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Radiation and Magneticfield Effects on Unsteady Mixed Convection Flow over a Vertical Stretching/Shrinking Surface with Suction/Injection

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Abstract
Present paper deals with the influence of radiation and magneticfiled on an unsteady mixed convection flow over a vertical stretching/shrinking sheet with suction/injection effects in presence of variable viscosity and viscous dissipation. The governing partial differential equations are transformed into system of nonlinear ordinary differential equations by using similarity transformation and solved numerically using bvp5c Matlab package. The effects of non-dimensional governing parameters on velocity, temperature profiles along with the local skin friction coefficient and Nusselt number are discussed and presented through graphs and tables.
Keywords: MHD, Radiation, Dissipation, Stretching/shrinking, Suction/injection.

1. Introduction

Boundary layer flows on a moving continuous surface have variety of applications in engineering and its allied areas like aerodynamic extrusion of plastic sheets, cooling of metallic sheets, wiring etc. The combined effects of non-uniform heat source/sink and thermal radiation on heat transfer over an unsteady stretching permeable surface was discussed by Pal (2011). Unsteady mixed convection heat transfer over a vertical stretching surface with variable viscosity and viscous dissipation was studied by Aziz (2014). Jayachandra Babu et al. (2015) discussed the influence of radiation and viscous dissipation on stagnation-point flow over a nonlinearly stretching surface with suction/injection effects. Radiation and magneticfield effects on unsteady natural convection flow of a nanofluid past an infinite vertical plate was analyzed by Mohankrishna et al. (2014). Sandeep and Sugunamma (2014) presented radiation and inclined magnetic field effects on unsteady MHD convective flow past an impulsively moving vertical plate in a porous medium.

Radiation and aligned magneticfield effects on the flow over a stretching surface was analyzed by Raju et al. (2015). Stagnation-point flow and heat transfer behavior of Cu-water nanofluid towards horizontal and exponentially stretching/shrinking cylinders was discussed by Sulochana and Sandeep. Boundary layer flow and heat transfer over a nonlinearly permeable stretching/shrinking sheet in a nanofluid was numerically presented by Zaimi et al. (2014). Sandeep et al.(2013) discussed the effect of radiation on an unsteady natural convective flow of a nanofluid past an infinite vertical plate. Thermal diffusion and chemical reaction effects on unsteady MHD dusty viscous flow was illustrated by Ramana Reddy et al. (2014). Unsteady hydromagnetic free convection flow of a dissipative and radiating fluid past a vertical plate through porous media with constant heat flux was analyzed by Sugunamma and Sandeep (2011). Sandeep et al. (2012) discussed the effect of radiation and chemical reaction on transient MHD free convective flow over a vertical plate through porous media.

Boundary layer flow of a nanofluid over a stretching sheet by considering convective boundary conditions was illustrated by Makinde and Aziz (2011). Bhattacharyya (2013) analyzed the heat transfer analysis in unsteady boundary layer stagnation-point flow towards a shrinking/stretching sheet. Chen (1998) discussed the laminar mixed convection flow adjacent to vertical continuously stretching sheet. Aziz (2010) illustrated flow and heat transfer over an unsteady stretching surface in presence of Hall effects. Sandeep and Sugunamma (2013) discussed the effect of inclined magnetic field on unsteady free convective flow of dissipative fluid past a vertical plate. Radiation effect due to natural convection flow between heated inclined plates under the influence of transverse magneticfield was studied by Sugunamma et al. (2011). Very recently, the researchers Sandeep and Sulochana (2015), Murshed et al. (2015), Johns et al. (2015), Mohankrishna et al. (2015), Raju et al. (2015) discussed the influence of radiation, magneticfield and chemical reaction by considering different types of flows through different channels. Malyand et al. (2014) discussed slip effects on unsteady stagnation point flow of a nanofluid over a stretching sheet.

In this paper we analyzed the influence of radiation and magneticfiled on an unsteady mixed convection flow over a vertical stretching/shrinking sheet with suction/injection effects in presence of variable viscosity and viscous dissipation. The governing partial differential equations are transformed into system of nonlinear ordinary differential equations by using similarity transformation and solved numerically using bvp5c Matlab package. The effects of non-dimensional governing parameters on velocity, temperature profiles along with the local skin friction coefficient and Nusselt number are discussed and presented through graphs and tables.

