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An Original New Statistical Chart: Polar Coordinate and Cumulative Distribution Function (CDF)-Based Individual and Central Tendency Deviation Statistical Chart Method

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Abstract
In empirical economic and managerial analysis, data visualization serves as a crucial bridge between theoretical modeling and policy decision-making. However, traditional descriptive tools often fail to present complex multi-dimensional distribution characteristics effectively. In descriptive statistics, traditional histograms, box plots, and density curves often suffer from a lack of dimensional intuitiveness or overlapping confusion when simultaneously displaying population central tendencies (mean, median, mode) and individual relative positions. This paper proposes a novel statistical chart representation method based on the polar coordinate system. This method uses central tendency measures as the "zero-angle reference line" (angle axis) in polar coordinates. By calculating the proportion of the area integral of individual values under the population cumulative frequency distribution function (CDF), it maps it into the deviation angle θ in polar coordinates. Additionally, by combining concentric circles corresponding to radii of different central tendency measures, it achieves a multi-dimensional visualization of the overall distribution shape and the relative positions of individuals.

Keywords: descriptive statistics; polar coordinate; cumulative distribution function (CDF); measures of central tendency; skewness; data visualization; empirical distribution function
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1. Overall Concept

The core purpose of descriptive statistics is to reveal the central tendency of the population distribution through central tendency measures (mean, median, mode) and describe individual differences through measures of dispersion (Pearson, 1895; Tukey, 1977; von Hippel, 2005). However, when displaying the relative positions of individuals within the population (such as income intervals, height, psychological test scores, corporate tax amounts, etc.), traditional one-dimensional scales struggle to simultaneously account for the overall skewness and the cumulative position of individuals.

Therefore, a new statistical chart method can be designed to intuitively represent the corresponding positions of individuals in descriptive statistics using polar coordinates. In this method, three colored solid straight line segments at a 0-degree angle in polar coordinates represent the mode, median, and mean, respectively. Taking the polar angle $\theta = 0$ and polar radius $r = 1$, respectively, they represent the baseline of "central tendency measures," which can also be called "angle axes."

In descriptive statistics, actual measured data reflected by specific individuals (such as individual economic income intervals, height, psychological measurement scores, types of diseases, genetic groups, sales volumes of different products of a company, tax amounts of different companies, total profit rates of different companies, etc.) are represented by solid line segments. The length of the line segment represents the proportion relative to the size of the central tendency measure obtained from descriptive statistics. Its deviation angle corresponds to the formula of the cumulative frequency distribution function (CDF) calculated through fitting based on the descriptive data of the parent population in descriptive statistics. Thus, compared with central tendency measures such as the mean, median, and mode, it shows how much cumulative frequency area integral the individual's position has "deviated." The size of this deviated area integral corresponds to the polar angle of the individual's own data in polar coordinates. Each central tendency measure and specific individual value, as two data points, form a set of two concentric polar coordinate circles. All calculations, including fitting the CDF from data and reasoning, can be left to the computer.

Since each central tendency measure has its own polar coordinate circle (e.g., the mean has its mean polar coordinate circle, and the median has its median polar coordinate circle), if one wishes to compare individual measured data with the central tendency measures of the parent population, it only requires creating a set of two concentric polar coordinate circles. The radii of these circles represent the central tendency measure and the individual measured value, respectively. When comparing the three central tendency measures, their respective polar radii and concentric circles use different colors. However, when comparing a single central tendency measure with individual sample data, the central tendency measure retains its original color, while the polar radius and concentric circle of the individual sample data are drawn in black. For convenience, the center of the concentric circles—that is, the pole of the polar coordinates—is uniformly drawn as a small black dot.

And since the median is always in the "center", when comparing the median, mean, and mode, the polar angle of the median is defined as 0 degrees. When comparing the mode, median, mean, and measured data of an individual sample, the polar angle of the measure of central tendency is also defined as 0 degrees.

