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Higher Order Duality for Vector Optimization Problem over Cones Involving Support Functions

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Abstract
In this paper, we consider a vector optimization problem over cones involving support functions in objective as well as constraints and associate a unified higher order dual to it. Duality result have been established under the conditions of higher order cone convex and related functions. A number of previously studied problems appear as special cases.
Keywords: Vector optimization, Cones, Support Functions, Higher Order Duality.

1 Introduction

Multiobjective or vector optimization problems are the ones in which more than one objective is involved. Vector optimization problems have a very wide range of applications in fields like operations research, economics, finance, product and process design, aircraft, automobile design, mechanical engineering and the list is far from being over (see [10], [12], [13]). Study of vector optimization problems involving general ordering cones has gained popularity because in real-world multiobjective problems, it is not only the coordinate-wise ordering (induced by the positive orthant as ordering cone) which appears. Examples are available in portfolio optimization, semidefinite programming where the cone of positive semidefinite symmetric matrices is of practical interest. It may be possible that the decision maker is not interested in the whole efficiency set. Varying the size of the cone he can reduce or extend the set of efficient solutions he is interested in.

Support functions play a significant role in optimization theory. The presence of support functions in any optimization problem makes it nonsmooth, hence we have to use subdifferentials. Since, subidfferentials of support function have a neat representation, they are used to study the non smooth optimization problems. The motivation behind the study of such problems arises from the fact that even though the objective function and/or constraint function of the primal problem are non smooth, we can associate dual problems which are differentiable. Generally it is easier to solve a differentiable problem than to solve a nondifferentiable problem. Researchers like Schechter [11], Husain et al [7] have studied optimization problems involving support functions. They have established optimality and duality results for Wolfe and Mond-Weir type duals under suitable convexity and generalized convexity assumptions.

Duality plays a crucial role in mathematical programming and is very useful both theoretically and practically. As a unified framework for the study of both Mond-Wier and Wolfe type duality, unified type duality has been introduced by Cambini and Carosi for multiobjective optimization problems over cones [2]. Recently, Suneja and Louhan [16] derived optimality conditions under cone convexity and its generalizations for vector optimization problem over cones involving support functions in objective as well as constraints. They also proposed a unified dual for the same and established first order duality results. Mangasarian ([8]) introduced the concept of second and higher order duality for nonlinear problems. The study of higher order duality is important due to computational advantage over first order duality as it provides better bounds for the value of the objective function when approximations are used because there are more parameters involved. Many researchers like Mond and Weir [9], Suneja et al [14], Bhatia [1] have studied second and higher order duality for various vector optimization problems.

In this paper we introduce a unified higher order dual to vector optimization problem over cones involving support functions in objective as well as constraints and prove duality results under the conditions of higher order cone convexity,pseudoconvexity,quasiconvexity introduced in [1]. Special cases of our study are also discussed.

2 Notations and Definitions

Let $K \subseteq R^m$ be a closed convex pointed ($K \cap (-K) = \{0\}$) cone with vertex at the origin such that $\text{int}K = \emptyset$ where intK denotes interior of K. The positive dual cone K^+ is defined as follows:

$$K^+ = \{y \in R^m : z^T y \geq 0, \forall z \in K\}$$

Since the cone under consideration is closed and convex, by bipolar theorem $(K^+)^+ = K$. In this case,

$$x \in K \Leftrightarrow \lambda^T x \geq 0, \forall \lambda \in K^+ .$$

As given by Flores-Bazan et al [3], we have

$$x \in \text{int}K \Leftrightarrow \lambda^T x > 0, \forall \lambda \in K^+ \setminus \{0\}.$$

Let $f = (f_1, f_2, \ldots, f_m)^T : R^n \to R^m$, $H = (H_1, H_2, \ldots, H_m)^T : R^n \times R^n \to R^m$ be differentiable vector valued functions. All vectors shall be considered as column vectors. We rewrite the following definitions on the lines of Suneja et al [15].

