Estimates of Bull and Bear parameters in Smooth Threshold Parameter Nonlinear Market model: A Comparative study between Nigerian and Foreign Stock Markets

O. S. Yaya[1*] O. I. Shittu[1] M. M. Tumala[2]
[1]Department of Statistics, University of Ibadan, Ibadan, Nigeria
[2]Department of Statistics, Central Bank of Nigeria, Abuja, Nigeria
Corresponding email: os.yaya@ui.edu.ng[1*]

Abstract

This study seeks to study the two phases of the financial market using the estimates of betas and nonlinearity in a smooth threshold parameter model. The model considered is an adaptation of the STAR, which is often applied in financial econometric modelling. The parameters, used as yardstick are able to detect and classify the behaviour of stocks into bull and bear. Stocks returns are found to stay longer in the up-market than in the down-market, therefore it is riskier for investors to keep portfolios when the market is at bull phase. The time of global financial crisis is correctly detected in the model and the results further shows that most stocks, except in Nigeria have undergone one or more market cycles over the years. The results further indicate that Nigeria, unlike other advanced nation cannot quickly recover from the effect of the shock. This research work therefore serves as guide for the concerned financial agency in the country.

Keywords: Bull and bear, Financial market, Stock market, Smooth Transition model.

1. Introduction

The Capital Asset Pricing Model (CAPM) has long been tested using simple linear market model on return series of financial assets. Though there are other models like Autoregressive Conditionally Heteroscedastic (ARCH) models of Engle (1982) and Bollerslev (1986), for the generalized case, which have been proposed in the literature to study the stability (volatility) in returns, the CAPM model is still much more relevant in financial analysis. The model assumed stability of the beta coefficient in the market model over the two phases of the market-bull and bear.

The relationship between beta risk and stock market conditions have been investigated in many empirical studies. Individual securities have been studied in Clinebell et al. (1993); mutual funds in Fabozzi and Francis (1979) and risk based portfolio is studied in Spiceland and Trapnell (1983). Most of these studies discovered the possibility of variation of beta with market conditions.

In Nigeria context, Nigerian stocks have not been investigated along this line of thought. Previous works have applied the volatility models (ARCH) to study the behaviours (see Shittu, Yaya and Oguntade, 2009). The Nigerian Stock Exchange (NSE) with many registered companies, publishes the All Share Index (ASI), and is the only stock index for Nigerian under the NSE. The index is calculated by summing the entire share and dividing the total by the official market average. The stock market index is regarded as an important indicator by the investors; this can be used as a benchmark by which the investors or fund managers compare the returns of their own portfolio. In this work, we identify the bull and bear periods of the Nigerian ASI and examine the stability behaviour of its beta over the bull and bear periods. We deploy the LSTM to assess the transition between the two regimes. Results are compared with those of the US, UK and the Asian indices. In US stock market, we have three stock indices (Standard and Poor 500; Dow Jones and Nasdaq). There are three European markets (FTSE100; CAC40 and DAX) and three Asian (Nikkei225, Hang Seng and STI) indices. The Nigerian stock market is then compared with these markets based on the description of the indices and beta estimate in the LSTM model.

The Logistic Smooth Threshold Market (LSTM), which is an adaptation of Logistic Smooth Transition Autoregressive (LSTAR) model of Teräsvirta (1994) is chosen as the best model to solve the problems at hand. This model is known to possess the possibility of allowing for smooth and continuous transition between regimes.

The remaining part of the paper is structured as follows: Section 2 explains the bull and bear phases of financial market, and as well gives the overview of the LSTM model as well as nonlinearity and specification tests. Section 3 presents the results while Section 4 renders the conclusion.

2. The Market Phases

Financial markets have been classified to be in the period of 'up' and 'down' market phases which implies 'bull' and 'bear' periods (Maheu and McCurdy, 2000). Wiggins (1992) defines 'up' months as months when the (excess) market return is greater than 0, while 'down' months as months when the market return is less than 0. Granger and Silvapulle (2001) define market into bullish and bearish periods by using quantiles of return distributions. With the current concern as to classify markets between these two phases (regimes), Cohen et al. (1987) has published some market dates which are being applied in the literatures. Dukes et al. (1987) used the S&P500 index to define bull markets as periods in which the index increased by at least 20% from a trough to a peak and bear markets as periods in which the index decreased by at least 20% from a peak to a trough.

Recently, Pagan and Sossounov (2003) and Lunde and Timmermann (2004) proposed algorithms to classify market based on the definitions of the phases in terms of movements between peaks and troughs given in Duke et al. (1987) and both papers find that bull market last longer than the bear markets. Gonzalez, Powell and Shi (2002) define bull and bear markets in relation to a simplified regime-switching model. In the model, definition of two turning point detection methods is applied to examine whether two centuries of stock index returns can be separated into economically and statistically significant bull and bear phases. Cunado, Gil-Alana and Perez de Gracia (2008) and Gursakal (2010) classify stock markets in S&P500 index into bull and bear phases using the algorithm of Pagan and Sossounov (2003). They further detect long memory in each of the market phases. More recently, Gil-Alana, Shittu and Yaya (2013) consider the stocks in Europe, America and Asia and applied 20%'s rule to classify the stocks into phases and obtain similar results as obtained in Cunado, Gil-Alana and Perez de Gracia (2008).

Due to the difficulty of interpreting the Pagan and Sossounov (2003) algorithm, researchers are trying to look for a model-based classification which is simpler to interprete and apply. Apart from the Markov Switching (MS) model used in Maheu and McCurdy (2000), continuous regime switching model is proposed in Granger and Teräsvirta (1993) and Teräsvirta (1994). Woodward and Anderson (2009) apply an endogenous threshold parameter nonlinear market model top investigate bull and bear states in Australian markets using the estimates of the betas from the model as proxies. Their model is an adaptation of the one earlier proposed in Granger and Teräsvirta (1993).

