Industrial Engineering Letters
ISSN 2224-6096 (Paper) ISSN 2225-0581 (online)
Vol.6, No.10, 2016

Revisiting the Evolution and Application of Assignment Problem: A Brief Overview

Dr Sarbapriya Ray
Assistant Professor, Dept. of Commerce,
Vivekananda College, Under University of Calcutta, Kolkata, India
&Guest Faculty, Dept. of Commerce, University of Calcutta, India

Abstract:
The assignment problem (AP) is incredibly challenging that can model many real-life problems. This paper provides a limited review of the recent developments that have appeared in the literature, meaning of assignment problem as well as solving techniques and will provide a review on a lot of research studies on different types of assignment problem taking place in present day real life situation in order to capture the variations in different types of assignment techniques.
Keywords: Assignment problem, Quadratic Assignment, Vehicle Routing, Exact Algorithm, Bound, Heuristic etc.

1. Introduction

There are several important areas of economic analysis in which progress depends on the advancement of methods for solving or analyzing problems in the efficient allocation of individual resources. In the subject ‘operation research’, the assignment problem is very challenging and interesting that can represents many real-life problems.

The optimal assignment problem is a classical combinatorial optimization problem. It entails optimally matching the elements of two or more sets, where the dimension of the problem refers to the number of sets of elements to be matched. A simple explanatory example of the problem is matching a number of persons and objects on a one-to-one basis where each person has a specific benefit associated with being matched to a certain object. The optimal matching corresponds to an assignment of persons to objects that maximizes the sum of benefits. When there are only two sets, as will be the case for most of the variations we will consider, they may be referred to as ‘‘tasks’’ and ‘‘agents’’. Therefore, ‘‘tasks’’ may be jobs to be done and ‘‘agents’’ the people or machines that can do them. Assignment Problem is a method for matching the "Tasks" (jobs) to the "Agent" (man, machine of facility) which can produce the most efficient outcome. AP can be categorized in to three groups – AP model with at most one task per agent, AP model with multiple tasks per agent, AP model for multi-dimensional assignment problem.

The assignment problem is special case of transportation problem and a linear zero-one programming problem in which the objective is to assign a number of resources to the equal number of activities at a minimum cost (or maximum profit).The assignment problem deals in allocating the various resources (items) to various activities (receivers) on a one to one basis in such a way that the resultant effectiveness is optimized. These types of problems are linear programming applications that can be solved using the simplex method. It is a one of the well-studied optimization problems in Management Science and has been widely applied in both manufacturing and service systems. Assignment problems can also be solved by transportation method .Possibly, the first documented algorithmic method for finding a solution to the assignment problem aside from the brute force approach is the Hungarian method. The method was developed by H. W. Kuhn (1955) and based on the work of two Hungarian mathematicians, Evgary and Konig in the honour of which Kuhn named the algorithm the Hungarian algorithm although the name ‘‘assignment problem’’ seems to have first appeared in a 1952 paper by Votaw and Orden(1952), what is generally recognized to be the beginning of the development of practical solution methods for and variations on the classic assignment problem.

The fundamental idea behind the Hungarian method is that the optimal assignment is conserved by the addition of a scalar to any row or column. This can easily be understood when considering that any assignment contains only one element from each row and one element from each column. The translation of a whole column (respective row) by a constant corresponds to a translation of the optimal assignment by that same constant as any assignment must still contain one element from that column (respective row). The Hungarian method uses this principle to transform a matrix into a sparse matrix by subtracting the minimal elements from each row and column, resulting in multiple zero entries while not affecting the optimal assignment of the matrix. When the matrix is sufficiently sparse ,an assignment of zeros entries that correspond to the minimal assignment would be possible and choosing such an assignment would solve the assignment problem for the matrix of interest. Konig’s theorem gives an algorithmic method to check whether the matrix has been reduced to a sufficiently sparse matrix.

König’s Theorem suggests that the recursive subtraction of smallest elements from the rows and columns of a matrix will clearly result in the presence of a large number of zeros in the matrix (at least one in each row and column). If a selection of zeros from the matrix can be made corresponding to an assignment (a selection of zeros such that the position of each zero corresponds to a one in an associated permutation matrix) the algorithm terminates. We can consider this as covering all zero entries in the matrix by drawing lines through the rows and columns. The minimum number of lines needed to cover all zeros in the matrix is then known as its cover and when the cover equals the order of the matrix an assignment of zeros is possible. Determining whether such an assignment is possible is simple enough to see when matrices are small but could be very difficult for large matrices. Obviously, there exists a need for an algorithmic method to find the cover and Konig's theorem allows the development of exactly such an algorithm. The Theorem suggests that for every rectangular matrix containing zeros as some of the entries, the number of zeros in a largest independent set of zeros is equal to the number of lines in a smallest cover of the zeros.

There are a number of decision making situations where assignment technique can be successfully used. For example, assignment of available sales-force to different regions; vehicles to routes; product to factories; contracts to bidders; machines to jobs; development engineers to several construction sites and so on. Management generally makes assignment on a one to one basis in such a manner that the group maximizes the revenue from the sales, the vehicles are deployed to various routes in such a way that the transportation cost is minimum and so on.

In assignment problems, supply in each row represents the availability of a resource such as a man, vehicle, product, salesman etc and demand in each column represents different activities to be performed, such as jobs, routes, factories, areas etc for each of which only one man or vehicle or product or salesman respectively is required. Entries in the square being costs, times or distances. The essential characteristic of the assignment problem is: *n* resources are to be assigned to n activities such that each resource is allocated to each activity and each activity is performed by one resource only. The allocation is to be done in such a way so as to maximize the resultant effectiveness.

Assignment problem is completely degenerate form of a transportation problem. The units available at each origin (resource) and units demanded at each destination (activity) are all equal to one. That means exactly one occupied cell in each row and each column of the transportation table, i.e only n occupied cells in place of required n+n-1=(2n-1).The assignment problem is concerned with the concept of finding an optimal one-to-one matching between two sets. Therefore, analysts aware of what these variations are so that they can more easily determine which variation most closely match their current problem. An assignment is obtained when we match an element from set 1 to an element from set 2 in a one-to-one fashion. Mathematically the assignment problem can be viewed as a bijective mapping of a finite set into itself, i.e. a permutation. This can be represented by a permutation matrix or an element from any set isomorphic to the set of permutation matrices such as the set of all possible permutations of the set of numbers $\{1,2,...,N\}$.

Various approaches have been developed to find efficient solutions to the assignment problem. To name a few of the more important ones: the Hungarian method by Kuhn and Munkres was of the first. Dantzig solved the problem using linear programming and more recently Bertsekas (1990) developed the Auction Algorithm that solves the problem very efficiently. Another interesting approach by Kosowsky and Yuille(1991) manages to find a solution to the assignment problem using an approach related to statistical physics. It is referred to as the Invisible hand algorithm. The assignment problem is of great theoretical importance as it embodies a fundamental linear programming structure. It is possible to reformulate important types of linear programming problems such as the linear network flow and shortest path problems to take the form of an assignment problem.

The assignment problem finds many applications; the most obvious being that of matching such as the matching of operators and machines or delivery vehicles and deliveries. There are however numerous other interesting applications. Van Wyk(2001) shows that the graph matching problem can be rewritten to take the form of an optimal assignment, it turns out that many such optimizations procedures that are concerned with finding a closest approximation of an element in some vector space can be wholly or partly reformulated as assignment problems. Another such example by Imai et.al.(1986) is the approximation of piecewise linear functions or human face detection by Ying,Z,et.al(1999). Wästlund,J(2003) shows that the shortest path problem can be reformulated as an assignment problem. Such algorithms are used in the determination of routing tables for telecommunication networks or optimal routes in GPS navigation systems. Some other applications of the assignment problem include tracking moving objects in space [Burkard, R.E et.al(1998)], the matching of moving objects[Brogan, W(1989)] and scheduling of an input queued switch[S. Chuang(2002)]. The importance of the assignment problem is quite clear from the above and various very successful solutions to the problem already exist.

In this article, we will briefly discuss the meaning of assignment problem, solving techniques and present a survey of some developments and researches in the said field.

General Mathematical model of Linear Assignment problems:

The linear assignment problem (LAP) is one of the basic and fundamental models in operations research, computer science, and discrete mathematics. In its most well-known interpretation, it answers the question of finding an assignment of n workers to n jobs that has the lowest total cost, if the cost of assigning worker i to task j equals cij . Apart from the straightforward applications, such as personnel assignment problems, the LAP frequently arises as a part of other optimization problems, such as quadratic assignment problem, multidimensional assignment problem, traveling salesman problem, etc. Other applications of the LAP, including earth-satellite systems with TDMA protocol, and tracking objects in space are considered in Burkard (1985) and Brogan (1989).

With the information about the number of assignment i(i=1,2,3………..,n) performing the same number of jobs j (j=1,2,3,………. n) and the pay off measure c_{ij} available for each assignment ,the objective is to determine the strategy that minimizes the total cost or maximizes the total utility.

The general data matrix for assignment problem is shown in table-1. It may be noted that this data matrix is the same as the transportation cost matrix except that the supply (or availability) of each of the resources and the demand at each of the destinations is taken to be one. It is due to this fact that the assignments are on a one to one basis.

The assignment problem can be conveniently represented in the form of n x n matrix $[c_{ij}]_{nxn}$ known as cost or effective ;where x_{ij} represents the assignment of resource (facility) i to activity(job) j and c_{ij} is the cost associated with assigning i th facility (person) to j th job.

Table-1

Resources(workers)	Activity(Jobs)				Supply (Available)
	J_1	J_2	…..	J_n	1
W_1	C_{11}	C_{12}	…….	C_{1n}	1
W_2	C_{21}	C_{22}	…………	C_{2n}	1
…	……	…..	……	…..	…..
W_n	C_{n1}	C_{n2}	……	C_{nn}	1
Demand(Required)	1	1		1	n

Further, it is also assumed that x_{ij}=1, if i th resources (person) is assigned to j th activity (job) and x_{ij}=0, otherwise.

The unique version of the assignment problem is discussed in almost either management science/operations research or production and operations management. Pentico (2007) proposes a survey of what appear to be the most useful variations of the assignment problem that have developed in the literature over the past 50 years. As usually described, the problem is to find a one-to-one matching between n tasks and n agents, the objective being to minimize the total cost of the assignments.

In this section, we will briefly present the papers that take the competency constraint into account.

2. Evolution of assignment problem:

Classical Assignment Problem:

The mathematical model of the assignment problem can be stated as :

$$\text{Minimize Total Cost}=Z=\sum_{i=1}^{n}\sum_{j=1}^{n} = c_{ij}\, x_{ij}$$

subject to constraints:

(i) $\sum_{j=1}^{n} x_{ij} = 1$ for all i(resource availability)

i.e each person should be assigned to one and only one job.i.e

$x_{i1}+x_{i2}+\ldots\ldots\ldots..+x_{in}=1;\ i=1,2,\ldots\ldots\ldots n$

(ii) $\sum_{i=1}^{n} x_{ij} = 1$ for all j(activity requirement)

i.e each job must be assigned to one and only one person i.e

$x_{1j}+x_{2j}+\ldots\ldots\ldots..+x_{nj}=1;\ j=1,2,\ldots\ldots\ldots n$

(iii) x_{ij} =0 or 1, for all i and j.

