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The Application of Conditional Value-at-Risk in Coal Purchasing Decision Considering Price and Demand Uncertainty

Minfang Huang[a,b], Jinge Fan[a], Ruiyang Liu[a], Shen Luo[a,b*], Luhao Shen[a]

[a]School of Economics and Management, North China Electric Power University, Beijing 102206, China

[b]Beijing Key Laboratory of New Energy and Low-Carbon Development, North China Electric Power University, Beijing 102206, China

*E-mail of the corresponding author: 50602570@ncepu.edu.cn

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Abstract

Coal procurement is the core business part of coal trading enterprises. The effectiveness of the decisions directly affects their profitability. With the adjustment of China's energy structure, coal market faces the supply-demand imbalance and great uncertainty, which brings great challenge to the procurement decision of coal trading enterprises. How to make good purchasing decisions in a fluctuating market environment has become the most important task for coal trading enterprises. In this paper, with the goal of improving the profit of coal trading enterprises, we establish a coal procurement decision model in an environment where the contract market and spot market coexist by considering the risk preference of procurement decision makers and the uncertainty of coal demand and price in the process of procurement. Furthermore, Conditional Value-at-Risk (CVaR) is introduced to quantify the risk return and optimize the coal procurement decision. Case study is conducted to verify the effectiveness of the algorithm. The results show that the proposed coal procurement decision method considering demand and price uncertainty proposed in this paper is more practical, scientific and reasonable than the traditional and empirical method, which can effectively improve the enterprise's profitability and provide theoretical guidance for the optimization of coal procurement decision-making under market volatility.

Keywords: coal procurement, uncertainty, conditional value-at-risk, futures market

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1. Introduction

The supply and security of energy are critical to both economic and social development. Accelerating the planning and construction of a new energy system is essential for ensuring national energy security, addressing global climate change, and supporting high-quality economic growth. Statistics indicate that coal still accounts for over 50% of energy production and consumption, as shown in Fig. 1 and Fig. 2, respectively, over the past decade.

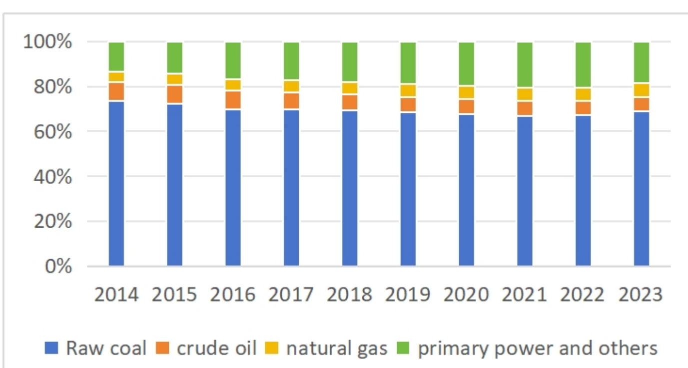

Fig. 1. Structure of energy production from 2014 to 2023

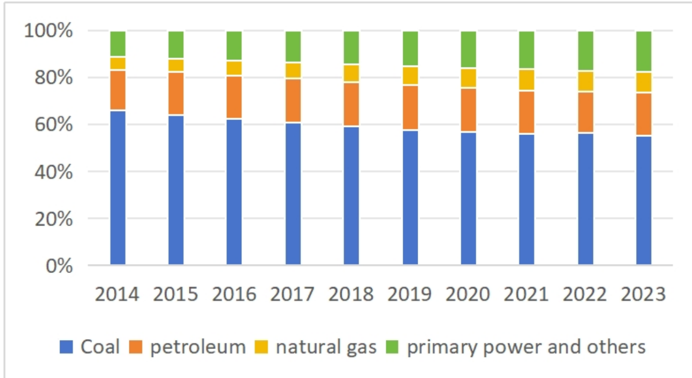

Fig. 2. Structure of energy consumption from 2014 to 2023

A new power system, characterized by a high proportion of renewable energy integration, faces challenges related to randomness, volatility, and intermittency. Thermal power generation continues to act as the stabilizing factor in ensuring a secure and stable energy supply, with installed capacity still exceeding 40%. Detailed data for the past ten years is illustrated in Fig. 3. China's energy resource endowment determines that the supply elasticity of coal, the stability and flexibility of coal-fired power, and the diversity of the coal chemical industry contribute significantly to the security and stability of the energy supply chain.

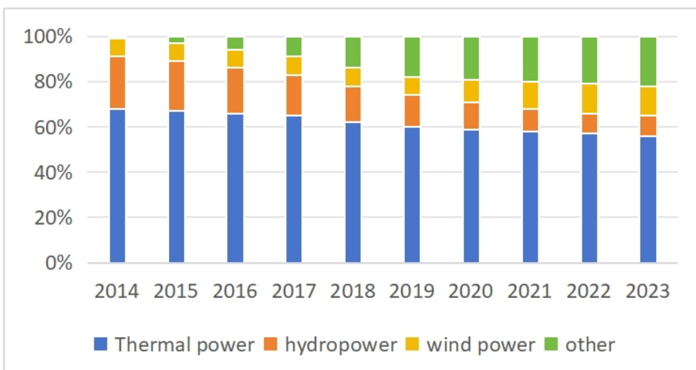

Fig. 3. National installed power structure from 2014 to 2023

The coal industry currently faces dual challenges arising from the uncertainty of renewable energy generation and fluctuations in the demand-side market, making procurement decisions for coal enterprises a critical issue requiring urgent attention. Variations in risk preferences and bargaining power among enterprises have a significant impact on procurement decisions. Presently, the primary procurement methods for Chinese coal enterprises include contract procurement, primarily through medium- and long-term contracts, and spot procurement, mainly based on short-term transactions. The diversity of procurement methods and the volatility of spot market prices further complicate the decision-making process in an increasingly complex and uncertain market environment. This paper investigates the procurement decision-making process under the conditions of coal market price fluctuations and demand uncertainty, with a strong emphasis on procurement risk.

The existing coal procurement decision-making processes in enterprises involve the collection of data (coal demand and supply, prices, policies, etc.), analysis of actual supply and demand considering factors such as coal inventory and transportation capacity, and reliance on human experience for decision-making. While this approach can produce the necessary procurement decisions, the process is influenced by numerous factors, including fluctuations in coal demand and price, inefficiency in human decision-making, underestimation of risks, and a failure to account for relevant indicators. As a result, it is difficult to formulate decisions that consider all relevant factors and optimize benefits while maintaining controllable risks. Moreover, the effectiveness of decision-making is limited by the capabilities of procurement personnel, making it increasingly challenging to achieve scientific, efficient, and economical coal purchasing decisions. Therefore, this paper focuses on improving coal procurement decision-making under conditions of uncertain demand and price, aiming to enhance the performance of coal trading enterprises.