2.1 The Logistic Smooth Threshold Market (LSTM) model

An unconditional beta model for stocks as defined in Woodward and Anderson (2009) is given by,

$$r_{it} = \alpha + \beta r_{it-1} + \varepsilon_t \tag{1}$$

where r_{it} is the return series on stock/portfolio i for period t, r_{it} is the return on the market index for period t. The α and β are the parameters in the model; the α is the constant and β is the slope, which measures the correlation between r_{it} and r_{it-1}. The ε_t is the disturbance term assumed to be a white noise process. Using the Smooth Threshold Market (STM) model given by,

$$r_{it} = \alpha^D + \beta^D r_{it-1} + \left(\alpha^U + \beta^U r_{it-1}\right) F\left(s_t;\gamma,c\right) + \varepsilon_t \tag{2}$$

with

$$F\left(s_t;\gamma,c\right) = \left(1+\exp\left[-\gamma\left(s_t - c\right)\right]\right)^{-1} \tag{3}$$

for Logistic STM (LSTM) and

$$F\left(s_t;\gamma,c\right) = 1-\exp\left(-\gamma\left(s_t - c\right)^2\right) \tag{4}$$

for Exponential STM (ESTM) with $\gamma > 0$ in both models. The superscript D and U signify the 'up' and 'down' market values for the parameter α and β respectively. The $F(.)$ are the transition functions, with transition variable s_t and threshold value c and $\varepsilon_t \approx N\left(0,\sigma^2\right)$. The LSTM classifies the market into a 'bear' phase when $s_t < c$ and in a bull phase when $s_t > c$ while the ESTM classifies the market into a 'bull' phase at $-c < s_t < c$, excluding point $s_t = 0$ when the model suddenly assumes 'bear' state. It is very rare for the ESTM to be in the 'bear' state, even if it does not stay longer in the state if it suddenly gets there. The ESTM cannot quite classify the market into the two phases, therefore LSTM is often applied to classify market into phases. In the LSTM model, the value β^U measures the difference between the 'up' and 'down' market values of the slope coefficient and therefore presents the up-market value of beta as $\beta^D + \beta^U$, whereas the down market value is β^D. The equation (3) as used in (2) show that beta changes monotonically with the transition variable s_t due to the fact that $F(.)$ in (3) is a smooth and continuous increasing function of s_t. Note, the value of s_t is determined during the estimation and this is a function of the endogenous variable r_t (van Dijk, et al., 2002). As the transition function $F(.)$ changes monotonically between 0 and 1, it assumes the extreme values 0 and 1 occasionally depending on the value of $\left(s_t - c\right)$. When $\left(s_t - c\right) \to -\infty$, $F(.) = 0$ and the model in (2) becomes the linear model $r_{it} = \alpha^D + \beta^D r_{it-1} + \varepsilon_t$ and therefore the stock market is in the 'bearish' state. On the other hand, when $\left(s_t - c\right) \to +\infty$, the model is nonlinear and r_t is generated as

$r_{it} = \left(\alpha^D + \alpha^U\right) + \left(\alpha^D + \beta^U\right) r^*_{it-1} + \varepsilon_t$ which is a 'bullish' state for the stock. In most situations, $\left(s_t - c\right)$ gives values that set the market between the two extreme regimes. The parameter γ determines the speed of transitioning between the two regimes and this also measures the degree of nonlinearity. A high value of the nonlinear parameter implies an instantaneous switch between the market states-'bull' and 'bear' while a low value of the parameter implies a slow transitioning between the market phases.

2.2 Linearity testing against LSTM model

Teräsvirta (1994) provides a step-by-step procedure for specifying the STR model. The LSTM model as an adaptation of Logistic STAR (LSTAR) model follows the same model specification procedure as Smooth Transition Autoregressive (STAR) model. The important components of both LSTM and LSTAR models are the nonlinear component which controls nonlinearity. This also makes the clear difference between the classical Autoregressive (AR) model.

As given in the literature, start by fitting the initial AR model to the return series, r_t. We then test for independency of the residuals ε_t in the model and this test implies linearity in the returns. Linearity is achieved by setting the null $H_0 : \gamma = 0$ against $H_1 : \gamma > 0$. If the null $H_0 : \gamma = 0$ is accepted, then we conclude that the constant risk market model in (1) adequately represent the model. If H_0 is rejected, we accept $H_1 : \gamma > 0$ and proceed to estimating the nonlinear LSTM model using the Nonlinear Least Squares (NLS). Tests based on $H_0 : \gamma = 0$ is not standard and parameters of model in (2 with 3) are only identified when $\gamma > 0$, then Luukkonen, Saikkonen and Teräsvirta (1988) suggest a way out. They apply a first and third orders Taylor series linear approximation to the logistic transition function in the STR model. The resulting approximation expanded in the full STR model leads to the auxiliary regression model

$$r_t = \tilde{\beta}_0 + \tilde{\beta}_1 r^*_t + \tilde{\beta}_2 s_t + \tilde{\beta}_3 s_t^2 + \tilde{\beta}_4 s_t^3 + \tilde{\beta}_5 r^*_t s_t + \tilde{\beta}_6 r^*_t s_t^2 + \tilde{\beta}_7 r^*_t s_t^3 + \tilde{\varepsilon}_t \quad (5)$$

with the last six variables in the equation acting as proxies for the nonlinearity. The null hypothesis then becomes

$$H_0 : \tilde{\beta}_j = 0\left(j = 2, \ldots, 7\right).$$

This hypothesis follows a standard Wald test and the test statistic is denoted as S_3 with the *F*-distribution, with (v_1=6, v_2=*N*-7) degree of freedom, given as,

$$F = \frac{\left(SSE_0 - SSE_1\right)/6}{SSE_1 / \left(N - 7\right)} \quad (6)$$

where *N* is the sample size and SSE_0 and SSE_1 are the errors sum of squares in the linear model (2) and auxiliary regression model (5) respectively. The S_3 is derived as Lagrange Multiplier (LM) statistic with an asymptotic χ^2 distribution in Luukkonen et al. (1988). It is imperative to find evidence of nonlinearity because such evidence justifies nonlinear form, particularly the two market phases under investigation. The acceptance of linearity $H_0 : \gamma = 0$ provides a warning that identification of two regime parameter is not possible.