This mathematical model of assignment problem is a particular case of the transportation problem for two reasons: (i) the cost matrix is a square matrix and (ii) the optimal solution table(matrix) for the problem would have only one assignment in a given row or a column.

Minor modifications of the basic problem structure are easily handled with the standard solution procedures. A problem in which the objective function is to be maximized can be easily converted into a minimization problem by either (a) multiplying all of the c_{ij}s by -1, or (b) replacing each c_{ij} by c_{max} - c_{ij}, where c_{max} is the maximum of the c_{ij}s, thus converting the problem to one of minimizing ‘‘regret’’. A problem that is not balanced (i.e., one for which the numbers of tasks and agents differ) can be easily converted into a balanced problem by adding a sufficient number of ‘‘dummy’’ tasks or agents (whichever is in shorter supply) with costs of 0. One may also use non-zero costs for assignments using dummy tasks or agents to reflect differences based on which agents or tasks are not assigned. It is worth noting that the classic AP is mathematically identical to the weighted bipartite matching problem from graph theory, so that results from that problem have been used in constructing efficient solution procedures for the classic AP.

2.1. The classic assignment problem recognizing agent qualification

Caron et al.(1999) take an initiative to interest in the classic assignment problem recognizing agent qualification. In their work on a particular version of the assignment problem with side constraints, Caron et al. use a mathematical model for a variation of the classic assignment problem in which there are m agents and n tasks, not every agent is qualified to do every task, and the objective is utility maximization.

$$\text{Maximize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} = c_{ij}\, x_{ij}$$

$$\text{Subject to :} \quad \sum_{i=1}^{m} q_{ij}\, x_{ij} \le 1, \quad j = 1 \ldots\ldots\ldots n\,.$$

$$\sum_{i=1}^{n} q_{ij}\, x_{ij} \le 1, \quad i = 1 \ldots\ldots\ldots m$$

$$x_{ij} = 0 \text{ or } 1$$

where x_{ij} = 1 if agent i is assigned to task j, 0 if not, q_{ij} = 1 if agent i is qualified to perform task j, 0 if not, and c_{ij} = the utility of assigning agent i to task j (with c_{ij} = 0 if q_{ij} = 0). The first set of constraints ensures that no more than one qualified agent is assigned to any task and the second set of constraints ensures that no agent is assigned to more than one task. Note that even if m is greater than or equal to n it may not be possible to assign a qualified agent to every task or to give every agent a task for which it is qualified. Minimization problems can be handled using either of the methods suggested above for handling maximization problems for the standard classic AP.

2.2. The k-cardinality assignment problem

Mauro Dell Amico and Silvano Martello(1997) have taken into consideration a generalization of the assignment problem in which an integer *k* is given and one wants to assign *k* rows to *k* columns so that the sum of the corresponding costs is a minimum. The problem can be seen as a 2-matroid intersection, hence is solvable in polynomial time; immediate algorithms for it can be obtained from transformation to min-cost flow or from classical shortest augmenting path techniques. They introduce original preprocessing techniques for finding optimal solutions in which g≤ *k* rows are assigned, for determining rows and columns which must be assigned in an optimal solution and for reducing the cost matrix. A specialized primal algorithm is finally presented. The average computational efficiency of the different approaches is evaluated through computational experiments. Therefore, Mauro Dell Amico and Silvano Martello (1997) explain a variation on the classic AP in which there are m agents and n tasks, but only k of the agents and tasks are to be assigned, where k is less than both m and n. The mathematical model for the k-cardinality assignment problem is:

$$\text{Minimize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} = c_{ij}\, x_{ij}$$

Subject to:

$$\sum_{i=1}^{m} x_{ij} \le 1\,,\ j = 1,2,\ldots\ldots n$$

$$\sum_{j=1}^{n} x_{ij} \le 1\,,\ i = 1,2,\ldots\ldots m$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = k$$

$x_{ij} = 0$ or 1

where $x_{ij} = 1$ if agent i is assigned to task j, 0 if not, c_{ij} = the cost of assigning agent i to task j, and k is the number of agent-task assignments that must be made. Dell Amico and Martello suggest as a potential application for this model the situation in which workers are to be assigned to machines, but only a subset of the workers and machines need to be assigned. They also suggest that it can be used to solve sub-problems in the problem of assigning time slots on a communications satellite being used to transmit information from m earth stations to n different earth stations.

The k-cardinality assignment problem is an interesting generalization of the assignment problem, having applications in personnel scheduling and as a sub-problem in the solution of more complex problems, such as the SS/TDMA time slot assignment problem. We have developed, for the first time, efficient preprocessing techniques and a primal algorithm. Their computational tests show that the proposed approach is the fastest method available for the problem solution. It solves dense instances having up to 250000 entries, for all values of k, with very short running times. Future developments could concern the specialization of our algorithmic techniques to the case of sparse matrices, in order to solve instances with much higher values of *m* and *n*.

2.3 The Generalized Koopamans- Beckmann quadratic assignment problem (QAP)

The Quadratic Assignment Problem (QAP) was originally introduced in 1957 by Tjalling C. Koopmans and Martin Beckman who were trying to model a facilities location problem. Since then, it has been among the most studied problems in all combinatorial optimization. Many scientists including mathematicians, computer scientists, operations research analysts, and economists have used the QAP to model a variety of optimization problems. They first introduced the quadratic assignment problem (QAP) as a mathematical model for the location of indivisible economical activities. The QAP stated as a facility location problem is to assign N facilities to N locations such that the total interaction cost of all possible flow-distance products between the locations to which the facilities are assigned plus the allocation costs of facilities to locations are minimized. Given the flow matrix:

$F = [f_{ik}] \in R^{N\times N}$ where f_{ik} is the flow from facility i to facility k, the distance matrix $D = [d_{jn}] \in R^{N\times N}$ where d_{jn} is the distance from location j to location n, and the cost matrix $B = [b_{ij}] \in R^{N\times N}$ where b_{ij} is the allocation cost of placing facility i at location j, the QAP in Koopmans-Beckmann form can be modeled as:

$$\min \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{n=1}^{N} f_{ik} d_{jn} x_{ij} x_{kn} + \sum_{i=i}^{N} \sum_{j=1}^{N} b_{ij} x_{ij}$$

Subject to:

$$\sum_{i=1}^{N} x_{ij} = 1 \qquad j=1,\ldots,N$$

$$\sum_{j=1}^{N} x_{ij} = 1 \qquad i=1,\ldots,N$$

$$x_{ij} \in \{0,1\} \qquad i,j=1,\ldots,N$$

Each assignment of facilities to locations is represented by an $N \times N$ solution matrix $X = [x_{ij}]$ where $x_{ij} = 1$ if facility i is being placed at location j or $x_{ij} = 0$ otherwise.

Notice that $X = [x_{ij}]$ is a permutation matrix.

A natural application in location theory was used by Dickey and Hopkins (1972) in a campus planning model. The problem consists of planning the sites of n buildings on a campus, where b_{kl} is the distance from site k to site l, and a_{ij} is the traffic intensity between buildings i and j. The objective is to minimize the total weekly walking distance between the buildings. In addition to facility location, QAPs appear in applications such as layout problems, backboard wiring, computer manufacturing, scheduling, process communications and turbine balancing. In the field of ergonomics, Burkard and Offermann (1977) showed that QAPs can be applied to typewriter keyboard design. The problem is to arrange the keys on a keyboard such as to minimize the time needed to write some text.

In contrast to linear assignment problems, quadratic assignment problems remain among the hardest combinatorial optimization problems. The inherent difficulty for solving QAPs is reflected by their computational complexity. Sahni and Gonzalez [Sah1976] showed that QAP is *NP*-hard and that even finding an approximate solution within some constant factor from the optimum value cannot be done in polynomial time. These results hold even for Koopmans-Beckmann QAPs with coefficient matrices fulfilling the triangle inequality [Queyranne (1986)].

From the theoretical point of view, it is because of the high computational complexity: QAP is NP-hard, and even finding an approximate solution is a hard problem. Moreover, many well-known classical combinatorial optimization problems such as the traveling salesman problem, the graph partitioning problem, the maximum clique problem can be reformulated as special cases of the QAP. From the practical point of view, it is because of the diversified applications of the QAP. The techniques which can be used to find the optimal solution are limited to branch and bound and cutting planes methods: with current hardware, problems of order greater than 20 cannot be solved in an acceptable time (Burkard et al., 1994). For this reason, in recent years many heuristic algorithms have been proposed which, though not ensuring that the solution found is the best one, give good results in an acceptable computation time (Maniezzo *et al.,* 1994).

2.4 The generalized quadratic assignment problem (GQAP)

The generalized quadratic assignment problem (GQAP) studies a class of problems that optimally assign M entities to N destinations subject to the resource limitation at each destination. These problems arise naturally in yard management, where containers are to be located in the storage areas with limited capacity, and in distributed computing where processing tasks are to be assigned to processors with limited computing resources. The GQAP is a generalization of the QAP that multiple entities can be assigned to a single destination if only such assignment does not violate the resource capacity at destinations.

Lee and Ma (2004) proposed the first formulation of GQAP. Their study involves a facility location problem in manufacturing where M facilities to be located among N fixed locations given the space constraint at each possible location, with the objective to minimize the total installation and interaction transportation cost. The formulation of the GQAP is then:

$$\min \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{M}\sum_{n=1}^{N} f_{ik} d_{jn} x_{ij} x_{kn} + \sum_{i=1}^{M}\sum_{j=1}^{N} b_{ij} x_{ij}$$

subject to:

$$\sum_{i=1}^{M} S_{ij} x_{ij} \le S_j \qquad j=1,\ldots,N \qquad \ldots\ldots \quad (1)$$

$$\sum_{j=1}^{N} x_{ij} = 1 \qquad i=1,\ldots,M, \quad . \quad \ldots. \ (2)$$

$$x_{ij} \in \{0,1\} \qquad =1,\ldots,M; j=1,\ldots,N, \quad \ldots\ldots(3)$$

where

M = the number of facilities,

N =the number of locations,

f_{ik} =the commodity flow from facility i to facility k ,

d_{jn} =the distance from location j to location n ,

b_{ij} = the cost of installing facility i at location j ,

s_{ij} =the space requirement if facility i is installed at location j ,

s_j = the space available at location j ,

x_{ij} = binary variable, x_{ij} =1, if facility i is installed at location j .

The objective function sums the costs of installation and quadratic interactivity. The knapsack constraints (1) impose space limitations at each location, and the multiple choice constraints (2) ensure that each facility is to be installed at exactly one location.