Procurement decisions are typically based on information about key purchasing indicators, such as price, demand, and risk. To align procurement decisions with the business objectives of the enterprise, these indicators are derived from current market conditions and historical industry data following established rules. However, the coal spot market is highly volatile, and relevant indicators such as coal prices, demand, and market risk are often uncertain. This makes it particularly important to make effective procurement decisions in response to changes in these indicators.

Research on procurement decisions under uncertain environments, both domestically and internationally, has primarily focused on three aspects: procurement price uncertainty, demand uncertainty, and risk management. Regarding price uncertainty, most scholars have modeled prices as regular probability distribution functions. Whitin [1] was the first to study procurement decision-making under price fluctuations, developing a model that links demand means with prices, and solving for optimal inventory and procurement decisions to maximize expected purchasing and sales benefits under varying prices. Sanker [2] examined the situation where procurement demand is a function of price changes, constructing an optimization model to maximize purchasing and sales profits. In the coal industry, scholars have quantitatively predicted uncertain coal prices [3]. Tang et al. [4] proposed that a robust method for forecasting coal procurement costs can enhance coal procurement strategies for coal-consuming companies. Wang et al. [5] addressed the issue of imbalanced coal market supply and demand by introducing game theory into coal transactions, proposing a decision-making method for coal supply-demand order prices and procurement volume based on forecasts. Girish et al. [6] combined the autoregressive moving average (ARIMA) and generalized autoregressive conditional heteroskedasticity (GARCH) algorithms to predict India's coal prices on an hourly basis. Wang et al. [7] applied an integrated Bayesian network to forecast coal trading prices, with their model effectively identifying key factors influencing carbon trading prices in emerging markets. In recent years, machine learning [8] and deep learning [9]-[10] methods have increasingly been used to improve the accuracy of price forecasting.

Coal prices and demand are closely related. Mo et al. [11] noted that fluctuations in both domestic and international coal prices significantly impact China’s domestic coal demand, with imported coal offering irreplaceable advantages in terms of price and quality. In terms of procurement demand uncertainty, scholars generally quantify demand as a probability distribution function. Blanchini et al. [12] were the first to examine procurement demand uncertainty, analyzing and proposing methods for procurement decision-making under supply-demand balance constraints. Anh [13] explored procurement decisions for companies whose demand follows a Poisson distribution, though with partially known distribution parameters. Wang et al. [14] constructed a combined forecast model for coal demand in the chemical, building materials, steel, and thermal power industries by integrating data-driven characteristics and decomposition-integration ideas, identifying factors influencing demand and their evolution trends. Feng et al. [15] analyzed trends in coal demand under varying conditions of long-term income and price elasticity, providing a basis for coal policy choices. Yang et al. [16], considering uncertainties in coal demand, economic factors, environmental concerns, and energy security, employed intertemporal optimization methods to determine China's optimal coal production capacity for 2018-2025. Ren et al. [17] explored China's coal production demand and fluctuation range under the dual-carbon policy goals. Efforts to improve the accuracy of coal demand forecasts have focused on time series forecasting methods [18], machine learning [19], and hybrid forecasting methods [20]-[21], which have been widely applied in specific fields.

In traditional supply chain procurement research, procurement decision-makers are often assumed to be risk-neutral coal traders, aiming to maximize purchasing and sales profits or minimize procurement costs. However, Schweitzer et al. [22] noted that actual procurement decisions often deviate significantly from expectations, largely because decision-makers overlook the impact of procurement risk. Recent research has shifted towards studying procurement decisions from a risk management perspective. Woo et al. [23] suggested that local distribution companies (LDCs) could meet part of their future electricity demand through the spot market, while securing the remainder through long-term contracts to reduce the risk of spot market volatility. Arellano et al. [24] explored procurement decisions involving contracts with renewable energy power plants and the installation of renewable energy self-production facilities. In the coal sector, procurement risks are primarily concentrated in planning, bidding, contract performance, and supplier management. Key factors in the procurement process must be controlled to reduce risks fundamentally. Mo et al. [11] developed a coal procurement model based on the coal procurement practices of southeastern coastal Chinese power plants, simulating the impact of factors such as demand indices, seasons, and weather on power plants’ risk resilience. Wu et al. [25] used input-output structure analysis to investigate the drivers of coal demand in China from 1997 to 2012. Common risk measurement methods in procurement include mean-variance, downside risk, Value at Risk (VaR), and CVaR. Katariya et al. [26]compared the effectiveness of these methods in optimizing procurement decisions, demonstrating CVaR's advantages in both profit and loss risk measurement. Zhou et al. [27] aimed to maximize CVaR returns by constructing and solving a dual index coordinated optimal procurement decision model, analyzing how retailer risk aversion and procurement demand influence procurement decisions, and providing recommendations for optimal procurement.

These research findings provide a foundation and reference for the study presented in this paper on procurement decision-making methods for coal enterprises. However, most research has focused on internal procurement process control within coal enterprises, rarely incorporating the volatility of external coal markets into decision-making frameworks. Additionally, decisions have often relied on manual experience without adequately considering market uncertainties, making them less scientifically sound. Furthermore, the risk aversion of decision-makers is rarely addressed, and the use of VaR analysis in coal procurement is uncommon. This paper aims to address these gaps by fully considering the uncertainties in external market prices and demand, taking CVaR returns as a key decision-making criterion, and performing quantitative analysis to develop coal procurement decisions, thereby improving the relevant theoretical research and optimizing business practices.

This study focuses on coal procurement, adhering to the principle of “procurement based on sales” and aims at optimizing the overall profit and risk associated with final coal sales. It fully accounts for the uncertainties of the external coal market and seeks to develop optimal coal procurement decisions for enterprises. First, a coal procurement decision-making model is constructed based on CVaR method, followed by its derivation. A case study is then conducted to apply the model to a coal enterprise and verify its effectiveness. Finally, a sensitivity analysis is performed to explore the impact of key indices on procurement decision-making.

2. Problem analysis and solution principles

2.1 Problem analysis

In China's coal supply market, two primary procurement models are prevalent. The first is medium- to long-term procurement, which primarily operates through coal supply contracts. In this model, coal traders and sellers agree on future coal prices and volumes through contractual agreements. This approach helps stabilize market expectations and mitigates price fluctuations. The second model involves short-term transactions through coal spot procurement, where coal is traded at real-time market prices. This method offers high flexibility, immediacy, and adaptability, reflecting current supply and demand conditions for various types of coal. It enables traders to quickly adjust their purchasing strategies based on market changes. However, unlike the contract market, the spot market is more volatile, often exposing traders and sellers to higher risks. Given the advantages and limitations of both models, most coal enterprises adopt a hybrid procurement approach, combining contracts and spot market transactions. This paper will explore decision optimization strategies for such a combined procurement model. The business process for coal procurement is shown in Fig. 4.