2. Mathematical analysis

Consider an unsteady mixed convection viscous incompressible boundary layer flow past unsteady stretching sheet. The positive *x*-coordinate is measured along the stretching sheet with the slot as the origin and the *y*-coordinate is measured normal to the sheet in the outward direction toward the fluid. A variable magneticfield $B = B_0 / (1-\alpha t)$ is applied to the flow direction. Radiation effect and suction/injection effects are taken into account. T_w and u_w are respectively denotes the surface temperature and velocity, which are defined as $T_w = T_\infty + \frac{T_0}{(1-\alpha t)^2}\left(\frac{bx^2}{2v_\infty}\right)$ and $u_w = \frac{bx}{1-\alpha t}$ .Where T_0, T_∞ are the reference and ambient temperatures, b, α are the positive constants and v_∞ is the kinematic viscosity of the fluid. Using Boussinesq approximation for incompressible viscous fluid environment in addition to that, the fluid viscosity is assumed to vary as an inverse linear function of temperature $\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1+\gamma(T-T_\infty)\right].$

The boundary layer equations as per above assumptions can be take in the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + g\,\beta(T-T_\infty) - \frac{\sigma B^2}{\rho_\infty}u, \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{c_p\rho_\infty}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{c_p\rho_\infty}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{c_p\rho_\infty}\frac{\partial q_r}{\partial y}, \tag{3}$$

with the boundary conditions

$$\begin{aligned} &u = \varepsilon u_w, v = v_w, T = T_w \quad at \quad y = 0, \\ &u \to 0, T \to T_\infty \qquad as \quad y \to \infty, \end{aligned} \tag{4}$$

where *t* is the time, u, v are the velocity components in *x, y* directions, μ is dynamic viscosity, ρ_∞ is the fluid density, *T* is the fluid temperature, c_p is the specific heat, g is the acceleration due to gravity, β volumetric coefficient of thermal expansion, $q_r = \frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial T}{\partial y}$ is the linearized radiative heat flux, with k^* is the mean absorption coefficient, σ^* is the Stefan-Boltzmann constant, $v_w = -v_0 / 1-\alpha t$ is the suction/injection velocity and ε is the stretching/shrinking parameter with $\varepsilon > 0$ for stretching sheet and $\varepsilon < 0$ for shrinking sheet.

we now introducing the similarity transformations

$$\eta = \left(\frac{b}{v_\infty(1-\alpha t)}\right)^{1/2} y, \psi = \left(\frac{bv_\infty}{(1-\alpha t)}\right)^{1/2} xf(\eta), T = T_\infty + \frac{T_0}{(1-\alpha t)^2}\left(\frac{bx^2}{2v_\infty}\right)\theta(\eta), \tag{5}$$

where ψ is the stream function satisfying the continuity equation (1) with $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$.

By using equation (5) equations (2) and (3) along with the boundary conditions (4) reduced to

$$\frac{\theta_r}{\theta_r - \theta}f''' + \frac{\theta_r}{(\theta_r-\theta)^2}f''\theta' - A\left(f' + \frac{\eta}{2}f''\right) + ff'' - f'^2 + \lambda\theta - Mf' = 0, \tag{6}$$

$$\frac{1}{\Pr}\left(1 + \frac{4}{3}R\right)\theta'' - \frac{A}{2}(4\theta + \eta\theta') + f\theta' - 2f'\theta + \frac{\theta_r}{\theta_r - \theta}Ecf''^2 = 0, \tag{7}$$

with the transformed boundary conditions

$$\begin{aligned} &f = S, f' = \varepsilon, \theta = 1 \quad at \quad \eta = 0, \\ &f \to 0, \theta \to 0 \qquad as \quad \eta \to \infty, \end{aligned} \tag{8}$$

Where prime denotes the differentiation with respect to η, $M = \sigma B_0^2 / \rho_\infty b$ is the magneticfiled parameter, $\theta = T - T_\infty / T_w - T_\infty$ is the non-dimensional temperature, $\theta_r = T_r - T_\infty / T_w - T_\infty$ is the variable viscosity parameter, $A = \alpha / b$ is the unsteadiness parameter, $Ec = 2bv_\infty / T_0 c_p$ is the viscous dissipation parameter, $\Pr = \mu_\infty c_p / k$ is the Prandtl number, $\lambda = Gr_x / \mathrm{Re}_x^2$ is the mixed convection parameter with $Gr_x = g\beta(T_w - T_\infty)x^3 / v_\infty^2$ is the Grashof number, $R = \dfrac{4\sigma^* T_\infty^3}{kk^*}$ is the radiation parameter and $S = -v_0 / \sqrt{v_\infty b}$ is the suction/injection parameter with $S > 0$ for suction and $S < 0$ for injection.

The physical quantities of interest the local skin friction coefficient C_f and local Nusselt number Nu_x are given by

$$\mathrm{Re}_x^{1/2} C_f = \frac{2\theta_r}{\theta_r - 1} f''(0), Nu_x = -\frac{\mathrm{Re}_x^{3/2}}{2(1-\alpha t)} \theta'(0), \tag{9}$$

where $\mathrm{Re}_x = u_w x / v_\infty$ is the local Reynolds number.