Definition 2.1. The function f is said to be higher order K-convex at $u \in R^n$ with respect to H, if for every $(x, p) \in R^n \times R^n$,

$$f(x) - f(u) - [\nabla f(u) + \nabla_p H(u, p)](x - u) - H(u, p) + \nabla_p H(u, p)p \in K.$$

where $\nabla f(u) = [\nabla f_1(u), \nabla f_2(u), \ldots, \nabla f_m(u)]^T$ and $\nabla_p H(u, p) = [\nabla_p H_1(u, p), \nabla_p H_2(u, p), \ldots, \nabla_p H_m(u, p)]^T$ are $m \times n$ Jacobian matrices of f at u. For each i= {1, 2… m}, $\nabla f_i = (\frac{\partial f_i}{\partial x_1}, \ldots, \frac{\partial f_i}{\partial x_1})$ is $1 \times n$ Gradient vector of f_i at u.

Definition 2.2. The function f is said to be higher order K -pseudoconvex at $u \in R^n$ with respect to H, if for every $(x, p) \in R^n \times R^n$

$$-[f(x) - f(u) - H(u, p) + \nabla_p H(u, p)p] \in \text{int}K$$
$$\Rightarrow -[\nabla f(u) + \nabla_p H(u, p)](x - u) \in \text{int}K.$$

Definition 2.3. The function f is said to be higher order strongly K -pseudoconvex at $u \in R^n$ with respect to H, if for every $(x, p) \in R^n \times R^n$,

$$-[\nabla f(u) + \nabla_p H(u, p)](x - u) \notin \text{int}K$$
$$\Rightarrow f(x) - f(u) - H(u, p) + \nabla_p H(u, p)p \in K.$$

Definition 2.4. The function f is said to be higher order K -quasiconvex at $u \in R^n$ with respect to H, if for every $(x, p) \in R^n \times R^n$,

$$f(x) - f(u) - H(u, p) + \nabla_p H(u, p)p \notin \text{int}K$$
$$\Rightarrow -[\nabla f(u) + \nabla_p H(u, p)](x - u) \in K.$$

We consider the following vector optimization problem:

$$K\text{-Minimize } f(x) + s(x \mid C)k \qquad \text{(NVP)}$$
$$\text{Subject to } -(g(x) + s(x \mid D)q) \in Q,$$

where

(i) $f(x) + s(x \mid C)k = (f_1(x) + s(x \mid C)k_1, f_2(x) + s(x \mid C)k_2, \ldots, f_m(x) + s(x \mid C)k_m)^T$ and
$g(x) + s(x \mid D)q = (g_1(x) + s(x \mid D)q_1, g_2(x) + s(x \mid D)q_2, \ldots, g_p(x) + s(x \mid D)q_p)^T$.

(ii) $f: R^n \to R^m$, $g: R^n \to R^p$ are differentiable vector valued functions.

(iii) C, D are nonempty compact convex subsets of R^n.

(iv) K and Q are closed convex pointed cones with nonempty interiors in R^m and R^p respectively.

(v) $k = (k_1, k_2, \ldots, k_m)^T \in \text{int}K$, $q = (q_1, q_2, \ldots, q_p)^T \in Q$ are any arbitrary but fixed vectors.

$S^0 = \{x \in R^n : -(g(x) + s(x \mid D)q) \in Q\}$ denotes the set of all feasible solutions of (NVP).

3 Unified higher order duality

In this section, we associate the following unified higher order dual (UHD) to (NVP) by considering a 0 − 1 parameter $\delta \in \{0, 1\}$.

$$\begin{aligned}
K\text{-Maximize } & f(u) + (u^T z)k + \delta[H(u, p) - \nabla_p H(u, p)p] \\
& + \delta[(\mu^T g)(u) + (\mu^T G)(u, p) - \nabla_p(\mu^T G)(u, p)p + u^T w(\mu^T q)]k \qquad \text{(UHD)} \\
\text{Subject to } & [\nabla(\lambda^T f)(u) + z^T(\lambda^T k) + \nabla_p(\lambda^T H)(u, p) \\
& + \nabla(\mu^T g)(u) + \nabla_p(\mu^T G)(u, p) + w^T(\mu^T q)] = 0, \\
& (1 - \delta)[(\lambda^T H)(u, p) - \nabla_p(\lambda^T H)(u, p)p] \geq 0, \\
& (1 - \delta)[(\mu^T g)(u) + (\mu^T G)(u, p) - \nabla_p(\mu^T G)(u, p)p + u^T w(\mu^T q)] \geq 0. \\
& \delta(\lambda^T k) = \delta \\
& \lambda \in K^+ \setminus \{0\}, \mu \in Q^+, u \in R^n, z \in C, w \in D, p \in R^n.
\end{aligned}$$

where,

$H : R^n \times R^n \longrightarrow R^m$ and $G: R^n \times R^n \longrightarrow R^p$ are differentiable vector valued functions.