The estimation of the LSTM model follows the Nonlinear Least Squares (NLS). Consistent estimates will then be obtained when errors, ε_t are independently and identically distributed with mean zero and variance σ^2. Using normality assumption, NLS is seen to be equivalent to MLE. The estimation of LSTM poses much difficulty because of the nonlinear component, which often gives flat sum of squares error (or likelihood) with respect to γ and c (Teräsvirta, 1994; Maringer and Mayer, 2008; Chan and Theoharakis, 2009). Another problem is that of convergence which fails often. These problems can be overcome by sequentially condition estimates of $\alpha^D, \alpha^U, \beta^D$ and β^U on each value of γ and c as given by the Ordinary Least Squares (OLS) estimator,

$$\left(\hat{\alpha}^D, \hat{\alpha}^U, \hat{\beta}^D, \hat{\beta}^U\right)' = \frac{\sum_{t=1}^{N} (\gamma, c) r_t}{\sum_{t=1}^{N} r_t (\gamma, c) r_t (\gamma, c)'} . \tag{7}$$

Maringer and Mayer (2008) give a procedure for setting out a grid search over likely values for (γ, c) to determine the set value of $(\hat{\gamma}, \hat{c})$ that minimizes the residual sum of squares.

3. The Data, Empirical Results and Discussion

The data used in this study are the monthly Nigerian stocks (All Share Index), American stocks (S&P500; Dow Jones and Nasdaq), European stocks (FTSE100; CAC40 and DAX) and Asian stocks (Nikkei 225, Hang Seng and STI) indices. The data span from January 2000 to December 2011 giving a total of 144 data points.

We start by giving the plots of the stock indices with ASI of Nigeria plotted on each of one the other stock indices. The logarithm of each of the stock index has been taken in order to ease series comparism in the plots and the raw values are used in subsequent analyses.

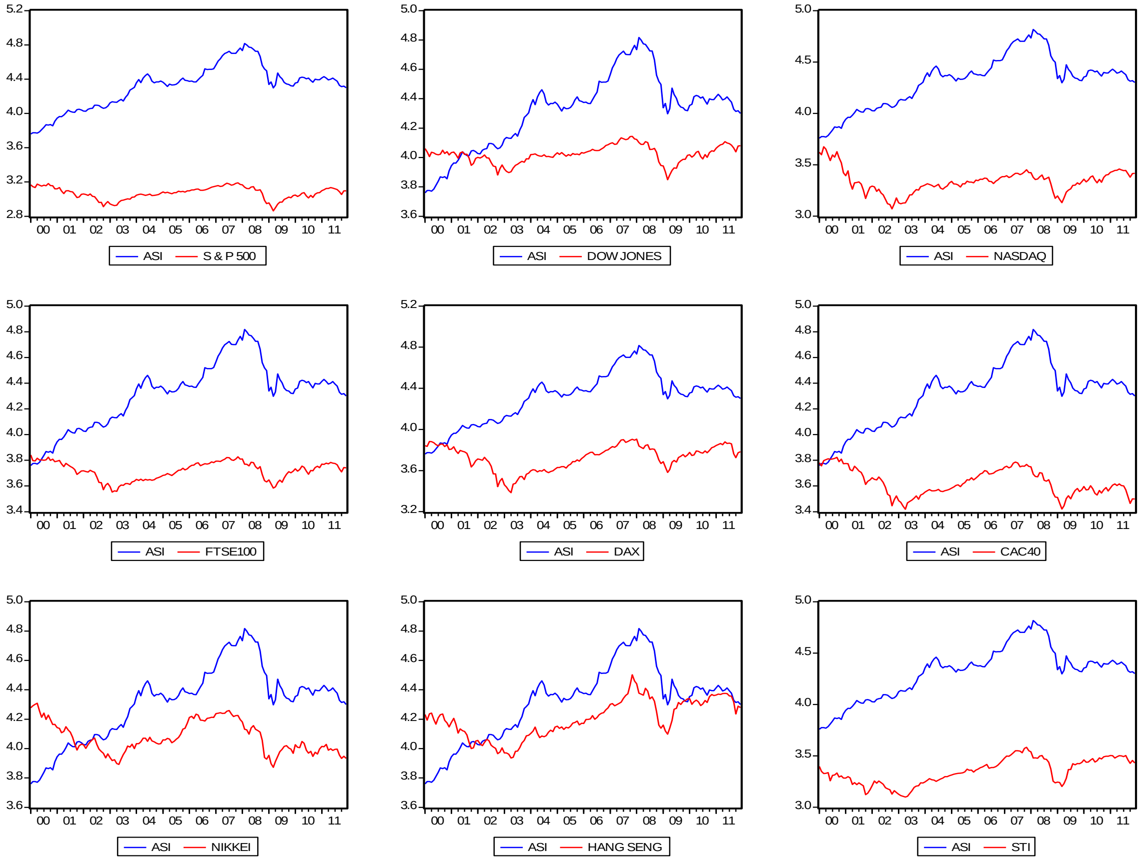

Figure 1: Time plot of ASI on each of the Foreign Stocks in UK, US and Asia

Since the concern is on the market phases of the stocks using return series as proxy, there is need to examine the possible nonlinearity and transition in the stock indices before these are transformed into returns. Table 1 presents the summary of the LSTM models estimated for the stock indices under investigation. Most of the indices showed autoregression of up to order 3 initially. The transition between regimes in ASI, S&P500, Dow Jones, Nasdaq, FTSE100, DAX and STI are quicker than that of the remaining indices (CAC40, Nikkei225 and Hang Seng) as indicated by the estimates $\hat{\gamma}$. The estimates for intercept ($\hat{c}$) of the give the 'jump' expected when the index moves between 'bull' ('bear') and 'bear' ('bear') phases. The coefficient of multiple determination, R^2 is quite high for all the indices indicating the possibility of the regime switching models.

