

On Restricted Least Squares: the Cobb-Douglas Production Function for the Nigerian Economy

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Abstract
The Cobb-Douglas production function ($Y = AL^{\beta}K^{\theta}$) is still today the most ubiquitous form in theoretical and empirical analyses of growth and productivity *(Felipe and Adams, 2005)*. This paper examines the Nigerian economy from 1990 to 2009 and uses the "F-test" approach of Restricted Least Square (RLS) to find out if the linear restriction of Cobb-Douglas production function's parameters $(\beta + \theta) = 1$ is significant to Nigeria economy. Using data on Nigeria's Gross Domestic Product (GDP), Capital and total labour force, it was found that the Nigerian economy is probably characterized by constant returns to scale over the sample period and therefore, using the restricted regression as stipulated by Cobb-Douglas function may not be misleading. Hence, if Capital/Labour ratio increased by 1 percent, on average, labour productivity went up by about 1 percent.
Keywords: Regression, Restricted Least Square, Cobb-Douglas, Returns to Scale

1. Introduction

In economic, the Cobb-Douglas functional form of production is widely used to represent the relationship of output and two inputs. It was developed and tested against statistical evidence by Charles Cobb and Paul Douglas during 1900 – 1947, (*Douglas, 1976)*. The origin of the Cobb-Douglas form is dated back to the original work of *(Cobb and Douglas, 1928)*, who used data for the U.S. manufacturing sector for 1899 – 1922. Although, researchers like *(Brown, 1966)*; *(Sandelin, 1976)*; and *(Samuelson, 1979)* had indicated that the credit should have been given to Wicksell for its discovery since he started working on it in the 19th century. *(Cobb and Douglas, 1928)* were not the first economists to use the functional form named after them. *(Weber, 1998)* notes that Wicksell employed the Cobb-Douglas functional form in production analysis, twenty years earlier. Weber also notes that Pareto used the Cobb-Douglas functional form to represent Utility in 1892. The estimation of the parameters of aggregate production functions is central to many of today's works on growth, technological change, productivity, and labour. Empirical estimates of aggregate production functions are essential tools of analysis in macroeconomics, and important theoretical constructs, such as potential output, technical change, or the demand for labour, are based on them, *(Felipe and Adams, 2005)*.

In its most standard form for production of a single good (output) with two factors (inputs), the Cobb-Douglas function is: $\mathbf{Y = AL^{\beta}K^{\theta}}$

Y = total production (like the monetary value of all goods in a year)
A = Total factor productivity
L = Labour input
K = Capital input
β and **θ** = output elasticities of Labour and Capital respectively

Cobb and Douglas were influenced by statistical evidence that applied to show that labour and capital shares of total output were constant over time in developed countries. They explained this by statistically fitting least squares regression of their production function, *(Gujarati and Sangeetha, 2007)*.

In stochastic form, Cobb-Douglas function becomes:

$$Y_i = \alpha X_{1i}^{\beta} X_{2i}^{\theta} e^{u_i} \quad \textbf{(i)}$$

Y = output of a production process (like GDP of a country)
X_{1i} = input 1 (one of the inputs that contributes to the output Y e.g. labour)
X_{2i} = input 2 (the second input that contributes to the output Y e.g. capital)
u = stochastic disturbance term
e = base of natural logarithm

2. Properties of Cobb-Douglas Production Function

- **β** is the (partial) elasticity of output with respect to the labour input, that is, it measures the percentage change in output for, say, a 1% change in the labour input, holding the capital input constant.
- **θ** is the (partial) elasticity of output with respect to the capital input, holding the labour input constant.
- The sum (**β + θ**) gives information about the returns to scale, that is, the response of output to a proportionate change in the inputs.

- If the sum is 1, then, there are constant returns to scale, that is, doubling the inputs will double the output, tripling the input will triple the output, and so on.
- If the sum is less than 1, there are decreasing returns to scale – doubling the inputs will give less double the output.
- If the sum is greater than 1, there are increasing returns to scale – doubling the inputs will give more double of the output.

