24}^{(2)} \right| e^{-\left( \overline{G}_{24}^{(4)} \right)} s(24) \left( \overline{G}_{24}^{(4)} \right) S(24) +$$

$$G_{24}^{(2)} \left( (a_{24}^{(4)}), (T_{25}^{(2)}), s(24) \right) - (a_{24}^{(4)}) \left( \overline{T}_{25}^{(2)}), s(24) \right) \left| e^{-\left( \overline{G}_{24}^{(4)} \right)} s(24) \right( \overline{G}_{24}^{(4)} \right) S(24) dS(24)$$

Where $s(24)$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left| (G_{27})^{(1)} - (G_{27})^{(2)} \right| e^{-\left( \overline{G}_{24}^{(4)} \right)} t \leq$$

$$\frac{1}{(M_{24})^{(4)}} \left( a_{24}^{(4)} \right) + (a_{24}^{(4)}) + (\overline{G}_{24})^{(4)} + (\overline{T}_{25})^{(4)} \left( (G_{27})^{(1)}, (T_{27})^{(1)}, (G_{27})^{(2)}, (T_{27})^{(2)} \right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a_{24}^{(4)})$ and $(b_{24}^{(4)})$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition
necessary to prove the uniqueness of the solution bounded by \((P_{24})^{(4)}e^{(\tilde{\mu}_{24})^{(4)t}}\) and \((Q_{24})^{(4)}e^{(\tilde{\nu}_{24})^{(4)t}}\) respectively of \(\mathbb{R}_+\).

If instead of proving the existence of the solution on \(\mathbb{R}_+\), we have to prove it only on a compact then it suffices to consider that \((a_i')^{(4)}\) and \((b_i')^{(4)}\), \(i = 24, 25, 26\) depend only on \(T_{25}\) and respectively on \((G_{27})(\text{not on } t)\) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \(t\) where \(G_i(t) = 0\) and \(T_i(t) = 0\)

From 19 to 24 it results
\[
G_i(t) \geq G_i^0 e^{-\int_0^t [a_i' \tilde{\mu}_{24} - \tilde{\mu}_{24}(t)] dt} \geq 0
\]
\[
T_i(t) \geq T_i^0 e^{-(b_i' t)} > 0 \quad \text{for } t > 0
\]

**Definition of** \(((\bar{M}_{24})^{(4)}),(\bar{M}_{24})^{(4)}_2\) and \(((\bar{M}_{24})^{(4)}_3)\):

**Remark 3:** if \(G_{24}\) is bounded, the same property have also \(G_{25}\) and \(G_{26}\). Indeed if
\[
G_{24} < (\bar{M}_{24})^{(4)}\) it follows \(\frac{dG_{25}}{dt} \leq ((\bar{M}_{24})^{(4)}_1 - (a_{25})^{(4)}G_{25}\) and by integrating
\[
G_{25} \leq ((\bar{M}_{24})^{(4)}_2 = G_0^0 + 2(a_{25})^{(4)}((\bar{M}_{24})^{(4)}_1)/(a_{25})^{(4)}
\]
In the same way, one can obtain
\[
G_{26} \leq ((\bar{M}_{24})^{(4)}_3 = G_0^0 + 2(a_{26})^{(4)}((\bar{M}_{24})^{(4)}_2)/(a_{26})^{(4)}
\]

If \(G_{25}\) or \(G_{26}\) is bounded, the same property follows for \(G_{24}\), \(G_{26}\) and \(G_{24}\), \(G_{25}\) respectively.

**Remark 4:** If \(G_{24}\) is bounded, from below, the same property holds for \(G_{25}\) and \(G_{26}\). The proof is analogous with the preceding one. An analogous property is true if \(G_{25}\) is bounded from below.

**Remark 5:** If \(T_{24}\) is bounded from below and \(\lim_{t \to \infty}((b_i'')^{(4)}((G_{27})(t), t)) = (b_{25}')^{(4)}\) then \(T_{25} \to \infty\).

**Definition of** \((m)^{(4)}\) and \(\varepsilon_4\):

Indeed let \(t_4\) be so that for \(t > t_4\)
\[
(b_{25})^{(4)} - (b_{25}')^{(4)}((G_{27})(t), t) < \varepsilon_4 T_{24}(t) > (m)^{(4)}
\]

Then \(\frac{dG_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}\) which leads to
\[
T_{25} \geq \frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}(1 - e^{-\varepsilon_4 t} + T_{25}^0 e^{-\varepsilon_4 t}\) If we take \(t\) such that \(e^{-\varepsilon_4 t} = \frac{1}{2}\) it results
\[
T_{25} \geq \frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}^\frac{1}{2}\) By taking now \(\varepsilon_4\) sufficiently small one sees that \(T_{25}\) is unbounded.

The same property holds for \(T_{26}\) if \(\lim_{t \to \infty}(b_{26}'')^{(4)}((G_{27})(t), t) = (b_{26}')^{(4)}\)

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for \(G_{29}, G_{30}, T_{28}, T_{29}, T_{30}\)
It is now sufficient to take \( \frac{(a_l)^{(5)}}{(M_{2b})^{(5)}} \), \( \frac{(b_l)^{(5)}}{(M_{2b})^{(5)}} < 1 \) and to choose

\( (\tilde{P}_{2b})^{(5)} \) and \( (\tilde{Q}_{2b})^{(5)} \) large to have

\[
\frac{(a_l)^{(5)}}{(M_{2b})^{(5)}} \left[ (\tilde{P}_{2b})^{(5)} + ((\tilde{P}_{2b})^{(5)} + G_l^0) e^{-\frac{(P_{2b}^{(5)} + G_l^0)}{\tau_f}} \right] \leq (\tilde{P}_{2b})^{(5)}
\]

\[
\frac{(b_l)^{(5)}}{(M_{2b})^{(5)}} \left[ (\tilde{Q}_{2b})^{(5)} + \tau_f^0 e^{-\frac{(Q_{2b}^{(5)} + \tau_f^0)}{\tau_f}} + (\tilde{Q}_{2b})^{(5)} \right] \leq (\tilde{Q}_{2b})^{(5)}
\]

In order that the operator \( \mathcal{A}^{(5)} \) transforms the space of sextuples of functions \( G_l, T_l \) into itself

The operator \( \mathcal{A}^{(5)} \) is a contraction with respect to the metric

\[
d \left( (G_{31})^{(1)}, (T_{31})^{(1)}), (G_{31})^{(2)}, (T_{31})^{(2)} \right) = \sup_{t \in \mathbb{R}^+} \left| G_l^{(1)}(t) - G_l^{(2)}(t) \right| e^{-(\tilde{M}_{2b})^{(5)} t} \max_{t \in \mathbb{R}^+} \left| T_l^{(1)}(t) - T_l^{(2)}(t) \right| e^{-(\tilde{M}_{2b})^{(5)} t}
\]

Indeed if we denote

\textbf{Definition of} \((\hat{G}_{31}), (\hat{T}_{31}) : (\overline{G}_{31}), (\overline{T}_{31}) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))\)

It results

\[
\left| \tilde{G}_{2b}^{(1)} - \tilde{G}_{2b}^{(2)} \right| \leq \int_0^t \left| (a_{2b})^{(5)} \right| \left| G_{2b}^{(1)} - G_{2b}^{(2)} \right| e^{-(\tilde{M}_{2b})^{(5)} x_{(2b)} t} e^{(\tilde{M}_{2b})^{(5)} x_{(2b)} t} dS_{(2b)} + \\
\int_0^t \left| (a_{2b}''')^{(5)} \right| \left| G_{2b}^{(1)} - G_{2b}^{(2)} \right| e^{-(\tilde{M}_{2b})^{(5)} x_{(2b)} t} e^{(\tilde{M}_{2b})^{(5)} x_{(2b)} t} + \\
\left| (a_{2b}''')^{(5)} \right| \left| T_{2b}^{(1)}(s_{(2b)}) - T_{2b}^{(2)}(s_{(2b)}) \right| e^{-(\tilde{M}_{2b})^{(5)} x_{(2b)} t} e^{(\tilde{M}_{2b})^{(5)} x_{(2b)} t} + \\
\left| (a_{2b}'')^{(5)} \right| \left| T_{2b}^{(1)}(s_{(2b)}) - T_{2b}^{(2)}(s_{(2b)}) \right| e^{-(\tilde{M}_{2b})^{(5)} x_{(2b)} t} e^{(\tilde{M}_{2b})^{(5)} x_{(2b)} t} dS_{(2b)}
\]

Where \( s_{(2b)} \) represents integrand that is integrated over the interval \([0,t]\)

From the hypotheses it follows

\[
\left| (G_{31})^{(1)} - (G_{31})^{(2)} \right| e^{-(\tilde{M}_{2b})^{(5)} t} \leq \frac{1}{(M_{2b})^{(5)}} \left[ \left( (a_{2b})^{(5)} + (a_{2b}''')^{(5)} + (\tilde{M}_{2b})^{(5)} + (\tilde{P}_{2b})^{(5)} (\tilde{Q}_{2b})^{(5)} \right) d \left( ((G_{31})^{(1)}, (T_{31})^{(1)}); (G_{31})^{(2)}, (T_{31})^{(2)} \right) \right]
\]

And analogous inequalities for \( G_l \) and \( T_l \). Taking into account the hypothesis (35,35,36) the result follows

\textbf{Remark 1}: The fact that we supposed \((a_{2b}'')^{(5)}\) and \((b_{2b}'')^{(5)}\) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\tilde{P}_{2b})^{(5)} e^{(\tilde{M}_{2b})^{(5)} t} \) and \((\tilde{Q}_{2b})^{(5)} e^{(\tilde{M}_{2b})^{(5)} t}\) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it
suffices to consider that \((a_i^{(5)})_i^{(5)}\) and \((b_i^{(5)})_i^{(5)}\), \(i = 28, 29, 30\) depend only on \(T_{29}\) and respectively on \((G_{31})(\text{not on } t)\) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2**: There does not exist any \(t\) where \(G_t(t) = 0\) and \(T_t(t) = 0\)

From GLOBAL EQUATIONS it results

\[
G_i(t) \geq G_i^0 e^{\left[-\int_{t_0}^{t} \left| (a_i^{(5)} - (a_i^{(5)}_i^{(5)})(G_{31}(t), t)) \right| dt \right]} \geq 0
\]

\[
T_i(t) \geq T_i^0 e^{-(b_i^{(5)})_i^{(5)}} > 0 \quad \text{for } t > 0
\]

**Definition of** \(\left(\bar{M}_{28}\right)_i^{(5)}, \left(\bar{M}_{29}\right)_i^{(5)}\) and \(\left(\bar{M}_{30}\right)_i^{(5)}\):

**Remark 3**: if \(G_{28}\) is bounded, the same property have also \(G_{29}\) and \(G_{30}\) . indeed if

\[
G_{28} < \left(\bar{M}_{28}\right)_i^{(5)}\text{ it follows } \frac{dG_{28}}{dt} \leq \left(\bar{M}_{28}\right)_i^{(5)} - (a_{28}^{(5)})G_{29}\text{ and by integrating}
\]

\[
G_{29} = \left(\bar{M}_{28}\right)_i^{(5)}_2 = \left(\bar{M}_{28}\right)_i^{(5)}_1 + 2(a_{29}^{(5)})(\bar{M}_{28})_i^{(5)}_1/(a_{29}^{(5)})
\]

In the same way , one can obtain

\[
G_{30} = \left(\bar{M}_{28}\right)_i^{(5)}_3 = \left(\bar{M}_{28}\right)_i^{(5)}_2 + 2(a_{30}^{(5)})(\bar{M}_{28})_i^{(5)}_2/(a_{30}^{(5)})
\]

If \(G_{29}\) or \(G_{30}\) is bounded, the same property follows for \(G_{28}, G_{30}\) and \(G_{28}, G_{29}\) respectively.

**Remark 4**: If \(G_{28}\) is bounded, from below, the same property holds for \(G_{29}\) and \(G_{30}\) . The proof is analogous with the preceding one. An analogous property is true if \(G_{29}\) is bounded from below.

**Remark 5**: If \(T_{28}\) is bounded from below and \(\lim_{t \to \infty}((b_i^{(5)})_i^{(5)}(G_{31})_i(t), t)) = (b_{29}^{(5)})(\text{then } T_{29} \to \infty)

**Definition of** \((m)^{(5)}\) and \(\epsilon_5\):

Indeed let \(t_5\) be so that for \(t > t_5\)

\[
(b_{29}^{(5)})(G_{31})_i(t), t) < \epsilon_5, T_{28}(t) > (m)^{(5)}
\]

Then \(\frac{dT_{29}}{dt} \geq (a_{29}^{(5)})(m)^{(5)}_i - \epsilon_5 T_{29}\) which leads to

\[
T_{29} \geq \left(\frac{(a_{29}^{(5)})(m)^{(5)}_i)}{\epsilon_5}\right) (1 - e^{-\epsilon_5 t}) + T_{29}^0 e^{-\epsilon_5 t}\text{ If we take } t \text{ such that } e^{-\epsilon_5 t} = \frac{1}{2} \text{ it results}
\]

\[
T_{29} \geq \left(\frac{(a_{29}^{(5)})(m)^{(5)}_i)}{2}\right), \quad t = log \frac{2}{\epsilon_5}\text{ By taking now } \epsilon_5 \text{ sufficiently small one sees that } T_{29} \text{ is unbounded.}
\]

The same property holds for \(T_{30}\) if \(\lim_{t \to \infty}((b_{30}^{(5)})(G_{31})_i(t), t) = (b_{30}^{(5)})(\text{then } T_{30} \to \infty)

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for \(G_{33}, G_{34}, T_{32}, T_{33}, T_{34}\)
It is now sufficient to take \( \frac{(a_1)^{(6)}}{(M_{32})^{(6)}} \), \( \frac{(b_1)^{(6)}}{(M_{32})^{(6)}} \) < 1 and to choose

\( (\tilde{P}_{32})^{(6)} \) and \( (\tilde{Q}_{32})^{(6)} \) large to have

\[
\frac{(a_1)^{(6)}}{(M_{32})^{(6)}} \left[ (\tilde{P}_{32})^{(6)} + ((\tilde{P}_{32})^{(6)} + G_j^0) e^{-\left(\frac{(P_{32})^{(6)} + G_j^0}{\tau_j}\right)} \right] \leq (\tilde{P}_{32})^{(6)}
\]

\[
\frac{(b_1)^{(6)}}{(M_{32})^{(6)}} \left[ (\tilde{Q}_{32})^{(6)} + \tau_j^0 e^{-\left(\frac{(Q_{32})^{(6)} + G_j^0}{\tau_j}\right)} + (\tilde{Q}_{32})^{(6)} \right] \leq (\tilde{Q}_{32})^{(6)}
\]

In order that the operator \( \mathcal{A}^{(6)} \) transforms the space of sextuples of functions \( G_i, T_i \) into itself

The operator \( \mathcal{A}^{(6)} \) is a contraction with respect to the metric

\[
d \left( \left((G_{33})^{(1)}, (T_{33})^{(1)}\right), \left((G_{33})^{(2)}, (T_{33})^{(2)}\right) \right) =
\]

\[
sup_{i \in \mathbb{N}^+} \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-\left(\mathcal{R}_{32}\right)^{(6)} t}, \max_{i \in \mathbb{N}^+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-\left(\mathcal{R}_{32}\right)^{(6)} t}\}
\]

Indeed if we denote

**Definition of** \( (\overline{G_{35}}, \overline{T_{35}}) : \ ( (\overline{G_{35}}), (\overline{T_{35}}) ) = \mathcal{A}^{(6)}((\overline{G_{35}}), (\overline{T_{35}})) \)