3. Results and Discussion

The coupled nonlinear ordinary differential equations (6) and (7) subjects to the boundary conditions (8) are solved numerically by using Matlab bvp5c package. Results show the influence of non-dimensional governing parameters namely magneticfiled parameter *M*, Radiation parameter *R*, Viscous dissipation parameter *Ec*, Suction/injection parameter *S*, Buoyancy parameter λ and Unsteadiness parameter *A* on velocity, temperature profiles along with skin friction coefficient and Nusselt number. For numerical results we considered $\theta_r = 3, \Pr = 0.71, M = \lambda = 1, A = 1.5, R = 0.5, Ec = 0.1, \eta = 6$ and $S = 2$. These values are kept as common in entire study except the varied values as shown in respective figures and tables.

Figs. 1 and 2 display the influence of radiation parameter on velocity and temperature profiles of the flow for stretching and shrinking cases. It is evident that an increase in radiation parameter enhances the velocity and temperature profiles of the flow in both cases. This is due to the fact that the enhancement in radiation releases the heat energy to the flow. This causes to develop the velocity and thermal boundary layer thicknesses. Figs. 3 and 4 depict the influence of magneticfield parameter on velocity and temperature profiles of the flow for stretching and shrinking cases. We observed an interesting result that the enhancement in magneticfiled parameter depreciates the velocity profiles in stretching case and enhances the velocity profiles in shrinking case. This shows the domination of Lorentz force on stretching surface. But we noticed opposite results to the above in case of temperature profiles as displayed in Fig.4.

The influence of unsteadiness parameter on velocity and temperature profiles of the flow for stretching and shrinking cases is displayed in Figs. 5 and 6. It is observed that the enhancement in unsteadiness parameter increases the velocity profiles in stretching case and depreciates the velocity profiles in shrinking case. It also observed that an increase in the unsteadiness parameter enhances the temperature profiles of the flow in both cases. It is due to the fact that the enhancement in unsteadiness parameter improves the velocity and thermal boundary layer thickness. Fig. 7 shows the effect of viscous dissipation parameter on temperature profiles of the flow for stretching and shrinking cases. It is clear that a raise in the value of Eckert number enhances the thermal boundary layer in both cases. This is may be the reason that the enhancement in viscous dissipation develops the thermal conductivity of the flow.

Figs. 8 and 9 illustrate the effect of suction/injection parameter on velocity and temperature profiles of the flow for stretching and shrinking cases. It is noticed that an increase in the suction/injection parameter depreciates the temperature profiles of the flow in both cases and enhances the velocity profiles in shrinking surface and declines the velocity profiles in stretching surface. The influence of buoyancy parameter on velocity profiles for both stretching/shrinking cases is displayed in Fig. 10. It is evident that the enhancement in buoyancy parameter increases the velocity profiles in both cases.

Table 1 depicts the influence of non-dimensional governing parameters on skin friction coefficient and Nusselt number for stretching case. It is evident from the table that a raise in the values of radiation parameter, viscous dissipation parameter and unsteadiness parameter enhances the coefficient of skin friction and declines the heat transfer rate. An increase in the magneticfield parameter depreciates the friction factor and Nusselt number. The enhancement in buoyancy parameter shows reverse results to that of magneticfield parameter. A raise in the value of suction/injection parameter deprecates the friction factor and enhances the heat transfer rate.