Table 1: Estimates of Nonlinear Parameters for Stcok Indices

Index	$\hat{\gamma}$	$\hat{c}$	R^2
ASI	23.71820	58364.65	0.9748
S&P 500	18.91150	978.3954	0.9229
DOW JONES	1064.2749	8825.795	0.9110
NASDAQ	37.36927	3976.956	0.9210
FTSE 100	24.09792	6470.554	0.9350
DAX	108.8046	7681.320	0.9441
CAC 40	7.263620	7974.919	0.9542
NIKKEI 225	7.810150	19598.532	0.9496
HANG SENG	6.714710	41876.187	0.9554
STI	304.39243	3467.507	0.9637

Gil-Alana, Shittu and Yaya (2013) are able to identify four market phases in UK, US and Asia using Pagan and Sossouvnov's (2003) algorithm. The results in Table 1 further confirms the possibility of two market phases ('bull' and 'bear'), as detected in Gil-Alana, Shittu and Yaya (2013). In Nigeria ASI (Table 1), we can identify one significant peak in February 2008 with market index 65652.38. We can therefore classify the Nigerian market to have undergone two market phases, with the bull phase from January 2000 to February 2008. The bear phase started from peak point till December 2011 time as covered in this work.

We can also notice that most Foreign markets (US, UK and Asia) reached their peak point as well around February 2008 and the markets crashed and this is confirmed from the time plots. Figures 2a-2j below show the behaviour of the transition functions for the stock indices across the sampled data. The transition function confirms movement to the peak (when $F(.)=1$) for the ASI and this quickly revert back to its stable value at $F(.)=0$. Most of these markets display one or more movement between the 'down' state $F(.)=0$ and 'up' state $F(.)=1$. Stocks indices stay longer at down-market state than up-market state as indicated in the plots of the transition functions in Figures 2a-2j below.