It results

\[
\left| \overline{G}_{32} - \overline{G}_{1}^{(2)} \right| \leq \int_{0}^{t} (a_{32})^{(6)} |G_{33}^{(1)} - G_{33}^{(2)}| e^{-\left(\mathcal{R}_{32}\right)^{(6)} x_{32} e^{-\left(\mathcal{R}_{32}\right)^{(6)} x_{32}}} ds_{32} + 
\]

\[
\int_{0}^{t} (a_{32}^{(6)} |G_{32}^{(1)} - G_{32}^{(2)}| e^{-\left(\mathcal{R}_{32}\right)^{(6)} x_{32} e^{-\left(\mathcal{R}_{32}\right)^{(6)} x_{32}}} + 
\]

\[
(\mathcal{A}_{32}^{(6)} |T_{33}^{(1)}(s_{32})|G_{32}^{(1)} - G_{32}^{(2)}| e^{-\left(\mathcal{R}_{32}\right)^{(6)} x_{32} e^{-\left(\mathcal{R}_{32}\right)^{(6)} x_{32}}} + 
\]

\[
G_{32}^{(2)} |(a_{32}^{(6)} |T_{33}^{(1)}(s_{32})| - (a_{32}^{(6)} |T_{33}^{(2)}(s_{32})| e^{-\left(\mathcal{R}_{32}\right)^{(6)} x_{32} e^{-\left(\mathcal{R}_{32}\right)^{(6)} x_{32}}} ds_{32}
\]

Where \( s_{32} \) represents integrand that is integrated over the interval \([0,t] \)

From the hypotheses it follows

\[
\left| (G_{33})^{(1)} - (G_{33})^{(2)} \right| e^{-\left(\mathcal{R}_{32}\right)^{(6)} t} \leq 
\]

\[
\frac{1}{(M_{32})^{(6)}} \left( (a_{32})^{(6)} + (a_{32}^{(6)} + (\mathcal{A}_{32}^{(6)} |T_{33}^{(1)}| + (P_{32})^{(6)}(\mathcal{A}_{32}^{(6)} |T_{33}^{(2)}| d \left( \left((G_{33})^{(1)}, (T_{33})^{(1)}; (G_{33})^{(2)}, (T_{33})^{(2)} \right) \right) \right) \right)
\]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis the result follows

**Remark 1**: The fact that we supposed \( (a_{32}^{(6)})^{(6)} \) and \( (b_{32}^{(6)})^{(6)} \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( (\tilde{P}_{32})^{(6)} e^{-\left(\mathcal{R}_{32}\right)^{(6)} t} \) and \( (\tilde{Q}_{32})^{(6)} e^{-\left(\mathcal{R}_{32}\right)^{(6)} t} \) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \( (a_i^{(6)})^{(6)} \) and \( (b_i^{(6)})^{(6)} \), \( i = 32, 33, 34 \) depend only on \( T_{33} \) and respectively on
(G_{35}) (and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any t where \( G_i(t) = 0 \) and \( T_i(t) = 0 \)

From 69 to 32 it results

\[
G_i(t) \geq G_i^0 \exp \left[ -\int_t^0 [(a_i''(s) - (a_i''(s)(T_i(s), T_i(s))) ds] \right] \geq 0
\]

\[
T_i(t) \geq T_i^0 e^{(-b_i'(t))/t} > 0 \quad \text{for } t > 0
\]

**Definition of** \((M_{32}(6))_1, (M_{32}(6))_2 \) and \((M_{32}(6))_3\):

**Remark 3:** If \( G_{32} \) is bounded, the same property also have \( G_{33} \) and \( G_{34} \). Indeed if

\[
G_{32} < (M_{32}(6)) \text{ it follows } \frac{dG_{33}}{dt} \leq ((M_{32}(6))_1 - (a_{33}) (G_{33}) \text{ and by integrating}
\]

\[
G_{33} \leq ((M_{32}(6))_2 = G_{33}^0 + 2(a_{33})((M_{32}(6))_1/(a_{33})^6)
\]

In the same way, one can obtain

\[
G_{34} \leq ((M_{32}(6))_3 = G_{34}^0 + 2(a_{34})((M_{32}(6))_2/(a_{34})^6)
\]

If \( G_{33} \) or \( G_{34} \) is bounded, the same property follows for \( G_{32}, G_{34} \) and \( G_{32}, G_{33} \) respectively.

**Remark 4:** If \( G_{32} \) is bounded, from below, the same property holds for \( G_{33} \) and \( G_{34} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{33} \) is bounded from below.

**Remark 5:** If \( T_{32} \) is bounded from below and \( \lim_{t \to \infty} (b_{33}'(6)((G_{33})(t), t)) = (b_{33}'(6)) \) then \( T_{33} \to \infty \).

**Definition of** \((m(6)) \) and \( \varepsilon_6 \):

Indeed let \( t_6 \) be so that for \( t > t_6 \)

\[
(b_{33}(6)) - (b_{33}'(6)((G_{33})(t), t)) \leq \varepsilon_6, T_{32}(t) > (m(6))
\]

Then \( \frac{dT_{33}}{dt} \geq (a_{33}) (m(6)) - \varepsilon_6 T_{33} \) which leads to

\[
T_{33} \geq \left( \frac{(a_{33}) (m(6))}{\varepsilon_6} \right) (1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_6 t} = \frac{1}{2} \text{ it results}
\]

\[
T_{33} \geq \left( \frac{(a_{33}) (m(6))}{\varepsilon_6} \right), \quad t = \log \frac{2}{\varepsilon_6} \text{ By taking now } \varepsilon_6 \text{ sufficiently small one sees that } T_{33} \text{ is unbounded.}
\]

The same property holds for \( T_{34} \) if \( \lim_{t \to \infty} (b_{34}'(6)((G_{35})(t), t)) = (b_{34}'(6)) \)

We now state a more precise theorem about the behaviors at infinity of the solutions

**Behavior of the solutions**

If we denote and define

**Definition of** \((\sigma_1(1), (\sigma_2(1), (\tau_1(1), (\tau_2(1)\) :
(a) \( \sigma_1^{(1)}, (\sigma_2^{(1)}, (\tau_1^{(1)}, (\tau_2^{(1)} \) four constants satisfying

\[-(\sigma_2^{(1)}) \leq -(a_{14}^{(1)} + (a_1^{(1)}) - (a_{13}^{(1)})(T_{14} + t) + (a_{14}^{(1)})(T_{14} + t) \leq -(\sigma_1^{(1)})
\]

\[-(\tau_2^{(1)}) \leq -(b_{13}^{(1)} + (b_1^{(1)}) - (b_{13}^{(1)})(G, t) - (b_{14}^{(1)})(G, t) \leq -(\tau_1^{(1)})
\]

**Definition of** \( (v_1^{(1)}, (v_2^{(1)}, (u_1^{(1)}, (u_2^{(1)}), (v^{(1)}), (u^{(1)}) \):

By \( (v_1^{(1)}) > 0, (v_2^{(1)}) < 0 \) and respectively \( (u_1^{(1)}) > 0, (u_2^{(1)}) < 0 \) the roots of the equations

\[ (a_{14}^{(1)})(v^{(1)}) + (\sigma_1^{(1)})(v^{(1)}) - (a_{13}^{(1)}) = 0 \) and \( (b_{14}^{(1)})(u^{(1)}) + (\tau_1^{(1)})(u^{(1)}) - (b_{13}^{(1)}) = 0 \]

**Definition of** \( (\tilde{v}_1^{(1)}, (\tilde{v}_2^{(1)}, (\tilde{u}_1^{(1)}, (\tilde{u}_2^{(1)}))\):

By \( (\tilde{v}_1^{(1)}) > 0, (\tilde{v}_2^{(1)}) < 0 \) and respectively \( (\tilde{u}_1^{(1)}) > 0, (\tilde{u}_2^{(1)}) < 0 \) the roots of the equations

\[ (a_{14}^{(1)})(v^{(1)}) + (\sigma_1^{(1)})(v^{(1)}) - (a_{13}^{(1)}) = 0 \) and \( (b_{14}^{(1)})(u^{(1)}) + (\tau_1^{(1)})(u^{(1)}) - (b_{13}^{(1)}) = 0 \]

**Definition of** \( (m_1^{(1)}, (m_2^{(1)}, (\mu_1^{(1)}, (\mu_2^{(1)}), (v_0^{(1)})) :-

(c) If we define \( (m_1^{(1)}, (m_2^{(1)}, (\mu_1^{(1)}, (\mu_2^{(1)}), (v_0^{(1)}) \) by

\[ (m_2^{(1)} = (v_0^{(1)}), (m_1^{(1)} = (v_1^{(1)}), if (v_0^{(1)}) < (v_1^{(1)})
\]

\[ (m_2^{(1)} = (v_1^{(1)}), (m_1^{(1)} = (\tilde{v}_1^{(1)}), if (v_1^{(1)}) < (v_0^{(1)}) < (\tilde{v}_1^{(1)})
\]

and \[ (v_0^{(1)}) = \frac{v_1^{(1)}}{v_0^{(1)}} \]

\[ (m_2^{(1)} = (v_1^{(1)}), (m_1^{(1)} = (v_0^{(1)}), if (\tilde{v}_1^{(1)}) < (v_0^{(1)}) \]

and analogously

\[ (\mu_2^{(1)} = (u_0^{(1)}), (\mu_1^{(1)} = (u_1^{(1)}), if (u_0^{(1)}) < (u_1^{(1)})
\]

\[ (\mu_2^{(1)} = (u_1^{(1)}), (\mu_1^{(1)} = (\tilde{u}_1^{(1)}), if (u_1^{(1)}) < (u_0^{(1)}) < (\tilde{u}_1^{(1)})
\]

and \[ (u_0^{(1)}) = \frac{u_1^{(1)}}{u_0^{(1)}} \]

\[ (\mu_2^{(1)} = (u_1^{(1)}), (\mu_1^{(1)} = (u_0^{(1)}), if (\tilde{u}_1^{(1)}) < (u_0^{(1)}) \]

are defined respectively

Then the solution satisfies the inequalities

\[ G_{13}^0 e^{((S_1^{(1)} - (P_{13}^{(1)}))t} \leq G_{13}^0 \leq G_{13}^0 e^{(S_1^{(1)} t}
\]

where \( (p_2^{(1)}) \) is defined

\[ \frac{1}{(m_2^{(1)})^2} \leq G_{14}^0 \leq \frac{1}{(m_1^{(1)})^2} \]

\[ \left[ e^{((S_1^{(1)} - (P_{13}^{(1)}))t} - e^{((S_2^{(1)} - (P_{13}^{(1)}))t} \right] + G_{15}^0 e^{-(S_2^{(1)} t)} \]
\[
\frac{1}{(\mu_1 t)^{\frac{1}{2}}}T_{13}^0 e^{(R_{13}) t} \leq T_{13}(t) \leq \frac{1}{(\mu_2 t)^{\frac{1}{2}}}T_{13}^0 e^{(R_{13}) t} + T_{13}^0 e^{-(R_{13}) t} \leq T_{13}(t) \leq \frac{1}{(\mu_2 t)^{\frac{1}{2}}}T_{13}^0 e^{(R_{13}) t} + T_{13}^0 e^{-(R_{13}) t} + T_{13}^0 e^{-(R_{13}) t}.
\]

**Definition of** \((S_1) \), \((S_2) \), \((R_1) \), \((R_2) \):

Where \((S_1) = (a_{13}) (m_2) - (a_{13}) (1)

\(S_2) = (a_{13}) (1) - (p_{13}) (1)

\((R_1) = (b_{13}) (1)(\mu_2)(1) - (b_{13}) (1)

\((R_2) = (b_{13}) (1) - (r_{13}) (1)

**Behavior of the solutions**

If we denote and define

**Definition of** \((\sigma_1) \), \((\sigma_2) \), \((\tau_1) \), \((\tau_2) \):

\((d) (\sigma_2) \), \((\sigma_2) \), \((\tau_1) \), \((\tau_2) \)

four constants satisfying

\[-(\sigma_2) \leq -(a_{16}) + (a_{17}) - (a_{10}) (T_{17}, t) + (a_{17}) (T_{17}, t) \leq -(\sigma_1) \]

\[-(\tau_2) \leq -(b_{16}) + (b_{17}) - (b_{10}) (G_{19}, t) - (b_{17}) (G_{19}, t) \leq -(\tau_1) \]

**Definition of** \((v_1) \), \((v_2) \), \((u_1) \), \((u_2) \):

By \((v_1) > 0, (v_2) < 0 \) and respectively \((u_1) > 0, (u_2) < 0 \) the roots of the equations

\[(a_{17}) (v_1)^2 + (\sigma_1) (v_2) - (a_{16}) = 0 \]

\[and (b_{14}) (u_1)^2 + (\tau_1) (u_2) - (b_{16}) = 0 \]

**Definition of** \((\tilde{v}_1) \), \((\tilde{v}_2) \), \((\tilde{u}_1) \), \((\tilde{u}_2) \):

By \((\tilde{v}_1) > 0, (\tilde{v}_2) < 0 \) and respectively \((\tilde{u}_1) > 0, (\tilde{u}_2) < 0 \) the roots of the equations

\[(a_{17}) (v_1)^2 + (\sigma_2) (v_2) - (a_{16}) = 0 \]

\[and (b_{17}) (u_1)^2 + (\tau_2) (u_2) - (b_{16}) = 0 \]

**Definition of** \((m_1) \), \((m_2) \), \((\mu_1) \), \((\mu_2) \):

\((f) \) If we define \((m_1) \), \((m_2) \), \((\mu_1) \), \((\mu_2) \) by

\[(m_2) = (v_1) \), \((m_1) = (v_1) \), \(i f (v_1) < (v_1) \]

\[(m_2) = (v_1) \), \((m_1) = (v_1) \), \(i f (v_1) < (v_1) < (v_1) \]

\[and \(v_0) = \frac{c_0}{c_0} \]
\( (m_2)^{(2)} = (v_2)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}, \ if \ (\bar{v}_1)^{(2)} < (v_0)^{(2)} \)

and analogously

\( (\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_2)^{(2)}, \ if \ (u_0)^{(2)} < (u_2)^{(2)} \)

\( (\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \ if \ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)} \),

and \( (u_0)^{(2)} = \frac{T^2_{16}}{T_{17}} \)

\( (\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \ if \ (\bar{u}_1)^{(2)} < (u_0)^{(2)} \)

Then the solution satisfies the inequalities

\[ G_{16}^0 e^{((S_1)^{(2)} - (P_{16})^{(2)})t} \leq G_{16}^0 e^{(S_1)^{(2)}t} \leq G_{16}^0 e^{((S_1)^{(2)} - (P_{16})^{(2)})t} \]

\( (p_2)^{(2)} \) is defined

\[ \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{((S_2)^{(2)} - (P_{16})^{(2)})t} \leq G_{16}^0 e^{(S_2)^{(2)}t} \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{((S_2)^{(2)} - (P_{16})^{(2)})t} \]

\[ \frac{(\alpha_{10})^{(2)} G_{16}^0}{(m_2)^{(2)}} \left[ e^{((S_2)^{(2)} - (P_{16})^{(2)})t} - e^{-(S_2)^{(2)}t} \right] + G_{16}^0 e^{-(S_2)^{(2)}t} \leq G_{16}^0 e^{(S_2)^{(2)}t} \leq \frac{(\alpha_{10})^{(2)} G_{16}^0}{(m_2)^{(2)}} \left[ e^{((S_2)^{(2)} - (P_{16})^{(2)})t} - e^{-(a_{10}^2)^{(2)}t} \right] + G_{16}^0 e^{-(a_{10}^2)^{(2)}t} \]

\[ T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^2)t} \leq T_{16}^0 \leq T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^2)t} \]

\[ \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^2)t} \leq T_{16}^0 \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^2)t} \]

\[ \frac{(b_{10})^{(2)} T_{16}^0}{(\mu_2)^{(2)}} \left[ e^{((R_1)^{(2)} + (r_{16})^2)t} - e^{-(b_{10})^{(2)}t} \right] + T_{16}^0 e^{-(b_{10})^{(2)}t} \leq T_{16}^0 \leq \frac{(b_{10})^{(2)} T_{16}^0}{(\mu_2)^{(2)}} \left[ e^{((R_1)^{(2)} + (r_{16})^2)t} - e^{-(b_{10})^{(2)}t} \right] + T_{16}^0 e^{-(b_{10})^{(2)}t} \]