$\nabla_p(\lambda^T H)(u, p)$ and $\nabla_p(\mu^T G)(u, p)$ are $1 \times n$ Gradient vectors of $\lambda^T H$ and $\mu^T G$ with respect to p at u.

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Remark 3.1. (i) For $\delta = 0$, unified higher order dual gives following Mond Weir type higher order dual:

$$K\text{-Maximize} \quad f(u) + (u^T z)k \qquad \text{(MHD)}$$
$$\text{Subject to} \quad [\nabla(\lambda^T f)(u) + z^T(\lambda^T k) + \nabla_p(\lambda^T H)(u,p) + \nabla(\mu^T g)(u) + \nabla_p(\mu^T G)(u,p) + w^T(\mu^T q)] = 0,$$
$$[(\lambda^T H)(u,p) - \nabla_p(\lambda^T H)(u,p)p] \geq 0,$$
$$[(\mu^T g)(u) + (\mu^T G)(u,p) - \nabla_p(\mu^T G)(u,p)p + u^T w(\mu^T q)] \geq 0.$$

$\lambda \in K^+ \setminus \{0\}$, $\mu \in Q^+$, $u \in R^n$, $z \in C$, $w \in D$, $p \in R^n$.

(ii) For $\delta = 1$, unified higher order dual gives following Wolfe type higher order dual:

$$K\text{-Maximize} \quad f(u) + (u^T z)k + H(u,p) - \nabla_p H(u,p)p + [(\mu^T g)(u) + (\mu^T G)(u,p) - \nabla_p(\mu^T G)(u,p)p + u^T w(\mu^T q)]k \qquad \text{(WHD)}$$
$$\text{Subject to} \quad [\nabla(\lambda^T f)(u) + z^T(\lambda^T k) + \nabla_p(\lambda^T H)(u,p) + \nabla(\mu^T g)(u) + \nabla_p(\mu^T G)(u,p) + w^T(\mu^T q)] = 0$$
$$\lambda^T k = 1$$
$$\lambda \in K^+ \setminus \{0\},\ \mu \in Q^+,\ u \in R^n,\ z \in C,\ w \in D,\ p \in R^n.$$

We now prove the duality relations between (NVP) and (UHD).

Theorem 3.1 (Weak Duality). Let x be feasible for (NVP) and $(u, z, w, \lambda, \mu, p)$ be feasible for (UHD). Suppose that f is higher order K-convex at u with respect to H and g is higher order Q-convex at u with respect to G then

$$f(u)+(u^T z)k+\delta[H(u,p)-\nabla_p H(u,p)p]+\delta[(\mu^T g)(u)+(\mu^T G)(u,p)-\nabla_p(\mu^T G)(u,p)p+u^T w(\mu^T q)]k - f(x) - s(x \mid C)k \notin \operatorname{int}K.$$

Proof. Let if possible,

$$f(u) + (u^T z)k + \delta[H(u,p) - \nabla_p H(u,p)p] + \delta[(\mu^T g)(u) + (\mu^T G)(u,p) - \nabla_p(\mu^T G)(u,p)p + u^T w(\mu^T q)]k - f(x) - s(x \mid C)k \in \operatorname{int}K. \qquad (1)$$

Since $z \in C \Longrightarrow s(x \mid C) \geq x^T z$. Therefore,

$$(s(x \mid C) - (x^T z))k \in K \qquad (2)$$

Since f is higher order K-convex, therefore

$f(x) - f(u) - [\nabla f(u) + \nabla_p H(u,p)][x - u] - H(u,p) + \nabla_p H(u,p)p \in K$.