2.5. A Quadratic 0-1 Formulation

Quadratic 0-1 Formulation is one more variant formulation of QAP problem. Initially, this formulation was used by Koopmans-Beckman**.**It is formulated using an n x n matrix is a permutation matrix X= [x_{ij}] that represents permutations $\pi \in S_n$ by 0-1 form .To be considered a permutation matrix, it should satisfy the three following conditions:

$$\sum_{i=1}^{n} X_{ij} \qquad ,j=1,\ldots..,n.$$

$$\sum_{j=1}^{n} X_{ij} \qquad ,i= 1,\ldots..,n.$$

$$X_{ij} \in \{0,1\} \qquad , i,j =1,\ldots,n.$$

If the three conditions are satisfied, then QAP can be formulated as:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} f_{ij} d_{kl} X_{ik} X_{jl}$$

Subject to:

$$\sum_{i=1}^{n} X_{ij} \qquad ,j=1,\ldots,n.$$

$$\sum_{j=1}^{n} X_{ij} \qquad ,i=1,\ldots,n.$$

$$X_{ij} \in \{0,1\} \qquad ,i,j=1,\ldots,n.$$

There are massive numbers of realistic applications that can be modeled as QAPs. Koopmans and Beckmann (1957) first proposed QAP as a mathematical model regarding the economic activities. Since then , it has appeared in numerous practical applications: Steinberg (1961) used QAP to minimize the number of connections between components in a backboard wiring, Heffley (1972, 1980) applied it to economic problems, Francis and White (1974) formulated a decision framework for assigning a new facility (police posts, supermarkets, schools) for serving a given set of clients, Geoffrion and Graves (1976) concentrated on scheduling problems, Pollatschek et al. (1976**)** mentioned QAP to define the best design for typewriter keyboards and control panels, Krarup and Pruzan (1978) applied it to archeology**.** Hubert (1987) experimented it in statistical analysis, Forsberg et al. (1994) applied it in the reaction chemistry analysis and Brusco and Stahl (2000) used it in numerical analysis. Inspite of the fact that the facilities layout problem is the most popular application for QAP, Dickey and Hopkins (1972) applied QAP to the assignment of buildings in a University campus, Elshafei (1977) in a hospital planning and Bos (1993) in a problem related to forest parks. Benjaafar (2002) introduced a formulation of the facility layout design problem for minimizing work-in-process (WIP). Ben-David and Malah (2005) looked into a special case of QAP called index assignment in order to minimize channel errors in vector-quantization. Vector-quantization is used when mapping images or speech to digital signals. A similar mapping problem is also found when configuring the layout of micro arrays, which is a problem in bioinformatics presented as a QAP by de Carvalho Jr. and Rahmann,S.A (2006). A more modern application of the same problem is the design of keyboards on touch screen devices **(**Dell'Amico et al., 2009). The main difference in this approach is that on a touch screen only one finger is used, and the letters can be placed anywhere on the screen instead of in a rectangle as with normal keyboards.

The following table-2 presents the survey on quadratic assignment problem which has been collected from several theses as well as from published articles in journals.

Table 2: A Survey on Quadratic Assignment Problem

author	title	university	objective
T.A. Johnson(1992)	New linear programming-based solution procedures for the quadratic assignment problem	Clemson University, Clemson, USA,	Introduce new solution procedures based on linear programming. The linear formulation derived in this thesis theoretically dominates alternate linear formulations for QAP.
T. Mautor(1992)	Contribution to solving problems implanation: sequential and parallel algorithms for the quadratic assignment	Pierre et Marie Curie university, France	Focus on parallel implementations and exploits the metric structure of the Nugent instances to reduce the branching tree considerably.
Y. Li(1992)	Heuristic and exact algorithms for the quadratic assignment problem.	The Pennsylvania State University, USA	Introduce beside other ideas lower bounding techniques based on reductions, GRASP and a problem generator for QAP.
F.Malucelli(1993)	Quadratic assignment problems: solution methods and applications.	University of Pisa, Pisa, Italy,	Propose a lower bounding technique for QAP based on a reformulation scheme and implemented it in a branch and bound code. Some new applications of QAP in the field of transportation were also presented.
E. Cela(1995)	The quadratic assignment problem: special cases and relatives.	Graz University of Technology, Graz, Austria	Investigate the computational complexity of specially structured quadratic assignment problems, and consider a generalization of QAP, the so called biquadratic assignment problem
S.E. Karisch(1995)	Nonlinear approaches for the quadratic assignment and graph partition problems	Graz University of Technology, Graz, Austria	Present nonlinear approaches for QAP. These provide the currently strongest lower bounds for problems instances whose distance matrix contains distances of a rectangular grid and for smaller sized general problems.
M. Rijal(1995)	Scheduling, design and assignment problems with quadratic costs	New York University, New York, USA	Investigate structural properties of the QAP polytope. The starting point is the quadric Boolean polytope.
Q. Zhao(1996)	Semi definite programming for assignment and partitioning problems	University of Waterloo, Waterloo, Canada	Investigate semi definite programming approaches for the QAP. Tight relaxations and bounds are obtained by exploiting the geometrical structure of the convex hull of permutation matrices.
A. Bouras(1996)	quadratic assignment problem of small rank: models, complexity, and applications	Joseph Fourier university, Grenoble, France	Consider special cases of QAP where the coefficient matrices have a low rank, especially rank one, and propose a heuristic based on matrix approximations by matrices with low rank.
V. Kaibel(1997)	Polyhedral combinatorics of the quadratic assignment problem	University of Cologne, Cologne, Germany	Investigate the QAP polytope and derived the first large class of facet defining inequalities for these polytopes, the box inequalities..
Gunes Erdogan (2006)	Quadratic assignment problem: linearizations and polynomial time solvable cases	BILKENT UNIVERSITY, Turkey	Focus on “flow-based” formulations, strengthen the formulations with valid inequalities, and report computational experience with a branch-and-cut algorithm.
Thomas Stützle(2006)	Iterated local search for the quadratic assignment problem	Darmstadt, Germany	Present and analyze the application of ILS to the quadratic assignment problem (QAP).
Yi-Rong Zhu(2007)	Recent advances and challenges in quadratic assignment and related problems	the University of Pennsylvania	Contribute to the theoretical, algorithmic and applicable understanding of quadratic assignment and its related problems.

author	title	university	objective
Tao Huang(2008)	Continuous Optimization Methods for the Quadratic Assignment Problem	the University of North Carolina at Chapel Hill	Study continuous optimization techniques as they are applied in nonlinear 0-1 programming. Specically, the methods of relaxation with a penalty function have been carefully investigated.
Franklin Djeumou Fomeni(2011)	New Solution Approaches for the Quadratic Assignment Problem	University of the Witwatersrand	Propose two new solution approaches to the QAP, namely, a Branch-and-Bound method and a discrete dynamic convexized method.
Francesco Puglierin(2012)	A Bandit-Inspired Memetic Algorithm for Quadratic Assignment Problems	University of Utrecht	Propose a new metaheuristic for combinatorial optimization, with focus on the Quadratic Assignment Problem as the hard-problem of choice, a choice that is reflected in the name of the method, BIMA-QAP.
Wu et al. (2012)	Global optimality conditions and optimization methods for quadratic assignment problems	School of Science, Information Technology and Engineering, University of Ballarat , Victoria, Australia Department of Mathematics, Shanghai University,China	Discuss some global optimality conditions for general quadratic {0, 1} programming problems with linear equality constraints, and then some global optimality conditions for quadratic assignment problems (QAP) are presented.
Duman et al. (2012)	Migrating Birds Optimization: A new metaheuristic approach and its performance on quadratic assignment problem	Ozyegin University, Department of Industrial Engineering,Istanbul, Turkey Dogus University, Department of Computer Engineering, Istanbul, Turkey Marmara University, Department of Computer Engineering, Istanbul, Turkey	Propose a new nature inspired metaheuristic approach based on the V flight formation of the migrating birds which is proven to be an effective formation in energy saving. Its performance is tested on quadratic assignment problem instances arising from a real life problem and very good results are obtained.
Benlic et al.(2013)	Breakout local search for the quadratic assignment problem	University of Angers,france	Present breakout local search (BLS) for solving QAP. BLS explores the search space by a joint use of local search and adaptive perturbation strategies.
Klerk et al.(2014)	Symmetry in RLT-type relaxations for the quadratic assignment and standard quadratic optimization problems	Department of Econometrics and OR, Tilburg University, The Netherlands Centrum Wiskunde & Informatica (CWI), Amsterdam, The Netherlands	Show that, in the presence of suitable algebraic symmetry in the original problem data, it is sometimes possible to compute level two RLT bounds with additional linear matrix inequality constraints.
Axel Nyberg (2014)	Some Reformulations for the Quadratic Assignment Problem	Department of Chemical Engineering Abo Akademi University , Finland	Reformulate the Quadratic Assignment Problem for optimization

Source: Collected and compiled by author

2.6. Vehicle Routing Problem (VRP):

Vehicle routing problem (VRP) is a generic name specified to a whole class of problems involving the design of optimal routes for a fleet of vehicles to provide service to a set of customers subject to side constraints. The VRP is a vital problem in the physical delivery of goods and services. In reality, numerous variants of the VRP exist, depending on the nature of the transported goods, the quality of service required, and the characteristics of customers and vehicles. Some typical obstacles are heterogeneous vehicles located at different depots, customers mismatched with certain vehicle types, customers accepting delivery within specified time windows, multiple-day planning horizons and vehicles performing multiple routes. In all cases, the objective is to supply the customers at minimum cost.

The simplest and most studied member of the VRP family is the capacitated VRP (CVRP). In the CVRP, all customers must be satisfied, all demands are known, and all vehicles have identical, limited capacity and are based at a central depot. A fleet of identical vehicles located at a central depot has to be optimally routed to supply a set of customers with known demands. The objectives are to minimize the vehicle fleet and the sum of travel time while the total demand of commodities for each route may not exceed the capacity of the vehicle which serves that route. .Each vehicle can perform at most one route and the total demand of the customers visited by a route cannot exceed the vehicle capacity.

Another important variant of the VRP is the VRP with time windows (VRPTW) that generalizes the CVRP by imposing that each customer is visited within a specified time interval, called time window. The objective is to minimize the vehicle fleet with the sum of travel time and waiting time needed to supply all customers in their required hour. A variety of exact algorithms and efficient heuristics have already been proposed for VRPTW by various researchers as shown in Table 3. In addition, Table 2 represents the various methods applying in exact algorithm, classical heuristic algorithms and metaheuristic algorithms for various type of VRP.Therefore, in the last decade, some innovative exact approaches for vehicle routing problems have been proposed, producing a significant improvement on the size of the instances that can be solved to optimality.

Table 3: Review of Vehicle Routing Problem with Time Window

Authors	Year	Methodologies
Dumas et all	1995	Time constraint routing and scheduling
Liu and Shen	1999	Route-neighborhood-based metaheuristic
Bent et al.	2003	Two-stage hybrid algorithm
Kim, et al.	2006	Capacitated clustering
Lysgaard	2006	Precedence constraints
Russell and Chiang	2006	Robust solution methods
Chena, Hsueh and Chang	2009	An elaborated solution Algorithms
Li, et al.	2009	Lagrangian heuristic

Source: Collected and compiled by author

2.7. Heuristics:

A heuristic technique, often called simply a *heuristic*, is any approach to problem solving, learning, or discovery that employs a practical methodology not guaranteed to be optimal or perfect, but sufficient for the immediate goals. Where finding an optimal solution is impossible or impractical, heuristic methods can be used to speed up the process of finding a satisfactory solution. Heuristics can be mental shortcuts that ease the cognitive load of making a decision. Examples of this method include using a rule of thumb, an educated guess, an intuitive judgment, stereotyping, profiling, or sense. More precisely, heuristics are strategies using readily accessible, though loosely applicable, information to control problem solving in human beings and machines. Heuristic technique is the procedure committed to the search of good quality solutions. Due to obvious complexities experienced in the development of exact solution procedures, a wide variety of heuristic approaches has been developed for QAP. Heuristic algorithms do not offer a guarantee of optimality for the best solution obtained. As a matter of fact, it is usual to find approximate algorithms treated as heuristic algorithms in the Combinatorial Optimization literature, as in Osman and Laporte (1996).These approaches can be classified into the following categories: simulated annealing, Improvement methods, Construction methods, Greedy randomized adaptive search procedures Limited enumeration methods, Genetic algorithms, and Ant colonies, Metaheuristics, tabu search which are discussed below in a nut shell.