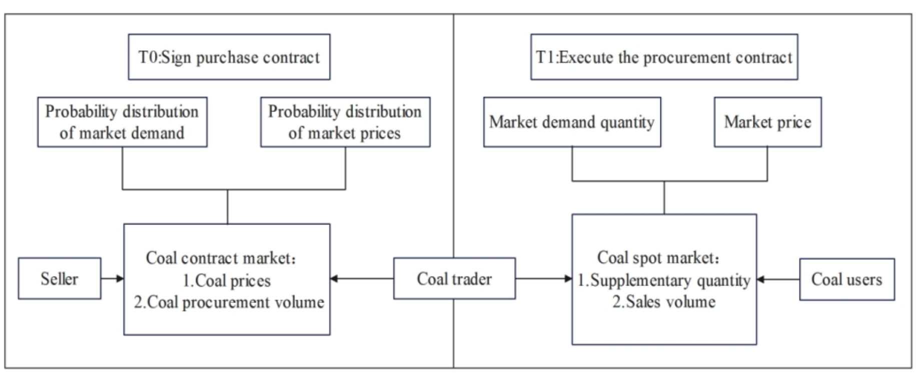

Fig. 4. Coal Procurement Process

As illustrated in Fig. 4, T_0 represents the first stage of procurement involves signing the coal procurement contract. This initial phase of the coal procurement process sees the coal trader and seller agree on key parameters, including the contract purchase price (p_0) and the purchase quantity (Q). In the second stage T_1, when the coal procurement contract is executed, the seller delivers the coal to the trader as per the contract terms. Because the coal procurement futures contract is fixed, the trader is obligated to purchase the specified quantity of coal at the agreed price (p_0). Based on the current coal spot market price (p) and their own demand quantity (q), the trader makes a final decision. If the contract quantity exceeds actual demand, the trader can sell the surplus on the spot market. Conversely, if the contract quantity falls short of actual demand, the trader will need to purchase additional coal on the spot market to meet user requirements.

2.2 Problem-solving principles

Uncertainty in coal procurement prices and demand within the coal spot market can result in high-risk, low-profit decisions for coal traders, increasing overall operational costs and risks for companies. To address this, this paper introduces and refines CVaR theory from the financial sector to effectively quantify the procurement risks associated with coal purchase and sale profits exceeding their respective profit intervals. Using this approach, we calculate the optimal procurement quantity for different coal types, ensuring the safety and economic efficiency of the enterprise's procurement process and ultimately optimizing decision-making.

The overall solution process is divided into three stages: 1. Indicator and Data Collection: In this initial stage, we identify the types and values of relevant indicators required for optimizing coal procurement, based on the procurement business process. 2. Decision Optimization Model Construction and Solving: Here, we establish an objective function grounded in CVaR theory, considering both procurement profits and risks, to determine the optimal procurement quantity. 3. Procurement Decision-Making: Finally, we determine the coal procurement volume in the contract market based on the results from the model, which serves as the final decision outcome. The problem-solving principles are illustrated in Fig. 5.

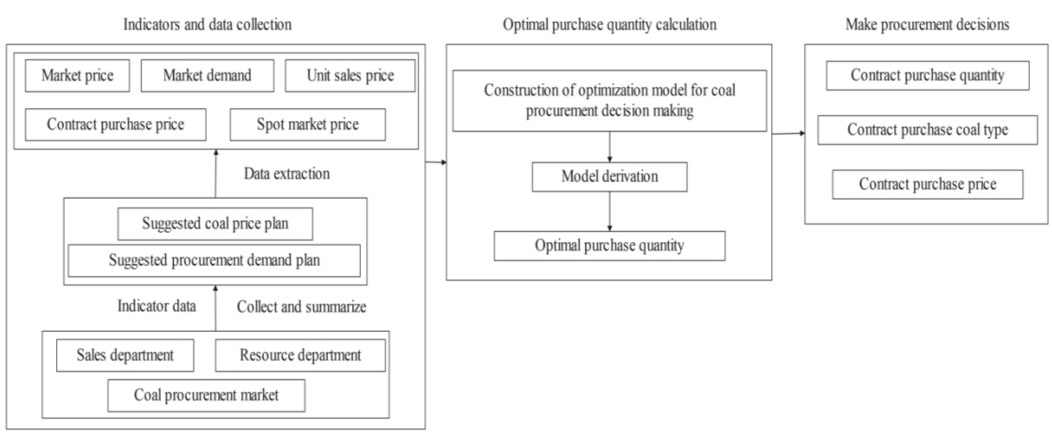

Fig. 5. The technical road map of coal procurement decision

3. Modelling

3.1 Assumptions and definitions

This study focuses on the external coal purchases made by coal traders. Based on problem analysis and actual conditions, the coal procurement market for enterprises encompasses both the coal contract market and the coal spot market. The following relevant assumptions are made:

(1) When purchasing and selling coal in the market, costs other than the purchase price are not considered.

(2) The coal spot market is a perfectly competitive market. Coal traders and sellers cannot control related procurement factors in the coal spot market. The purchase price in the coal spot market is influenced by natural factors.

(3) There are two possible purchase prices in the coal spot market: high price and low price, with the probability of high price being and low price being.

(4) Coal trader can obtain sufficient coal in the coal spot market, with no risk of supply interruption.

(5) The demand (q) faced by coal trader during the execution of coal procurement contracts is a random variable following an exponential distribution, with the probability density function $f(x)$ and the distribution function $F(x)$.

(6) If the selling price of coal per unit (s) exceeds the contract purchase price, i.e., ($s > p_h > p_0 > p_f$), the coal trader will continue to execute the purchase contract; otherwise, it will not.

(7) The procurement business is assumed to be conducted in a specific region for a specific coal supply enterprise.

(8) The coal spot market price has fluctuation risks. It is assumed that the spot market price fluctuates uniformly around the contract purchase price, with the same amplitude and probability, i.e. $p_h = p_0 - i$ 、 $p_f = p_0 - i$ 、 $z = 0.5$ (z is the probability of the high price in the spot market for a specific type of coal).

The variables and parameters involved in the model are shown in Table 1.