**Definition of \((S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}\):**

Where

\[ (S_1)^{(2)} = (a_{16}^2)^{(2)} (m_2)^{(2)} - (a_{10}^2)^{(2)} \]

\[ (S_2)^{(2)} = (a_{16}^2)^{(2)} - (P_{16})^{(2)} \]

\[ (R_1)^{(2)} = (b_{16}^2)^{(2)} (\mu_1)^{(2)} - (b_{16}^2)^{(2)} \]

\[ (R_2)^{(2)} = (b_{16}^2)^{(2)} - (r_{16})^2 \]

**Behavior of the solutions**

If we denote and define

**Definition of \((\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}\):**

(a) \((\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}\) four constants satisfying

\[-(\sigma_2)^{(3)} \leq -(a_{16}^2)^{(3)} + (a_{21}^2)^{(3)} - (a_{20}^2)^{(3)} (T_{21}, t) + (a_{21}^2)^{(3)} (T_{21}, t) \leq -(\sigma_1)^{(3)} \]
\[-(r_2)^{(3)} \leq -(b_{20}^{(3)})^2 \leq -(b_{20}^{(3)})^2 + (b_{21}^{(3)})^2 - (b_{21}^{(3)}(G, t) - (b_{21}^{(3)}(G, t) \leq -r_1^{(3)})\]

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$:

**(b)** By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21}^{(3)})(v)^{(3)} + (σ_1^{(3)})(v)^{(3)} - (a_{20}^{(3)}) = 0$ and $(b_{21}^{(3)})(u)^{(3)} + (r_1^{(3)})(u)^{(3)} - (b_{20}^{(3)}) = 0$ and

By $(v_2)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_2)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21}^{(3)})(v)^{(3)} + (σ_2^{(3)})(v)^{(3)} - (a_{20}^{(3)}) = 0$ and $(b_{21}^{(3)})(u)^{(3)} + (r_2^{(3)})(u)^{(3)} - (b_{20}^{(3)}) = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (μ_1)^{(3)}, (μ_2)^{(3)}$:

**(c)** If we define $(m_1)^{(3)}$, $(m_2)^{(3)}, (μ_1)^{(3)}, (μ_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, if $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_2)^{(3)}$, if $(v_1)^{(3)} < (v_0)^{(3)} < (v_2)^{(3)}$ and

$(v_0)^{(3)} = \frac{r_{20}^{(3)}}{r_{21}^{(3)}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, if $(v_1)^{(3)} < (v_0)^{(3)}$ and analogously

$(μ_2)^{(3)} = (u_0)^{(3)}, (μ_1)^{(3)} = (u_1)^{(3)}$, if $(u_0)^{(3)} < (u_1)^{(3)}$

$(μ_2)^{(3)} = (u_1)^{(3)}, (μ_1)^{(3)} = (u_0)^{(3)}$, if $(u_1)^{(3)} < (u_0)^{(3)} < (u_2)^{(3)}$, and $(u_0)^{(3)} = \frac{r_{20}^{(3)}}{r_{21}^{(3)}}$

$(μ_2)^{(3)} = (u_1)^{(3)}, (μ_1)^{(3)} = (u_0)^{(3)}$, if $(u_1)^{(3)} < (u_0)^{(3)}$

Then the solution satisfies the inequalities

$G_{20}^0e^{((s_1)^{(3)}-(p_{20})^{(3)})t} \leq G_{20}^0(t) \leq G_{20}^0e^{((s_1)^{(3)})t}$

$G_{20}^0e^{((s_1)^{(3)}-(p_{20})^{(3)})t} \leq G_{21}^0(t) \leq \frac{1}{(m_2)^{(3)}}G_{20}^0e^{((s_1)^{(3)}-(p_{20})^{(3)})t}$

$G_{21}^0(t) \leq \frac{1}{(m_2)^{(3)}}G_{20}^0e^{((s_1)^{(3)}-(p_{20})^{(3)})t}$

$\frac{(a_{22}^{(3)}g_{20}^0)}{(m_1)^{(3)}((s_1)^{(3)}-(p_{20})^{(3)})} \left[e^{((s_1)^{(3)}-(p_{22})^{(3)})t} - e^{-(s_2)^{(3)}t}\right] + g_{22}^0e^{-(s_2)^{(3)}t} \leq G_{22}^0(t) \leq T_{22}^0e^{((r_1)^{(3)}+(p_{20})^{(3)})t}$

$\frac{1}{(m_2)^{(3)}}T_{20}^0e^{((r_1)^{(3)})t} \leq T_{20}^0e^{((r_1)^{(3)})t} \leq T_{20}^0e^{((r_1)^{(3)}+(p_{20})^{(3)})t}$

$\frac{(b_{22}^{(3)}g_{20}^0)}{(μ_1)^{(3)}((r_1)^{(3)}-(b_{22}^{(3)})^{(3)})} \left[e^{(r_1)^{(3)}t} - e^{-(b_{22}^{(3)})^{(3)}t}\right] + T_{22}^0e^{-(b_{22}^{(3)})^{(3)}t} \leq T_{22}^0(t) \leq$
\[
\frac{(a_{22})^{(2)}\rho_{22}^{(2)}}{(b_{22})^{(2)}((R_1^{(2)})^2 + (R_2^{(2)})^2)} \left[ e^\{(R_1^{(2)})t + (R_2^{(2)})t\} - e^\{-R_2^{(2)}t\} \right] + \rho_{22}^{(2)} e^\{-R_2^{(2)}t\}
\]

**Definition of** \((S_2^{(3)}), (S_2^{(3)}), (R_2^{(3)}), (R_2^{(3)})\):-

Where \((S_2^{(3)}) = (a_{20^{(3)}}(m_2^{(3)}) - (a_{20^{(3)}})^{(3)})
(S_2^{(3)}) = (a_{22}^{(3)}) - (p_{22}^{(3)})
(R_1^{(3)}) = (b_{20}^{(3)})(\mu_2^{(3)}) - (b_{20}^{(3)})^{(3)}
(R_2^{(3)}) = (b_{22}^{(3)}) - (r_{22}^{(3)})

**Behavior of the solutions**
If we denote and define

**Definition of** \((\sigma_1^{(4)}), (\sigma_2^{(4)}), (\tau_1^{(4)}), (\tau_2^{(4)})\):

(d) \((\sigma_1^{(4)}), (\sigma_2^{(4)}), (\tau_1^{(4)}), (\tau_2^{(4)})\) four constants satisfying

\[-(\sigma_2^{(4)}) \leq -(a_{24}^{(4)}) + (a_{25}^{(4)}) - (a_{25}^{(4)})(T_{25}, t) + (a_{25}^{(4)})(T_{25}, t) \leq -(\sigma_1^{(4)})
- (\tau_2^{(4)}) \leq -(b_{24}^{(4)}) + (b_{25}^{(4)}) - (b_{25}^{(4)})(G_{27}, t) - (b_{25}^{(4)})(G_{27}, t) \leq -(\tau_1^{(4)})

**Definition of** \((v_1^{(4)}), (v_2^{(4)}), (u_1^{(4)}), (u_2^{(4)}), (u^{(4)}), (u^{(4)})\):

(e) By \((v_1^{(4)}) > 0, (v_2^{(4)}) < 0\) and respectively \((u_1^{(4)}) > 0, (u_2^{(4)}) < 0\) the roots of the equations

\[(a_{22}^{(4)})(v^{(4)})^2 + (\sigma_1^{(4)})(u^{(4)}) - (a_{24}^{(4)}) = 0\]

and \((b_{25}^{(4)})(u^{(4)})^2 + (\tau_1^{(4)})(u^{(4)}) - (b_{24}^{(4)}) = 0 \]

**Definition of** \((\tilde{\tau}_1^{(4)}), (\tilde{\tau}_2^{(4)}), (\tilde{\mu}_1^{(4)}), (\tilde{\mu}_2^{(4)})\):

By \((\tilde{\tau}_1^{(4)}) > 0, (\tilde{\tau}_2^{(4)}) < 0\) and respectively \((\tilde{\mu}_1^{(4)}) > 0, (\tilde{\mu}_2^{(4)}) < 0\) the roots of the equations

\[(a_{25}^{(4)})(v^{(4)})^2 + (\sigma_2^{(4)})(u^{(4)}) - (a_{24}^{(4)}) = 0\]

and \((b_{25}^{(4)})(u^{(4)})^2 + (\tau_2^{(4)})(u^{(4)}) - (b_{24}^{(4)}) = 0 \]

**Definition of** \((\mu_1^{(4)}), (\mu_2^{(4)}), (\mu_1^{(4)}), (\mu_2^{(4)}), (v_0^{(4)})\):

(f) If we define \((m_1^{(4)}), (m_2^{(4)}), (\mu_1^{(4)}), (\mu_2^{(4)})\) by

\[m_2^{(4)} = (v_0^{(4)}), (m_1^{(4)}), (\mu_1^{(4)}), (\mu_2^{(4}) = (v_0^{(4)}) < (v_1^{(4)})\]

\[m_0^{(4)} = (v_0^{(4)}), (m_1^{(4)}), (\mu_1^{(4)}), (\mu_2^{(4}}) = (\tilde{\tau}_1^{(4)}) < (\tilde{\tau}_2^{(4)})\]

and analogously

\[\mu_0^{(4)} = (u_0^{(4)}), (\mu_1^{(4)}), (\mu_2^{(4)}) = (u_0^{(4)}) < (u_1^{(4)})\]

\[\mu_0^{(4)} = (u_0^{(4)}), (\mu_1^{(4)}), (\mu_2^{(4}}) = (\tilde{\mu}_1^{(4)}) < (\tilde{\mu}_2^{(4)})\]

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\[
(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, tf (\bar{u}_1)^{(4)} < (u_0)^{(4)} \text{ where } (u_1)^{(4)}, (\bar{u}_1)^{(4)}
\]
are defined by 59 and 64 respectively.

Then the solution satisfies the inequalities

\[
G_2 e^{(S_1)^{(4)}-(p_{24})^{(4)}t} \leq G_{24} e^{(S_1)^{(4)}t} \leq G_2 e^{(S_2)^{(4)}t}
\]

where \((p_2)^{(4)}\) is defined

\[
\frac{1}{(m_2)^{(4)} - (a_{24})^{(4)}} \frac{G_0}{G_{24} e^{(S_1)^{(4)}-(p_{24})^{(4)}t}} \leq \frac{G_2}{G_{25} e^{(S_2)^{(4)}t}} \leq \frac{G_0}{G_{24} e^{(S_1)^{(4)}t}}
\]

\[
\frac{1}{(m_1)^{(4)} - (a_{24})^{(4)}} \frac{G_0}{G_{24} e^{(S_1)^{(4)}-(p_{24})^{(4)}t} - e^{-(S_2)^{(4)}t}} + G_2 e^{-(S_2)^{(4)}t} \leq G_{26} \leq \frac{G_0}{G_{24} e^{(S_1)^{(4)}t}}
\]

\[
T_{24} e^{(R_1)^{(4)}t} \leq T_{24} e^{(R_1)^{(4)}t} \leq T_{24} e^{(R_1)^{(4)}t} + \frac{1}{(m_2)^{(4)}} T_{24} e^{(R_{24})^{(4)}t}
\]

\[
\frac{1}{(m_2)^{(4)} - (b_{24})^{(4)}} \frac{G_0}{G_{24} e^{(R_{24})^{(4)}t} - e^{-(b_{24})^{(4)}t}} + T_{24} e^{-(b_{24})^{(4)}t} \leq T_{26} \leq \frac{G_0}{G_{24} e^{(R_{24})^{(4)}t}}
\]

**Definition of \((S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}\):**

Where \((S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a_{24})^{(4)}\)

\[(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}\]

\[(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b_{24})^{(4)}\]

\[(R_2)^{(4)} = (b_{26})^{(4)} - (r_{26})^{(4)}\]

**Behavior of the solutions**

If we denote and define

**Definition of \((\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}\):**

\(- (\sigma_2)^{(5)} \leq -(a_{28})^{(5)} + (a_{29})^{(5)} - (a_{28})^{(5)}(T_{29}, t) + (a_{29})^{(5)}(T_{29}, t) \leq -(\sigma_1)^{(5)}\)

\(- (\tau_2)^{(5)} \leq - (b_{28})^{(5)} + (b_{29})^{(5)} - (b_{28})^{(5)}(G_{31}, t) - (b_{29})^{(5)}(G_{31}, t) \leq -(\tau_1)^{(5)}\)

**Definition of \((v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}\):**

By \((v_1)^{(5)} > 0, (v_2)^{(5)} < 0\) and respectively \((u_1)^{(5)} > 0, (u_2)^{(5)} < 0\) the roots of the equations \((a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} -(a_{28})^{(5)} = 0\)
and \((b_{28}^{(5)}(u^{(5)})^2 + (r_1^{(5)}u^{(5)} - (b_{28}^{(5)}) = 0 \text{ and}

\textbf{Definition of} \( (\bar{v}_1^{(5)}, (\bar{v}_2^{(5)}, (\bar{u}_1^{(5)}, (\bar{u}_2^{(5)}) : \)

By \((\bar{v}_1^{(5)} > 0, (\bar{v}_2^{(5)} < 0\text{ and respectively } (\bar{u}_1^{(5)} > 0, (\bar{u}_2^{(5)} < 0 \text{ the roots of the equations } (a_{29}^{(5)}(v^{(5)})^2 + (r_2^{(5)}v^{(5)} - (a_{29}^{(5)}) = 0 \text{ and } (b_{29}^{(5)}(u^{(5)})^2 + (r_2^{(5)}u^{(5)} - (b_{29}^{(5)}) = 0.