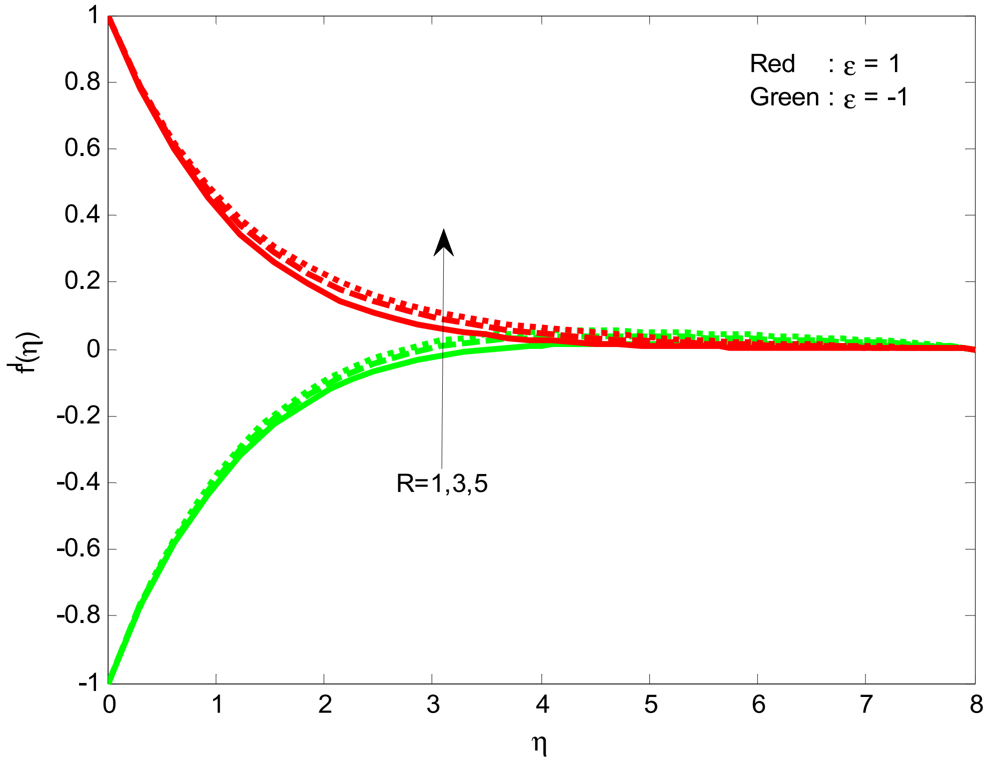

Fig.1 Velocity profiles for different values of R

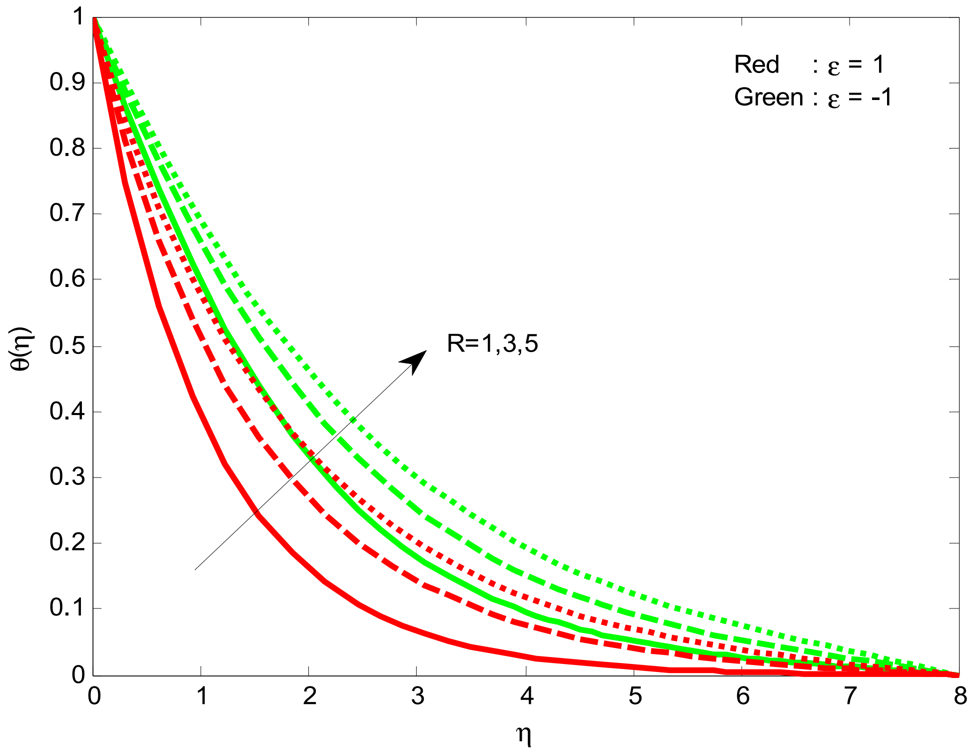

Fig.2 Temperature profiles for different values of R

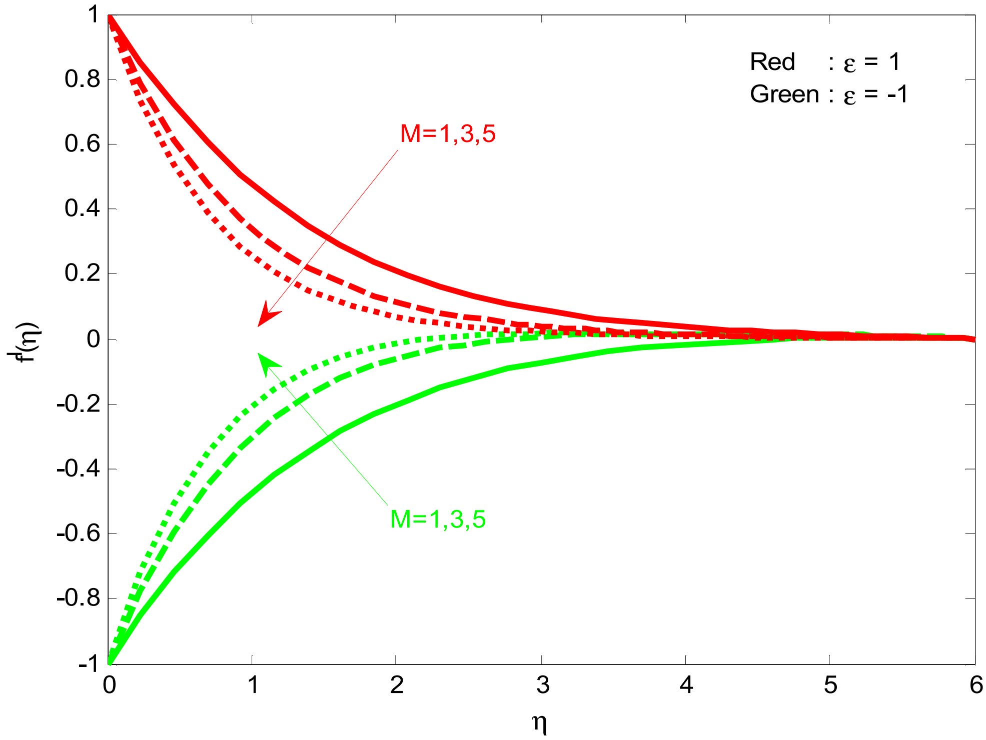

Fig.3 Velocity profiles for different values of *M*

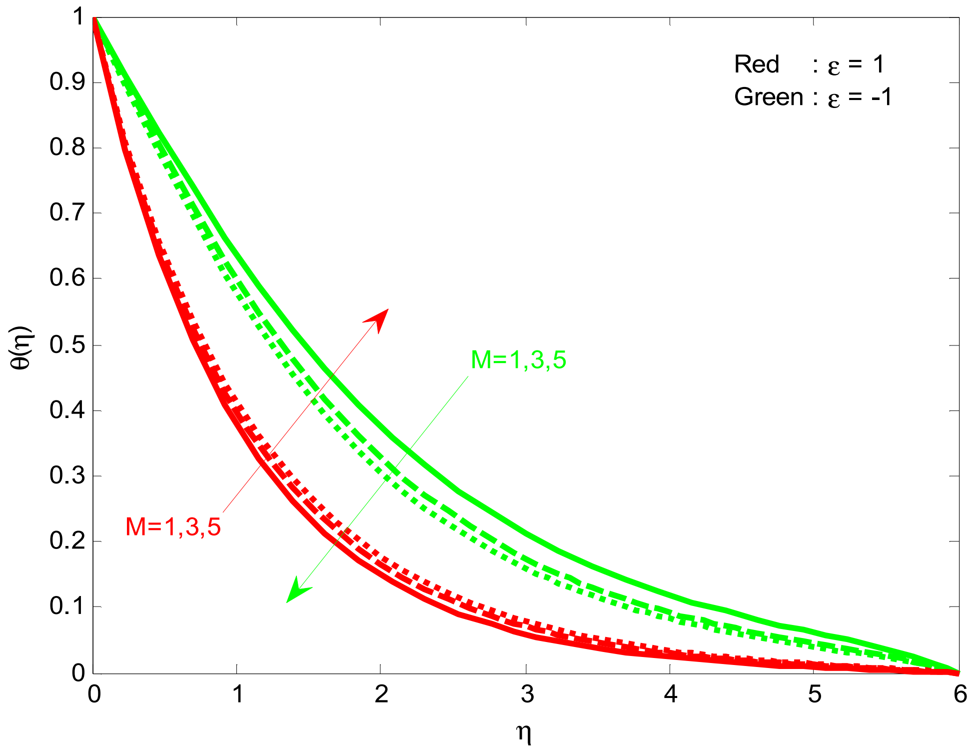

Fig.4 Temperature profiles for different values of *M*

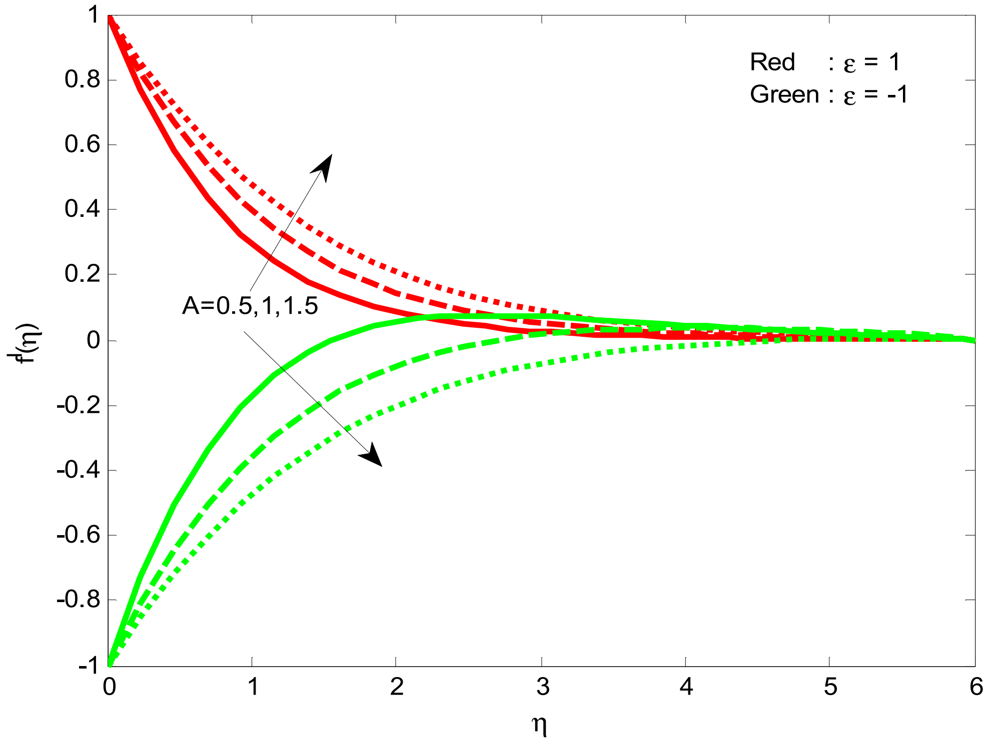

Fig.5 Velocity profiles for different values of *A*

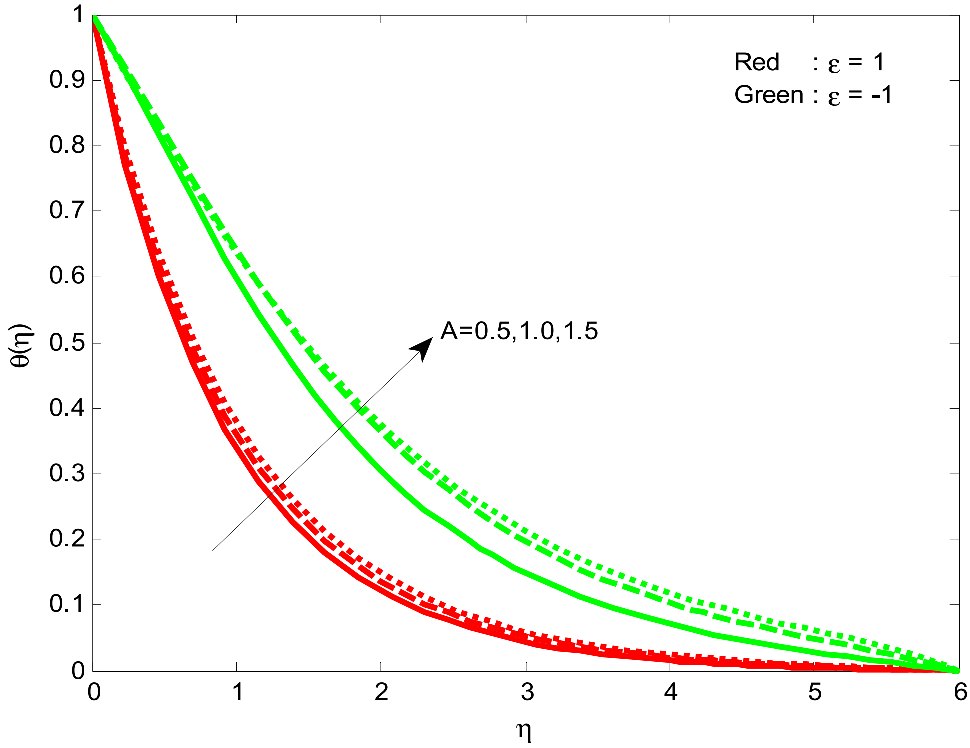

Fig.6 Temperature profiles for different values of *A*