Output elasticity measures the responsiveness of output to a change in levels of either labour or capital used in production, ceteris paribus. If $(\beta + \theta) = 1$, the production function has a constant returns to scale (i.e. the production output increases by a constant factor). This implies that doubling capital (K) and labour (L) will yield double output (Y). If $(\beta + \theta) < 1$, returns to scale are decreasing, and if $(\beta + \theta) > 1$, returns to scale are increasing.

3. *Literature Review*

(Elias, 1992) in *(Gujarati and Sangeetha, 2007)* used Restricted Least Squares to observe Mexican Economy from 1955 – 1974. Using data on the country's Gross Domestic Product, Employment (labour), and Fixed capital, they found that Mexican economy was probably characterized by constant returns to scale over the sampled period and concluded that using Restricted Least Squares obtained for the data set could not be harmful. They also observed that increasing capital/employment ratio by 1 percent, on the average will increase the labour productivity by about 1 percent.

(Effiong and Umoh, 2010) estimated the profit efficiency and the relevant indices determining efficiency levels for egg-laying industry in Akwa-Ibom State, Nigeria utilizing Cobb-Douglas production function based on stochastic profit frontier. With the aid of a structured questionnaire, they collected primary data from sixty poultry farms across the six agricultural zones of the state. Empirical results revealed the mean economic efficiency of 65.00% implying the need for increased resource use efficiency Their results further showed that variable inputs such as price of feeds, price of drugs and medication were statistically significant thus indicating that profit decreased with increase in input prices while fixed inputs such as capital inputs and farm size were statistically significant and had the right sign a-priori indicating that profit increased with increase in the level of its utilization. Their study suggested that policy that would enhance extension services, encourage membership in cooperative farming and enhanced good and adequate utilization of improved livestock inputs should be put in place.

(Abidemi, 2010) examines productivity in the banking sector by way of estimating two major production functions known in the economic literature. The result obtained from the Ordinary Least Square (OLS) estimates shows that substitution parameters α and β (substitution parameters for capital and labour, respectively) confirms the a priori expectation that the duo of α and β are positive values of less than one. He found that the addition of the values of α and β is greater than one, which indicates that as the banking sector doubles its inputs in terms of capital and labour, the output in terms of deposit will be more than doubled. He also observes that the substitution parameters in the Constant Elasticity of Substitution Production Function were equally positive, which supports the theory. In his final analysis, his study supports economic theory on the specification of both Cobb-Douglas and Constant Elasticity of Substitution production functions.

In *(Abiola, 2010), (Douglas, 1934) and (Solow, 1957)* in their study on capital-labour substitution concluded that labour is the single most important factor of production in a certain subtle sense. Both labour and capital are needed in production: they opined that when capital is taken away, or alternatively all labour and negligible total products will be left with. But they found that a one percent increase in labour seems to increase output about three times as much as would a one percent increase in capital. This largely corresponds with the widely known fact that wages are about three-fourth of the share of property incomes.

(Liedholm, 1964) was perhaps the first work to be done on productivity in Nigeria. An attempt was made at finding out between labour and capital, which input contributed more to the output of major industries in Eastern Nigeria and it was found that labours' contribution to the output of the selected manufacturing industries was larger than that of capital. This position was confirmed by *(Osagie and Odaro, 1975)*.

(Ekanem, 2003) provides estimates of Total Factor Productivity for the banking industry in Nigeria for the period 1986 – 2000. The methodology in the work involved the use of the Growth Accounting Model based on

aggregate production functions. In the study, the most appropriate production function that describes the production process of the industry in Nigeria was found to be the Cobb- Douglas. The parameters of the estimated Cobb-Douglas function were used to calibrate the Growth Accounting Model. The results showed that measured aggregate output grew at an average annual rate of 4.29%, while Total Factor Productivity grew at an average annual rate of 3.33%. The study analyzed that TFP provided 78% of the recorded growth in the industry during this period.