Adding (1), (2) and above relation and using the fact that $\lambda \in K^+ \setminus \{0\}$, we get

$$-[\nabla(\lambda^T f)(u)+\nabla_p(\lambda^T H)(u,p)+z^T(\lambda^T k)][x-u]-(1-\delta)[(\lambda^T H)(u,p) - \nabla_P(\lambda^T H)(u,p)p]+\delta[(\mu^T g)(u)+(\mu^T G)(u,p) - \nabla_p(\mu^T G)(u,p)p + u^T w(\mu^T q)] > 0.$$

Using feasibility of $(u, z, w, \lambda, \mu, p)$, we get

$$[\nabla(\mu^T g)(u) + \nabla_p(\mu^T G)(u,p) + w^T(\mu^T q)][x - u] + [(\mu^T g)(u) + (\mu^T G)(u,p) - \nabla_p(\mu^T G)(u,p)p + u^T w(\mu^T q)] > 0.$$

Since g is higher order Q-convex and $\mu \in Q^+$, therefore

$$(\mu^T g)(x) - (\mu^T g)(u) - [\nabla(\mu^T g)(u) + \nabla_p(\mu^T G)(u,p)][x - u] - (\mu^T G)(u,p) + \nabla_p(\mu^T G)(u,p)p \geq 0.$$

Adding above two inequalities, we get

$$(\mu^T g)(x) + x^T w(\mu^T q) > 0 \qquad (3)$$

Since x is feasible for (NVP), $w \in D$, $\mu \in Q^+$ therefore

$(\mu^T g)(x) + x^T w(\mu^T q) \leq 0$ which is a contradiction to (3) and hence the result.

Remark 3.2. (i) When $\delta = 0$, the corresponding Weak Duality Theorem for (MHD) can also be proved by assuming $f(.) + ((.)^T z)k$ to be higher order strongly K-pseudoconvex at u with respect to H and $g + ((.)^T w)q$ to be higher order Q-quasiconvex at u with respect to G.

(ii) When $\delta = 1$, the corresponding Weak Duality Theorem for (WHD) can also be proved by assuming $\mathcal{L}(., z, w, \mu)$ to be higher order K-pseudoconvex at u with respect to $H + k\mu^T G$ where $\mathcal{L}(x, z, w, \mu) = (f(x) + (x^T z)k + [\mu^T g(x) + (x)^T w(\mu^T q)]k$.

To prove Strong Duality we use the following Kuhn-Tucker type necessary optimality conditions for a point to be a weak minimum of (NVP) from Suneja et al [16].

Lemma 3.1. Let $\bar{x} \in S^o$ be a weak minimum of (NVP). Then there exist $\bar{\lambda} \in K^+$

, $\bar{\mu} \in Q+$ with $(\bar{\lambda},\bar{\mu}) = 0$ and $\bar{z} \in \partial s(\bar{x} \mid C)$, $\bar{w} \in \partial s(\bar{x} \mid D)$ such that

$$\nabla(\bar{\lambda}^T f)(\bar{x}) + \bar{z}(\bar{\lambda}^T k) + \nabla(\bar{\mu}^T g)(\bar{x}) + \bar{w}(\bar{\mu}^T q) = 0 \quad (4)$$

$$(\bar{\mu}^T g)(\bar{x}) + \bar{x}^T \bar{w}(\bar{\mu}^T q) = 0 \quad (5)$$

Moreover, if g is Q-convex at $\bar{x}$ and there exist $x^* \in Rn$ such that

$$(\bar{\mu}^T g)(x^*) + s(x^* \mid D)(\bar{\mu}^T q) < 0.$$

Then $\bar{\lambda} \neq 0$.

Theorem 3.2 (Strong Duality) Let $\bar{x} \in S_o$ be a weak minimum of (NVP).
Then there exist
$\bar{\lambda} \in K+$, $\bar{\mu} \in Q+$ with $(\bar{\lambda}, \bar{\mu}) \neq 0$ and $\bar{z} \in \partial s(\bar{x} \mid C)$, $\bar{w} \in \partial s(\bar{x} \mid D)$ such that (4) and (5) hold. Moreover, if $H(\bar{x}, 0) = 0 = G(\bar{x}, 0)$, $\nabla_p H(\bar{x}, 0) = 0 = \nabla_p G(\bar{x}, 0)$, g is Q-convex at $\bar{x}$ and we can find $x^* \in Rn$ such that $(\bar{\mu}^T g)(x*) + s(x* \mid D)(\bar{\mu}^T q) < 0$, then there exist $\bar{\lambda} \in K+ \setminus \{0\}$ and $\bar{\mu} \in Q+$ such that $(\bar{x}, \bar{z}, \bar{w}, \bar{\lambda}, \bar{\mu}, \bar{p} = 0)$ is feasible for (UHD). Suppose conditions of Weak Duality Theorem 3.1 is satisfied for each feasible solution (u, z, w, λ, μ, p) of (UHD), then $(\bar{x}, \bar{z}, \bar{w}, \bar{\lambda}, \bar{\mu}, \bar{p} = 0)$ is weak maximum for (UHD).