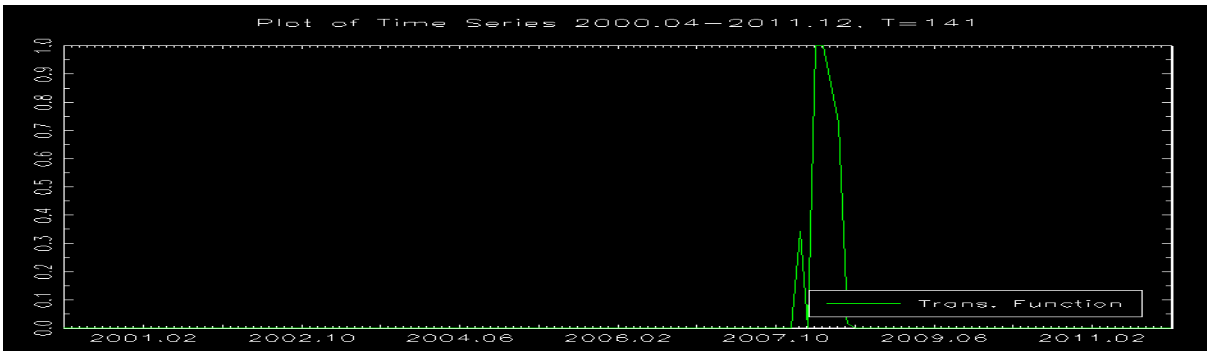

Figure 2a: ASI

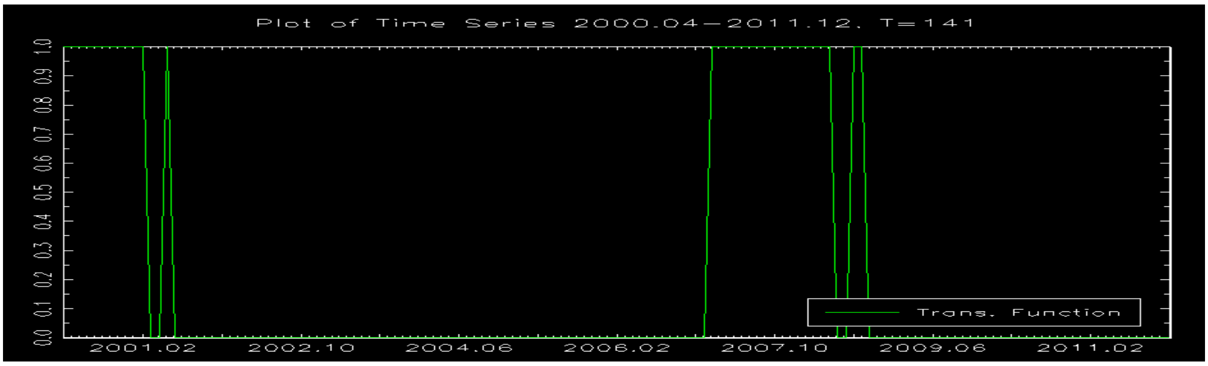

Figure 2b: S & P 500

Figure 2c: DOW JONES

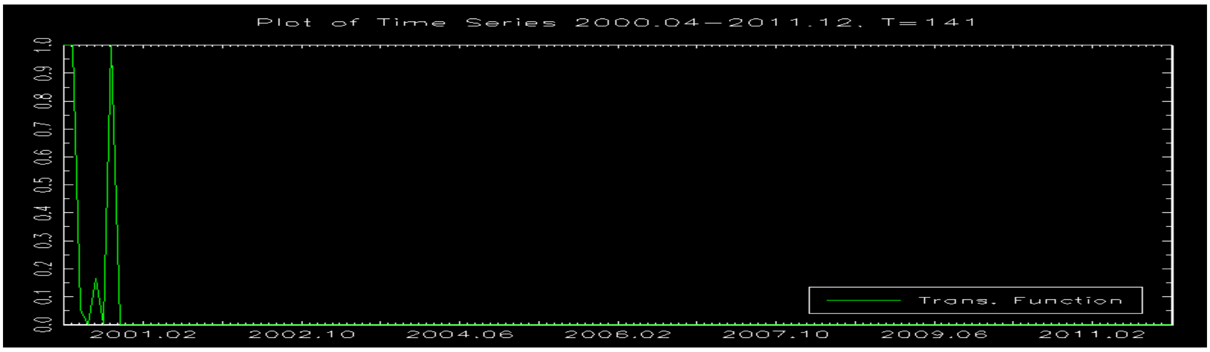

Figure 2d: NASDAQ

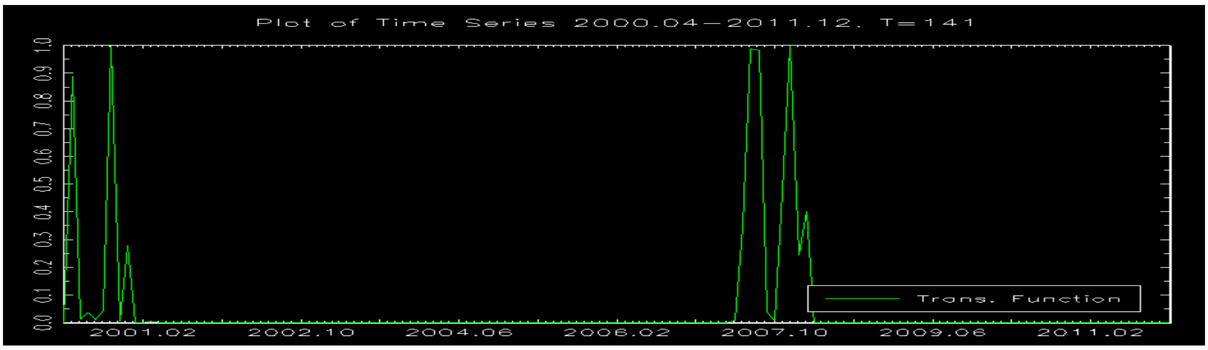

Figure 2e: FTSE 100

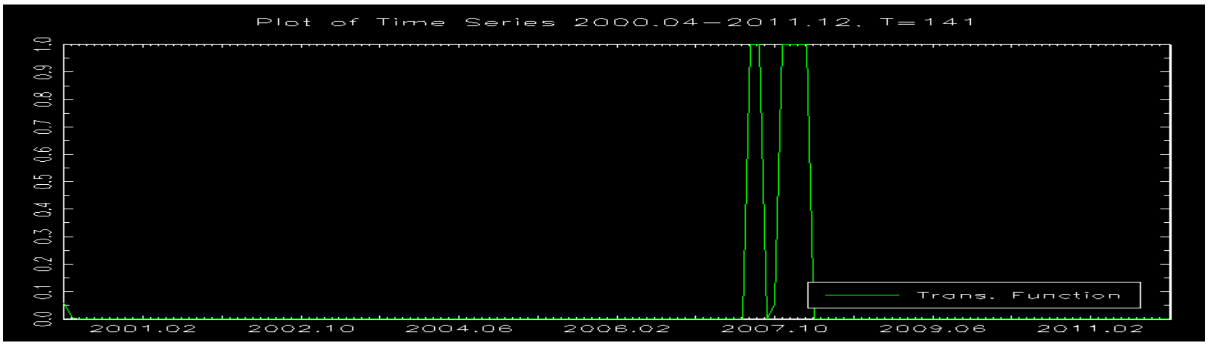

Figure 2f: DAX

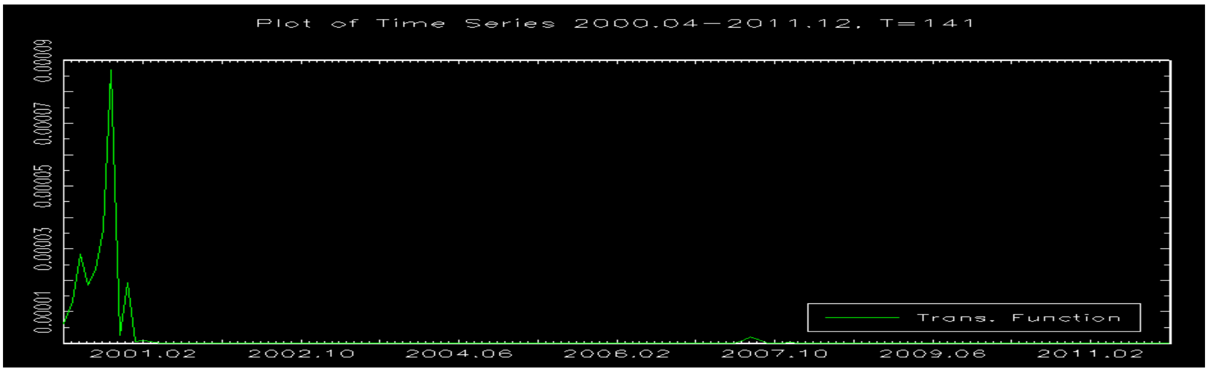

Figure 2g: CAC 40

Figure 2h: NIKKEI 225

Figure 2i: HANG SENG

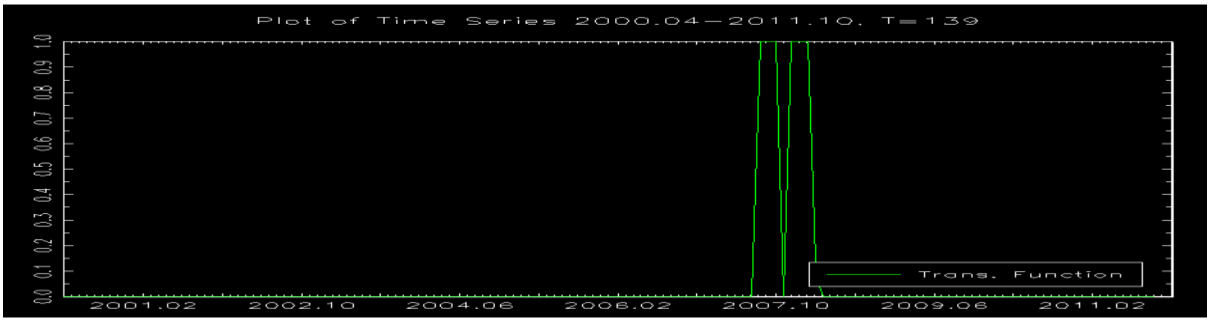

Figure 2j: STI

Figure 2: Plots of the Transition functions for the Indices of ASI and Foreign Stocks