Table 1. Model Parameters

Parameters and Variables	Description
s	Unit selling price of a specific type of coal
q	Market demand for a specific type of coal
p_0	Contract purchase price for a specific type of coal
Q	Contract purchase quantity for a specific type of coal
$Q*$	Optimal contract purchase quantity for a specific type of coal
p	Spot market price for coal
$f(x)$	Probability density function of the demand for a specific type of coal
$F(x)$	Probability distribution function of the demand for a specific type of coal
p_h	High price in the spot market for a specific type of coal, with probability z
p_f	Low price in the spot market for a specific type of coal, with probability 1−z
β	Risk aversion or pessimism degree of the decision maker
μ_q	Expected market demand for a specific type of coal
σ_q	Variance of market demand for a specific type of coal
T_0	Time of signing the procurement contract
T_1	Time of executing the procurement contract

3.2 Construction and solution of the coal procurement decision optimization model

3.2.1 Model construction

The problem studied in this paper is divided into two stages. The first stage involves signing the coal procurement contract, during which the spot market price and demand for coal are uncertain. However, the coal trader understands the probability distribution of spot market prices and demand, allowing them to determine the contract purchase price and quantity with the seller based on this information. The second stage is the execution of the coal procurement contract, where the trader can adjust procurement decisions according to user demand. If the contract purchase quantity exceeds actual demand, the trader can sell the surplus coal in the spot market. Conversely, if the contract quantity falls short, the trader will purchase additional coal in the spot market to meet user needs.

Given this process and considering the coexistence of coal procurement contracts and the spot market, we account for the uncertainty in the coal trader's external procurement demand and prices. First, we will construct a procurement decision optimization model aimed at maximizing profit. Then, we will develop another model focused on maximizing the value of CVaR：

When the market risk of coal is not considered, the procurement decision objective is to maximize the expected profit $E[r(Q)]$, the coal trader's profit $r(Q)$ is expressed as:

$$r(Q)=\begin{cases} sq-p_0Q+p_h(Q-q) & p=p_h \\ sq-p_0Q+p_f(Q-q) & p=p_f \end{cases} \tag{1}$$

$$r(Q)=\begin{cases} sq-p_0Q-p_h(q-Q) & p=\mathrm{p_h} \\ sq-p_0Q-p_f(q-Q) & p=\mathrm{p_f} \end{cases} \tag{2}$$

$$r(Q)=\begin{cases} sq-p_0Q+p_h(Q-q) & p=\mathrm{p_h} \\ sq-p_0Q+p_f(Q-q) & p=\mathrm{p_f} \end{cases} \tag{3}$$

The expected profit $E[r(Q)]$ is given by the formula(4):

$$E\left(r\left(Q\right)\right)=z[(s-p_h)u_q+(p_h-p_0)Q]+(1-z)[(s-p_f)u_q+(p_f-p_0)Q] \tag{4}$$

Due to demand uncertainty, there can be a discrepancy between the actual customer demand faced by coal users during the execution of the procurement contract and the coal contract purchase quantity. Two main scenarios arise: 1. Excess Supply Scenario: In the first scenario, the coal trader’s contract purchase quantity exceeds actual market demand. The trader will implement a strategy of immediate production and sale, selling the excess coal in the spot market at prevailing prices. The profit for the coal trader in this case can be expressed as Equation(1). 2. Insufficient Supply Scenario: In the second scenario, the coal trader’s contract purchase quantity is less than the actual market demand. The trader will need to purchase additional coal from the spot market to meet the required quantity. The profit for the trader in this case can be expressed as Equation(2). Considering these two scenarios, the overall profit function for the coal trader is outlined in Equation(3). Additionally, Equation(4) represents the expected profit under the uncertainty of market demand and prices.

When assessing risks in the coal market, the goal is to maximize CVaR of the returns. The uncertainty surrounding coal user demand and spot market prices can lead procurement decision-makers to make high-risk, low-profit decisions during actual operations. Market risk is a crucial factor for decision-makers during coal procurement. Considering the decision-maker's risk preferences, utilizing the CVaR of coal procurement returns as the objective for decision-making is of practical significance.

VaR was initially applied in the financial field as a scientific method for quantifying investment risk. It is defined as the maximum potential loss of a particular investment method over a specific investment period and given confidence level. The mathematical expression is shown in Equation(5). Here, ΔL represents the loss of the investment method, and β represents the confidence level. However, this measurement method can only provide the maximum potential loss value at that confidence level, lacking the measurement of tail risk beyond the confidence level, thereby affecting investors' decision-making. To address this shortcoming, scholars at home and abroad have developed CVaR theory based on VaR theory. CVaR is defined as the expected loss of a particular investment method over a specific investment period and given confidence level, when the loss exceeds the VaR at that confidence level. The mathematical expression is shown in Equation(6). Here, $f(X,\delta)$ is the loss function of the investment method, and δ is the random variable.

$$P\left\{\Delta L>VaR_\beta\right\}=1-\beta \tag{5}$$

$$CVaR_\beta(x)=E[f(X,\delta)\,|\,f(X,\delta)>VaR_\beta] \tag{6}$$

Based on CVaR-related theory, when measuring returns, the coal trader's coal procurement CVaR considers the average profit below the quantile β confidence level. This metric is used to measure the average profit below the quantile β confidence level. The definition is shown in Equation(7). It indicates that the decision-making objective of the coal trader is to ensure that the profit $r(Q)$ of a certain type of coal is below the average profit at the β quantile under a given confidence level. Here, v represents the VaR β value at confidence level β for the procurement decision, $r(Q)$ represents the random profit function of the coal trader, and $[v-r(Q)]^+$ represents $\max(v-r(Q),0)$.

$$CVaR_\beta[r(Q)]=\max_{v\in R}\left\{v+\frac{1}{\beta}E[r(Q)-v]^-\right\}=\max_{v\in R}\left\{v-\frac{1}{\beta}E[v-r(Q)]^+\right\} \tag{7}$$

3.2.2 Model solution

If the risk-averse characteristics of the decision-maker are not considered, meaning the decision-maker ignores market risks while making decisions, the contract procurement quantity that maximizes the expected profit can be determined. The optimal contract procurement quantity is shown in Equation(8), which represents the value of the coal procurement quantity when $E[r(Q)]$ reaches its maximum value. Equation(9) is derived to explore the functional form of $E[r(Q)]$. By differentiating, we obtain the optimal contract purchase quantity for this type of coal, as shown in Equation(10).

$$Q^* = \arg\max_{Q\geq 0} E[r(Q)] \tag{8}$$

$$\frac{dE[r(Q)]}{d(Q)} = z(p_h - p_0) + (1-z)(p_f - p_0) = zp_h + (1-z)\, p_f - p_0 \tag{9}$$

$$Q^* = \begin{cases} 0, & zp_h + (1-z)p_f - p_0 \leq 0 \\ \infty, & zp_h + (1-z)p_f - p_0 > 0 \end{cases} \tag{10}$$

From Equation(10), we observe that if the spot market price per unit of this type of coal exceeds the contract procurement price, it is assumed that a higher contract procurement quantity yields better outcomes. This assumption arises because the decision-maker overlooks procurement risk and focuses solely on maximizing expected profit. However, this perspective does not reflect real-world decision-making scenarios. Therefore, it is essential to account for the decision-maker's risk aversion to ensure more reasonable coal procurement decisions.