Of course, based on the fitting of the CDF curve, the area distribution of the CDF curve can also be used to determine how much the mean and mode deviate in polar angle relative to the median. Taking the median as the central baseline, its polar angle is set to 0 degrees, representing a "deviation amount of 0 degrees." Through computer calculation of the curve area integral formula obtained from CDF curve fitting, it compares how much cumulative frequency integral area the mode and mean have "deviated" relative to the median. Thus, a set of three concentric polar coordinate circles can be drawn, with their radii representing the median, mean, and mode, respectively. A radius (i.e., the polar radius) is drawn for each of the three circles, representing the respective values of the central tendency measures and their "deviation" amplitudes relative to the central value. At this point, individual measured data does not appear in the concentric polar coordinate circles, so as to avoid confusing the comparison of individual data's deviation degrees relative to these three central tendency measures. At this point, the extreme radius is represented by a solid line, while the concentric circles are represented by dashed lines. Meanwhile, when comparing the three sets of central tendency measures, the polar radius of the median is defined as 1, and the polar radii of the other central tendency measures are calculated based on their numerical ratios relative to the median; when comparing individual sample data with a specific central tendency measure, the polar radius of the central tendency measure is fixed at 1, so as to intuitively obtain the relative magnitude proportion of the individual sample data compared with the central tendency measure.

For nominal scales, since only the mode is applicable, there is no concept of "deviation." In the statistics of frequency (count) for the same nominal item, two line segments can be drawn directly, representing the descriptive statistical mode of the nominal scale and the individual's own measured value (or the count value of the category to which the individual belongs), respectively.

2. Qualitative Mathematical Expression and Derivation of the Statistical Chart

The fundamental idea behind the design of this chart method is as follows:

- **Angle Axis (Zero-Degree Reference):** The 0-degree angle in the polar coordinate system serves as the reference line for the "central tendency measures."
- **Polar Radius (r):** Represents the specific absolute magnitude of central tendency measures or individual measured values.
- **Polar Angle (Deviation Angle θ):** Determined by the proportional area integral of the fitted cumulative frequency distribution function (CDF) over different intervals, intuitively reflecting the percentile deviation degree of the individual within the population distribution.

To precisely map the cumulative frequency of data into the deviation angle in the polar coordinate system, a qualitative mathematical derivation process based on continuous fitting functions is introduced.

①**: Frequency Density Fitting Function t(x)** Assume that the measured data of the parent population is statistically fitted to obtain the population probability density / frequency density fitting function t(x), where x is the measured variable (such as economic income, height, sales volume, etc.), and the domain of x is [a, b]. The total cumulative quantity of the entire parent population (i.e., the total area under the distribution curve) can be expressed as a definite integral: Total area integral size = *∫[a to b] t(x) dx*

②**: Interval Cumulative Frequency Area Integral Function f(x)** Define the interval integral function f(x), which is used to calculate the cumulative frequency area from the distribution starting point a to a specific position x (such as an individual measured value or a specific central tendency measure): *f(x) = ∫[a to x] t(u)* du Here, u is a formal variable of integration (dummy variable), representing the continuously changing measured values within the integration interval [a, x], used to symbolically distinguish from the upper limit of integration (target position x). If it is necessary to calculate the cumulative frequency difference between two different positions x1 and x2 (such as the median and an individual value), the corresponding area integral difference is: "*Δf(x1, x2) = | f(x2) - f(x1) | = | ∫[x1 to x2] t(u) du |*"

③: Proportional Conversion Formula for Deviation Angle θ

The ratio of the interval area integral to the total area integral over the entire domain is calculated, and this ratio is proportionally mapped to the full 360-degree cycle of the polar coordinate system to obtain the deviation angle θ:

"*Deviation Angle θ = (f(x) / ∫[a to b] t(u) du) · 360°*"

If calculating the deviation angle θ_dev of an individual value x_ind relative to the median x_med:

"*θ_dev = (| ∫[x_med to x_ind] t(u) du | / ∫[a to b] t(u) du) · 360°*"

Note: In practical applications, the fitting, integration, and ratio operations for t(x) are all automatically performed by computer algorithms, and users only need to extract the final output of angle θ and radius r.

3. Chart Construction Rules and Concentric Circle Drawing Mechanism Based on differences in measurement scales and comparison requirements, this chart method is divided into the following three specific drawing modes:

①: "Dual Concentric Circle Mode" for Individual and Central Tendency Measures

- **Graph Structure:** Composed of two concentric circles. The inner circle radius r1 represents the population central tendency measure (e.g., mean), and the outer circle radius r2 represents the individual measured value (or vice versa).
- **Line Segments and Styles:**
  - **0-degree angle axis:** Uses a dashed straight line segment to represent the median or mean; uses a dashed wavy line segment to represent the mode.
  - **Deviation solid line:** A solid line segment radiates outward from the pole, with its length corresponding to the individual measured value radius r2, and its deviation angle being θ_dev calculated from the aforementioned integral ratio.