Statistical inference on stocks is usually carried out using the log return series. This is the logarithmic transformation of the difference of the stock index. This is carried out for all the 10 series under investigation and Table 2 presents the descriptive measures.

Table 2: Descriptive measures on the Returns Series of Stocks

	Mean	Median	Maximum	Minimum	Standard dev.	Skewness	Kurtosis	JB	Prob
ASI	0.0037	0.001428	0.140501	-0.1589	0.032911	-0.51684	8.54724	188.3889	0.0000
S&P 500	-0.00034	0.002728	0.043714	-0.07984	0.02099	-0.6206	3.903953	13.94988	0.0009
DOW JONES	0.000295	0.001416	0.043638	-0.06563	0.019785	-0.59328	3.844454	12.54931	0.0019
NASDAQ	-0.00127	0.001966	0.077307	-0.1151	0.033443	-0.55625	3.598713	9.443594	0.0089
FTSE 100	-0.0004	0.001421	0.036046	-0.0606	0.01884	-0.65057	3.51942	11.61305	0.0030
DAX	-0.00036	0.004399	0.083715	-0.1251	0.030439	-0.87717	5.191186	46.61759	0.0000
CAC 40	-0.00181	0.003291	0.056455	-0.07892	0.025183	-0.51809	3.137302	6.46415	0.0395
NIKKEI 225	-0.00252	-0.00076	0.058055	-0.11712	0.0265	-0.58015	4.227265	16.87721	0.0002
HANG SENG	0.000608	0.004678	0.07625	-0.09269	0.028983	-0.39702	3.60471	5.894056	0.0525
STI	0.000572	0.004754	0.08382	-0.11884	0.027073	-0.96386	6.252874	84.5923	0.0000

***significant at 1, 5 and 10% level

From the summary statistics, Nigerian All Share Index (ASI) gave the highest returns followed by Asian Hang Seng. The lowest returns were given by European FTSE100. The values of the standard deviation were observed to be very close to one another. All the returns series also showed signs of leptokurticity and as well negatively skewed, which is in line with findings from other financial time series. Hence, Jarque-Bera (JB) tests indicated significance at 5% for 9 of the stocks except Asian Hang Seng which was found to be significant at 6% level.

As part of LSTM model specification, there is need to first estimate the linear model from which the auxiliary model to test nonlinearity will be constructed. The linear AR(1) models for the returns are presented in Table 3 with the diagnostic checks, Akaike Information (AIC) and Schwarz Bayesian Information (SBIC) as well as Sum of Squares Error (SSE_0). All except the betas for FTSE100 is significant at 5% level in the initial linear market model.

Table 3: Initial Linear Market model

Return	$\hat{\alpha}$	$\hat{\beta}$	AIC	SBIC	SSE_0
ASI	0.003691 (0.003338)	0.176803 (0.083621)	-3.993359	-3.951533	0.147955
S & P 500	-0.000273 (0.002053)	0.142060 (0.083910)	-4.882796	-4.840970	0.060793
DOW JONES	0.000555 (0.001788)	0.073872 (0.082565)	-5.005945	-4.964118	0.053749
NASDAQ	-0.001896 (0.003118)	0.114262 (0.082565)	-3.983467	-3.941641	0.149426
FTSE 100	-0.000382 (0.001688)	0.054497 (0.084695)	-5.080350	-5.038524	0.049895
DAX	-0.000731 (0.002793)	0.087648 (0.083752)	-4.144061	-4.102235	0.127257
CAC 40	-0.002165 (0.002408)	0.129768 (0.083194)	-4.535614	-4.493788	0.086026
NIKKEI 225	-0.002635 (0.002522)	0.115362 (0.084242)	-4.410178	-4.368352	0.097523
HANG SENG	0.000201 (0.003005)	0.206257 (0.082334)	-4.276235	-4.234408	0.111501
STI	0.000733 (0.002619)	0.133226 (0.084094)	-4.375219	-4.333393	0.100993

The auxiliary regression model in (5) is then constructed and the error sum of squares SSE_1 obtained. This together with SSE_0 obtained from the linear model are used based on the F-statistic (6) to get the F-value with critical value obtained as $F_{6,N-7}$ at 5% level of significance. In Table 4, the estimate of the slope (nonlinearity) for ASI of Nigeria is very low for the return series indicating that Nigeria cannot quickly recover from the effect of the financial shock. For other countries, the values are very high, indicating quick and sharp transition between the two regimes. Out of the 10 LSTM models, 4 of $\hat{\beta}^U$ are negative and the remaining 6 are positive, and this supports the claim in the literature which says risk in the up-market is usually lower than risk in the down-market (Woodward and Anderson, 2009). For the ASI, risk in the up-market is higher than that of the down-market and the up-market beta is positive and higher than the down-market beta for the same stock returns series. In other 9 markets, 5 of the stocks have up-betas higher than the down-betas and these are the stocks in Europe and Asia. Closer look at the estimates of the threshold values ($\hat{c}$) as compared with the mean returns values in Table 2 indicate that though the values are negatives, these are very low compared to the mean of each of the returns series. This is contrary to the results of Woodward and Anderson (2009). Only FTSE100 gave positive threshold value, while others are negative. This negativity may be the reason for evidence of bull and bear markets differentials.