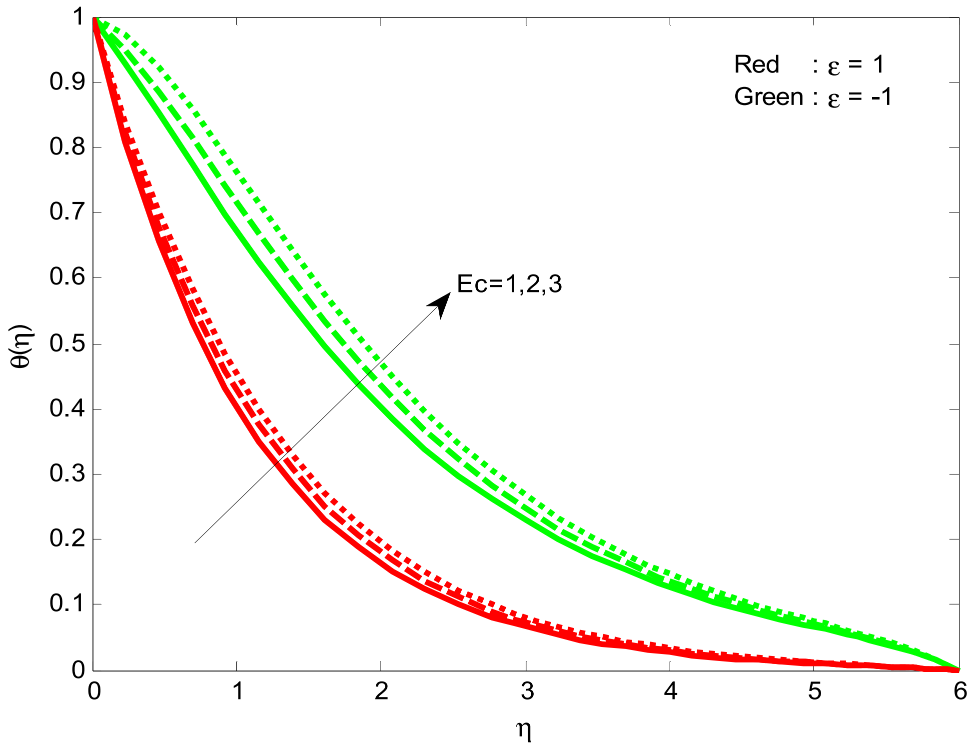

Fig.7 Temperature profiles for different values of *Ec*

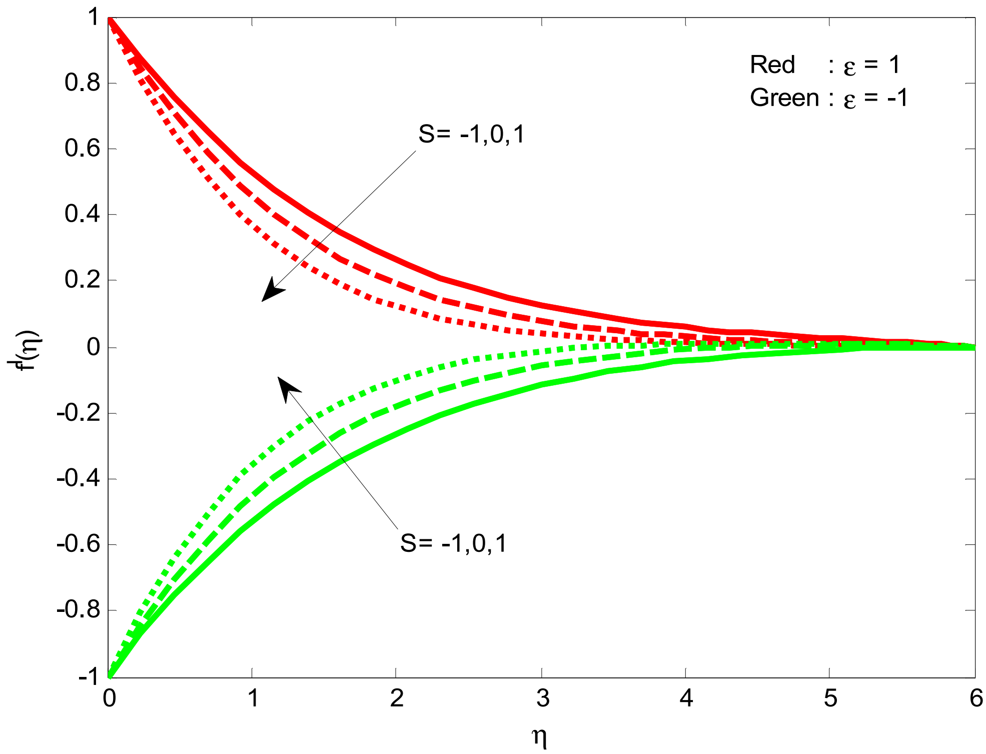

Fig.8 Velocity profiles for different values of *S*

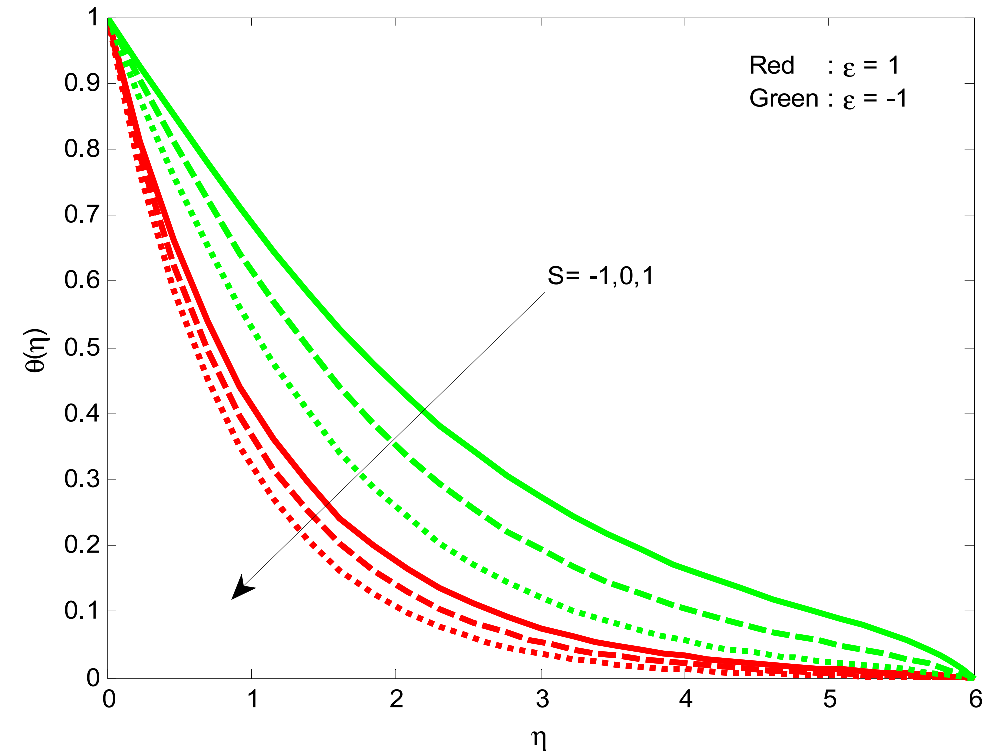

Fig.9 Temperature profiles for different values of *S*

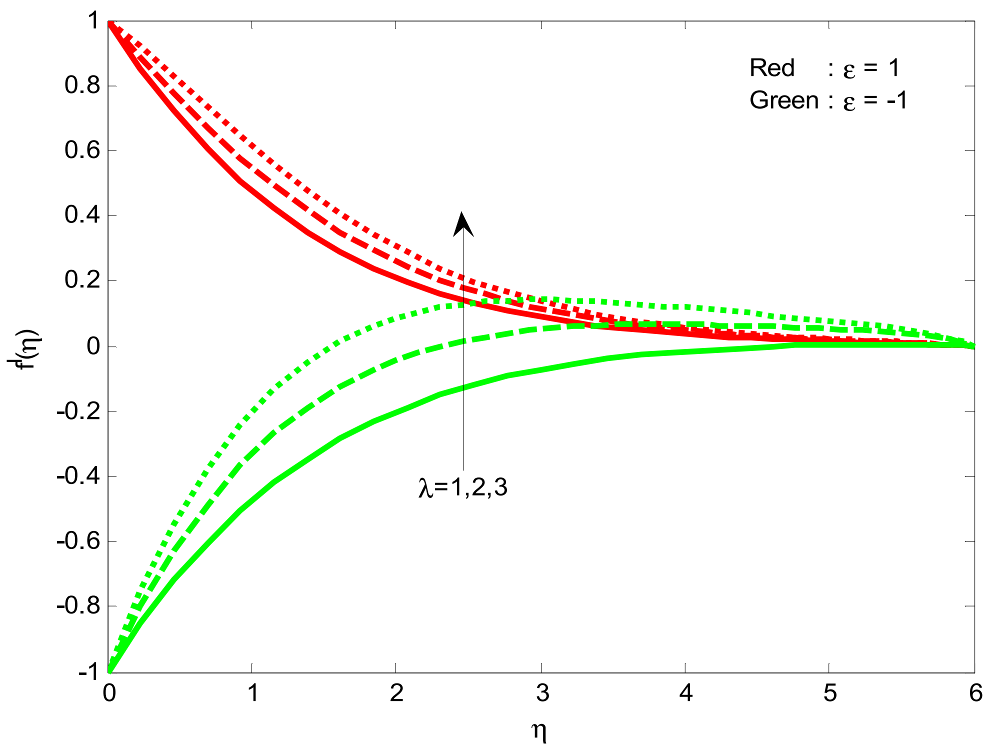

Fig.10 Velocity profiles for different values of λ

Table 1 Variation in friction factor and Nusselt number at different non-dimensional parameters.

S	R	M	Ec	A	λ	$f''(0)$	$-\theta'(0)$
-1						-1.717668	0.905489