\textbf{Definition of} \( (m_1^{(5)}, (m_2^{(5)}, (\mu_1^{(5)}, (\mu_2^{(5)}, (v_0^{(5)} \text{: -}

(i) \text{ If we define } (m_1^{(5)}, (m_2^{(5)}, (\mu_1^{(5)}, (\mu_2^{(5)} \text{ by }

\begin{align*}
(m_2^{(5)} &= (v_0^{(5)}, (m_1^{(5)} = (v_1^{(5)}, \text{ if } (v_0^{(5)} < (v_1^{(5)}), \\
(m_1^{(5)} &= (v_1^{(5)}, (m_1^{(5)} = (\bar{v}_1^{(5)}, \text{ if } (v_1^{(5)} < (\bar{v}_1^{(5)}) \text{, and }

(v_0^{(5)} &= \frac{c_{28}}{v_{29}^{(5)}}
\end{align*}

\begin{align*}
(m_2^{(5)} &= (v_0^{(5)}, (m_1^{(5)} = (v_0^{(5)}, \text{ if } (\bar{v}_1^{(5)} < (v_0^{(5)})
\end{align*}

and analogously

\begin{align*}
(\mu_2^{(5)} &= (u_0^{(5)}(\mu_1^{(5)} = (u_1^{(5)}, \text{ if } (u_0^{(5)} < (u_1^{(5)}), \\
(\mu_1^{(5)} &= (u_1^{(5)}(\mu_1^{(5)} = (u_1^{(5)}, \text{ if } (u_0^{(5)} < (u_1^{(5)}) \text{, and }

(u_0^{(5)} &= \frac{T_{28}}{T_{29}}
\end{align*}

(\mu_2^{(5)} &= (u_1^{(5)}, (\mu_1^{(5)} = (u_0^{(5)}, \text{ if } (u_1^{(5)} < (u_0^{(5)}) \text{ where } (u_1^{(5)}, (\bar{u}_1^{(5)} \text{ are defined respectively.}

Then the solution satisfies the inequalities

\begin{align*}
G_{28}^{(5)}e^{((S_2^{(5)} - (P_{28}^{(5}))t) \leq G_{28}^{(5)}e^{((S_1^{(5)})t}
\end{align*}

where \((p_{28}^{(5)}) \text{ is defined}

\begin{align*}
\frac{1}{(m_2^{(5)})}G_{28}^{(5)}e^{((S_2^{(5)} - (P_{28}^{(5)})t) \leq G_{28}^{(5)}e^{((S_1^{(5)})t}
\end{align*}

\begin{align*}
\frac{1}{(m_2^{(5)})}G_{28}^{(5)}e^{((S_2^{(5)} - (P_{28}^{(5)})t) \leq G_{28}^{(5)}e^{((S_1^{(5)})t}
\end{align*}

\begin{align*}
T_{28}^{(5)}e^{((R_1^{(5)} - (R_{28}^{(5)})t) \leq T_{28}^{(5)}e^{((R_1^{(5)})t}
\end{align*}

\begin{align*}
T_{28}^{(5)}e^{((R_1^{(5)} - (R_{28}^{(5)})t) \leq T_{28}^{(5)}e^{((R_1^{(5)})t}
\end{align*}

\begin{align*}
\frac{1}{(\mu_1^{(5)})}T_{28}^{(5)}e^{((R_1^{(5)} - (R_{30}^{(5)})t) \leq T_{28}^{(5)}e^{((R_1^{(5)})t}
\end{align*}

\begin{align*}
\frac{1}{(\mu_1^{(5)})}T_{28}^{(5)}e^{((R_1^{(5)} - (R_{30}^{(5)})t) \leq T_{28}^{(5)}e^{((R_1^{(5)})t}
\end{align*}

\begin{align*}
T_{30}^{(5)}e^{((R_1^{(5)} + (R_{28}^{(5)})t) \leq T_{30}^{(5)}e^{((R_1^{(5)})t}
\end{align*}

\begin{align*}
T_{30}^{(5)}e^{((R_1^{(5)} + (R_{28}^{(5)})t) \leq T_{30}^{(5)}e^{((R_1^{(5)})t}
\end{align*}

\begin{align*}
(\mu_1^{(5)})e^{((R_1^{(5)} + (R_{28}^{(5)})t) - e^{((R_2^{(5)})(t)} \leq N_{30}^{(5)}e^{((R_1^{(5)} + (R_{28}^{(5)})t)}
\end{align*}

\begin{align*}
(\mu_1^{(5)})e^{((R_1^{(5)} + (R_{28}^{(5)})t) - e^{((R_2^{(5)})(t)} \leq N_{30}^{(5)}e^{((R_1^{(5)} + (R_{28}^{(5)})t)}
\end{align*}

\textbf{Definition of} \( (S_1^{(5)}, (S_2^{(5)}, (R_1^{(5)}, (R_2^{(5)}) :-

Where \((S_1^{(5)} = (a_{28}^{(5)}(m_2^{(5)} - (a_{28}^{(5)}))\)


\[ (S_2)^(5) = (a_{30})^(5) - (p_{30})^(5) \]
\[ (R_1)^(5) = (b_{28})^(5)(\mu_2)^(5) - (b_{28}')^(5) \]
\[ (R_2)^(5) = (b_{30}')^(5) - (r_{30})^(5) \]

**Behavior of the solutions**

If we denote and define

**Definition of \((\sigma_1)^(6), (\sigma_2)^(6), (\tau_1)^(6), (\tau_2)^(6)\):**

(i) \((\sigma_1)^(6), (\sigma_2)^(6), (\tau_1)^(6), (\tau_2)^(6)\) four constants satisfying

\[-(\sigma_2)^(6) \leq -(a_{132})^(6) + (a_{33})^(6) - (a_{32}')^(6)(T_{33}, \tau) + (a_{32}'')^(6)(T_{33}, \tau) \leq -(\sigma_1)^(6) \]

\[-(\tau_2)^(6) \leq -(b_{32}')^(6) + (b_{33})^(6) - (b_{32}'')^(6)((G_{33})_5, \tau) - (b_{33}')^(6)((G_{35})_5, \tau) \leq -(\tau_1)^(6) \]

**Definition of \((\nu_1)^(6), (\nu_2)^(6), (u_1)^(6), (u_2)^(6), (v_6), u_6) :**

(k) By \((\nu_1)^(6) > 0, (\nu_2)^(6) < 0 \) and respectively \((u_1)^(6) > 0, (u_2)^(6) < 0 \) the roots of the equations

\[(a_{33})^(6)(v_6)^2 + (\sigma_1)^(6)v_6 - (a_{32})^(6) = 0 \]
\[ (b_{33})^(6)(u_6)^2 + (\tau_1)^(6)u_6 - (b_{32})^(6) = 0 \]

**Definition of \((\bar{\nu}_1)^(6), (\bar{\nu}_2)^(6), (\bar{u}_1)^(6), (\bar{u}_2)^(6) :**

By \((\bar{\nu}_1)^(6) > 0, (\bar{\nu}_2)^(6) < 0 \) and respectively \((\bar{u}_1)^(6) > 0, (\bar{u}_2)^(6) < 0 \) the roots of the equations

\[(a_{33})^(6)(\bar{v}_6)^2 + (\sigma_1)^(6)\bar{v}_6 - (a_{32})^(6) = 0 \]
\[ (b_{33})^(6)(\bar{u}_6)^2 + (\tau_1)^(6)\bar{u}_6 - (b_{32})^(6) = 0 \]

**Definition of \((m_1)^(6), (m_2)^(6), (\mu_1)^(6), (\mu_2)^(6), (\nu_6), (\nu_6) :**

(l) If we define \((m_1)^(6), (m_2)^(6), (\mu_1)^(6), (\mu_2)^(6)\) by

\[(m_2)^(6) = (v_6)^(6), (m_1)^(6) = (\nu_1)^(6), \text{ if } (v_6)^(6) < (\nu_1)^(6) \]
\[ (m_2)^(6) = (v_1)^(6), (m_1)^(6) = (\bar{v}_6)^(6), \text{ if } (v_1)^(6) < (\bar{v}_6)^(6) \]
\[ (v_6)^(6) = \frac{v_6^2}{v_6} \]
\[ (m_2)^(6) = (v_1)^(6), (m_1)^(6) = (v_6)^(6), \text{ if } (v_1)^(6) < (v_6)^(6) \]

and analogously

\[(\mu_1)^(6) = (u_6)^(6), (\mu_1)^(6) = (\nu_1)^(6), \text{ if } (u_6)^(6) < (\nu_1)^(6) \]
\[ (\mu_2)^(6) = (u_1)^(6), (\mu_2)^(6) = (\bar{u}_6)^(6), \text{ if } (u_1)^(6) < (\bar{u}_6)^(6) \]
\[ (u_6)^(6) = \frac{u_6^2}{u_6} \]
\[ (\mu_2)^(6) = (u_1)^(6), (\mu_2)^(6) = (u_6)^(6), \text{ if } (u_1)^(6) < (u_6)^(6) \]

where \((u_1)^(6), (\bar{u}_1)^(6)\) are defined respectively

Then the solution satisfies the inequalities

\[ G_{32}^0((S_1)^(6)-(p_{32})^(6))t \leq G_{32}(t) \leq G_{32}^0((S_1)^(6))t \]
where \((p_j)^{(6)}\) is defined

\[
\frac{1}{(m_2)^{(6)}t} G_{32}^0 e^{((S_1)^{(6)}-(p_{32})^{(6)})t} \leq G_{33}^{0} (t) \leq \frac{1}{(m_2)^{(6)}t} G_{32}^{0} e^{((S_j)^{(6)})t} \\
\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)(S_1)^{(6)}-(p_{32})^{(6)}-(S_j)^{(6)})} + \frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)(S_j)^{(6)}-(a_{34})^{(6)}}} \right) e^{((S_1)^{(6)}-(p_{32})^{(6)}t - e^{-(S_j)^{(6)}t})} + G_{34}^{0} e^{-(a_{34})^{(6)}t} \leq G_{34}^{0} (t) \leq \frac{1}{(m_2)^{(6)(S_j)^{(6)}-(a_{34})^{(6)}}} G_{32}^{0} e^{-(a_{34})^{(6)}t} \\
T_{32}^{0} e^{((R_1)^{(6)}+(R_2)^{(6)})t} \leq T_{32}^{0} (t) \leq T_{32}^{0} e^{((R_1)^{(6)}+(R_2)^{(6)})t} \\
\frac{1}{(m_2)^{(6)}t} T_{32}^{0} e^{((R_1)^{(6)}+(R_2)^{(6)})t} \leq T_{32}^{0} (t) \leq \frac{1}{(m_2)^{(6)}t} T_{32}^{0} e^{((R_1)^{(6)}+(R_2)^{(6)})t} \\
\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)(R_1)^{(6)}-(b_{34})^{(6)}}} e^{((R_1)^{(6)}t)} - e^{-(b_{34})^{(6)}t} \right) + T_{34}^{0} e^{-(b_{34})^{(6)}t} \leq T_{34}^{0} (t) \leq \frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)(R_1)^{(6)}+(R_2)^{(6)}+(b_{34})^{(6)}}} e^{((R_1)^{(6)}+(R_2)^{(6)})t} - e^{-(R_2)^{(6)}t} \right) + T_{34}^{0} e^{-(R_2)^{(6)}t} \\
\text{Definition of } (S_j)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}:

Where \((S_1)^{(6)} = (a_{34})^{(6)} - (m_2)^{(6)} - (a_{32})^{(6)}\)
\((S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}\)
\((R_1)^{(6)} = (a_{34})^{(6)} + (m_2)^{(6)} - (b_{34})^{(6)}\)
\((R_2)^{(6)} = (a_{34})^{(6)} - (m_2)^{(6)}\)

\textbf{Proof:} From GLOBAL EQUATIONS we obtain

\[
\frac{dv_1^{(1)}}{dt} = (a_{13})^{(1)} - \left( (a_{14})^{(1)} - (a_{14})^{(1)} + (a_{14})^{(1)} (T_{14}, t) \right) - (a_{14})^{(1)} (T_{14}, t) v_1^{(1)} - (a_{14})^{(1)} v_1^{(1)}
\]

\textbf{Definition of } v_1^{(1)}:\

\[
v_1^{(1)} = \frac{\bar{v}_1}{a_{14}}
\]

It follows

\[
- (a_{14})^{(1)} (v_1^{(1)})^2 + (\sigma_2)^{(1)} v_1^{(1)} - (a_{13})^{(1)} \leq \frac{dv_1^{(1)}}{dt} \leq - (a_{14})^{(1)} (v_1^{(1)})^2 + (\sigma_2)^{(1)} v_1^{(1)} - (a_{13})^{(1)}
\]

From which one obtains

\textbf{Definition of } (\bar{v}_1)^{(1)}, (v_0)^{(1)}:\

\[
(a) \text{ For } 0 < \left( \frac{(v_0)^{(1)}}{(v_1)^{(1)}} = \frac{\bar{v}_1}{a_{14}} \right) < (v_1)^{(1)} < (\bar{v}_1)^{(1)}
\]

\[
v_1^{(1)}(t) \geq \frac{\left( \frac{(v_0)^{(1)}}{(v_1)^{(1)}} + \frac{(v_0)^{(1)}}{(v_1)^{(1)}} \right) e^{-(a_{14})^{(1)} (v_1^{(1)} - (v_0)^{(1)}) t}}{1 + (C_1)^{(1)} e^{-(a_{14})^{(1)} (v_1^{(1)} - (v_0)^{(1)}) t}}
\]

\[
(C_1)^{(1)} = \frac{(v_1^{(1)} - (v_0)^{(1)})}{(v_0)^{(1)} - (v_2)^{(1)}}, \text{ it follows } (v_0)^{(1)} \leq v_1^{(1)}(t) \leq (v_1)^{(1)}
\]
In the same manner, we get

\[ v^{(1)}(t) \leq \frac{(v_1)^{(1)}(1)(v_2)^{(1)}(1)e^{-a(14)(t)(\bar{v}_1)^{(1)}(1)-v_2)^{(1)}(1)}]}{1+a(14)(t)(\bar{v}_1)^{(1)}(1)-v_2)^{(1)}(1)} \leq v^{(1)}(t) \leq \frac{(v_1)^{(1)}(1)(v_2)^{(1)}(1)e^{-a(14)(t)(\bar{v}_1)^{(1)}(1)-v_2)^{(1)}(1)}]}{1+a(14)(t)(\bar{v}_1)^{(1)}(1)-v_2)^{(1)}(1)} \leq (\bar{v}_1)^{(1)}(1) \]

From which we deduce \((v_0)^{(1)}(1) \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}(1)\)

(b) If \(0 < (v_1)^{(1)}(1) < (v_0)^{(1)}(1) = \frac{\alpha_2}{\sigma_2} < (\bar{v}_1)^{(1)}(1)\) we find like in the previous case,

\[ (v_1)^{(1)}(1) \leq \frac{(v_1)^{(1)}(1)(v_2)^{(1)}(1)e^{-a(14)(t)(\bar{v}_1)^{(1)}(1)-v_2)^{(1)}(1)}]}{1+a(14)(t)(\bar{v}_1)^{(1)}(1)-v_2)^{(1)}(1)} \leq v^{(1)}(t) \leq \frac{(v_1)^{(1)}(1)(v_2)^{(1)}(1)e^{-a(14)(t)(\bar{v}_1)^{(1)}(1)-v_2)^{(1)}(1)}]}{1+a(14)(t)(\bar{v}_1)^{(1)}(1)-v_2)^{(1)}(1)} \leq (v_0)^{(1)}(1) \]

(c) If \(0 < (v_1)^{(1)}(1) < (\bar{v}_1)^{(1)}(1)\), we obtain

\[ (v_1)^{(1)}(1) \leq v^{(1)}(t) \leq \frac{(v_1)^{(1)}(1)(v_2)^{(1)}(1)e^{-a(14)(t)(\bar{v}_1)^{(1)}(1)-v_2)^{(1)}(1)}]}{1+a(14)(t)(\bar{v}_1)^{(1)}(1)-v_2)^{(1)}(1)} \leq (v_0)^{(1)}(1) \]

And so with the notation of the first part of condition (c), we have

**Definition of** \(v^{(1)}(t)\):

\[ (m_2)^{(1)}(1) \leq v^{(1)}(t) \leq (m_1)^{(1)}(1), \quad v^{(1)}(t) = \frac{g_13(t)}{g_14(t)} \]

In a completely analogous way, we obtain

**Definition of** \(u^{(1)}(t)\):

\[ (\mu_2)^{(1)}(1) \leq u^{(1)}(t) \leq (\mu_1)^{(1)}(1), \quad u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)} \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case**:

If \((a_{13})^{(1)} = (a_{14})^{(1)}, then \(s_1)^{(1)} = (s_2)^{(1)}\) and in this case \((v_1)^{(1)} = (\bar{v}_1)^{(1)}\) if in addition \((v_0)^{(1)} = (v_1)^{(1)}\) then \(v^{(1)}(t) = (v_0)^{(1)}\) and as a consequence \(G_{13}(t) = (v_0)^{(1)}G_{14}(t)\) this also defines \((v_0)^{(1)}\) for the special case

Analogously if \((b_{11})^{(1)} = (b_{14})^{(1)}, then \(r_1)^{(1)} = (r_2)^{(1)}\) and then

\[ (u_1)^{(1)} = (\bar{u}_1)^{(1)}\] if in addition \((u_0)^{(1)} = (u_1)^{(1)}\) then \(T_{13}(t) = (u_0)^{(1)}T_{14}(t)\) This is an important consequence of the relation between \((v_1)^{(1)}\) and \((\bar{v}_1)^{(1)}\), and definition of \((u_0)^{(1)}\).