2.7.1. Simulated Annealing (SA)

This group of heuristics, which is also used for overcoming local optima, receives its name from the physical process which it imitates. This process, called *annealing* moves high energy particles to lower energy states with the lowering of the temperature, thus cooling a material to a steady state. Initially, in the initial state of the heuristic, the algorithm is lenient and capable of moving to a *worse* solution. However, with each iteration the algorithm becomes stricter requiring a better solution at each step. Simulated annealing (SA) is a generic probabilistic metaheuristic for the global optimization problem of locating a good approximation to the global optimum of a given function in a large search space. It is often used when the search space is discrete (e.g., all tours that visit a given set of cities). For certain problems, simulated annealing may be more efficient than exhaustive enumeration provided that the goal is merely to find an acceptably good solution in a fixed amount of time, rather than the best possible solution. The name and inspiration come from annealing in metallurgy, a technique involving heating and controlled cooling of a material to increase the size of its crystals and reduce their defects.

2.7.2. Improvement Methods (IM)

These methods belong to the larger class of local search algorithms. A local search procedure starts with an initial feasible solution and iteratively tries to improve the current solution. This is done by substituting the latter with a (better) feasible solution from its neighborhood. This iterative step is repeated until no further improvement can be found. Improvement methods are local search algorithm which allows only improvements of the current solution in each iteration. The most accepted improvement methods are the local search and the tabu search . Both methods work by starting with an initial basic feasible solution and then trying to improve it. The local search seeks an improved solution in the neighborhood of the current solution, terminating when no better solution exists within that neighborhood. The tabu search (TS) (initiated by SkorinKapov (1990), Taillard (1991) works similarly to the local search. However, the tabu search is sometimes more approving as it was designed to cope up with the problem of a heuristic getting trapped at local optima.

This group of heuristics, adopted by Burkard and Rendl (1984) ,Wilhelm and Ward (1987), which is also used for overcoming local optima, receives its name from the physical process that it imitates. This process, called annealing moves high energy particles to lower energy states with the lowering of the temperature, thus cooling a material to a steady state. Initially, in the initial state of the heuristic, the algorithm is lenient and capable of moving to a worse solution. With each iteration, the algorithm becomes stricter requiring a better solution at each step.

2.7.3. Construction Methods (CM)

Construction Methods create suboptimal permutations by starting with a partial permutation which is initially empty. The permutation is expanded by repetitive assignments based on set selection criterion until the permutation is complete. One of the oldest heuristics in use is a construction method algorithm [Buffa, Armour and Vollmann (1964)].

2.7.4. Greedy Randomized Adaptive Search Procedures (GRASP)

The greedy randomized adaptive search procedure (also known as GRASP) is a metaheuristic algorithm commonly applied to combinatorial optimization problems. GRASP typically consists of iterations made up from successive constructions of a *greedy randomized* solution and subsequent iterative improvements of it through a local search. The greedy randomized solutions are generated by adding elements to the problem's solution set from a list of elements ranked by a *greedy function* according to the quality of the solution they will achieve. To obtain variability in the candidate set of greedy solutions, well-ranked candidate elements are often placed in a *restricted candidate list* (also known as RCL), and chosen at random when building up the solution. This kind of greedy randomized construction method is also known as a semi-greedy heuristic, first described in Hart and Shogan (1987).

GRASP is a relatively new heuristic used for solving combinatorial optimization problems. At each iteration, a solution is computed. The final solution is taken as the one that is the best after all GRASP iterations are performed. The GRASP was first applied to QAP by Li, Pardalos, and Resende in 1994 . They applied the GRASP to 88 instances of QAP, finding the best known solution in almost every case, and improved solutions for a few instances.

2.7.5. Limited Enumeration Methods (LEM)

Limited enumeration is a procedure of operations research which corresponds to complete enumeration, but the algorithm is aborted when a time or node limit is reached. Enumeration methods can guarantee that the obtained solution is optimum only if they can go to the end of the enumerative process. However, it is possible that a good solution, or even an optimal solution, is found by the beginning of the process. It can be observed that the best the information used to guide the enumeration, the bigger the chances to find prematurely good quality solutions.

The method is based directly on the enumeration of sequentially organized full enumeration. Before the enumeration process starts, general heuristic methods are used. With them an initial solution is calculated. An upper bound of the initial costs is generated (enumeration limit). If, during the enumeration process, a solution is reached or exceeded, the calculation of this solution is stopped and we start to build another solution. It skips groups of branches of full enumeration. When during the enumeration process, a solution is found and the costs are below the existing bound, you continue with these new costs as the limit of the calculation. When the enumeration is done, the optimal solution is the last bound which was created. For the operation of the limited enumeration, it is very important to reduce that problem. It corresponds to the determination of costs lower limits on branching and bounding. By "reducing" of the problem, a separation of basic costs and solution-dependent costs is introduced. It should help to increase the cost during the enumerative structure of solution. Thus non-optimal solutions are seen early and large groups of branches of the full enumeration can be skiped.

The limited enumeration methods [West (1983), Burkard and Bonniger (1983)] are robustly related to exact methods like branch and bound and cutting planes. The inspiration behind these algorithms is that a good suboptimal solution may be produced early in an enumerative search. In addition, an optimal solution may be found earlier in the search while the rest of the time is spent on proving the optimality of this solution. There are several ways to limit enumeration of the search space; one approach is to impose a time limit. Enumeration stops when the algorithm reaches a time limit or no improvement has been made in a predetermined time interval. These pre-specified parameters can be problem specific. A second option is to decrease the requirement for optimality. For example, if no improvement has been made after a certain pre-specified time interval, then the upper bound is decreased by a certain percentage resulting in deeper cuts in the enumeration tree. Although the optional solution may be cut off, it differs from the obtained solution by the above percentage.

2.7.6. Genetic Algorithms (GA)

In the field of artificial intelligence, a genetic algorithm (GA) is a search heuristic that mimics the process of natural selection. This heuristic (also sometimes called a metaheuristic) is routinely used to generate useful solutions to optimization and search problems. Genetic algorithms belong to the larger class of evolutionary algorithms (EA), which generate solutions to optimization problems using techniques inspired by natural evolution, such as inheritance, mutation, selection, and crossover.

Genetic algorithms [Fleurent and Ferland (1994), Tate and Smith (1985), Ahuja, Orlin, and Tewari (1998), Drezner, Z (2003), Yongzhong Wu and Ping Li(2007)] receive their name from an intuitive explanation of the manner in which they behave. This explanation based on Darwin's theory of natural selection. Genetic algorithms store a group of solutions and then work to replace these solutions with better ones based on some fitness criterion, usually the objective function value. Genetic algorithms are parallel and supportive when applied in such an environment.

2.7.7 Ant Colonies (AC)

The ant colony optimization algorithm (ACO) is a probabilistic technique for solving computational problems which can be reduced to finding good paths through graphs. This algorithm is a member of the ant colony algorithms family, in swarm intelligence methods, and it constitutes some metaheuristic optimizations. Initially proposed by Marco Dorigo in 1992 in his PhD thesis, the first algorithm was aiming to search for an optimal path in a graph, based on the behavior of ants seeking a path between their colony and a source of food.Ant Colonies are based on the principle that using very simple communication mechanisms, an ant group is able to find the shortest path between any two points. During their trips, a chemical trail (pheromone) is left on the ground. The role of this trail is guiding the other ants towards the target point. For one ant, the path is selected according to the quantity of pheromone. Moreover, this chemical substance has a decreasing action over time, and the quantity left by one ant relies on the amount of food found and the number of ants using this trail. Applied on QAP, the AC is based on a hybridization of the ant system with a local search method, each ant being associated with an integer permutation. Modifications based on the pheromone trail are then applied to each permutation. The solutions (ants) found so far are then optimized using a local search method, update of the pheromone trail simulates the evaporation and takes into account the solutions produced in the search strategy.

2.7.8. Tabu search

Tabu search is a local search algorithm that was introduced by Glover to find good quality solutions for integer programming problems. Its main feature is an updated list of the best solutions that were found in the search process. Each solution receives a priority value or an aspiration criterion. Their basic ingredients are: a *tabu list*, used to keep the history of the search process evolution; a mechanism that allows the acceptance or rejection of a new allocation in the neighborhood, based on the tabu list information and on their priorities; and a mechanism that allows the alternation between neighborhood diversification and intensification strategies.

2.8. Exact Algorithms:

Fukasawa et al. (2006) described an exact algorithm based on the SP model where the variables correspond to the set of q-routes, introduced by Christofides et al. (1981), while the constraints correspond to the set partitioning constraints and valid inequalities , such as rounded capacity inequalities, framed capacity, strengthened comb, multistar, partial multistar, generalized large multistar and hypo tour inequalities, all presented in Lysgaard et al. (2004) for formulation R. Baldacci et al. (2008) proposed an exact algorithm based on model SP, strengthened by the following two types of valid inequalities.- Strengthened capacity inequalities, Clique inequalities. To attain optimality for QAP there are three main methods: the Branch and bound procedures, the cutting plane techniques, and the Dynamic programming

2.8.1 The Branch and Bound Method

The most important and used method to achieve optimality for QAP is the Branch and bound as it is more efficient technique. The Branch and bound procedures was firstly introduced by Gilmore in 1962 who solved a QAP of size 8 = n. The Branch and bound method was named as it is applied. All recent break-through in solving QAP are actually related to this approach. The main idea is as follows: we do not want to check all permutations; therefore we construct an optimal permutation step by step. We start with the empty permutation (i. e. no building is assigned) and successively extend it to a full (optimal) permutation. Throughout the procedure we need a good feasible solution for QAP (i. e. a good upper bound B). Therefore, to apply Branch and bound method; first, chose a heuristic procedure to get an initial feasible solution "suboptimal" and it is used as upper bound. Second, the problem is fragmented into sub- problems with a lower bound, then formulate search tree by repeating the fragmentation and lower bounding of each sub-problem .During iterations, an optimal permutation is being constructed. Branch and bound algorithms have been applied successfully to many hard combinatorial optimization problems, and they appear to be the most efficient exact algorithms for solving the QAP. The basic ingredients of branch and bound algorithms are bounding, branching, and the selection rule. Although many bounding techniques have been developed for the QAP the most efficient branch and bound algorithms for this problem employ the Gilmore-Lawler bound (GLB). The reason is that other bounds which outperform GLB in terms of bound quality are simply expensive in terms of computation time.