Table 4: Estimates of the Parameters of LSTM models for Returns Series

Index	$\hat{\alpha}^D$	$\hat{\beta}^D$	$\hat{\alpha}^U$	$\hat{\beta}^U$	$\hat{\gamma}$	$\hat{c}$	AIC	SBIC	SSE_1	F
ASI	-1.12142 (9.3843)	-5.04958 (32.2679)	1.69977 (13.1530)	1.93745 (20.1203)	0.36824 (1.0419)	-0.06026 (0.0012)	-6.8729	-6.7475	0.1341	2.341884**
S & P 500	0.25993 (0.2400)	3.71505 (3.0556)	-0.26050 (0.2400)	-3.53945 (3.0622)	7.80178 (9.2769)	-0.05348 (0.0025)	-7.7185	-7.5931	0.0497	5.059182**
DOW JONES	0.45312 (0.2704)	7.25496 (4.2674)	-0.45346 (0.2704)	-7.06753 (4.2662)	10.06531 (8.1451)	-0.05342 (0.0033)	-7.8757	-7.7502	0.0592	2.08709**
NASDAQ	-0.02214 (0.0288)	0.06000 (0.4224)	0.02383 (0.0290)	-0.16096 (0.4370)	3166.59 (5.8E08)	-0.04398 (57.2049)	-6.8243	-6.6988	0.1431	1.002022**
FTSE 100	0.00141 (0.0020)	0.13442 (0.1109)	-0.06251 (0.0191)	2.05929 (0.7017)	25.28656 (20.4598)	0.01549 (0.0012)	-7.9589	-7.8334	0.0459	1.97284**
DAX	-0.06992 (0.0257)	-0.83714 (0.3651)	0.07017 (0.0259)	0.89700 (0.3853)	25.31383 (43.5100)	-0.03931 (0.0034)	-6.9843	-6.8588	0.1208	1.211578**
CAC 40	-0.10521 (0.0384)	-1.77554 (0.6957)	0.10354 (0.0384)	1.91954 (0.7034)	21.89199 (31.4445)	-0.03753 (0.0018)	-7.3816	-7.2561	0.0811	1.376769**
NIKKEI 225	-0.15134 (0.1474)	-1.19002 (1.3115)	0.15044 (0.1476)	1.16983 (1.3015)	10.23372 (10.1035)	-0.05240 (0.0062)	-7.2970	-7.1716	0.0879	2.481471**
HANG SENG	-0.04695 (0.0259)	-0.54287 (0.4550)	0.04850 (0.0261)	0.68606 (0.4683)	33.04781 (60.6030)	-0.03746 (0.0022)	-7.0896	-6.9641	0.1089	0.541377**
STI	-0.00323 (0.0030)	0.37599 (0.1534)	0.00834 (0.0101)	-0.26086 (0.2604)	-9448.26 (4.1E12)	-0.1561 (57837.48)	-7.1841	-7.0587	0.0977	0.763985**

From the LSTM models estimated in Table 4, the plots of the transition functions as they assume values from 0 to 1 are presented in Figures 2a-2j for stock indices, as it was observed that stocks stayed more in the down (lower) regime than the up-market regime. Figures 3a-3j present the market transition behaviour of stocks returns for the indices under investigation. The results obtained here are contrary to the ones obtained for the stock indices in Figure 2a-2j above. Here, the returns of ASI neither for once falls in the 'bull' (up) or 'bear' (down) phase, but it migrates between the regimes assumes strictly the continuous nonlinearity. In the remaining 9 markets, larger proportion of the returns are found in the up-market phase and with relatively few found in the lower market and this further confirms that risk in the up-market is lower than risk in the down-market. For example, returns of S&P500, Dow Jones, Nikkei225 and STI only had fewer time to stay at lower phase while CAC40 and Hang Seng moves as many times to lower regime (down) but as well stayed most of the time in the up-market. Only FTSE100 seemed to stay at about 50% of the time at lower level.

Plots of the Transition functions for the Return Series of ASI and Foreign Stocks