we obtain
\[ \frac{dv_2(t)}{dt} = (a_{16})^{(2)} - (a_{17})^{(2)} + (a_{16})^{(2)}(T_{17}, t) - \frac{(a_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}}{a_{17}} \]

**Definition of \( v^{(2)} \):**

\[ v^{(2)} = \frac{a_{16}}{a_{17}} \]

It follows

\[ -((a_{17})^{(2)}v^{(2)})^2 + \sigma_2^{(2)}v^{(2)} - (a_{16})^{(2)} \leq \frac{dv_2(t)}{dt} \leq -((a_{17})^{(2)}v^{(2)})^2 + \sigma_1^{(2)}v^{(2)} - (a_{16})^{(2)} \]

From which one obtains

**Definition of \( \bar{v}\) \( ^{(2)} \), \( v_0^{(2)} \):**

(d) For \( 0 < v_0^{(2)} < \frac{a_{16}}{a_{17}} < v_1^{(2)} < \bar{v}_1^{(2)} \)

\[ v^{(2)}(t) \leq \frac{(\bar{v}_1^{(2)} - v_0^{(2)})e^{-[(a_{17})^{(2)}(v_1^{(2)} - v_2^{(2)})]t}}{1 + (\bar{C})^{(2)}e^{-[(a_{17})^{(2)}(v_1^{(2)} - v_2^{(2)})]t}} \]

it follows \( v_0^{(2)} \leq v^{(2)}(t) \leq v_1^{(2)} \)

In the same manner, we get

\[ v^{(2)}(t) \leq \frac{(v_1^{(2)} - v_0^{(2)})e^{-[(a_{17})^{(2)}(v_1^{(2)} - v_2^{(2)})]t}}{1 + (\bar{C})^{(2)}e^{-[(a_{17})^{(2)}(v_1^{(2)} - v_2^{(2)})]t}} \]

From which we deduce \( v_0^{(2)} \leq v^{(2)}(t) \leq \bar{v}_1^{(2)} \)

(e) If \( 0 < v_1^{(2)} < \bar{v}_1^{(2)} = \frac{a_{16}}{a_{17}} \) we find like in the previous case,

\[ v_0^{(2)} \leq \frac{(v_1^{(2)} - v_0^{(2)})e^{-[(a_{17})^{(2)}(v_1^{(2)} - v_2^{(2)})]t}}{1 + (\bar{C})^{(2)}e^{-[(a_{17})^{(2)}(v_1^{(2)} - v_2^{(2)})]t}} \leq \bar{v}_1^{(2)} \]

(f) If \( 0 < v_0^{(2)} < \bar{v}_1^{(2)} \leq v_0^{(2)} = \frac{a_{16}}{a_{17}} \), we obtain

\[ v_1^{(2)} \leq v^{(2)}(t) \leq \frac{(v_1^{(2)} - v_0^{(2)})e^{-[(a_{17})^{(2)}(v_1^{(2)} - v_2^{(2)})]t}}{1 + (\bar{C})^{(2)}e^{-[(a_{17})^{(2)}(v_1^{(2)} - v_2^{(2)})]t}} \leq v_0^{(2)} \]

And so with the notation of the first part of condition (c), we have

**Definition of \( v^{(2)}(t) \):**

\[ (m_2^{(2)} \leq v^{(2)}(t) \leq m_1^{(2)}), \quad v^{(2)}(t) = \frac{a_{16}(t)}{a_{17}(t)} \]

In a completely analogous way, we obtain

**Definition of \( u^{(2)}(t) \):**

\[ (\mu_2^{(2)} \leq u^{(2)}(t) \leq \mu_1^{(2)}), \quad u^{(2)}(t) = \frac{a_{14}(t)}{a_{17}(t)} \]
From which we deduce

Definition of

It follows

Definition of

Analogously if

Then

Particular case:

If

then

and in this case

if in addition

then

and as a consequence

If

for

This is an important consequence of the relation between

and

From GLOBAL EQUATIONS we obtain

Definition of

It follows

From which one obtains

(a) For

it follows

In the same manner, we get

Definition of

From which we deduce

(b) If

we find like in the previous case,
(c) If \( 0 < (v_1)^{(3)} \leq (v_2)^{(3)} \leq (v_0)^{(3)} = \frac{G_0}{G_21} \), we obtain
\[
(v_1)^{(3)} \leq v^{(3)}(t) \leq \frac{((v_1)^{(3)} + (C)^{(3)}((v_2)^{(3)} - (v_0)^{(3)})) e^{-[(a_{23})^{(3)}(v_1)^{(3)} - (v_2)^{(3)})] t}}{1 + (C)^{(3)} e^{[(a_{23})^{(3)}(v_1)^{(3)} - (v_2)^{(3)})] t}} \leq (v_0)^{(3)}
\]
And so with the notation of the first part of condition (c), we have

**Definition of** \( v^{(3)}(t) : \)
\[
(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}, \quad v^{(3)}(t) = \frac{G_{20}(t)}{G_{221}(t)}
\]
In a completely analogous way, we obtain

**Definition of** \( u^{(3)}(t) : \)
\[
(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad u^{(3)}(t) = \frac{T_{20}(t)}{T_{221}(t)}
\]
Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case:
If \( (a_{20}^n)^{(3)} = (a_{21}^n)^{(3)} \), then \( (\sigma_1)^{(3)} = (\sigma_2)^{(3)} \) and in this case \( (v_1)^{(3)} = (v_2)^{(3)} \) if in addition \( (v_0)^{(3)} = (v_1)^{(3)} \) then \( v^{(3)}(t) = (v_0)^{(3)} \) and as a consequence \( G_{20}(t) = (v_0)^{(3)} G_{221}(t) \)

Analogously if \( (b_{20}^n)^{(3)} = (b_{21}^n)^{(3)} \), then \( (\tau_1)^{(3)} = (\tau_2)^{(3)} \) and then
\[
(u_1)^{(3)} = (u_2)^{(3)} \text{ if in addition } (u_0)^{(3)} = (u_1)^{(3)} \text{ then } T_{20}(t) = (u_0)^{(3)} T_{221}(t) \text{ This is an important}
\]

consequence of the relation between \( (v_1)^{(3)} \) and \( (\tilde{v}_1)^{(3)} \)

\[ : \text{From GLOBAL EQUATIONS we obtain} \]
\[
\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left( (a_{24}'')^{(4)} - (a_{25}'')^{(4)} + (a_{24}^{(4)})(T_{25}, t) \right) - (a_{25}'')^{(4)}(T_{25}, t) v^{(4)} - (a_{25})^{(4)} v^{(4)}
\]

**Definition of** \( v^{(4)} : \)
\[
v^{(4)} = \frac{G_{24}}{G_{25}}
\]
It follows
\[
-\left( (a_{25})^{(4)} v^{(4)} \right)^2 + (\sigma_2)^{(4)} v^{(4)} - (a_{24})^{(4)} \leq \frac{dv^{(4)}}{dt} \leq -\left( (a_{25})^{(4)} v^{(4)} \right)^2 + (\sigma_4)^{(4)} v^{(4)} - (a_{24})^{(4)}
\]
From which one obtains

**Definition of** \( (\tilde{v}_1)^{(4)}, (v_0)^{(4)} : \)
\[
v^{(4)}(t) \geq \frac{(v_0)^{(4)} + (C)^{(4)}((v_0)^{(4)} - (v_1)^{(4)})) e^{-[(a_{23})^{(4)}(v_1)^{(4)} - (v_2)^{(4)})] t}}{1 + (C)^{(4)} e^{[(a_{23})^{(4)}(v_1)^{(4)} - (v_2)^{(4)})] t}} \cdot \frac{(C)^{(4)}}{(v_0)^{(4)} - (v_2)^{(4)}}
\]
It follows \( (v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)} \)
In the same manner, we get
\[ \nu^{(4)}(t) \leq \frac{(\nu_2^{(4)} + (\tilde{C})^{(4)})(\nu_2^{(4)} - (\tilde{C})^{(4)})}{4 + (\tilde{C})^{(4)}} e^{-(a_2^{(4)})(\nu_1^{(4)} - (\tilde{C})^{(4)}) t} \]

From which we deduce \( \nu_0^{(4)} \leq \nu^{(4)}(t) \leq (\tilde{\nu}_1^{(4)}) \)

(e) If \( 0 < (\nu_1^{(4)}) < (\nu_0^{(4)}) = \frac{G_24}{G_{25}} < (\tilde{\nu}_1^{(4)}) \) we find like in the previous case,

\[ \frac{(\nu_1^{(4)})}{1 + (\tilde{C})^{(4)} e^{-(a_2^{(4)})(\nu_1^{(4)} - (\tilde{C})^{(4)}) t}} \leq \nu^{(4)}(t) \leq \frac{(\nu_2^{(4)} + (\tilde{C})^{(4)})(\nu_2^{(4)} - (\tilde{C})^{(4)})}{1 + (\tilde{C})^{(4)} e^{-(a_2^{(4)})(\nu_1^{(4)} - (\tilde{C})^{(4)}) t}} \]

(f) If \( 0 < (\nu_1^{(4)}) \leq (\tilde{\nu}_1^{(4)}) \leq (\nu_0^{(4)}) = \frac{G_24}{G_{25}} \), we obtain

\[ (\nu_1^{(4)}) \leq \nu^{(4)}(t) \leq \frac{(\nu_2^{(4)} + (\tilde{C})^{(4)})(\nu_2^{(4)} - (\tilde{C})^{(4)})}{1 + (\tilde{C})^{(4)} e^{-(a_2^{(4)})(\nu_1^{(4)} - (\tilde{C})^{(4)}) t}} \]

And so with the notation of the first part of condition (c), we have

**Definition of** \( \nu^{(4)}(t) \) :-

\[ (m_2^{(4)}) \leq \nu^{(4)}(t) \leq (m_4^{(4)}) \]

In a completely analogous way, we obtain

**Definition of** \( u^{(4)}(t) \) :-

\[ (\mu_2^{(4)}) \leq u^{(4)}(t) \leq (\mu_1^{(4)}) \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case** :

If \( (a_2^{(4)}) = (a_2^{(4)}) \), then \( (\nu_1^{(4)}) = (\nu_2^{(4)}) \) and in this case \( (\nu_1^{(4)}) = (\tilde{\nu}_1^{(4)}) \) if in addition \( (\nu_0^{(4)}) = (\nu_1^{(4)}) \) then \( \nu^{(4)}(t) = (\nu_0^{(4)}) \) and as a consequence \( G_24(t) = (\nu_0^{(4)}) G_{25}(t) \). **This also defines** \( (\nu_0^{(4)}) \) for the special case.

Analogously if \( (b_2^{(4)}) = (b_2^{(4)}) \), then \( (\tau_1^{(4)}) = (\tau_2^{(4)}) \) and then \( (\nu_1^{(4)}) = (\tilde{\nu}_1^{(4)}) \) if in addition \( (u_0^{(4)}) = (u_1^{(4)}) \) then \( T_24(t) = (u_0^{(4)}) T_{25}(t) \). This is an important consequence of the relation between \( (\nu_1^{(4)}) \) and \( (\tilde{\nu}_1^{(4)}) \), and **definition of** \( (u_0^{(4)}) \).

From GLOBAL EQUATIONS we obtain

\[ \frac{dv^{(5)}}{dt} = \left( a_{28}^{(5)} - \left( a_{28}^{(5)} - a_{29}^{(5)} + a_{28}^{(5)} T_{29} T_{29} \right) \right) - a_{29}^{(5)} T_{29} v^{(5)} - a_{28}^{(5)} v^{(5)} \]

**Definition of** \( v^{(5)} \) :-

\[ v^{(5)} = \frac{G_{29}}{G_{29}} \]

It follows

\[ -\left( a_{29}^{(5)} v^{(5)} \right)^2 + \left( \sigma_2^{(5)} v^{(5)} - a_{28}^{(5)} \right) \leq \frac{dv^{(5)}}{dt} \leq -\left( a_{29}^{(5)} \right)^2 + \left( \sigma_1^{(5)} v^{(5)} - a_{28}^{(5)} \right) \]

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Analogously if

Then

For

In a completely analogous way, we obtain

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(5)}(t)$ :.

\[
(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}, \quad v^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}
\]

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$ :.

\[
(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}
\]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{28}^{(5)}) = (a_{29}^{(5)})$, then $(\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(v_1)^{(5)} = (v_2)^{(5)}$ if in addition $(v_0)^{(5)} = (v_1)^{(5)}$, then $v^{(5)}(t) = (v_0)^{(5)}$ and as a consequence $G_{28}(t) = (v_0)^{(5)}G_{29}(t)$ this also defines $(v_0)^{(5)}$ for the special case.

Analogously if $(b_{28}^{(5)}) = (b_{29}^{(5)})$, then $(\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important
we obtain
\[
\frac{dv(6)}{dt} = (a_{32}(6) - (a_{32}(6) - (a_{32}(6) + (a_{32}(6)T_{33}, t)) - (a_{33}(6)T_{33}, t))v(6) - (a_{33}(6)v(6)
\]

**Definition of** \(v(6)\) :-
\[
v(6) = \frac{\sigma_{33}}{\alpha_{33}}
\]

It follows
\[
-((a_{33}(6)v(6)) + (a_{32}(6))v(6) - (a_{32}(6)) \leq \frac{dv(6)}{dt} \leq -((a_{33}(6)v(6)) + (a_{32}(6))v(6) - (a_{32}(6))
\]

From which one obtains

**Definition of** \((\bar{v}_1)(6), (v_0)(6)\) :-

(j) For \(0 < \frac{(v_0)(6)}{\sigma_{33}} < (v_1)(6) < (\bar{v}_1)(6)\)
\[
v(6)(t) \geq \frac{(v_1)(6) + \bar{v}(6)}{\pi(6)(v_1)(6) - (v_2)(6)} \leq \frac{(v_1)(6) - (v_0)(6)}{(v_1)(6) - (v_2)(6)}
\]

it follows \((v_0)(6) \leq v(6)(t) \leq (v_1)(6)\)

In the same manner, we get
\[
v(6)(t) \leq \frac{(v_1)(6) + \bar{v}(6)}{\pi(6)(v_1)(6) - (v_2)(6)} \leq \frac{(v_1)(6) - (v_0)(6)}{(v_1)(6) - (v_2)(6)}
\]

From which we deduce \((v_0)(6) \leq v(6)(t) \leq (\bar{v}_1)(6)\)

(k) If \(0 < (v_1)(6) < (v_0)(6) = \frac{\sigma_{33}}{\alpha_{33}} < (\bar{v}_1)(6)\) we find like in the previous case,
\[
(v_1)(6) \leq \frac{(v_1)(6) + \bar{v}(6)}{\pi(6)(v_1)(6) - (v_2)(6)} \leq \frac{(v_1)(6) - (v_0)(6)}{(v_1)(6) - (v_2)(6)}
\]

(l) If \(0 < (v_2)(6) \leq (\bar{v}_2)(6) \leq \frac{(v_0)(6)}{\sigma_{33}}\), we obtain
\[
(v_2)(6) \leq v(6)(t) \leq \frac{(v_1)(6) + \bar{v}(6)}{\pi(6)(v_1)(6) - (v_2)(6)} \leq \frac{(v_1)(6) - (v_0)(6)}{(v_1)(6) - (v_2)(6)}
\]

And so with the notation of the first part of condition (c), we have

**Definition of** \(v(6)(t)\) :-
\[
(m_2)(6) \leq v(6)(t) \leq (m_1)(6), \quad \frac{v(6)(t)}{\sigma_{33}(t)} = \frac{\sigma_{33}(t)}{\alpha_{33}(t)}
\]

In a completely analogous way, we obtain

**Definition of** \(u(6)(t)\) :-
\( (\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad u^{(6)}(t) = \frac{\tau_{32}(t)}{\tau_{33}(t)} \)