2.8.2 The Cutting Plane Method

The Cutting Plane Method has two classes: traditional cutting plane methods and polyhedral cutting-plane or branch-and-cut methods. Traditional cutting plane algorithms for QAP first used by Kaufman and Broekx in 1978 , these algorithms make use of mixed integer linear programming (MILP) formulations for QAP . Generally, the time needed for these methods is too large, and hence these methods may solve to optimality only very small QAPs. However, heuristics derived from cutting plane approaches produce good suboptimal solutions in early stages of the search. The polyhedral cutting planes or branch and cut algorithms make use of MILP formulations of QAP. The polyhedral cutting plane is not widely used in QAP because of the scarcity of knowledge about QAP polytypes.

2.8.3.The Dynamic Programming

The thought behind the dynamic programming method is reasonably straightforward. In broad-spectrum, to solve a given problem, different parts of the problem "sub problems" need to be solved, and then the solutions of the sub problems should be combined to reach an overall solution. Christofides and Benavent(1989) used a dynamic programming approach to solve a special case of QAP. Dynamic programming solves a problem of size n by starting from sub problems of size 1,2,…,n-1. After solving sub problems of size k it upgrades the solutions to size k+1. Problems may arise in dynamic programming if the solution to a sub problem or the upgrade procedure cannot be performed in polynomial time.

Table 4: Representative Exact algorithms, Classical Heuristic Algorithms and Metaheuristic Algorithms for VRP.

Authors	Year	Methodologies
Christofides and Eilon	1969	Branch and Bound
G. Laporte et al	1988	Branch and Bound
Martin Desrochers et	1992	Column Generation
Miller	1995	Branch and Bound
Hadjiconstantinou et al	1995	Set-partitioning
Ralph, T.K.	2003	Parallel Branch and Cut
Baldacci et al.	2004	Branch and Cut
Jin et al.	2008	Column Generation

Source: Collected and compiled by author

Table 5: Classical Heuristic Algorithms

Authors	Year	Methodologies
Dantzig and Ramser	1959	Constructive algorithm. First approach
Clarke and Wright	1964	Saving. Concurrent & sequential
Wren and Holliday	1972	Sweep Algorithm. Multiple depots
Lin and Kernighan	1973	Single-route improvement Sequential k-exchange
Gillett and Miller	1974	Sweep Algorithm. Single depot
Foster and Ryan	1976	Petal algorithm. Optimal petal solution
Mole and Jameson	1976	Sequential Route-Building Insertion position check
Christofides et al.	1979	Sequential Route-Building Sequential & Parallel construction
Fisher and Jaikumar	1981	Cluster-First Route-Second Generalized Assignment + TSP
Beasley	1983	Route-First Cluster-Second
Altinkemer and Gavish	1991	Matching Algorithm. Matching clusters
Ryan et al.	1993	Petal algorithm
Thompson and Psaraftis	1993	Multiple-Route Improvement b-cyclic k-transfer
Potvin and Rousseau	1995	Single-route improvement. Based on 2-opt
Bramel and Simchi-Levi	1995	Cluster-First Route-Second
Renaud et al.	1996	Single-route improvement
Kindervater and Savelsbergh	1997	Multiple-Route Improvement

Source: Collected and compiled by author

2.9. Metaheuristics

In computer science and mathematical optimization, a metaheuristic is a higher-level procedure or heuristic designed to find, generate, or select a heuristic (partial search algorithm) that may provide a sufficiently good solution to an optimization problem, especially with incomplete or imperfect information or limited computation capacity. Metaheuristics sample a set of solutions which is too large to be completely sampled. Metaheuristics may make few assumptions about the optimization problem being solved, and so they may be usable for a variety of problems. There are properties that characterize most metaheuristics which are as follows: Metaheuristics are strategies that guide the search process.The goal is to efficiently explore the search space in order to find near–optimal solutions.Techniques which constitute metaheuristic algorithms range from simple local search procedures to complex learning processes.Metaheuristic algorithms are approximate and usually non-deterministic.Metaheuristics are not problem-specific.Compared to optimization algorithms and iterative methods, metaheuristics do not guarantee that a globally optimal solution can be found on some class of problems. Many metaheuristics implement some form of stochastic optimization, so that the solution found is dependent on the set of random variables generated. By searching over a large set of feasible solutions, metaheuristics can often find good solutions with less computational effort than optimization algorithms, iterative methods, or simple heuristics. As such, they are useful approaches for optimization problems. Metaheuristics are used for combinatorial optimization in which an optimal solution is sought over a discrete search-space. An example problem is the travelling salesman problem where the search-space of candidate solutions grows faster than exponentially as the size of the problem increases, which makes an exhaustive search for the optimal solution infeasible.

Table 6: Metaheuristic Algorithms

Authors	Year	Methodologies
Osman	1993	SA
Taillard	1993	TS
Gendreau et al.	1994	TS
Van Breedam	1995	SA
Rochat and Taillard	1995	TS
Xu and Kelly	1996	TS
Kawamura et al.	1998	ACO
Bullnheimer et al.	1999	ACO
Toth and Vigo	2003	TS
Baker and Ayechew	2003	GA
Mazzeo and Loiseau	2004	ACO

Source: Collected and compiled by author

2.10.The simultaneous assignment problem:

Yamada and Nasu(2000) discuss an application of a many-to-one three-dimensional AP that they call the simultaneous assignment problem. They discuss and provide both heuristics and algorithms for the problem of simultaneously assigning cadets at Japan's National Defense Academy to science/engineering departments and to branches of Japan's armed services, recognizing both the students' preferences (the objective function) and limits on the sizes of the classes and the numbers of cadets to be assigned to the different service branches.

2.11.Fuzzy assignment problem:

Fuzzy assignment problems have received great attention in recent years. Aggarwal et al(1987) developed two algorithms for solving bottleneck assignment problems. Costs in many real life applications are not deterministic numbers. The fuzzy assignment problem (FAP) is more realistic than the AP because most real environments are uncertain. In recent years, many researchers have begun to investigate AP and its variants under fuzzy environments. For instance, Chen, M.S(1985) solved a fuzzy assignment model that considers all individuals have same skills. He proposed a fuzzy assignment model that did not consider the differences of individuals, and also proved some theorems. Feng and Yang(2006)studied a two objective fuzzy k-cardinality AP. Sakawa et al.(2001),considered interactive fuzzy programming for two-level or multi level linear programming problems to obtain a satisfactory solution for decision making. Longsheng Huang and Guang-hui Xu(2005) proposed a solution procedure for the AP with restriction of qualification. By the max–min criterion suggested by Bellman and Zadeh(1970), the fuzzy assignment problem can be treated as a mixed integer nonlinear programming problem. Lin and Wen(2004) investigated a fuzzy assignment problem in which the cost depends on the quality of the job. Michéal ÓhÉigeartaigh(1982) and Chanas et al.(1984)solved transportation problems with fuzzy supply and demand values. An integer fuzzy transportation problem was solved in Tada and Ishii(1996). Chanas and Kuchta(1996) proposed the concept of the optimal solution of the transportation problem with fuzzy coefficients expressed as L-R fuzzy numbers, and developed an algorithm for determining the solution. Additionally, Chanas and Kuchta(1998) designed an algorithm for solving integer fuzzy transportation problem with fuzzy demand and supply values in the sense of maximizing the joint satisfaction of the fuzzy goal and the constraints. Pandian and Natarajan (2010a,2010b) have introduced two different methods for solving the fuzzy transportation problem. Fractional programming is a particular type of non-linear programming in which the objective function to be optimized is the ratio of two other objective function. Wang(1987) solved a similar model by graph theory. Dubois and Fortemps(1999)proposed a flexible assignment problem, which combines with fuzzy theory, multiple criteria decision-making and constraint-directed methodology. They also demonstrated and solved an example of fuzzy assignment problem. Sakawa et al.(2001) dealt with actual problems on production and work force assignment of a housing material manufacturer and formulated two-level linear and linear fractional programming problems according profitability maximization respectively. By applying interactive fuzzy programming for two-level linear and linear fractional programming problems, they derived satisfactory solutions to the problems and compared the results. Majumdar and Bhunia(2007) proposed an elitist genetic algorithm to solve generalized assignment problem with imprecise cost/time. Ye and Xu(2008) proposed an effective method on priority-based genetic algorithm to solve fuzzy vehicle routing assignment when there is no genetic algorithm which can give clearly procedure of solving it. Liu and Gao(2009) proposed an equilibrium optimization problem and extended the assignment problem to the equilibrium multi-job assignment problem, equilibrium multi-job quadratic assignment problem and used genetic algorithm to solve the proposed models. Bai et al.(2009) proposed a method for solving fuzzy generalized assignment problem.

2.12.Frequency Assignment Problems:

Wireless communication is used in many different situations such as mobile telephony, radio and TV broadcasting, satellite communication, and military operations. In each of these situations a frequency assignment problem arises with application specific characteristics. Researchers have developed different modeling ideas for each of the features of the problem, such as the handling of interference among radio signals, the availability of frequencies, and the optimization criterion.

Frequency assignment problems (FAPs) first appeared in the 1960s. The development of new wireless services such as the first cellular phone networks led to scarcity of usable frequencies in the radio spectrum. Frequencies were licensed by the government who charged operators for the usage of each single frequency separately. This introduced the need for operators to develop frequency plans that not only avoided high interference levels, but also minimized the licensing costs. It turned out that it was far from obvious to find such a plan. At this point, operations research techniques and graph theory were introduced. Metzger(1970) usually receives the credits for pointing out the opportunities to use mathematical optimization, especially graph coloring techniques, for this purpose.

The literature on frequency assignment problems, also called channel assignment problems, has grown quickly over the past years. This is mainly due to the fast implementation of wireless telephone networks (e.g., GSM networks) and satellite communication projects. But also the renewed interest in other applications like TV broadcasting and military communication problems inspired new research. These applications lead to many different models, and within the models to many different types of instances. However, all of them share two common features:

(i) A set of wireless communication connections (or a set of antennae) must be assigned frequencies such that data transmission between the two endpoints of each connection (the receivers)
is possible. The frequencies should be selected from a given set that may differ among connections.

(ii) The frequencies assigned to two connections may incur interference to one another, resulting in quality loss of the signal. Two conditions must be fulfilled in order to have interference of two signals:

(a) The two frequencies must be close on the Electromagnetic band (or harmonics(Doppler effects of one another).

(b) The connections must be geographically close to each other, so that the interfering signal is powerful enough to disturb the original signal.

Until the early 1980s, most contributions on frequency assignment used heuristics based on the related graph coloring problem. First lower bounds were derived by Gamst and Rave(1982) for the most used problem of that time . The development of the digital cellular phone standard GSM (General System for Mobile Communication) in the late 1980s and 1990s led to a rapidly increasing interest for frequency assignment for a discussion of the typical frequency planning problems in GSM networks). But also projects on other applications such as military wireless communication and radio/TV broadcasting contributed to the literature on frequency assignment in recent years. So far, we only discussed Fixed Channel Assignment (FCA), i.e., static models where the set of connections remains stable over time. Opposite to FCA, Dynamic Channel Assignment (DCA) deals with the problem, where the demand for frequencies at an antenna varies over time. Hybrid Channel Assignment (HCA) combines FCA and DCA: a number of frequencies have to be assigned beforehand, but space in the spectrum has to be reserved for the online assignment of frequencies upon request.

Wireless communication is used in many different situations such as mobile telephony, radio and TV broadcasting, satellite communication, and military operations. In each of these situations a frequency assignment problem arises with application specific characteristics. Researchers have developed different modeling ideas for each of the features of the problem, such as the handling of interference among radio signals, the availability of frequencies, and the optimization criterion.