(Ekanem and Oyefusi, 2000) estimated the Cobb-Douglas and the Constant Elasticity of Substitution (CES) production functions for the manufacturing industry in Nigeria for the period 1980 – 1997, taking into consideration the phenomenon of idle capacity that has characterized the industry at the time. The results of the models when compared with the work of *(Liedholm, 1964)* and *(Osagie and Odaro, 1975)* gave satisfactory results in terms of goodness of fit. Of the two production functions estimated, the Cobb-Douglas Production Function performs better considering all the relevant econometric test criteria. This then showed that the Cobb-Douglas Production Function gives a better explanation of the aggregate production process in the manufacturing industry in Nigeria for the period studied.

4. Materials and Method

The dataset used for this research work are Real Gross Domestic Product and Capital expenditure has obtained from Central Bank of Nigeria's Statistical Bulletin, 2011 and the total labour force of Nigeria 1990 – 2009 obtained from World Bank national accounts data, and OECD National Accounts data files.

Written (i) in log form, the equation becomes:

$$\ln Y_i = \ln \alpha + \beta \ln X_{1i} + \theta \ln X_{2i} + u_i \quad \textbf{(ii)}$$

$$\ln Y_i = \varphi + \beta \ln X_{1i} + \theta \ln X_{2i} + u_i \ (\ln \alpha = \varphi) \quad \textbf{(iii)}$$

If there are constant returns to scale (equiproportional change in output for an equiproportional change in the input), economic theory suggests that β + θ = 1. This is a form of linear equality restriction. In order to examine the validity of (ii), there are two possible approaches.

The t-Test Approach (The unrestricted/unconstrained Regression)

This procedure involves estimating parameters of (iii) without considering the linear equality restriction (β + θ = 1) explicitly by using Ordinary Least Squares (OLS) method. Having estimated β and θ, a test of hypothesis/restriction is conducted using t-test where the test statistic is given by:

$$t_0 = \frac{(\hat{\beta}+\hat{\theta})-1}{\sqrt{Var(\hat{\beta})+Var(\hat{\theta})+2\,Cov(\hat{\beta}\hat{\theta})}} \quad \textbf{(iv)}$$

The test statistic is compared with $t_{\frac{\alpha}{2}}(n-k)$ where α is the level of significance, n is the number of observation and k is the number of estimated parameters in (iii) and appropriate decision is taken on the set hypothesis $\{H_0: (\beta+\theta) = 1\}$

The F-Test Approach (Restricted Least Square, RLS)

The t-test approach is post-mortem in that it only finds out whether the linear restriction is satisfied after estimating the unrestricted regression. However, the F-test approach incorporates the linear equality restriction into the estimating procedure at the outset.

Since β + θ = 1, hence β = 1 – θ. Substituting (1 – θ) for β in (iii) gives:

$$\ln Y_i = \varphi + (1-\theta)\ln X_{1i} + \theta \ln X_{2i} + u_i$$

$$\ln Y_i = \varphi + \ln X_{1i} - \theta \ln X_{1i} + \theta \ln X_{2i} + u_i$$

$$(\ln Y_i - \ln X_{1i}) = \varphi + \theta\,(\ln X_{2i} - \ln X_{1i}) + u_i$$

$$\ln(Y_i/X_{1i}) = \varphi + \theta \ln(X_{2i}/X_{1i}) + u_i \quad \textbf{(v)}$$

Variables (Y_i/X_{1i}) and (X_{2i}/X_{1i}) are of great economic importance since (Y_i/X_{1i}) measures the ratio of the output with one of the inputs and (X_{2i}/X_{1i}) compares one of the input variables with the other. Once θ can be estimated from (v), β can be estimated using β = 1 – θ.