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If \( (a_{12}^{(6)} = (a_{33}^{(6)}, \text{ then } (\sigma_1)^{(6)} = (\sigma_2)^{(6)} \text{ and in this case } (v_1)^{(6)} = (\bar{\nu}_1)^{(6)} \text{ if in addition } v_0)^{(6)} = (v_0)^{(6}) \text{ then } v^{(6)}(t) = (v_0)^{(6)} \text{ and as a consequence } \sigma_{32}(t) = (v_0)^{(6)} \sigma_{33}(t) \text{ this also defines } (v_0)^{(6)} \text{ for the special case .}

Analogously if \( (b_{12}^{(6)} = (b_{33}^{(6)}), \text{ then } (\tau_1)^{(6)} = (\tau_2)^{(6)} \text{ and then } (u_1)^{(6)} = (\bar{u}_1)^{(6)} \text{ if in addition } (u_0)^{(6)} = (u_0)^{(6}) \text{ then } \tau_{32}(t) = (u_0)^{(6)} \tau_{33}(t) \text{ This is an important consequence of the relation between } (v_1)^{(6)} \text{ and } (\bar{v}_1)^{(6)}, \text{ and definition of } (u_0)^{(6)}.\)

We can prove the following

**Theorem 3:** If \( (a_i^{(1)}) \text{ and } (b_i^{(1)}) \text{ are independent on } t, \text{ and the conditions}

\[
(a_{13}^{(1)} a_{14}^{(1)} - (a_{12}^{(1)})(a_{14})^{(1)}) < 0
\]

\[
(a_{13}^{(1)} a_{14}^{(1)} - (a_{12}^{(1)})(a_{14})^{(1)} + (a_{13}^{(1)})(p_{13})^{(1)} + (a_{14}^{(1)})(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)}) > 0
\]

\[
(b_{13}^{(1)} b_{14}^{(1)} - (b_{12}^{(1)})(b_{14})^{(1)}) > 0 ,
\]

\[
(b_{13}^{(1)} b_{14}^{(1)} - (b_{12}^{(1)})(b_{14})^{(1)} - (b_{13})^{(1)}(r_{13})^{(1)} - (b_{14})^{(1)}(r_{14})^{(1)} + (r_{12})^{(1)}(r_{14})^{(1)}) < 0
\]

with \( (p_{12})^{(1)}, (r_{14})^{(1}) \text{ as defined, then the system}

If \( (a_i^{(2)}) \text{ and } (b_i^{(2)}) \text{ are independent on } t, \text{ and the conditions}

\[
(a_{16}^{(2)} a_{17}^{(2)} - (a_{16}^{(2)})(a_{17})^{(2)}) < 0
\]

\[
(a_{16}^{(2)} a_{17}^{(2)} - (a_{16}^{(2)})(a_{17})^{(2)} + (a_{16}^{(2)})(p_{16})^{(2)} + (a_{16}^{(2)})(p_{16})^{(2)} + (a_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)}) > 0
\]

\[
(b_{16}^{(2)} b_{17}^{(2)} - (b_{16}^{(2)})(b_{17})^{(2)}) > 0 ,
\]

\[
(b_{16}^{(2)} b_{17}^{(2)} - (b_{16}^{(2)})(b_{17})^{(2)} - (b_{16}^{(2)})(r_{16})^{(2)} - (b_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0
\]

with \( (p_{16})^{(2)}, (r_{17})^{(2)} \text{ as defined are satisfied , then the system}

If \( (a_i^{(3)}) \text{ and } (b_i^{(3)}) \text{ are independent on } t, \text{ and the conditions}

\[
(a_{20}^{(3)} a_{21}^{(3)} - (a_{20}^{(3)})(a_{21})^{(3)}) < 0
\]

\[
(a_{20}^{(3)} a_{21}^{(3)} - (a_{20}^{(3)})(a_{21})^{(3)} + (a_{20}^{(3)})(p_{20})^{(3)} + (a_{21}^{(3)})(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)}) > 0
\]

\[
(b_{20}^{(3)} b_{21}^{(3)} - (b_{20}^{(3)})(b_{21})^{(3)}) > 0 ,
\]

\[
(b_{20}^{(3)} b_{21}^{(3)} - (b_{20}^{(3)})(b_{21})^{(3)} - (b_{20}^{(3)})(r_{20})^{(3)} - (b_{21}^{(3)})(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0
\]

with \( (p_{20})^{(3)}, (r_{21})^{(3)} \text{ as defined are satisfied , then the system}

If \( (a_i^{(4)}) \text{ and } (b_i^{(4)}) \text{ are independent on } t, \text{ and the conditions}
(\(a'_{24}(4)\))(\(a'_{25}(4)\)) - (\(a_{24}(4)\))(\(a_{25}(4)\)) < 0

(\(a'_{24}(5)\))(\(a'_{25}(5)\)) - (\(a_{24}(5)\))(\(a_{25}(5)\)) + (\(a_{24}(4)\))(\(p_{24}(4)\)) + (\(a'_{25}(4)\))(\(p_{25}(4)\)) + (\(p_{24}(4)\))(\(p_{25}(4)\)) > 0

(\(b'_{24}(4)\))(\(b'_{25}(4)\)) - (\(b_{24}(4)\))(\(b_{25}(4)\)) > 0,

(\(b'_{24}(5)\))(\(b'_{25}(5)\)) - (\(b_{24}(5)\))(\(b_{25}(5)\)) - (\(b_{24}(4)\))(\(r_{25}(4)\)) - (\(b'_{25}(4)\))(\(r_{25}(4)\)) + (\(r_{24}(4)\))(\(r_{25}(4)\)) < 0

with (\(p_{24}(4)\))(\(r_{25}(4)\)) as defined are satisfied, then the system

If \((a''(5))\) and \((b''(5))\) are independent on \(t\), and the conditions

(\(a'_{28}(5)\))(\(a'_{29}(5)\)) - (\(a_{28}(5)\))(\(a_{29}(5)\)) < 0

(\(a'_{28}(5)\))(\(a''_{29}(5)\)) - (\(a_{28}(5)\))(\(a_{29}(5)\)) + (\(a_{28}(5)\))(\(p_{28}(5)\)) + (\(a'_{29}(5)\))(\(p_{29}(5)\)) + (\(p_{28}(5)\))(\(p_{29}(5)\)) > 0

(\(b'_{28}(5)\))(\(b'_{29}(5)\)) - (\(b_{28}(5)\))(\(b_{29}(5)\)) > 0,

(\(b'_{28}(5)\))(\(b''_{29}(5)\)) - (\(b_{28}(5)\))(\(b_{29}(5)\)) - (\(b_{28}(5)\))(\(r_{29}(5)\)) - (\(b'_{29}(5)\))(\(r_{29}(5)\)) + (\(r_{28}(5)\))(\(r_{29}(5)\)) < 0

with (\(p_{28}(5)\))(\(r_{29}(5)\)) as defined are satisfied, then the system

If \((a''(6))\) and \((b''(6))\) are independent on \(t\), and the conditions

(\(a'_{32}(6)\))(\(a'_{33}(6)\)) - (\(a_{32}(6)\))(\(a_{33}(6)\)) < 0

(\(a'_{32}(6)\))(\(a''_{33}(6)\)) - (\(a_{32}(6)\))(\(a_{33}(6)\)) + (\(a_{32}(6)\))(\(p_{32}(6)\)) + (\(a'_{33}(6)\))(\(p_{33}(6)\)) + (\(p_{32}(6)\))(\(p_{33}(6)\)) > 0

(\(b'_{32}(6)\))(\(b'_{33}(6)\)) - (\(b_{32}(6)\))(\(b_{33}(6)\)) > 0,

(\(b'_{32}(6)\))(\(b''_{33}(6)\)) - (\(b_{32}(6)\))(\(b_{33}(6)\)) - (\(b_{32}(6)\))(\(r_{33}(6)\)) - (\(b'_{33}(6)\))(\(r_{33}(6)\)) + (\(r_{32}(6)\))(\(r_{33}(6)\)) < 0

with \((p_{32}(6)\))(\(r_{33}(6)\)) as defined are satisfied, then the system

\(a_{13}(1)G_{14} - [a_{13}(1) + (a''_{13}(1))(T_{14})]G_{13} = 0\)

\(a_{14}(1)G_{13} - [(a_{14}(1) + (a''_{14}(1))(T_{14})]G_{14} = 0\)

\(a_{15}(1)G_{14} - [(a'_{15}(1) + (a''_{15}(1))(T_{14})]G_{15} = 0\)

\(b_{13}(1)T_{14} - [(b'_{13}(1) - (b''_{13}(1))(G)]T_{13} = 0\)

\(b_{14}(1)T_{13} - [(b_{14}(1) - (b''_{14}(1))(G)]T_{14} = 0\)

\(b_{15}(1)T_{14} - [(b'_{15}(1) - (b''_{15}(1))(G)]T_{15} = 0\)

has a unique positive solution, which is an equilibrium solution for the system

\(a_{16}(2)G_{17} - [(a_{16}(2) + (a''_{16}(2))(T_{17})]G_{16} = 0\)

\(a_{17}(2)G_{16} - [(a_{17}(2) + (a''_{17}(2))(T_{17})]G_{17} = 0\)

\(a_{18}(2)G_{17} - [(a'_{18}(2) + (a''_{18}(2))(T_{17})]G_{18} = 0\)

\(b_{16}(2)T_{17} - [(b'_{16}(2) - (b''_{16}(2))(G_{19})]T_{16} = 0\)
has a unique positive solution, which is an equilibrium solution for

\[(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})]T_{17} = 0\]

\[(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})]T_{18} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})]G_{20} = 0\]

\[(a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})]G_{21} = 0\]

\[(a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})]G_{22} = 0\]

\[(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})]T_{20} = 0\]

\[(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})]T_{21} = 0\]

\[(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})]T_{22} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})]G_{24} = 0\]

\[(a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})]G_{25} = 0\]

\[(a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})]G_{26} = 0\]

\[(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27})]T_{24} = 0\]

\[(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27})]T_{25} = 0\]

\[(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27})]T_{26} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})]G_{28} = 0\]

\[(a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})]G_{29} = 0\]

\[(a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})]G_{30} = 0\]

\[(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0\]

\[(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0\]

\[(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})]G_{32} = 0\]
(a) Indeed the first two equations have a nontrivial solution \( G_{13}, G_{14} \) if
\[
F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0
\]

(b) Indeed the first two equations have a nontrivial solution \( G_{16}, G_{17} \) if
\[
F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0
\]

(c) Indeed the first two equations have a nontrivial solution \( G_{20}, G_{21} \) if
\[
F(T_{23}) = (a''_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0
\]

(d) Indeed the first two equations have a nontrivial solution \( G_{24}, G_{25} \) if
\[
F(T_{27}) = (a''_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a''_{24})^{(4)}(a'_{25})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0
\]

(e) Indeed the first two equations have a nontrivial solution \( G_{28}, G_{29} \) if
\[
F(T_{31}) = (a''_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a''_{28})^{(5)}(a'_{29})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0
\]

(f) Indeed the first two equations have a nontrivial solution \( G_{32}, G_{33} \) if
\[
F(T_{35}) =
\]
\[(a_{32})^6(a_{33})^6 - (a_{32})^6(a_{33})^6 + (a_{32})^6(a_{33})^6(T_{33}) + (a_{32})^6(a_{33})^6(T_{33}) + (a_{32})^6(T_{33})(a_{33})^6(T_{33}) = 0 \]

**Definition and uniqueness of** \(T_{14}^\ast \):-

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \((a_{14}''')^{(1)}(T_{14})\) being increasing, it follows that there exists a unique \( T_{14}^\ast \) for which \( f(T_{14}^\ast) = 0 \). With this value, we obtain from the first three equations

\[ G_{13} = \frac{(a_{13})^3G_{14}}{[(a_{13})^3(a_{14})^3](T_{14})} \quad G_{15} = \frac{(a_{15})^3G_{14}}{[(a_{15})^3(a_{14})^3](T_{14})} \]

**Definition and uniqueness of** \(T_{17}^\ast \):-

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \((a_{14}''')^{(2)}(T_{17})\) being increasing, it follows that there exists a unique \( T_{17}^\ast \) for which \( f(T_{17}^\ast) = 0 \). With this value, we obtain from the first three equations

\[ G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a_{16})^{(2)}(a_{17})^{(2)}](T_{17})} \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a_{18})^{(2)}(a_{17})^{(2)}](T_{17})} \]

**Definition and uniqueness of** \(T_{21}^\ast \):-

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \((a_{14}''')^{(1)}(T_{21})\) being increasing, it follows that there exists a unique \( T_{21}^\ast \) for which \( f(T_{21}^\ast) = 0 \). With this value, we obtain from the first three equations

\[ G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a_{20})^{(3)}(a_{21})^{(3)}](T_{21})} \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a_{22})^{(3)}(a_{21})^{(3)}](T_{21})} \]

**Definition and uniqueness of** \(T_{25}^\ast \):-

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \((a_{14}''')^{(4)}(T_{25})\) being increasing, it follows that there exists a unique \( T_{25}^\ast \) for which \( f(T_{25}^\ast) = 0 \). With this value, we obtain from the first three equations

\[ G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a_{24})^{(4)}(a_{25})^{(4)}](T_{25})} \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a_{26})^{(4)}(a_{25})^{(4)}](T_{25})} \]

**Definition and uniqueness of** \(T_{29}^\ast \):-

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \((a_{14}''')^{(5)}(T_{29})\) being increasing, it follows that there exists a unique \( T_{29}^\ast \) for which \( f(T_{29}^\ast) = 0 \). With this value, we obtain from the first three equations

\[ G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a_{28})^{(5)}(a_{29})^{(5)}](T_{29})} \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a_{30})^{(5)}(a_{29})^{(5)}](T_{29})} \]

**Definition and uniqueness of** \(T_{33}^\ast \):-

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \((a_{14}''')^{(6)}(T_{33})\) being increasing, it follows that there exists a unique \( T_{33}^\ast \) for which \( f(T_{33}^\ast) = 0 \). With this value, we obtain from the first three equations

\[ G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a_{32})^{(6)}(a_{33})^{(6)}](T_{33})} \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a_{34})^{(6)}(a_{33})^{(6)}](T_{33})} \]

(e) By the same argument, the equations 92, 93 admit solutions \(G_{13}, G_{14}\) if

\[ \varphi(G) = (b_{13})^{(1)}(b_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - \]

\[ [(b_{13})^{(1)}(b_{14})^{(1)}(G) + (b_{14})^{(1)}(b_{13})^{(1)}(G)] + (b_{13})^{(1)}(G)(b_{14})^{(1)}(G) = 0 \]

Where in \( G(G_{13}, G_{14}, G_{15}) \), \( G_{13}, G_{15} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{14} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there
exists a unique $G_{14}^*$ such that $\varphi(G^*) = 0$

(f) By the same argument, the equations 92, 93 admit solutions $G_{16}, G_{17}$ if

$$\varphi(G_{19}) = (b_{16}')(2)(b_{17}')(2) - (b_{16})(2)(b_{17})(2) - $$

$$[b_{16}')(2)(b_{17}')(2)(G_{19}) + (b_{17}')(2)(b_{16})(2)(G_{19})] + (b_{16}')(2)(G_{19})(b_{17}')(2)(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^*$ such that $\varphi((G_{19})') = 0$

(g) By the same argument, the concatenated equations admit solutions $G_{20}, G_{21}$ if

$$\varphi(G_{23}) = (b_{20})(3)(b_{21})(3) - (b_{20})(3)(b_{21})(3) - $$

$$[b_{20}')(3)(b_{21}')(3)(G_{23}) + (b_{21}')(3)(b_{20})(3)(G_{23})] + (b_{20}')(3)(G_{23})(b_{21}')(3)(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{23}$ taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique $G_{23}^*$ such that $\varphi((G_{23})') = 0$