This survey gives an overview of the models and methods that the literature provides on the topic. We present a broad description of the practical settings in which frequency assignment is applied. We also present a classification of the different models and formulations described in the literature, such that the common features of the models are emphasized. The solution methods are divided in two parts. Optimization and lower bounding techniques on the one hand, and heuristic search techniques on the other hand. The literature is classified according to the used methods. Again, we emphasize the common features, used in the different papers. The quality of the solution methods is compared, whenever possible, on publicly available benchmark instances.

2.13. The Multicommodity Multilevel Bottleneck Assignment Problem

The Multilevel Bottleneck Assignment Problem is defined on a weighted graph of L levels and consists in finding L−1L−1 complete matchings between contiguous levels, such that the heaviest path formed by the arcs in the matchings has a minimum weight. The problem, introduced by Carraresi and Gallo (1984) to model the rostering of bus drivers in order to achieve an even balance of the workload among the workers, though frequently cited, seems to have never been applied or extended to more general cases.

2.14.Range Assignment Problem in Ad Hoc Networks:

Recent emergence of affordable, portable, wireless communication and computation devices, and concomitant advances in the communication infrastructure, have resulted in the rapid growth of mobile wireless networks. Among these, *ad hoc networks*, i.e. networks of mobile, untethered units communicating with each other via radio transceivers, are receiving increasing attention in the scientific community. *Ad hoc* networks, also called *multi-hop packet radio networks*, can be used wherever a wired backbone is not viable, e.g. in mobile computing applications in areas where other infrastructure is unavailable, or to provide communications during emergencies. When designing protocols for *ad hoc* networks, the following characteristics peculiar to these networks have to be taken into account:

- *shared communications*: since the stations in the network communicate via radio transceivers, the most natural communication paradigm is one-to-many: when a unit transmits, all the units within its transmitting range receive the message. On the contrary, wired networks use selective transmission (one-to-one) as the natural communication paradigm.
- *energy constraints*: since the stations are equipped with limited energy supplies, one of the primary goals is to reduce the overall energy consumption of the network, thus increasing its lifetime. Routing, broadcast and clustering protocols explicitly designed for ad hoc networks have been recently proposed in the literature. Some of these protocols are designed for energy-efficient operation in an existing network topology, while others attempt to deal with the effects of mobility, and still others consider both of these aspects. It should be observed that further energy can be saved if the network topology itself is energy-efficient, i.e. if the transmitting ranges of the units are set in such a way that a target property (e.g. strong connectivity1) of the resulting network topology is guaranteed, while the global energy consumption is minimal. For this reason, *topology control* protocols have been recently introduced in the literature. Informally speaking, a topology control protocol is an algorithm in which units adjust their transmitting ranges in order to achieve a desired topological property, while optimizing energy consumption. The problem of ensuring strong connectivity while minimizing some measure of energy consumption has also been considered in a more theoretical framework, where it is referred to as the *range assignment problem*. In particular, it has been shown that determining an optimal range assignment is solvable in polynomial time in the one-dimensional case, while it is NP-hard in the two and three-dimensional cases .

2.15.Logic Cuts for the Multilevel Generalized Assignment Problem

In the multilevel generalized assignment problem (MGAP), agents can perform tasks at more than one efficiency level (María A. Osorio et.al, 2003). Important manufacturing problems, such as lot sizing, can be easily formulated as MGAPs; however, the large number of variables in the related 0-1 integer program makes it hard to find optimal solutions to these problems, even when using powerful commercial optimization packages. The MGAP includes a set of knapsack constraints, one per agent, that can be useful for generating simple logical constraints or logic cuts. The method exploit the fact that logic cuts can be generated in linear time and can be easily added to the model before solving it with classical branch and bound methodology.

2.16.A Branch-and-Cut Algorithm for the Multilevel Generalized Assignment Problem

The multilevel generalized assignment problem (MGAP) consists of minimizing the assignment cost of a set of jobs to machines, each having associated therewith a capacity constraint (Pasquale Avella, Maurizio Boccia, and Igor Vasilyev, 2013). Each machine can perform a job with different efficiency levels that entail different costs and amount of resources required. The MGAP was introduced in the context of large manufacturing systems as a more general variant of the well-known generalized assignment problem, where a single efficiency level is associated with each machine. In this method, a branch-and-cut algorithm is proposed whose core is an exact separation procedure for the multiple-choice knapsack polytope induced by the capacity constraints and single-level execution constraints.

3. Conclusion

The focal impetus behind this research presentation is the constant interest in assignment problem shown by a great deal of researchers worldwide for the theory, applications and solution techniques of this problem. This article depicts fundamentals of different kinds of assignment problem and some solving techniques, highlights some efforts on solving AP. This paper can be directed towards assisting potential researchers in this field to develop new assignment solution technique as well as modeling real world applications by getting an over view about assignment problem and developments in the field of optimization.

References

[1] A. Gamst , W. Rave (1982), On Frequency Assignment in Mobile Automatic Telephone Systems. *Proc. Globecom'82*, B3.1.1-B3.1.7.
[2] Altinkemer, K. and B. Gavish(1991), "Parallel savings based heuristics for the delivery problem". *Operations*

Research 39(3), pp.456–469.
[3] A.Bouras, Problème d'affectation quadratique de petit rang; modèles, compléxite, et applications, PhD Thesis, L'Université Joseph Fourier, Grenoble, France,1996.
[4]Axel Nyberg , Some Reformulations for the Quadratic Assignment Problem, Department of Chemical Engineering Abo Akademi University , Finland,2014.
[5] Agrawal,A, and M.M.Klawe,S.Moran, P.Shor and R.Wilber(1987), Geometric Applications of a matrix searching algorithm,*Algorithmica*,vol.2(2),pp.195-208.
[6]Abdel Nasser H. Zaied , Laila Abd El-Fatah Shawky(2014), A Survey of the Quadratic Assignment Problem**,** *International Journal of Computer Applications, Vol. 101, No.6, September,* pp.28 -36.
[7]Ahuja, R., Orlin, J.B., Tiwari, A(2000), A greedy genetic algorithm for the quadratic assignment problem, *Computers and Operations Research* ,vol.27 (10), pp.917–934.
[8]David W. Pentico(2007), “Assignment problems: A golden anniversary survey”, *European Journal of Operational Research*,vol. 176 ,pp. 774–793.
[9]Baldacci, R., Christofides, N., Mingozzi, A., 2008. An exact algorithm for the vehicle routing problem based on the set partitioning formulation with additional cuts, *Mathematical Programming Series A* ,vol.115 (2),pp.351–385.
[10]Baldacci, R., E. Hadjiconstantinou, and A. Mingozzi(2004), “An exact algorithm for the capacitated vehicle routing problem based on a two-commodity network flow formulation”,*Operations Research* ,vol.*52*(5), pp.723–738.
[11]Baker, B. M. and M. A. Ayechew(2003), “A genetic algorithm for the vehicle routing problem”. *Computers & Operations Research*,vol. *30*(5), pp.787–800.
[12]Beasley, J. E(1983), “Route-first cluster-second methods for vehicle routing, *Omega* ,vol.11(4), pp. 403–408.
[13]Bos(1993), quadratic assignment problem solved by simulated annealing, *Journal of Environmental Management* ,vol. 37 (2), pp.127–145.
[14]Buffa, E.S., Armour, G.C., Vollmann, T.E(1964), Allocating facilities with CRAFT, *Harvard Business Review*,vol. 42 (2), pp.136–158.
[15]Burkard, R.E., Bonniger, T(1983), A heuristic for quadratic Boolean programs with applications to quadratic assignment problems, *European Journal of Operation Research*,vol. 13, pp.374–386.
[16]Brusco, M.J., Stahl, S(2000), Using quadratic assignment methods to generate initial permutations for least-squares unidimensional scaling of symmetric proximity matrices, *Journal of Classification*,vol. 17 (2), pp.197–223.
[17]Bramel, J. and D. Simchi-Levi(1995), “A location based heuristic for general routing problems”, *Operations Research* ,vol.*43*(4), pp.649–660.
[18]Burkard, R.E., Rendl, F(1984), A thermodynamically motivated simulation procedure for combinatorial optimization problems, *European Journal of Operational Research*, vol.17 (2), pp.169–174.
[19]Burkard, R. E. (1985), “Time-slot assignment for TDMA systems,” *Computing*, vol.35 (2), pp.99–112.
[20] Benjaafar, S(2002), Modeling and analysis of congestion in the design of facility layouts, Management Science ,vol.48 (5), pp.679–704.
[21]Benjaafar, S(2002), Modeling and analysis of congestion in the design of facility layouts, Management Science ,vol.48 (5), pp.679–704.
[22]Bullnheimer, B., R. Hartl, and C. Strauss (1999), “An improved ant system algorithm for the vehicle routing problem”. *Annals of Operations Research*,vol. *89*, pp.319–328.
[23]Bertsekas ,Dimitri P, "The auction algorithm for assignment and other network flow problems: A tutorial", *Interfaces*, 1990
[24]Ben-David, G., Malah, D(2005), Bounds on the performance of vector-quantizers under channel errors, IEEE Transactions on Information Theory, vol.51 (6), pp.2227–2235.
[25]Bellman, R. and Zadeh, L. (1970), Decision-making in a fuzzy environment, *Management Science*,vol.17(4),pp. 141–164.
[26] Brogan,W. L. (1989), “Algorithm for ranked assignments with applications to multiobject tracking,” *Journal of Guidance, Control, and Dynamics,* vol.12 (3), pp.357–364.
[27]Christofides, N. and S. Eilon (1969, “September). An algorithm for the vehicle-dispatching problem”. *Operational Research Quarterly*,vol. 20(3), pp.309–318.
[28]Christofides, N., Mingozzi, A., Toth, P(1981), Exact algorithms for the vehicle routing problem based on spanning tree and shortest path relaxations, *Mathematical Programming*,vol. 20 (1), pp.255–282.
[29]Clarke G & Wright JW(1964), Scheduling of vehicles from a central depot to a number of delivery points, *Operations Research*, vol.12(4), pp. 568{581.
[30]Christofides, N., Benavent, E(1989), An exact algorithm for the quadratic assignment problem, *Operations Research*,vol. 37 (5),pp. 760–768.
[31]C.J. Lin and U.P. Wen(2004), The labeling algorithm for the fuzzy assignment problem, *Fuzzy Sets and*