If the decision-maker's risk aversion characteristic, denoted as $0<\beta<1$, is considered, the optimal contract procurement quantity is the quantity that maximizes the expected profit of the procurement decision CVaR model. The optimal contract procurement quantity is shown in Equation(11), representing the value of the coal procurement quantity when the Conditional CVaR of the procurement decision reaches its maximum value. Equation(12) is derived to facilitate the solution of the procurement decision CVaR model by transforming its functional form. Consequently, the optimal contract purchase quantity for this type of coal at this time can be derived as shown in Equation(13). To solve Equation(13), $\max_{v\subseteq R} h(Q,v)$ must first be determined, and the solving process and result are shown in Equation(14).

$$Q^* = \arg\max_{Q\geq 0} CVaR_\beta[r(Q)] \tag{11}$$

$$h(Q,v) = v - \frac{1}{\beta}E[v - r(Q)]^+ \tag{12}$$

$$Q^* = \arg\max_{Q\geq 0}\max_{v\in R}[h(Q,v)] \tag{13}$$

$$\begin{aligned} h(Q,v) &= v - \frac{1}{\beta}E[v - r(Q)]^+ \\ &= v - \frac{1}{\beta}z\int_0^\infty [v - sq + p_0 Q - p_h(Q-q)]^+ dF(q) \\ &\quad - \frac{1}{\beta}(1-z)\int_0^\infty [v - p_f(Q-q) + p_0 Q - sq]^+ dF(q) \\ &= v - \frac{1}{\beta}z\int_0^\infty [v - (s-p_h)q - (p_h - p_0)Q]^+ dF(q) \\ &\quad - \frac{1}{\beta}(1-z)\int_0^\infty [v - (s-p_f)q - (p_f - p_0)Q]^+ dF(q) \end{aligned} \tag{14}$$

According to equation(14), there are three cases: if $v \leq (p_f - p_0)\, Q$, the result is as shown in equation(15); If $(p_f - p_0)\, Q < v \leq (p_h - p_0)Q$, the result is as shown in equation(16). From equation(16), it can be seen that when $v = (p_f - p_0)\, Q$ and $v = (p_h - p_0)Q$, the derivative of $h(Q,v)$ with respect to v has a special property, as shown in equation(17). Therefore, if $1 - \frac{1}{\beta}(1-z)F(\frac{(p_h - p_f)Q}{s - p_f}) \leq 0$ holds, there exists a first-order condition v_1 such that $\frac{dh(Q,v)}{dv} = 0$ holds, and the value of v_1 is as shown in equation(18); If $v > (p_f - p_0)Q$, the result is as shown in equation(19). From equation(19), it can be seen that when $v = (p_f - p_0)Q$ and $v = \infty$, $h(Q,v)$ has special characteristics with respect to v, and the result is as shown in equation(20). Analysis shows that if $1-\frac{1}{\beta}(1-z)F(\frac{(p_h-p_f)Q}{s-p_f})>0$ holds, there exists a first-order condition v_2 such that

$\frac{dh(Q,v)}{dv}=1-\frac{1}{\beta}zF(\frac{v-(p_h-p_0)Q}{s-p_h})-\frac{1}{\beta}(1-z)F(\frac{v-(p_f-p_0)Q}{s-p_f})=0$ is satisfied.

Considering the above situations, the solution for $\max_{v\in R} h(Q,v)$ can be summarized as shown in equation(21), where v^* is the value of v when $h(Q,v)$ reaches its maximum.

$$\begin{cases} h(Q,v)=v \\ \frac{dh(Q,v)}{dv}=1 \end{cases} \tag{15}$$

$$\begin{cases} h(Q,v)=v-\frac{1}{\beta}(1-z)\int_0^{\frac{v-(p_f-p_0)Q}{s-p_f}}[v-(s-p_h)q-(p_f-p_0)Q]dF(q) \\ \frac{dh(Q,v)}{dv}=1-\frac{1}{\beta}(1-z)[F(\frac{v-(p_f-p_0)Q}{s-p_f})] \end{cases} \tag{16}$$

$$\begin{cases} \frac{dh(Q,v)}{dv}|_{v=(p_f-p_0)Q}=1 \\ \frac{dh(Q,v)}{dv}|_{v=(p_h-p_0)Q}=1-\frac{1}{\beta}(1-z)F(\frac{(p_h-p_f)Q}{s-p_f}) \end{cases} \tag{17}$$

$$v_1=(s-p_f)F^{-1}(\frac{\beta}{1-z})+(p_f-p_0)Q \tag{18}$$

$$\begin{cases} h(Q,v)=v-\frac{1}{\beta}z\int_0^{\frac{v-(p_h-p_0)Q}{s-p_h}}[v-(s-p_h)q-(p_h-p_0)Q]\,dF(q) \\ \qquad -\frac{1}{\beta}(1-z)\int_0^{\frac{v-(p_f-p_0)Q}{s-p_h}}[v-(p_f-p_0)Q-(s-p_f)q]\,dF(q) \\ \frac{dh(Q,v)}{dv}=1-\frac{1}{\beta}zF(\frac{v-(p_h-p_0)Q}{s-p_h})-\frac{1}{\beta}(1-z)F(\frac{(p_h-p_f)Q}{s-p_f}) \end{cases} \tag{19}$$

$$\begin{cases} \frac{dh(Q,v)}{dv}|_{v=(p_f-p_0)Q}=1-\frac{1}{\beta}(1-z)F(\frac{(p_h-p_f)Q}{s-p_f}) \\ \frac{dh(Q,v)}{dv}|_{v=\infty}=1-\frac{1}{\beta}<0 \end{cases} \tag{20}$$

$$v^*=\arg\max_{v\in R} h(Q,v)=\begin{cases} v_1\ , & F(\frac{(p_h-p_f)Q}{s-p_f})\geq\frac{\beta}{(1-z)} \\ v_2\ , & F(\frac{(p_h-p_f)Q}{s-p_f})<\frac{\beta}{(1-z)} \end{cases} \tag{21}$$