(h) By the same argument, the equations of modules admit solutions $G_{24}, G_{25}$ if

$$\varphi(G_{27}) = (b_{24})(4)(b_{25})(4) - (b_{24})(4)(b_{25})(4) - $$

$$[b_{24}')(4)(b_{25}')(4)(G_{27}) + (b_{25}')(4)(b_{24})(4)(G_{27})] + (b_{24}')(4)(G_{27})(b_{25}')(4)(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}, G_{24}, G_{26})$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{27}$ taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^*$ such that $\varphi((G_{27})') = 0$

(i) By the same argument, the equations (modules) admit solutions $G_{26}, G_{29}$ if

$$\varphi(G_{31}) = (b_{29})(5)(b_{28})(5) - (b_{29})(5)(b_{28})(5) - $$

$$[b_{29}')(5)(b_{28}')(5)(G_{31}) + (b_{28}')(5)(b_{29})(5)(G_{31})] + (b_{29}')(5)(G_{31})(b_{28}')(5)(G_{31}) = 0$$

Where in $(G_{31})(G_{29}, G_{29}, G_{30}, G_{29}, G_{30})$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique $G_{26}^*$ such that $\varphi((G_{31})') = 0$

(j) By the same argument, the equations (modules) admit solutions $G_{32}, G_{33}$ if

$$\varphi(G_{33}) = (b_{32})(6)(b_{33})(6) - (b_{32})(6)(b_{33})(6) - $$

$$[b_{32}')(6)(b_{33}')(6)(G_{33}) + (b_{33}')(6)(b_{32})(6)(G_{33})] + (b_{32}')(6)(G_{33})(b_{33}')(6)(G_{33}) = 0$$

Where in $(G_{33})(G_{32}, G_{33}, G_{34}, G_{32}, G_{34})$ must be replaced by their values It is easy to see that $\varphi$ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^*$ such that $\varphi((G_{33})') = 0$

Finally we obtain the unique solution of 89 to 94
Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{14}^* \] given by \( \varphi(G^*) = 0 \), \( T_{14}^* \) given by \( f(T_{14}^*) = 0 \) and

\[
G_{13}^* = \frac{\left( a_{13} \right)^{(3)} G_{13}^*}{\left[ a_{13}^{(2)} + (a_{13}^{(2)} T_{14}^*) \right]}, \quad G_{15}^* = \frac{\left( a_{13} \right)^{(4)} G_{15}^*}{\left[ a_{15}^{(3)} + (a_{15}^{(3)} T_{16}^*) \right]}
\]

\[
T_{13}^* = \frac{\left( b_{13} \right)^{(3)} T_{16}^*}{\left[ b_{13}^{(3)} - (b_{13}^{(3)} G^*) \right]}, \quad T_{15}^* = \frac{\left( b_{13} \right)^{(4)} T_{16}^*}{\left[ b_{13}^{(4)} - (b_{13}^{(4)} G^*) \right]}
\]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{17}^* \] given by \( \varphi(G_{17}^*) = 0 \), \( T_{17}^* \) given by \( f(T_{17}^*) = 0 \) and

\[
G_{16}^* = \frac{\left( a_{16} \right)^{(2)} G_{16}^*}{\left[ a_{16}^{(2)} + (a_{16}^{(2)} T_{17}^*) \right]}, \quad G_{18}^* = \frac{\left( a_{18} \right)^{(2)} G_{18}^*}{\left[ a_{18}^{(2)} + (a_{18}^{(2)} T_{17}^*) \right]}
\]

\[
T_{16}^* = \frac{\left( b_{16} \right)^{(2)} T_{17}^*}{\left[ b_{16}^{(2)} - (b_{16}^{(2)} G_{19}^*) \right]}, \quad T_{18}^* = \frac{\left( b_{18} \right)^{(2)} T_{17}^*}{\left[ b_{18}^{(2)} - (b_{18}^{(2)} G_{19}^*) \right]}
\]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{21}^* \] given by \( \varphi(G_{21}^*) = 0 \), \( T_{21}^* \) given by \( f(T_{21}^*) = 0 \) and

\[
G_{20}^* = \frac{\left( a_{20} \right)^{(3)} G_{20}^*}{\left[ a_{20}^{(3)} + (a_{20}^{(3)} T_{21}^*) \right]}, \quad G_{22}^* = \frac{\left( a_{22} \right)^{(3)} G_{22}^*}{\left[ a_{22}^{(3)} + (a_{22}^{(3)} T_{21}^*) \right]}
\]

\[
T_{20}^* = \frac{\left( b_{20} \right)^{(3)} T_{21}^*}{\left[ b_{20}^{(3)} - (b_{20}^{(3)} G_{23}^*) \right]}, \quad T_{22}^* = \frac{\left( b_{22} \right)^{(3)} T_{21}^*}{\left[ b_{22}^{(3)} - (b_{22}^{(3)} G_{23}^*) \right]}
\]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{25}^* \] given by \( \varphi(G_{25}^*) = 0 \), \( T_{25}^* \) given by \( f(T_{25}^*) = 0 \) and

\[
G_{24}^* = \frac{\left( a_{24} \right)^{(4)} G_{24}^*}{\left[ a_{24}^{(4)} + (a_{24}^{(4)} T_{25}^*) \right]}, \quad G_{26}^* = \frac{\left( a_{26} \right)^{(4)} G_{26}^*}{\left[ a_{26}^{(4)} + (a_{26}^{(4)} T_{25}^*) \right]}
\]

\[
T_{24}^* = \frac{\left( b_{24} \right)^{(4)} T_{25}^*}{\left[ b_{24}^{(4)} - (b_{24}^{(4)} G_{27}^*) \right]}, \quad T_{26}^* = \frac{\left( b_{26} \right)^{(4)} T_{25}^*}{\left[ b_{26}^{(4)} - (b_{26}^{(4)} G_{27}^*) \right]}
\]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{29}^* \] given by \( \varphi(G_{29}^*) = 0 \), \( T_{29}^* \) given by \( f(T_{29}^*) = 0 \) and

\[
G_{28}^* = \frac{\left( a_{28} \right)^{(5)} G_{28}^*}{\left[ a_{28}^{(5)} + (a_{28}^{(5)} T_{29}^*) \right]}, \quad G_{30}^* = \frac{\left( a_{30} \right)^{(5)} G_{30}^*}{\left[ a_{30}^{(5)} + (a_{30}^{(5)} T_{29}^*) \right]}
\]

\[
T_{28}^* = \frac{\left( b_{28} \right)^{(5)} T_{29}^*}{\left[ b_{28}^{(5)} - (b_{28}^{(5)} G_{31}^*) \right]}, \quad T_{30}^* = \frac{\left( b_{30} \right)^{(5)} T_{29}^*}{\left[ b_{30}^{(5)} - (b_{30}^{(5)} G_{31}^*) \right]}
\]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution
Obviously, these values represent an equilibrium solution.

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions \((a_{ii}^{(r)})^{(1)}\) and \((b_{ii}^{(r)})^{(1)}\) belong to \(C^{(1)}(\mathbb{R}^+)\) then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** \(G_i, T_i\) :-

\[
G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i
\]

\[
\frac{\partial (a_{ii}^{(r)})^{(1)}}{\partial r_{ij}} \left(T_{14}^{ij}\right) = (q_{14})^{(1)} \quad , \quad \frac{\partial (b_{ii}^{(r)})^{(1)}}{\partial G_j} \left(G_j^*\right) = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{13}}{dt} = -((a_{13}^{(1)} + (p_{13})^{(1)})G_{13} + (a_{13}^{(1)})G_{14} - (q_{13})^{(1)}G_{13}T_{14}
\]

\[
\frac{dG_{14}}{dt} = -((a_{14}^{(1)} + (p_{14})^{(1)})G_{14} + (a_{14}^{(1)})G_{13} - (q_{14})^{(1)}G_{14}T_{14}
\]

\[
\frac{dG_{15}}{dt} = -((a_{15}^{(1)} + (p_{15})^{(1)})G_{15} + (a_{15}^{(1)})G_{14} - (q_{15})^{(1)}G_{15}T_{14}
\]

\[
\frac{dT_{13}}{dt} = -((b_{13}^{(1)} - (r_{13})^{(1)})T_{13} + (b_{13}^{(1)})T_{14} + \sum_{j=13}^{15}(s_{j13}(j)T_{14}^*G_j)
\]

\[
\frac{dT_{14}}{dt} = -((b_{14}^{(1)} - (r_{14})^{(1)})T_{14} + (b_{14}^{(1)})T_{13} + \sum_{j=13}^{15}(s_{j14}(j)T_{14}^*G_j)
\]

\[
\frac{dT_{15}}{dt} = -((b_{15}^{(1)} - (r_{15})^{(1)})T_{15} + (b_{15}^{(1)})T_{14} + \sum_{j=13}^{15}(s_{j15}(j)T_{15}^*G_j)
\]

If the conditions of the previous theorem are satisfied and if the functions \((a_{ii}^{(r)})^{(2)}\) and \((b_{ii}^{(r)})^{(2)}\) belong to \(C^{(2)}(\mathbb{R}^+)\) then the above equilibrium point is asymptotically stable.

**Denote**

**Definition of** \(G_i, T_i\) :-

\[
G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i
\]

\[
\frac{\partial (a_{ii}^{(r)})^{(2)}}{\partial r_{ij}} \left(T_{17}^{ij}\right) = (q_{17})^{(2)} \quad , \quad \frac{\partial (b_{ii}^{(r)})^{(2)}}{\partial G_j} \left(G_j^*\right) = s_{ij}
\]

Taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{16}}{dt} = -((a_{16}^{(1)} + (p_{16})^{(1)})G_{16} + (a_{16}^{(2)})G_{17} - (q_{16})^{(2)}G_{16}T_{17}
\]

\[
\frac{dG_{17}}{dt} = -((a_{17}^{(1)} + (p_{17})^{(1)})G_{17} + (a_{17}^{(2)})G_{16} - (q_{17})^{(2)}G_{17}T_{17}
\]
If the conditions of the previous theorem are satisfied and if the functions \((a'_i)^{(3)}\) and \((b'_i)^{(3)}\) Belong to \(C^{(3)}(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

Denote

**Definition of \(G_i, T_i \):**

\[
G_i = G^*_i + G_i, \quad T_i = T^*_i + T_i
\]

\[
\frac{\partial (a'_i)^{(3)}}{\partial \mathcal{P}_21}(T^*_i) = (q_{21})^{(3)}, \quad \frac{\partial (b'_i)^{(3)}}{\partial \mathcal{G}_j}(G_{27})^* = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{20}}{dt} = -((a'_i)^{(3)} + (p_{20})^{(3)})G_{20} + (a_{20})^{(3)}G_{21} - (q_{20})^{(3)}G^*_i T_{21}
\]

\[
\frac{dG_{21}}{dt} = -((a'_i)^{(3)} + (p_{21})^{(3)})G_{21} + (a_{21})^{(3)}G_{20} - (q_{21})^{(3)}G^*_i T_{21}
\]

\[
\frac{dG_{22}}{dt} = -((a'_i)^{(3)} + (p_{22})^{(3)})G_{22} + (a_{22})^{(3)}G_{21} - (q_{22})^{(3)}G^*_i T_{21}
\]

\[
\frac{dT_{20}}{dt} = -((b'_i)^{(3)} - (r_{20})^{(3)})T_{20} + (b_{20})^{(3)}T_{21} + \sum_{j=20}^{21} (s_{20}(j), T_{20}) G_{21}
\]

\[
\frac{dT_{21}}{dt} = -((b'_i)^{(3)} - (r_{21})^{(3)})T_{21} + (b_{21})^{(3)}T_{20} + \sum_{j=20}^{21} (s_{21}(j), T_{21}) G_{21}
\]

\[
\frac{dT_{22}}{dt} = -((b'_i)^{(3)} - (r_{22})^{(3)})T_{22} + (b_{22})^{(3)}T_{21} + \sum_{j=20}^{21} (s_{22}(j), T_{22}) G_{21}
\]

If the conditions of the previous theorem are satisfied and if the functions \((a''_i)^{(4)}\) and \((b''_i)^{(4)}\) Belong to \(C^{(4)}(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

Denote

**Definition of \(G_i, T_i \):**

\[
G_i = G^*_i + G_i, \quad T_i = T^*_i + T_i
\]

\[
\frac{\partial (a''_i)^{(4)}}{\partial \mathcal{P}_25}(T^*_i) = (q_{25})^{(4)}, \quad \frac{\partial (b''_i)^{(4)}}{\partial \mathcal{G}_j}(G_{27})^* = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{24}}{dt} = -((a''_i)^{(4)} + (p_{24})^{(4)})G_{24} + (a_{24})^{(4)}G_{25} - (q_{24})^{(4)}G^*_i T_{25}
\]

\[
\frac{dG_{25}}{dt} = -((a''_i)^{(4)} + (p_{25})^{(4)})G_{25} + (a_{25})^{(4)}G_{24} - (q_{25})^{(4)}G^*_i T_{25}
\]
If the conditions of the previous theorem are satisfied and if the functions \( (a_i''(5)) \) and \( (b_i''(5)) \) Belong to \( C^{(5)}(\mathbb{R}_+) \) then the above equilibrium point is asymptotically stable

Denote

**Definition of \( G_i, T_i \):**

\[
G_i = G_i' + G_i , \quad T_i = T_i' + T_i
\]

\[
\frac{\partial (a_i''(5)(T_{29}))}{\partial T_{29}} = (q_{29})^{(5)} , \quad \frac{\partial (b_i''(5)(G_{33}))}{\partial G_i} ( (G_{33})^* ) = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{28}}{dt} = -((a_2''(5) + (p_{28})^{(5)})G_{28} + (a_{28})^{(5)}G_{29} - (q_{28})^{(5)}G_{28}^* T_{29}
\]

\[
\frac{dG_{29}}{dt} = -((a_2''(5) + (p_{29})^{(5)})G_{29} + (a_{29})^{(5)}G_{28} - (q_{29})^{(5)}G_{29}^* T_{29}
\]

\[
\frac{dG_{30}}{dt} = -((a_3''(5) + (p_{30})^{(5)})G_{30} + (a_{30})^{(5)}G_{29} - (q_{30})^{(5)}G_{30}^* T_{29}
\]

\[
\frac{dT_{28}}{dt} = -((b_2''(5) - (r_{28})^{(5)})T_{28} + (b_{28})^{(5)}T_{29} + \sum_{j=28}^{30}(s_{28}(j)T_{29}^* G_j)
\]

\[
\frac{dT_{29}}{dt} = -((b_2''(5) - (r_{29})^{(5)})T_{29} + (b_{29})^{(5)}T_{28} + \sum_{j=29}^{30}(s_{29}(j)T_{29}^* G_j)
\]

\[
\frac{dT_{30}}{dt} = -((b_3''(5) - (r_{30})^{(5)})T_{30} + (b_{30})^{(5)}T_{29} + \sum_{j=30}^{30}(s_{30}(j)T_{30}^* G_j)
\]

If the conditions of the previous theorem are satisfied and if the functions \( (a_i''(6)) \) and \( (b_i''(6)) \) Belong to \( C^{(6)}(\mathbb{R}_+) \) then the above equilibrium point is asymptotically stable