0						-2.103256	1.029037
1						-2.642396	1.173992
	1					-2.342632	0.959024
	3					-2.324471	0.689377
	5					-2.314157	0.566951
		1				-1.994562	0.996229
		3				-3.053422	0.964156
		5				-3.964499	0.938446
			1			-1.992560	0.919552
			2			-1.990360	0.835734
			3			-1.988187	0.753320
				0.5		-3.287418	1.139129
				1.0		-2.448080	1.059965
				1.5		-1.994562	0.996229
					1	-1.994562	0.996229
					2	-1.462025	1.011247
					3	-0.937694	1.024886

4. Conclusions

This paper presents the numerical solution to analyze the influence of radiation and magneticfiled on an unsteady mixed convection flow over a vertical stretching/shrinking sheet with suction/injection effects in presence of variable viscosity and viscous dissipation. The effects of non-dimensional governing parameters on velocity, temperature profiles along with the local skin friction coefficient and Nusselt number are discussed and presented through graphs and tables. Conclusions of the present study are made as follows:

- A raise in the value of suction/injection parameter enhances the heat transfer rate and depreciates the friction factor.
- An increase in radiation parameter enhances the velocity and temperature profiles of the flow and declines the friction factor along with heat transfer rate.
- Increase in magneticfield parameter depreciates the temperature profiles in shrinking case and enhances the temperature profiles in shrinking case.
- Unsteadiness parameter have tendency to increase the friction factor and reduce the heat transfer rate.
- An increase in viscous dissipation parameter enhances the friction factor and thermal boundary layer thickness.

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