After determining $v^*=\arg\max_{v\in R} h(Q,v)$, it is necessary to find the optimal purchase procurement quantity specified by the coal procurement contract, as given by equation(13). Substituting equation(21) into this, the result is shown in equation(22). Now, differentiating Q with respect to $\max_{v\in R} h(Q,v)$, as shown in equation(21), it can be divided into two cases: when $F(\frac{(p_h-p_f)Q}{s-p_f})\geq\frac{\beta}{(1-z)}$, the result is as shown in equation(23); when $F(\frac{(p_h-p_f)Q}{s-p_f})<\frac{\beta}{(1-z)}$, the result is as shown in equation(24). Therefore, from equation(24), it can be concluded that there exists a Q^* such that the values of $\frac{d\max_{v\in R} h(Q,v)}{dQ}=0$ and Q^* are as shown in equation(25).

$$Q^* = \arg\max_{Q\geq 0} \max_{v\in R}[h(Q,v)] = \arg\max_{Q\geq 0} h(Q,v^*) \tag{22}$$

$$\frac{d\max_{v\in R} h(Q,v)}{dQ} = \frac{dh(Q,v_1)}{dQ} = p_{\mathrm{f}} - p_0 < 0 \tag{23}$$

$$\begin{cases} \dfrac{d\max_{v\in R} h(Q,v)}{dQ} = \dfrac{dh(Q,v_2)}{dQ} \\ = -\dfrac{1}{\beta} z\int_0^{\frac{v_2-(p_h-p_0)Q}{s-p_h}} [-(p_h-p_0)]dF(q) - \dfrac{1}{\beta}(1-z)\int_0^{\frac{v_2-(p_{\mathrm{f}}-p_0)Q}{s-p_{\mathrm{f}}}} (p_0-p_{\mathrm{f}})\, dF(q) \\ = \dfrac{1}{\beta} z(p_h-p_0)F(\dfrac{v_2-(p_h-p_0)Q}{s-p_h}) + \dfrac{1}{\beta}(1-z)(p_f-p_0)F(\dfrac{v_2-(p_{\mathrm{f}}-p_0)Q}{s-p_f}) \\ = \dfrac{p_h-p_f}{\beta} zF(\dfrac{v_2-(p_h-p_0)Q}{s-p_h}) + p_f - p_0 \end{cases} \tag{24}$$

$$Q^* = \frac{1}{p_h-p_0}\left\{ v_2 - (s-p_h)F^{-1}[\frac{(p_0-p_f)\beta}{(p_h-p_f)z}] \right\} \tag{25}$$

Based on the above derivation, and considering equations(19) and (24) together, set their values to zero, as shown in equation(26). The solution is then shown in equation(27).

$$\begin{cases} \dfrac{p_h-p_{\mathrm{f}}}{\beta} zF(\dfrac{v_2-(p_h-p_0)Q}{s-p_h}) + p_{\mathrm{f}} - p_0 = 0 \\ 1 - \dfrac{1}{\beta} zF(\dfrac{v_2-(p_h-p_0)Q}{s-p_h}) - \dfrac{1}{\beta}(1-z)F(\dfrac{(p_h-p_{\mathrm{f}})\ Q}{s-p_f}) = 0 \end{cases} \tag{26}$$

$$v_2 = (s-p_f)F^{-1}[\frac{(p_0-p_f)\beta}{(p_h-p_f)(1-z)}] + (p_f-p_0)Q^* \tag{27}$$

By organizing Equations (25) and(27), the optimal purchase coal procurement quantity under the conditions of uncertain procurement prices and demand, considering market risk, can be obtained as shown in Equation(28).

$$Q^* = \frac{p_{\mathrm{h}}-s}{p_h-p_{\mathrm{f}}}F^{-1}[\frac{(p_0-p_f)\beta}{(p_h-p_f)z}] + \frac{s-p_f}{p_h-p_f}F^{-1}[\frac{(p_h-p_0)\beta}{(p_h-p_f)(1-z)}] \tag{28}$$

Based on the model solution results, the optimal coal procurement decision—when the decision-maker ignores procurement risk—is as follows: If the spot market purchase price for this type of coal exceeds the contract purchase price, it is deemed advantageous to increase the contract purchase quantity. However, this decision carries significant risk and does not accurately reflect real-world conditions. This outcome arises from a complete disregard for procurement risk, focusing solely on the expected profit from coal procurement. Therefore, it is crucial to consider coal procurement decisions under the framework of risk aversion, using the CVaR of procurement returns as a measure of risk. By applying a weighted combination of both upward and downward risks associated with the spot market procurement price, the coal procurement decision in this scenario can be effectively optimized.

4. Case study

4.1 Background of the case

A Company is a large-scale coal enterprise in China, primarily involved in coal mining, transportation, and sales, categorizing it within the midstream segment of the coal industry. The company does not produce or consume coal itself; rather, it serves as a "transit station" for coal operations, conducting centralized procurement and sales. A Company trades in two main categories of coal: "self-produced coal" and "purchased coal." Procurement is primarily conducted through medium- to long-term contracts in the contract market, with the spot market serving as a supplementary source. Historical data indicates that the purchased coal accounted for 60%-70% of the total resources in its affiliated system. This background forms the basis for a case study to verify the effectiveness of the proposed model and solution methods.

4.2 Case calculations and comparative studies

The experimental data and solution algorithms in this article are implemented in the MATLAB R2022a environment on a computer equipped with an Intel Core i7-9750H @ 2.60GHz six-core processor and dual-channel 32GB memory. We first analyze a single coal type for A Company, using purchased coal type g as an example. The actual execution date for the procurement business is a specific date in a particular month of 2023, with the procurement location being a key coal resource area in western China. According to the model solution results, the coal procurement decision is influenced by the decision-maker's risk aversion level, coal sales price, spot market price, and contract procurement price. To determine the optimal coal procurement quantity, numerical experiments are conducted using MATLAB. It is assumed that the market demand for coal type g on that day follows an exponential distribution. Based on actual business data and expert insights, the expected market demand is set at 14,000 tons, with both high and low-price probabilities of the spot market set at 0.5 (z=0.5). Other relevant parameter values are detailed in Table 2.

Table 2. Single coal purchasing parameter settings

Relevant parameters	Descriptions	Value
s	Selling price of coal type g (Yuan/Ton)	1317
p_0	Contract procurement price (Yuan/Ton)	1046
p_h	Coal type g spot market high price with probability z above futures contract procurement price (Yuan/Ton)	1076
p_f	Coal type g spot market low price with probability 1-z below futures contract procurement price (Yuan/Ton)	1016
β	Risk aversion of decision makers (confidence level)	0.50

Using the method proposed in this article and based on Equation(28), the optimal coal procurement quantity is calculated to be 9,704.06 tons, resulting in a procurement profit of 3.6651 million yuan. In contrast, if the procurement quantity is determined solely by expert experience, the decision-maker would conclude a procurement quantity of 9,500 tons, leading to a profit of 2.5745 million yuan. Thus, the proposed method can enhance the company's profit by 1.0906 million yuan.