②: "Triple Concentric Circle Mode" for Mutual Deviation Among Central Tendency Measures When it is necessary to observe the skewness degree of the population distribution (i.e., the degree to which the mode and mean deviate from the median), three concentric circles are used for comparison:

- **Baseline Setting:** The median is used as the central reference, with its polar angle set to 0 degrees (i.e., "deviation amount of 0 degrees").
- **Radius Allocation:** The radii of the three concentric circles, r_med, r_mean, and r_mode, represent the numerical values of the median, mean, and mode, respectively.
- **Angle Mapping:** By calculating the ratios of the integrals "*∫[x_med to x_mean] t(u)* du" and "*∫[x_med to x_mode] t(u) du*" to the total area, the deviation angles θ_mean (deviation angle of the mean relative to the median) and θ_mode (deviation angle of the mode relative to the median) are obtained, respectively.
- **Anti-Confusion Mechanism:** In this mode, individual measured data does not appear inside the chart. Only three radial lines representing the central tendency measures are retained, specifically to present the tilt and skewness characteristics of the population distribution itself.

③: Special Handling for Nominal Scale For nominal scale data (such as disease types, genetic groups,

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etc.), since the data lacks order and continuity, only the mode has statistical significance, and there is no concept of continuous CDF fitting or "area deviation."

- **Drawing Rules:** Directly draw two line segments: one representing the statistical mode (or the most frequent category) of the nominal scale, and the other representing the count value of the category to which the individual belongs. There is no need to calculate the deviation angle (the angle deviation is fixed or discretely arranged by category).

4. Example

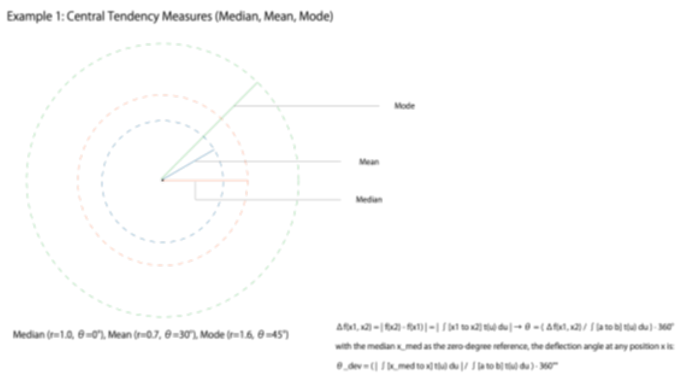

Figure 1

(Comparison of concentric circles of three lumped quantities)

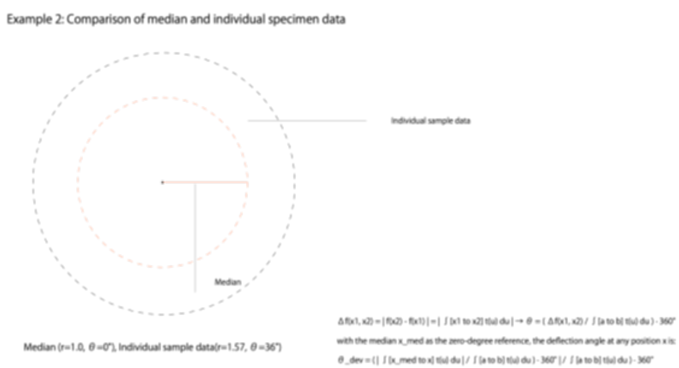

Figure 2

(Example: Comparison of median and individual specimen data)

5. Conclusion

The polar coordinate cumulative frequency deviation-based statistical chart method proposed in this paper organically integrates the "magnitude" (radius) of central tendency measures with the "positional value" (CDF integral deviation angle) of individuals or various central tendency measures within the population. This charting method not only intuitively presents the percentile deviation status of a single individual within the population, but also demonstrates the skewness degree of the population distribution through a multi-concentric circle structure, providing a new approach for the visualization of descriptive statistics that is both information-dense and structurally elegant. Future research could explore the empirical implementation of this polar chart method in open-source statistical packages, such as R (e.g., ggplot2 extensions) or Python (matplotlib/seaborn). Additionally, applying this visualization framework to real-world high-dimensional economic datasets, such as corporate income distribution and regional wealth disparities, represents a promising domain for further investigation.

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