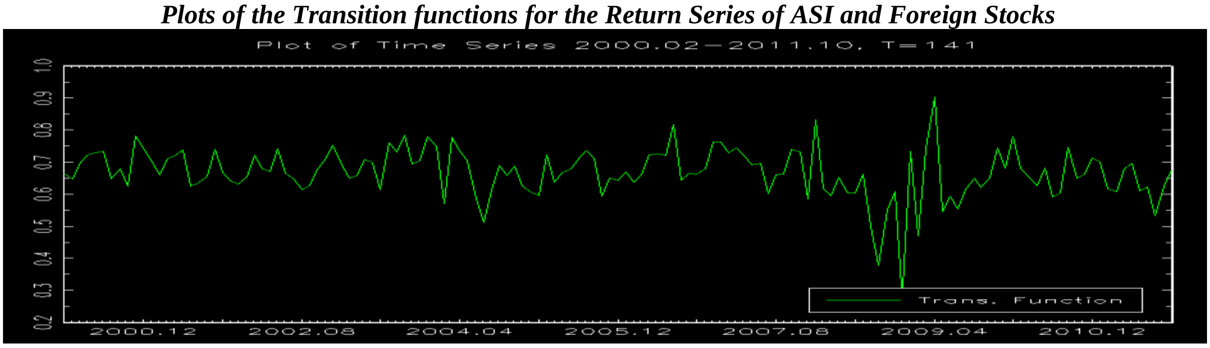

Figure 3a: ASI

Figure 3b: S&P500

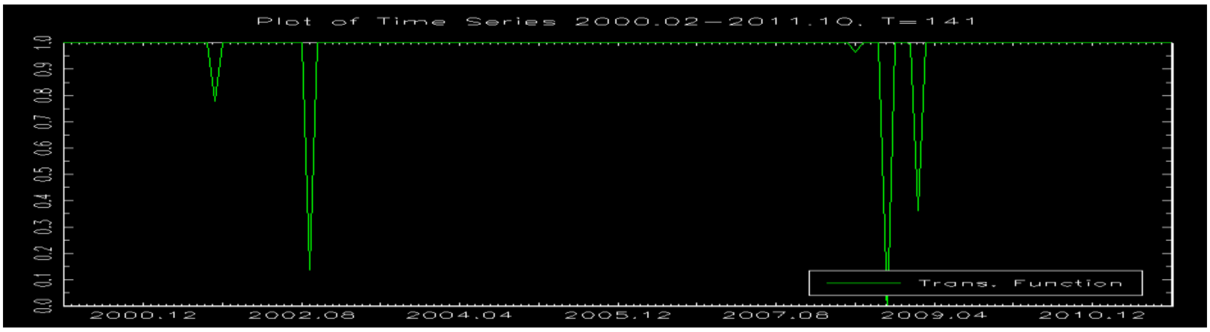

Figure 3c: DOW JONES

Figure 3d: NASDAQ

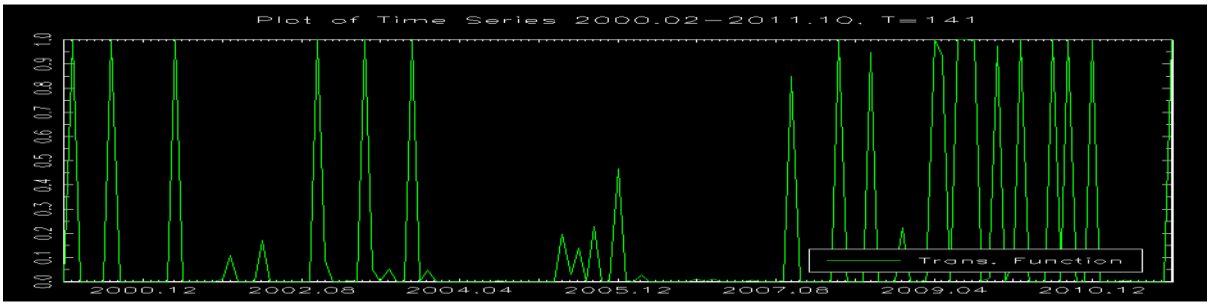

Figure 3e: FTSE 100

Figure 3f: DAX

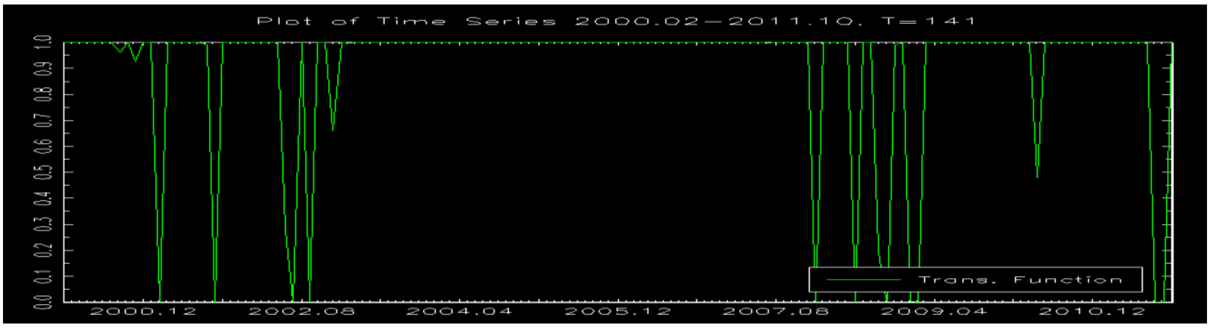

Figure 3g: CAC 40

Figure 3h: NIKKEI 225

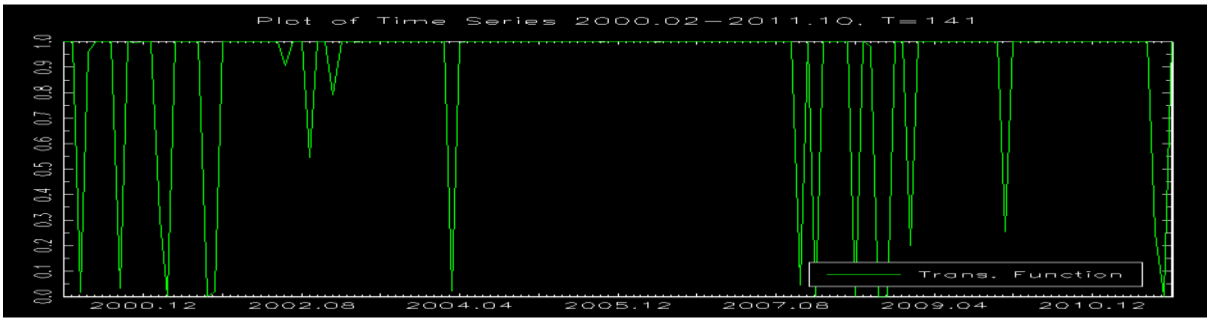

Figure 3i: HANG SENG

Figure 3j: STI

4. Concluding remarks

This study considers the Nigerian stock market with other foreign markets in Europe, America and Asian. We try to explore the possibility of the two market phases, 'bull' and 'bear' in the stocks data. The LSTM, which is an adaptation of LSTAR model of Teräsvirta (1994) is used. We obtain results which strongly confirms the existence of the two phases of markets: 'bull' and 'bear' in our data. The Peak period detected in the model coincides with the period of global financial crisis and this is uniform in all the stocks including Nigeria All Share Index. In the LSTM models, the estimates of betas vary from stock to stock, even stocks in the same country. Though, we have only ASI for Nigeria but the value obtained is still very different from others. Stocks returns are found to stay longer in the up-market than in the down-market, therefore it is riskier for investors to keep portfolios when the market is at bull phase.

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