Building on the single coal type decision-making and considering actual business conditions, a comprehensive procurement decision optimization is conducted for multiple main coal types contracted by A Company. This analysis focuses on six main coal types (from coal type g to coal type l). The optimal procurement quantities for each coal type are determined, with index values presented in Table 3.

Table 3. Parameter settings

Coal type	Risk aversion of decision makers β	Contract procurement price p_0	High price of spot market p_h	Low price of spot market p_f	Selling price s	Expectations of market demand μ_q
g coal	0.50	1046	1076	1016	1317	14000
h coal	0.64	1110	1130	1090	1285	11000
i coal	0.54	1102	1122	1082	1277	9000
j coal	0.61	1015	1035	995	1190	11000
k coal	0.63	1032	1062	1002	1190	13000
l coal	0.59	902	932	872	1060	6500

Based on the parameters listed in Table 2 and the model results from Equation(28), the optimal procurement quantities for the main purchased coal types contracted by A Company are presented in Table 4.

Table 4. Contractual optimal procurement volume for multiple coal

Coal type	g coal	h coal	i coal	j coal	k coal	l coal
Optimal procurement volume Q^*	9704.06	11238.20	6988.76	10357.70	12925.30	6241.19

In the actual execution of procurement activities, personnel responsible for coal procurement decisions at A Company often rely on their personal work experiences and the procurement plans provided by the affiliated group, adhering to the following decision-making criteria:

(1) Distribute the coal procurement plan issued by the group as evenly as possible.

(2) Maximize profits from the purchased coal.

(3) Minimize procurement risks.

Based on these criteria, the actual procurement volume and corresponding benefits for purchased coal defined by A Company in the area are shown in Table 5.

Table 5. Actual purchases of multiple coal types and the benefits

Coal type	g coal	h coal	i coal	j coal	k coal	l coal
Actual procurement volume (ton)	9500	11500	8300	9500	14000	6500
Actual benefit (ten thousand yuan)	257.45	201.25	145.25	166.25	221.20	102.70

A comparison between A Company's results from manual decision-making and those from the optimized procurement decision using the model is illustrated in Fig. 6.

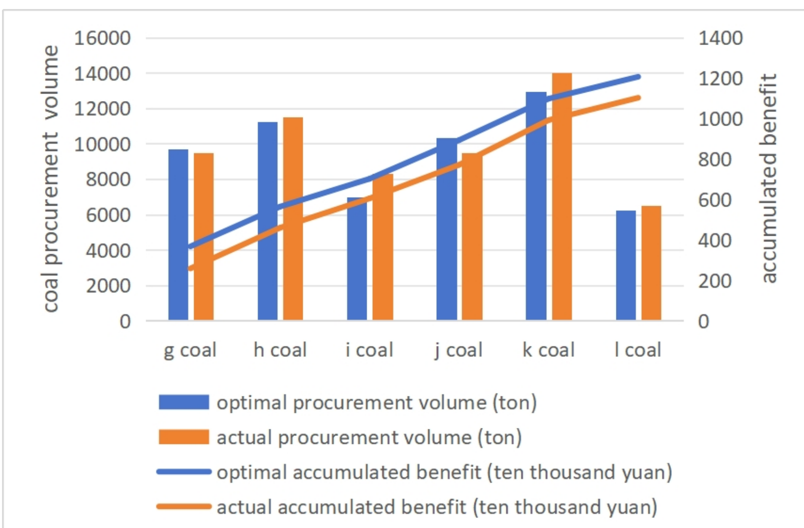

Fig. 6. Comparison of procurement volume and accumulated benefit before and after optimization

The summarized results indicate that the optimized total procurement quantity is 1,844.79 tons lower than the total procurement quantity determined through manual decision-making. In terms of total procurement benefits, the optimized procurement decision yields 1.0364 million yuan more than the benefits from manual decision-making. This suggests that the optimized coal procurement approach can generate greater profits while minimizing the total procurement and transportation of coal. Additionally, the individual profits for optimized coal types h, i, k, and l are lower than those from manual decision-making, primarily due to short-term gains associated with the manual experience-based criteria. In summary, the procurement decision method proposed in this article, which considers demand and price uncertainty while accounting for the coexistence of contract and spot markets for coal, offers significant advantages over A Company's actual manual experience-based procurement approach. This method enhances procurement transportation efficiency and profits, providing valuable insights for A Company's procurement decision-makers.

4.3 Sensitivity analysis

In the actual execution of the coal procurement business, decision-makers are unlikely to proceed with contract procurement if the contract procurement price is deemed too high. Consequently, contract pricing significantly influences the final coal contract procurement volume. Additionally, procurement decisions are subject to a degree of subjectivity, primarily due to the risk aversion characteristics of the decision-makers, which is another crucial factor affecting procurement volume. Moreover, the volatility of the coal market and the uncertainty of customer demand greatly impact the company's procurement volume decisions. In summary, the key indicators that significantly affect coal procurement volume include: the contract procurement price, the degree of risk aversion of decision-makers, and the expected market demand for coal. To illustrate this, a sensitivity analysis will be conducted on these three indicators using coal type g procured by Company A, exploring how changes in these factors affect the optimal procurement volume of coal type g.

(1) The impact of contract procurement price on optimal procurement volume

Referring to the historical data of the company, the coal price for contract signing is set at [1000, 1110]. As the procurement price of the coal contract increases, the change in the optimal procurement volume of coal type g is shown in Fig. 7.

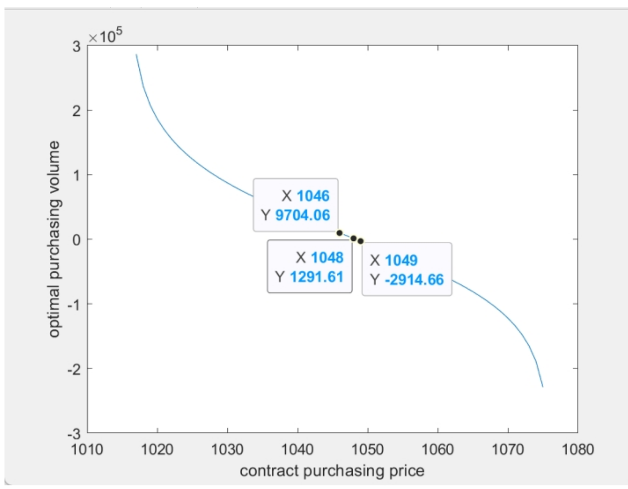

Fig. 7. Impact of contract procurement price on optimal procurement volume

Fig. 7 illustrates that, with other decision indicators held constant, the contract procurement price of coal negatively impacts the contract procurement volume. Specifically, as the contract price increases, the contract procurement volume determined by Company A at time T_0 decreases. Conversely, when the contract procurement price decreases, the procurement volume increases, potentially exceeding expected demand to build inventory or facilitate spot sales at time T_1.When the coal price set by both the trader and the seller exceeds 1,048 yuan/ton, Company A will refrain from ordering coal in the contract market, opting instead to procure entirely from the spot market. For instance, with an actual contract price of 1,046 yuan/ton, model results based on formula (28) indicate that the optimal procurement volume for coal type g at time T_0 should be 9,704.06 tons. This volume is less than the expected demand of 14,000 tons, necessitating supplementation from the spot market to cover the shortfall.