Systems, vol.142, pp.373–391.
[32]Chanas, S. , W. Kolodziejczyk and A. Machaj(1984), A fuzzy approach to the transportation problem, *Fuzzy Sets and Systems*, vol.13 ,pp. 211–221.
[33]Chanas, S and D. Kuchta(1996), A concept of the optimal solution of the transportation problem with fuzzy cost coefficients, *Fuzzy Sets and Systems,* vol. 82,pp. 299–305.
[34]Christofides, N., A. Mingozzi, and P. Toth, “The vehicle routing problem”. In N. Christofides, A. Mingozzi, P. [35]Toth, and C. Sandi (Eds.), *Combinatorial optimization*, Chapter 11, 1979, pp. 315–338. Chichester, England: John Wiley & Sons Ltd.
[36]Dumas. Yvan, Jacques Desrosiers, , Marius M Solomon, François Soumis(1995), Time constrained routing and scheduling, *Handbooks in operations research and management science,* vol.8,pp.35-139, Elsevier.
[37]Drezner, Z(2003), A new genetic algorithm for the quadratic assignment problem, Informs Journal on Computing,vol. 15 (3), pp.320–330.
[38]Dell’Amico, M., Díaz, J. C. D., Iori, M., & Montanari, R. (2009), The single-finger keyboard layout problem, *Computers & Operations Research*,vol. *36*(11), pp.3002-3012.
[39]Dubois, D. & Fortemps, P. (1999), Computing improved optimal solutions to max-min flexible constraint satisfaction problems, *European J. Op. R.* 118, 95-126.
[40]De Carvalho Jr., S.A., Rahmann, S. (2006): Microarray Layout and the Quadratic Assignment Problem. (poster) 14th Annual International Conference on Intelligent Systems for Molecular Biology (ISMB), Fortaleza, Brazil.
[41]D. Kőnig, "Gráfok és Mátrixok," *Matematikai és Fizikai Lapok*, vol. 38, 1931, pp. 116–119.
[42]Dickey, J.W., Hopkins, J.W(1972), “*Campus building arrangement using Topaz”, Transportation Research* ,vol.6, pp.59–68.
[43]Elshafei, A.N(1977), Hospital layout as a quadratic assignment problem, *Operations Research Quarterly*,vol. 28 (1), pp.167–179.
[44]E. ÇELA, *The quadratic assignment problem: special cases and relatives*. PhD thesis, Graz University of Technology, Graz, Austria, 1995[Cela:98].
[45]E. Duman, M. Uysal, A. F. Alkaya. Migrating Birds Optimization: A new metaheuristic approach and its performance on quadratic assignment problem, Information Sciences, Volume 217, 25 December 2012.
[46]E. .Klerk, M.Nagy, R. Sotirov, U. Truetsch(2014), Symmetry in RLT-type relaxations for the quadratic assignment and standard quadratic optimization problems, *European Journal of Operational Research*, Volume 233, Issue 3, 16 March 2014.
[47]F. Malucelli, *Quadratic assignment problems: solution methods and applications*. PhD thesis, University of Pisa, Pisa, Italy, 1993[Maniezzo:97] .
[48]Feng, Y.and L.Yang(2006), A two objective fuzzy k-cardinality assignment problem, *Journal of Computational and Applied Mathematics*, vol.1,pp.233-244.
[49]Francis, R. L., & White, J. A(1974), *Facility Layout and Location: An Analytical Approach*. Prentice-Hall, Englewood Cliffs, NJ.
[50]Forsberg, J.H., Delaney, R.M., Zhao, Q., Harakas, G., Chandran, R., “*Analyzing lanthanide-included shifts in the NMR spectra of lanthanide (III) complexes derived from 1,4,7,10-tetrakis (N,N-diethylacetamido)-1,4,7,10-tetraazacyclododecane*”, Inorganic Chemistry 34, 3705–3715, 1994.
[51]Franklin Djeumou Fomeni, New Solution Approaches for the Quadratic Assignment Problem , University of the Witwatersrand,2011, URI: http://hdl.handle.net/10539/11074.
[52]Francesco Puglierin, A Bandit-Inspired Memetic Algorithm for Quadratic Assignment Problems, University of Utrecht, unpublished master’s thesis, 2012.
[53]Fleurent, C., Ferland, J.A(1994), Genetic hybrids for the quadratic assignment problem. In: Pardalos, P.M.,
[54]Wolkowicz, H. (Eds.), Quadratic Assignment and Related Problems, DIMACS Series in Discrete Mathematics and Theoretical Computer Science, vol. 16. AMS, Rhode Island, pp. 173–187.
[55]Fisher, M. and R. Jaikumar(1981), “A generalized assignment heuristic for vehicle routing”. *Networks,* vol.11(2), pp.109–124.
[56]Fukasawa, R., Longo, H., Lysgaard, J., Poggi de Aragão, M., Reis, M., Uchoa, E., Werneck, R(2006), Robust branch-and-cut-and-price for the capacitated vehicle routing problem. Mathematical Programming Series A 106, pp.491–511.
[57]Foster, B. A. and D. M. Ryan(1976), “An integer programming approach to the vehicle scheduling problem”. *Operational Research Quarterly*,vol. *27*(2), pp. 367–384.
[58]Geoffrion,A.M and Graves,G.W (1976), Scheduling Parallel Production Lines with Changeover Costs: Practical Applications of a Quadratic Assignment/LP ,Operation Research,vol.24,no.4,pp.595-610.
[59]G.B. Dantizig, J.H. Ramser(1959), ”The Truck Dispatching Problem” Management Science, vol. 6, pp 81-91. Gunes Erdogan, Quadratic assignment problem: linearizations and polynomial time solvable cases, BILKENT UNIVERSITY, Turkey, 2006.

[60]Gillett, B. & Miller L. (1974), A Heuristic for the Vehicle Dispatching Problem. *Operations Research*, vol.22, pp.340-349.

[61]G. Laporte, Y. Nobert, S. Taillefer(1988), Solving a family of multi-depot vehicle routing and location-routing problems, Transportation Science,vol. 22 ,pp.161–172.

[62]G. Caron, P. Hansen, B. Jaumard(1999), The assignment problem with seniority and job priority constraints, *Operations Research,*vol. 47 (3),pp. 449–454.

[63]Hart & Shogan(1987), Hart J.P. and A.W. Shogan, "Semigreedy heuristics: An empirical study", *Operations Research Letters*, 6:107-114.

[64]Gendreau, M., A. Hertz, and G. Laporte, "A tabu search heuristic for the vehicle routing problem", *Management Science* ,vol.*40*(10), pp. 1276–1290.

[65]Hadjiconstantinou, E., N. Christofides, and A. Mingozzi(1995), "A new exact algorithm for the vehicle routing problem based on q-paths and k-shortest paths relaxation". *Annals of Operations Research*,vol. 61(1-4), pp.21–43.

[66]H. Imai and M. Iri(1986), "Computational-Geometric Methods for Polygonal Approximations of a Curve," *Computer Vision, Graphics and Image Processing*, vol. 36, pp. 31-41.

[67a]Heffley, D. R(1972), The quadratic assignment problem: A note. *Econometrica*, vol.40 (6), pp.1155–1163.

[67b]Hubert, L., "*Assignment methods in combinatorial data analysis*", Statistics: Textbooks and Monographs Series, vol. 73. Marcel Dekker, 1987.

[68]Huey-Kuo Chen, Che-Fu Hsueh, Mei-Shiang Chang(2009), "Production Scheduling and Vehicle Routing with Time Windows for Perishable Food Products", *Computers & Operations Research*,vol. 36 ,pp. 2311 – 2319.

[69]Heffley, D. R(1980), Decomposition of the Koopmans–Beckmann problem. *Regional Science and Urban Economics*, vol.10 (4), pp.571–580.

[70]Jin, M., Liu, K., & Ekşioğlu, B(2008), A column generation approach for the split delivery vehicle routing problem, *Operations Research Letters,*vol. 36,pp. 265–270.

[71]Jing-Quan Li , Pitu B. Mirchandanib, Denis Borenstein(2009), Real-time Vehicle Rerouting Problems With Time Windows, *European Journal of Operational Research*,vol. 194,pp.711–727.

[72]J. Kosowsky and A. Yuille, *Solving the Assignment Problem with Statistical Physics*, IEEE, 1991.

[73] J. Wastlund(2003), "Random Assignment and Shortest Path Problems," *Discrete Mathematics And Theoretical Computer Science*, pp. 1-11.

[74]J. W. Dickey and J. W. Hopkins(1972), Campus building arrangement using TOPAZ, *Transportation Research*, vol.6, pp.59–68.

[75]Krarup, J., Pruzan, P.M.(1978), "*Computer-aided layout design*", *Mathematical Programming Study*,vol. 9,pp. 75–94.

[76]Kaufman, L., Broeckx, F(1978), An algorithm for the quadratic assignment problem using Bender's decomposition, *European Journal of Operation Research*,vol. 2,pp. 204–211.

[77]Kindervater, G. A. P. and M. W. P. Savelsbergh, "Vehicle routing: handling edge exchanges", 1997.

[78]Kawamura, H., M. Yamamoto, T. Mitamura, K. Suzuki, and A. Ohuchi, "Cooperative search based on pheromone communication for vehicle routing problems", *IEICE transactions on fundamentals of electronics, communications and computer sciences E81-A*(6), 1998, pp.1089–1096.

[79]Kim ,B, I., S. Kim, and S. Sahoo(2006), "Waste collection vehicle routing problem with time windows," *Computers & Operations Research,* vol. 33, pp. 3624-3642.

[80] Kuhn, H. (1955), "The Hungarian Method for the Assignment Problem," *Naval Research Logistics Quarterly*, vol. 2, , pp. 83-97.

[81]Lysgaard, J., Letchford, A.N., Eglese, R.W., 2004. A new branch-and-cut algorithm for the capacitated vehicle routing problem, *Mathematical Programming Series A* ,100, pp.423–445.

[82]Long-sheng Huang and Guang-hui Xu(2005) , Solution of assignment problem of restriction of qualification, *Operations Research and Management Science*, vol.14 ,pp.28-31.

[83]Lee, C.-G., and Z. Ma(2004), The generalized quadratic assignment problem, Research Report, Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, Ontario M5S 3G8, Canada.

[84]Lin, S., Kernighan, B.W(1973), An effective heuristic algorithm for the traveling salesman problem, *Operations Research*,vol. 21, pp.498–516.

[85]Li, Y., Pardalos, P.M., Resende, M.G.C(1994), A greedy randomized adaptive search procedure for the quadratic assignment problem. In: Pardalos, P.M., Wolkowicz, H. (Eds.), Quadratic Assignment and Related Problems, DIMACS Series in Discrete Mathematics and Theoretical Computer Science, vol. 16. AMS, Rhode Island, pp. 237–261.

[86]Liu, L.and X. Gao(2009), Fuzzy weighted equilibrium multi-job assignment problem and genetic algorithm, *Applied Mathematical Modeling*, vol. 33, pp. 3926-3935.

[87]Liu. Fuh-Hwa Franklin , Shen. Sheng-Yuan(1999), A route-neighborhood-based metaheuristic for vehicle

routing problem with time windows, *European Journal of Operational Research*,vol. 118,pp. 485-504.

[88]Lysgaard, J. (2006), Reachability cuts for the vehicle routing problem with time windows, *European Journal of Operational Research*, vol.175,pp.210–233.

[89]Martin Desrochers, Jacques Desrosiers, Marius Solomon, A New Optimization Algorithm for the Vehicle Routing Problem with Time Windows, *Operations Research*, vol. 40, No. 2,1992.

[90]Miller, D. L(1995), "A matching based exact algorithm for capacities vehicle routing problems". *ORSA Journal on Computing 7*(1), pp.1–9.

[91]M.S.Chen(1985), On a fuzzy assignment problem, *Tamkang J*, vol.22,pp. 407-411.

[92]Metzger, B. H. ,Spectrum management technique, Fall 1970, Presentation at 38th National ORSA meeting (Detroit, MI).