Denote

**Definition of \( G_i, T_i \):**

\[
G_i = G_i' + G_i , \quad T_i = T_i' + T_i
\]

\[
\frac{\partial (a_i''(6)(T_{33}))}{\partial T_{33}} = (q_{33})^{(6)} , \quad \frac{\partial (b_i''(6)(G_{33}))}{\partial G_i} ( (G_{33})^* ) = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{32}}{dt} = -((a_{32}''(6) + (p_{32})^{(6)})G_{32} + (a_{32})^{(6)}G_{33} - (q_{32})^{(6)}G_{32}^* T_{33}
\]
\[ \frac{dG_{33}}{dt} = -(a_{33})^6 + (p_{33})^6 - (q_{33})^6 \cdot G_{33}^* \cdot T_{33} \]
\[ \frac{dG_{34}}{dt} = -(a_{34})^6 + (p_{34})^6 - (q_{34})^6 \cdot G_{34}^* \cdot T_{33} \]
\[ \frac{dT_{32}}{dt} = -(b_{32})^6 - (r_{32})^6 \cdot T_{32} + (b_{32})^6 \cdot T_{33} + \sum_{j=32}^{a_{32}} (a_{32})^j \cdot T_{32}^* \cdot G_j \]
\[ \frac{dT_{33}}{dt} = -(b_{33})^6 - (r_{33})^6 \cdot T_{33} + (b_{33})^6 \cdot T_{33} + \sum_{j=33}^{a_{33}} (a_{33})^j \cdot T_{33}^* \cdot G_j \]
\[ \frac{dT_{34}}{dt} = -(b_{34})^6 - (r_{34})^6 \cdot T_{34} + (b_{34})^6 \cdot T_{34} + \sum_{j=34}^{a_{34}} (a_{34})^j \cdot T_{34}^* \cdot G_j \]

The characteristic equation of this system is
\[ \left( (\lambda)^1 + (b_{13})^1 - (r_{13})^1 \right) \left( (\lambda)^1 + (a^1_{15}) + (p_{15})^1 \right) \]
\[ \left( (\lambda)^1 + (a^1_{13}) + (p_{13})^1 \right) \left( (\lambda)^1 + (a^1_{14}) + (p_{14})^1 \right) \]
\[ \left( (\lambda)^1 + (b^1_{13}) - (r_{13})^1 \right) S_{(14),(14)} T_{14} + \left( b_{14}^1 \right) S_{(13),(13)} T_{14} \]
\[ \left( (\lambda)^1 + (a^1_{14}) + (p_{14})^1 \right) \left( q_{13}^1 \right) G_{13}^* + \left( a_{13}^1 \right) \left( q_{14}^1 \right) G_{14}^* \]
\[ \left( (\lambda)^1 + (b^1_{13}) - (r_{13})^1 \right) S_{(14),(14)} T_{13} + \left( b_{14}^1 \right) S_{(13),(13)} T_{13} \]
\[ \left( (\lambda)^1 \right)^2 + \left( a^1_{13} \right) + \left( a^1_{14} \right) + \left( p_{13}^1 \right) + \left( p_{14}^1 \right) \left( \lambda \right) \]
\[ \left( (\lambda)^1 \right)^2 + \left( b^1_{13} \right) - \left( r_{13} \right) + \left( r_{14} \right) \left( \lambda \right) \]
\[ \left( (\lambda)^1 \right)^2 + \left( a^1_{13} \right) + \left( a^1_{14} \right) + \left( p_{13}^1 \right) + \left( p_{14}^1 \right) \left( \lambda \right) \left( q_{13} \right) \left( G_{13}^* \right) \]
\[ \left( (\lambda)^1 + (b^1_{13}) - (r_{13})^1 \right) S_{(14),(15)} T_{14} + \left( b_{14}^1 \right) S_{(13),(15)} T_{13} \]
\[ \left( (\lambda)^1 \right)^2 + \left( a^1_{13} \right) + \left( a^1_{14} \right) + \left( p_{13}^1 \right) + \left( p_{14}^1 \right) \left( \lambda \right) \left( q_{13} \right) \left( G_{13}^* \right) \]

\[ + \]
\[ \left( (\lambda)^2 + (b^1_{18})^2 - (r_{18})^2 \right) \left( (\lambda)^2 + (a^1_{18})^2 + (p_{18})^2 \right) \]
\[ \left( (\lambda)^2 + (a^1_{16})^2 + (p_{16})^2 \right) \left( q_{17} \right) \left( G_{17}^* \right) + \left( a_{17} \right) \left( q_{18} \right) \left( G_{18} \right) \]
\[ \left( (\lambda)^2 + (b^1_{16})^2 - (r_{16})^2 \right) S_{(17),(17)} T_{17} + \left( b_{17}^1 \right) S_{(16),(17)} T_{17} \]
\[ + \left( (\lambda)^2 + (a^1_{17})^2 + (p_{17})^2 \right) \left( q_{16} \right) \left( G_{16}^* \right) + \left( a_{16} \right) \left( q_{17} \right) \left( G_{17} \right) \]
\[ \left( (\lambda)^2 + (b^1_{16})^2 - (r_{16})^2 \right) S_{(17),(16)} T_{17} + \left( b_{17}^1 \right) S_{(16),(16)} T_{16} \]
\[ \left( (\lambda)^2 \right)^2 + \left( a^1_{16} \right) + \left( a^1_{17} \right) + \left( p_{16}^1 \right) + \left( p_{17} \right) \left( \lambda \right) \left( \lambda \right) \]
\[
\left( (\lambda)^{(2)} \right)^2 + \left( (b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)} \right) (\lambda)^{(2)}
\]
\[
+ \left( (\lambda)^{(2)} \right)^2 + \left( (a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} \right) (\lambda)^{(2)} (q_{18})^{(2)} G_{18}
\]
\[
+ (\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)} \left( (a_{19})^{(2)} (q_{17})^{(2)} G_{17} + (a_{17})^{(2)} (a_{18})^{(2)} (q_{16})^{(2)} G_{16} \right)
\]
\[
\left( (\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)} S_{(17),(18)} T_{17}^* + (b_{17})^{(2)} S_{(16),(18)} T_{16}^* \right) = 0
\]

\[
+ 
\left( (\lambda)^{(3)} + (b_{22})^{(3)} \right) - (r_{22})^{(3)} \left( (\lambda)^{(3)} + (a_{22})^{(3)} + (p_{22})^{(3)} \right)
\]
\[
\left[ \left( (\lambda)^{(3)} + (a_{20})^{(3)} + (p_{20})^{(3)} \right) (q_{21})^{(3)} G_{21}^{*} + (a_{21})^{(3)} (q_{20})^{(3)} G_{20}^{*} \right]
\]
\[
\left( (\lambda)^{(3)} + (b_{20})^{(3)} \right) - (r_{20})^{(3)} S_{(21),(22)} T_{21}^* + (b_{21})^{(3)} S_{(20),(22)} T_{21}^*
\]
\[
+ \left( (\lambda)^{(3)} + (a_{21})^{(3)} + (p_{21})^{(3)} \right) (q_{20})^{(3)} G_{20}^{*} + (a_{20})^{(3)} (q_{21})^{(1)} G_{21}^{*}
\]
\[
\left( (\lambda)^{(3)} + (b_{20})^{(3)} \right) - (r_{20})^{(3)} S_{(21),(22)} T_{21}^* + (b_{21})^{(3)} S_{(20),(22)} T_{21}^*
\]
\[
\left( (\lambda)^{(3)} \right)^2 + \left( (a_{20})^{(3)} + (a_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda)^{(3)}
\]
\[
\left( (\lambda)^{(3)} \right)^2 + \left( (b_{20})^{(3)} + (b_{21})^{(3)} - (r_{20})^{(3)} + (r_{22})^{(3)} \right) (\lambda)^{(3)}
\]
\[
+ \left( (\lambda)^{(3)} \right)^2 + \left( (a_{20})^{(3)} + (a_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda)^{(3)} \left( q_{22} \right)^{(3)} G_{22}
\]
\[
+ \left( (\lambda)^{(3)} + (a_{20})^{(3)} + (p_{20})^{(3)} \right) \left( (a_{22})^{(3)} (q_{21})^{(3)} G_{21}^{*} + (a_{21})^{(3)} (a_{22})^{(3)} (q_{20})^{(3)} G_{20}^{*} \right)
\]
\[
\left( (\lambda)^{(3)} + (b_{20})^{(3)} - (r_{20})^{(3)} S_{(21),(22)} T_{21}^* + (b_{21})^{(3)} S_{(20),(22)} T_{21}^* \right) = 0
\]

\[
+ 
\left( (\lambda)^{(4)} + (b_{20})^{(4)} - (r_{20})^{(4)} \right) \left( (\lambda)^{(4)} + (a_{20})^{(4)} + (p_{20})^{(4)} \right)
\]
\[
\left[ \left( (\lambda)^{(4)} + (a_{20})^{(4)} + (p_{20})^{(4)} \right) (q_{25})^{(4)} G_{25}^{*} + (a_{25})^{(4)} (q_{24})^{(4)} G_{24}^{*} \right]
\]
\[
\left( (\lambda)^{(4)} + (b_{24})^{(4)} \right) - (r_{24})^{(4)} S_{(25),(25)} T_{25}^* + (b_{25})^{(4)} S_{(24),(25)} T_{25}^*
\]
\[
+ \left( (\lambda)^{(4)} + (a_{25})^{(4)} + (p_{25})^{(4)} \right) (q_{24})^{(4)} G_{24}^{*} + (a_{24})^{(4)} (q_{25})^{(4)} G_{25}^{*}
\]
\[
\left( (\lambda)^{(4)} + (b_{24})^{(4)} \right) - (r_{24})^{(4)} S_{(25),(24)} T_{25}^* + (b_{25})^{(4)} S_{(24),(24)} T_{25}^*
\]
\[
\left( (\lambda)^{(4)} \right)^2 + \left( (a_{24})^{(4)} + (a_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} \right) (\lambda)^{(4)}
\]
\[
\left( (\lambda)^{(4)} \right)^2 + \left( (b_{24})^{(4)} + (b_{25})^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)} \right) (\lambda)^{(4)}
\]

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\[ + \left( (\lambda)^{(4)} + (a_{24}^{(4)} + (a_{25}^{(4)}) + (p_{24}^{(4)} + (p_{25}^{(4)})) (\lambda)^{(4)} q_{26}^{(4)} G_{26} \right) \\
+ \left( (\lambda)^{(4)} + (a_{25}^{(4)} + (p_{24}^{(4)})) (a_{26}^{(4)} q_{25}^{(4)} G_{25} + (a_{25}^{(4)} a_{26}^{(4)} q_{24}^{(4)} G_{24}^{*}) \\
\left( (\lambda)^{(4)} + (b_{24}^{(4)} - (r_{24}^{(4)})) s_{(25),(26)}^{(4)} T_{25}^{*} + (b_{25}^{(4)} s_{(24),(26)}^{(4)} T_{24}^{*}) \right) = 0 \]

\[ + \]
\[ (\lambda)^{(5)} + (b_{30}^{(5)} - (r_{30}^{(5)})) \left( \left( (\lambda)^{(5)} + (a_{30}^{(5)} + (p_{30}^{(5)})) \right) \right) \]
\[ \left( (\lambda)^{(5)} + (a_{28}^{(5)} + (p_{28}^{(5)})) (q_{29}^{(5)} G_{29} + (a_{29}^{(5)} q_{28}^{(5)} G_{28}^{*}) \right) \]
\[ \left( (\lambda)^{(5)} + (b_{28}^{(5)} - (r_{28}^{(5)})) s_{(29),(29)}^{(5)} T_{29}^{*} + (b_{29}^{(5)} s_{(28),(28)}^{(5)} T_{28}^{*} \right) \]
\[ + \left( (\lambda)^{(5)} + (a_{29}^{(5)} + (p_{29}^{(5)})) (q_{30}^{(5)} G_{30} + (a_{30}^{(5)} q_{29}^{(5)} G_{29}^{*} \right) \]
\[ \left( (\lambda)^{(5)} + (b_{29}^{(5)} - (r_{29}^{(5)})) s_{(30),(30)}^{(5)} T_{30}^{*} + (b_{30}^{(5)} s_{(29),(29)}^{(5)} T_{29}^{*} \right) \] = 0

\[ + \]
\[ (\lambda)^{(6)} + (b_{34}^{(6)} - (r_{34}^{(6)})) \left( \left( (\lambda)^{(6)} + (a_{34}^{(6)} + (p_{34}^{(6)})) \right) \right) \]
\[ \left( (\lambda)^{(6)} + (a_{32}^{(6)} + (p_{32}^{(6)})) (q_{33}^{(6)} G_{33} + (a_{33}^{(6)} q_{32}^{(6)} G_{32}^{*}) \right) \]
\[ \left( (\lambda)^{(6)} + (b_{32}^{(6)} - (r_{32}^{(6)})) s_{(33),(33)}^{(6)} T_{33}^{*} + (b_{33}^{(6)} s_{(32),(33)}^{(6)} T_{33}^{*} \right) \]
\[ + \left( (\lambda)^{(6)} + (a_{33}^{(6)} + (p_{33}^{(6)})) (q_{34}^{(6)} G_{34} + (a_{34}^{(6)} q_{33}^{(6)} G_{33}^{*} \right) \]
\[ \left( (\lambda)^{(6)} + (b_{34}^{(6)} - (r_{34}^{(6)})) s_{(33),(34)}^{(6)} T_{33}^{*} + (b_{33}^{(6)} s_{(32),(34)}^{(6)} T_{33}^{*} \right) \] = 0

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\begin{align*}
&+ \left( (\lambda)^6 + (a_3^{13})^6 + (a_3^{12})^6 + (p_{32})^6 + (p_{33})^6 \right) (\lambda)^6 (q_{34})^6 (q_{33})^6 G_{34} \\
&+ \left( (\lambda)^6 + (a_3^{13})^6 + (p_{32})^6 \right) \left( (a_3^{14})^6 (q_{33})^6 G_{33}^* + (a_3^{13})^6 (a_3^{14})^6 (q_{32})^6 (q_{33})^6 G_{32}^* \right) \\
&\left( (\lambda)^6 + (b_3^{12})^6 - (r_{32})^6 \right) s_{33,34} T_{33} + (b_3^{13})^6 s_{32,34} T_{32} \right] = 0
\end{align*}

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

Acknowledgments:
The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Journal Reviews, Nature ‘s Letters, Article Abstracts, Research papers, Abstracts Of Research Papers, Stanford Encyclopedia, Web Pages, Ask a Physicist Column, Deliberations with Professors, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, we regret with great deal of compunction, contrition, regret, trepidation and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive

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numerically simulated nonprecipitating stratocumulus” J. Atmos. Sci., 53, 980-1006


(13)^ Note that the relativistic mass, in contrast to the rest mass $m_0$, is not a relativistic invariant, and that the velocity is not a Minkowski four-vector, in contrast to the quantity, where is the differential of the proper time. However, the energy-momentum four-vector is a genuine Minkowski four-vector, and the intrinsic origin of the square-root in the definition of the relativistic mass is the distinction between $d\tau$ and $dt$.


(20)^ [2] Cockcroft-Walton experiment

(21)^a^b^c Conversions used: 1956 International (Steam) Table (IT) values where one calorie ≡ 4.1868 J and one BTU ≡ 1055.05585262 J. Weapons designers’ conversion value of one gram TNT ≡ 1000 calories used.
1. 

(22)^ Assuming the dam is generating at its peak capacity of 6,809 MW.

(23)^ Assuming a 90/10 alloy of Pt/Ir by weight, a $C_p$ of 25.9 for Pt and 25.1 for Ir, a Pt-dominated average $C_p$ of 25.8, 5.134 moles of metal, and 132 J.K$^{-1}$ for the prototype. A variation of ±1.5 picograms is of course, much smaller than the actual uncertainty in the mass of the international prototype, which are ±2 micrograms.


(25)^* # Earth's gravitational self-energy is $4.6 \times 10^{-10}$ that of Earth's total mass, or 2.7 trillion metric tons. Citation: The Apache Point Observatory Lunar Laser-Ranging Operation (APOLLO), T. W. Murphy, Jr. et al. University of Washington, Dept. of Physics (132 kB PDF, here).

(26)^ There is usually more than one possible way to define a field energy, because any field can be made to couple to gravity in many different ways. By general scaling arguments, the correct answer at everyday distances, which are long compared to the quantum gravity scale, should be minimal coupling, which means that no powers of the curvature tensor appear. Any non-minimal couplings, along with other higher order terms, are presumably only determined by a theory of quantum gravity, and within string theory, they only start to contribute to experiments at the string scale.


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484.


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