(2) The impact of decision makers' risk aversion on optimal procurement volume

In the actual execution of procurement activities, the risk aversion characteristics of procurement decision-makers significantly influence the determination of coal procurement volume. If the decision-makers' risk aversion level is set within the range [0, 1], the optimal procurement volume will vary according to their risk aversion level, as illustrated in Fig. 8.

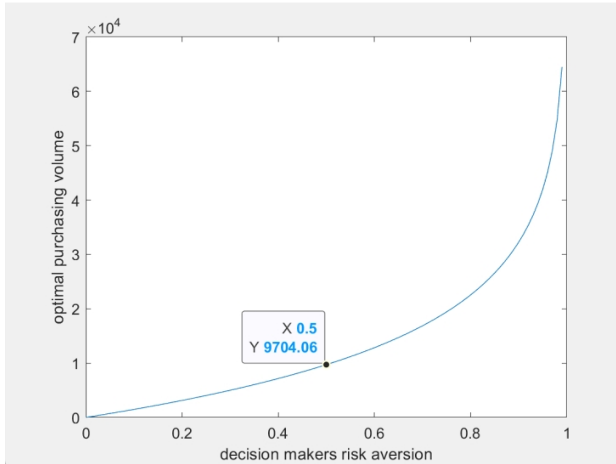

Fig. 8. Impact of decision makers' risk aversion on optimal procurement volume

From Fig. 8, it is evident that, with other procurement decision indicators held constant, lower risk aversion (higher β) among coal procurement decision-makers leads to a higher contract procurement volume determined by Company A at time T_0 . Conversely, higher risk aversion (lower β) results in a lower contract procurement volume. This correlation aligns with practical observations. For example, if Company A's expected demand for coal type g in the region is 14,000 tons, the model solution indicates that with a risk aversion level of 0.5, the optimal procurement quantity for coal type g should be 9,704.06 tons.

(3) The impact of market demand expectations for coal type g on the optimal procurement volume
Based on practical business experience and historical data of the company, the expected variation range of coal market demand is set to [0, 15,000]. According to the model solution results, the optimal procurement volume of coal for the company changes with the expected market demand, as shown in Fig. 9.

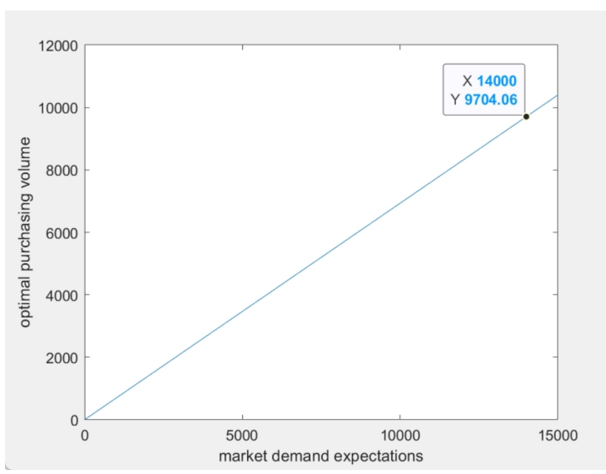

Fig. 9. Impact of market demand expectations for coal type g on optimal procurement volume

Fig. 9 demonstrates that, under constant conditions of other procurement decision indicators, the market demand for coal has a positive linear impact on the contract procurement volume. Specifically, as the market demand for coal increases, the contract procurement volume determined by Company A at time T_0 also increases, and vice versa. Taking coal type g as an example, given the initial parameter settings for other indicators, the optimal procurement volume has an approximate linear ratio of 1.44 to the expected market demand. Therefore, if the expected market demand is 14,000 tons, the contract procurement volume that Company A should sign is 9,704.06 tons.

5. Conclusions

This paper primarily addresses the uncertainties in coal market demand and price within the realm of coal procurement. Under risk management conditions, we have developed a model aimed at maximizing coal procurement profits. By integrating CVaR theory from the financial sector into the coal procurement decision-making process, we solve the model to enhance decision-making. This approach offers procurement decision-makers a new perspective, improving both the scientific rigor and practical applicability of coal procurement strategies. In practice, these theoretical advancements can provide valuable insights for guiding the operations of coal enterprises, ultimately helping them enhance operational efficiency and increase profitability.

The achievements and innovations of this study can be summarized as follows:

(1) By integrating CVaR theory from the financial sector into coal procurement, we emphasize the decision-makers' risk aversion as a crucial indicator. The model is constructed and solved with the objective of maximizing coal procurement profits under market risk conditions, enhancing the scientific accuracy and adaptability of decision-making.

(2) The study develops probabilistic distributions for market demand and price, incorporating these as essential indicators in the model-building process. By addressing uncertainties in coal market demand and price, the paper ensures the model's robustness and relevance to actual market conditions.

(3) In a model where both the coal contract market and the spot market coexist, the paper recommends prioritizing procurement from the contract market, supplemented by spot market purchases. This strategy enables rational decision-making regarding coal quantities from both markets, stabilizing market expectations, reducing risks from supply disruptions due to volatility, and allowing for real-time adjustments based on actual customer demands, thereby ensuring flexibility in procurement operations.

This paper focuses exclusively on the enterprise's procurement decisions for externally purchased coal, excluding the company's coal inventory from consideration. Future research could explore a combined decision-making analysis that integrates both existing inventory and external purchases to optimize the use of internal and external coal resources. Additionally, the study assumes a single coal supplier; future models could enhance practicality by incorporating multiple suppliers. Lastly, this paper presumes that the spot market consistently offers sufficient coal supply, and that customer demand can always be met. Future research should also consider scenarios where procurement errors lead to unmet customer demand, enabling a more comprehensive optimization of enterprise coal procurement decisions.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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