[93]M. Queyranne(1986), Performance ratio of heuristics for triangle inequality quadratic assignment problems. *Operations Research Letters*,vol. 4, pp.231–234.

[94]M.Sakawa, I.Nishizaki and Y.Ucmura(2001), Interactive fuzzy programming for two level linear and linear fractional production and assignment problems: a case study, *European Journal of Operational Research*, vol.135 ,pp. 142-157.

[95]M. Sakawa, I. Nishizaki and Y. Uemura(2001), Interactive fuzzy programming for two-level linear and linear fractional production and assignment problems: a case study, *European Journal of Operational Research*, Vol. 135, pp. 142-157.

[96]Majumdar, J. and A.K. Bhunia(2007), Elitist genetic algorithm for assignment problem with imprecise goal, *European Journal of Operational Research*, vol. 177, pp.684-692.

[97]Mazzeo, S. and I. Loiseau(2004), "An ant colony algorithm for the capacitated vehicle routing". *Electronic Notes in Discrete Mathematics,vol. 18*(1), pp.181–186.

[98]M. Dell_Amico, S. Martello(1997), The k-cardinality assignment problem, *Discrete Applied Mathematics* ,vol.76 (1–3) ,pp. 103–121.

[99]Mole, R. H. and S. R. Jameson(1976), "A sequential route-building algorithm employing a generalized savings criterion". *Operational Research Quarterly,*vol. *27*(2), pp.503–511.

[100]María A. Osorio , Manuel Laguna(2003), Logic Cuts for the Multilevel Generalized Assignment Problem, *European Journal of Operational Research,*vol.151(1),pp.238-246.

[101]M. Van Wyk and J. Clark(2001), "An Algorithm for Approximate Least-Squares Attributed Graph Matching," *Problems in Applied Math. and Computational Intelligence*, pp. pp. 67-72.

[102]Osman, I.H. and Laporte, G(1996), Metaheuristics: A bibliography, *Annals of Operations Research*,vol. 63, pp.513–623.

[103]M. Dorigo, *Optimization, Learning and Natural Algorithms*, PhD thesis, Politecnico di Milano, Italy, 1992.

[104]M. Rijal, *Scheduling, design and assignment problems with quadratic costs*, PhD thesis, New York University, New York, USA, 1995,[Rodriguez:04] .

[105]Michéal ÓhÉigeartaigh(1982), A fuzzy transportation algorithm, *Fuzzy Sets and Systems*, vol.8 ,pp.235–243.

Potvin, J.-Y. and J.-M. Rousseau(1995), "An exchange heuristic for routing problems with time windows. *Journal of the Operational Research Society,vol. 46*(12), pp.1433–1446.

[106]Osman, I. H(1993), "Metastrategy simulated annealing and tabu search algorithms for the vehicle routing problem, *Annals of Operations Research*,vol. *41*(1-4), pp.421–451.

[107]Pasquale Avella, Maurizio Boccia, and Igor Vasilyev (2013), A Branch-and-Cut Algorithm for the Multilevel Generalized Assignment Problem, *IEEE ACCESS Digital Object Identifier 10.1109/ACCESS.2013.2273268*.

[108]Pentico, David W. (2007), Discrete Optimization Assignment problems: A golden anniversary survey, *European Journal of Operational Research,* vol. 176, pp. 774–793.

[109]P. Carraresi and G. Gallo(1984), A multi-level bottleneck assignment approach to the bus drivers' rostering Problem, *European Journal of Operational Research*., vol.16, pp.163-173.

[110]Pollatschek, M., Gershoni, N., & Radday, Y(1976), Optimization of the typewriter keyboard by simulation. *Angewandte Informatik,*vol. *17*, pp.438-439.

[111]Q. Zhao, *Semi definite programming for assignment and partitioning problems,* PhD thesis, University of Waterloo, Waterloo, Canada, 1996,[ZhKaReWo:96] .

[112]Ralphs, T. K. (2003). Parallel branch and cut for capacitated vehicle routing. Parallel Computing, 29, 607–629.

[113]R. Bent, P.V. Hentenryck(2003), "A Two-Stage Hybrid Algorithm For Pickup and Delivery Vehicle Routing Problems With Time Windows", Computers & Operations Research ,vol.33,pp. 875–893.

[114]Russell .Robert A, Wen-Chyuan Chiang(2006), "Scatter Search For The Vehicle Routing Problem With Time Windows", *European Journal of Operational Research*, vol.169,pp. 606–622.

[115]Renaud, J., F. F. Boctor, and G. Laporte(1996), "A fast composite heuristic for the symmetric traveling

salesman problem". *INFORMS Journal on Computing*,vol. *8*(2), pp.134–143.
[116]R. E. Burkard and J. Offermann(1977), Entwurf von Schreibmaschinentastaturen mittels quadratischer Zuordnungsprobleme, Z. *Operations Research*,vol.21, B121–B132.
[117]Ryan, D. M., C. Hjorring, and F(1993), "Extensions of the petal method for vehicle routing:, *The Journal of the Operational Research Society*,vol. *44*(3), pp.289–296.
[118]Rochat, Y. and E. Taillard(1995), "Probabilistic diversification and intensification in local search for vehicle routing". *Journal of Heuristics*,vol. *1*, pp.147–167.
[119]R.E. Burkard and E. Çela, "Linear Assignment Problems and Extensions," *Handbook of Combinatorial Optimization*, 1998.
[120]S. Sahni and T. Gonzalez(1976), P-complete approximation problems, *Journal of the Association for Computing Machinery*, vol.23, pp.555–565.
[121]Skorin-Kapov, J(1990), Tabu search applied to the quadratic assignment problem, ORSA Journal on Computing ,vol. 2 (1), pp.33–45.
[122]S. Chanas and D. Kuchta(1998), Fuzzy integer transportation problem, *Fuzzy Sets and Systems*, vol.98 ,pp. 291–298.
[123] Pandian, P. and G. Natarajan(2010a), An appropriate method for real life fuzzy transportation problems, *International Journal of Information Sciences andApplications*, vol.2 ,pp. 75-82.
[124]Stutzle, T(2006), Iterated local search for the quadratic assignment problem. European Journal of Operational Research,Vol. 174(1), pp.1519–1539 .
[125]S.E. Karisch, *Nonlinear approaches for the quadratic assignment and graph partition problems*. PhD thesis, Graz University of Technology, Graz, Austria, 1995,[KaCeClEs:98].
[126]Steinberg, L(1961), The backboard wiring problem: A placement algorithm, *SIAM Review*, vol.3(1), pp.37–50.
[127]Taillard, E(1991), Robust taboo search for the quadratic assignment problem, vol. Parallel Computing,vol. 17,pp. 443–455.
[128]Tate, D.E., Smith, A.E(1995), A genetic approach to the quadratic assignment problem, Computers and Operations Research,vol. 22, pp.73–83.
[129]T.C. Koopmans and M. Beckman. Assignment Problems and the Location of Economic Activities. Econometric ,1957.
[130]Taillard, E. D(1994), "Parallel iterative search methods for vehicle routing problem. *Networks* ,vol.23(8), 1993, pp.661–673.
[131]Toth, P. and D. Vigo (2003, Fall), "The granular tabu search and its application to the vehicle routing problem". *INFORMS Journal on Computing 15*(4), pp.333–346.
[132]T.A. Johnson, *New linear programming-based solution procedures for the quadratic assignment problem*. PhD thesis, Clemson University, Clemson, USA, 1992.
[133]T. Mautor, *Contribution à la résolution des problèmes d'implanation: algorithmes séquentiels et parallèles pour l'affectation quadratique*. PhD thesis, Université Pierre et Marie Curie, Paris, France, 1992.
[134]Tao Huang, Continuous Optimization Methods for the Quadratic Assignment Problem, University of North Carolina at Chapel Hill,USA,2008.
[135]Tada, M.and H. Ishii(1996), An integer fuzzy transportation problem, *Comput. Math. Appl.*, vol.31, pp.71–87.
[136]Thompson, P. and H. Psaraftis(1993), "Cyclic transfer algorithms for multivehicle routing and scheduling problems". *Operations Research 41*(5), pp. 935–946.
[137]Van Breedam, A(1995), "Improvement heuristics for the vehicle routing problem based on simulated annealing". *European Journal of Operational Research, vol.86*(3), pp. 480–490.
[138]U. Benlic, J.Hao(2013), Breakout local search for the quadratic assignment problem, Applied Mathematics and Computation, Volume 219, Issue 9, 1 January 2013.
[139]V. Kaibel, Polyhedral combinatorics of QAPs with less objects than locations. Technical Report Nr. 97-297, Angewandte Mathematik und Informatik, Universitaet zu Koeln, Cologne, Germany, 1997.
[140]Votaw, D.F. and A. Orden(1952), The personnel assignment problem, Symposium on Linear Inequalities and Programmng, SCOOP 10, US Air Force, , pp. 155–163.
[141]Wilhelm, M.R., Ward, T.L(1987), Solving quadratic assignment problems by simulated annealing, IEEE Transactions,vol. 19, pp.107–119.
[142]Wang, X. (1987), Fuzzy optimal assignment problem, *Fuzzy Math*, vol. 3,pp. 101-108.
[143]West, D.H(1983), Algorithm 608: Approximate solution of the quadratic assignment problem, *ACM Transactions on Mathematical Software*, vol. 9, pp.461–466.
[144]Wren A and Holliday A (1972) Computer scheduling of vehicles from one or more depots to a number of delivery points. *Operational Research Quarterly*, vol.**23**(1),pp.333-344.
[145]Xu, J. and J. Kelly(1996) , "A network flow-based tabu search heuristic for the vehicle routing problem".

Transportation Science,vol. *30*(4), pp. 379–393.
[146]Y. Li, *Heuristic and exact algorithms for the quadratic assignment problem*. PhD thesis, The Pennsylvania State University, USA, 1992 [LiPa:92] .
[147]Yi-Rong Zhu, "Recent advances and challenges in quadratic assignment and related problems" (January 1, 2007), the University of Pennsylvania, Dissertations *available from Pro Quest.* Paper AAI3292096, http://repository.upenn.edu/dissertations/AAI3292096 .
[148]Yongzhong Wu and Ping Li(2007), Solving the Quadratic Assignment Problems by a Genetic Algorithm with a New Replacement Strategy, Proceedings of World Academy of Science: Engineering & Technolog;2007, Vol. 24, p310.
[149]Yamada,T and Nasu,Y(2000), Heuristic and exact algorithms for the simultaneous assignment problem, *European Journal of Operational Research*, vol. 123, issue 3, pp. 531-542
[150]Ye ,X and J. Xu(2008), A fuzzy vehicle routing assignment model with connection network based on priority-based genetic algorithm, *World Journal of Modeling and Simulation*, vol. 4, pp. 257-268.
[151]Z. Wu, Y. Yang, F. Bai, J. Tian. Global optimality conditions and optimization methods for quadratic assignment problems , School of Science, Information Technology and Engineering, University of Ballarat , Victoria, Australia Department of Mathematics, Shanghai University,China ,vol.218, Issue 11, 5 February 2012.
[152]Z. Ying and D. Castanon, "Statistical Model for Human Face Detection Using Multi-Resolution Features," *International Conference on Information Intelligence and Systems (ICIIS'99)*, IEEE, 1999, p. 560.