Proof. Since $\bar{x}$ is a weak minimum of (NVP), therefore using Lemma 3.1 and proceeding on the lines of Suneja et al [16] there exist $\bar{\lambda} \in K+ \setminus \{0\}$, $\bar{\mu} \in Q+$, $\bar{z} \in \partial s(\bar{x} \mid C)$, $\bar{w} \in \partial s(\bar{x} \mid D)$ such that $(\bar{x}, \bar{z}, \bar{w}, \bar{\lambda}, \bar{\mu}, \bar{p} = 0)$ is feasible for (UHD). Suppose the condition of Weak Duality Theorem is satisfied for each feasible point of (UHD) and let if possible $(\bar{x}, \bar{z}, \bar{w}, \bar{\lambda}, \bar{\mu}, \bar{p} = 0)$ be not a weak maximum of (UHD), then there exist a feasible point (u, z, w, λ, μ, p) of (UHD) such that
$f(u)+(u^T z)k+\delta[H(u,p) - \nabla_p H(u,p)]+\delta[(\mu^T g)(u)+(\mu^T G)(u,p) - \nabla_p(\mu^T G)(u,p)p + (u^T w)(\mu^T q)k - f(\bar{x}) - s(\bar{x}|C)k \;\epsilon$ intK, because $H(\bar{x}, 0) = 0 = G(\bar{x}, 0)$, $\nabla_p H(\bar{x}, 0) = 0 = \nabla_p G(\bar{x}, 0)$ and $\bar{z} \in \partial s(\bar{x} \mid C)$. But this is a contradiction to Weak Duality.
Hence $(\bar{x}, \bar{z}, \bar{w}, \bar{\lambda}, \bar{\mu}, \bar{p} = 0)$ is weak maximum for (UHD).

4 Special Cases

In this section, we discuss few special cases of our dual problems.
(i) If H (u, p) ≡ 0, G (u, p) ≡ 0, then (UHD) reduces to (UD) considered by Suneja and Louhan in [16]. Consequently, all special cases discussed in [16] are also particular cases of the primal-dual model discussed by us.
(ii) If C=D= {0}, p=q, K=$\mathbb{R}_+^m, Q = \mathbb{R}_+^q$ and f, g are twice differentiable functions then (NVP) reduces to (MOP) discussed in [6]. When H (u,p)=$\frac{1}{2} p^T \nabla^2 f(u) p$ and G(u,p)= $\frac{1}{2} p^T \nabla^2$g(u)p then (WHD) reduces to second order dual (XMOP) with $J_2 = \emptyset$ considered in [6]. In addition if m=1 and p= q= m then (NVP) reduces to (1.1) and (WHD) redcues to (1.7) considered in [8].
(iii) If C=D= {0}, p=q,K=$\mathbb{R}_+^m, Q = \mathbb{R}_+^q$ then (NVP) reduces to (MOP) discussed in [5]. When H (u, p)≡ 0, G (u, p)≡ 0 then (MHD) reduces to first order dual (XMOP) with $J_1 = \emptyset$ and (WHD) reduces to first order dual (XMOP) with $J_2 = \emptyset$ considered in [5].
(iv) If C = D = {0}, m = 1, p = m, K = R+, Q =$\mathbb{R}_+^m$, then (NVP) reduces to (NP) considered in [4]. When H (u, p) ≡ 0, G(u, p) ≡ 0 then (MHD) reduces to (MD) and (WHD) reduces to (WD) considered in [4].

5 Conclusions

A unified higher order dual (UHD) has been proposed for the vector optimization problem (NVP). Unified dual enables us to study Mond-Weir and Wolfe type duals together. Many known duals are particular cases of (UHD). It will be interesting to see how the optimality conditions and dual shape up when (NVP) is considered with support functions on distinct sets Ci , i ∈ {1, · · · , m} and Dj , j ∈ {1, · · · , p}.

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