The overall implication of this is that the procedure ensures that sum of the estimated coefficients of the two inputs equal 1 and hence, the name "Restricted Least Squares, RLS". The generalization of this procedure into more than one linear equality restriction is explained by *(Theil, 1971).*

Examining the validity of RLS

The validity of the Restricted Least Squares can be examined by applying F-test where

$$F_{cal} = \frac{(SSE_R - SSEg_{UR})/a}{SSE_{UR}/(n-k)} \quad \textbf{(vi)}$$

SSE_R is the sum of squares of error for restricted regression in (v)
SSE_{UR} is the sum of squares of error for unrestricted regression in (iii)
a is number of linear restriction (a = 1 in Cobb-Douglas function)
k is the number of estimated parameters in (iii)
n is the number of observations
The F_{cal} is compared with F_{table} with **[*a, (n – k)*]** degree of freedom at specified level of significance with the null hypothesis, H_0: The Restricted Least Squares (RLS) is not significant. If the RLS is not significant, the economy is probably characterized by constant returns to scale over the sampled period and hence, there may be no harm in using the restricted regression in (v).

5. Results and Discussion

Fitting the Cobb-Douglas production function to the data on Nigeria's Gross Domestic Product (GDP), Capital and total labour force from 1990 to 2009 gives:

$$\ln \widehat{GDP}_t = -129.439208 + 8.095468 \ln Labour_t + 0.253359 \ln Capital_t \quad \textbf{(vii)}$$

t =	*(-7.609275)*	*(7.644071)*	*(1.951656)*
P-value =	*(0.000001)*	*(0.000001)*	*(0.067654)*
Std Err. =	*(17.010715)*	*(1.059052)*	*(0.129817)*

$R^2 = 0.978207$ $^*RSS_{UR} = 38.525827$ $^*SSE_{UR} = 0.858280$

The equation above (vii) shows that the output/labour elasticity is about 8.10 and the output/capital elasticity is about 0.25. if we add these coefficients, we obtain 8.36, indicating that the Nigeria economy has a very high increasing returns to scale. As high as this value, it is very essential to subject this finding to statistical hypothesis. Imposing the restriction of constant returns to scale gives the following result:

$$\ln(\widehat{GDP/Labour})_t = 3.785507 + 1.190436 \ln(Capital/Labour)_t \quad \textbf{(viii)}$$

t =	*(6.845303)*	*(11.321751)*
P-value =	*(0.000002)*	*(0.000000)*
Std Err. =	*(0.553008)*	*(0.105146)*

$R^2 = 0.876866$ $\hat{}RSS_R = 28.169922$ $\hat{}SSE_R = 3.955770$

* UR → Unrestricted Regression ^ → Restricted Regression
Since the independent variables in (vii) and (viii) differ, F_{cal} in (vi) is used to obtain F-value

$$F_{cal} = \frac{(SSE_R - SSE_{UR})/a}{SSE_{UR}/(n-k)} = \frac{(3.955770 - 0.858280)/1}{0.858280/17} = 3.60895$$

The F_{cal} follows the *F-distribution* with (1, 17) degree of freedom. Hence, F_{cal} is significant at both 1% (8.40) and 5% (4.45) level.

6. Conclusion

The conclusion then is that the Nigerian economy is probably characterized by constant returns to scale over the sample period and therefore, using the restricted regression as stipulated by Cobb-Douglas function may not be harmful. Hence, if Capital/Labour ratio increased by 1 percent, on average, labour productivity went up by about 1 percent over the sampled period.

7. Policy Implications

This study examined Cobb-Douglas coefficient's restriction on Nigeria economy from 1990 to 2009 by way of estimating the mostly utilized production functions in the economics literature. The result obtained from the OLS estimates shows that substitution parameters, β and θ do not support economic theory of the duo being positive values of less than one. The addition of the values of β and θ is greater than one which suggests that as the Nigeria economy doubles its inputs in terms of labour and capital, the output in terms of GDP will be more than doubled but a test of significance revealed otherwise. The study supports economic theory in the specification of the Cobb-Douglas and production functions in researching into inputs/output relationship. The study, therefore, recommends for Nigerian government and those involve in decision making that for desired increase productivity in terms of output like the GDP, more units of both labour and capital should be employed.

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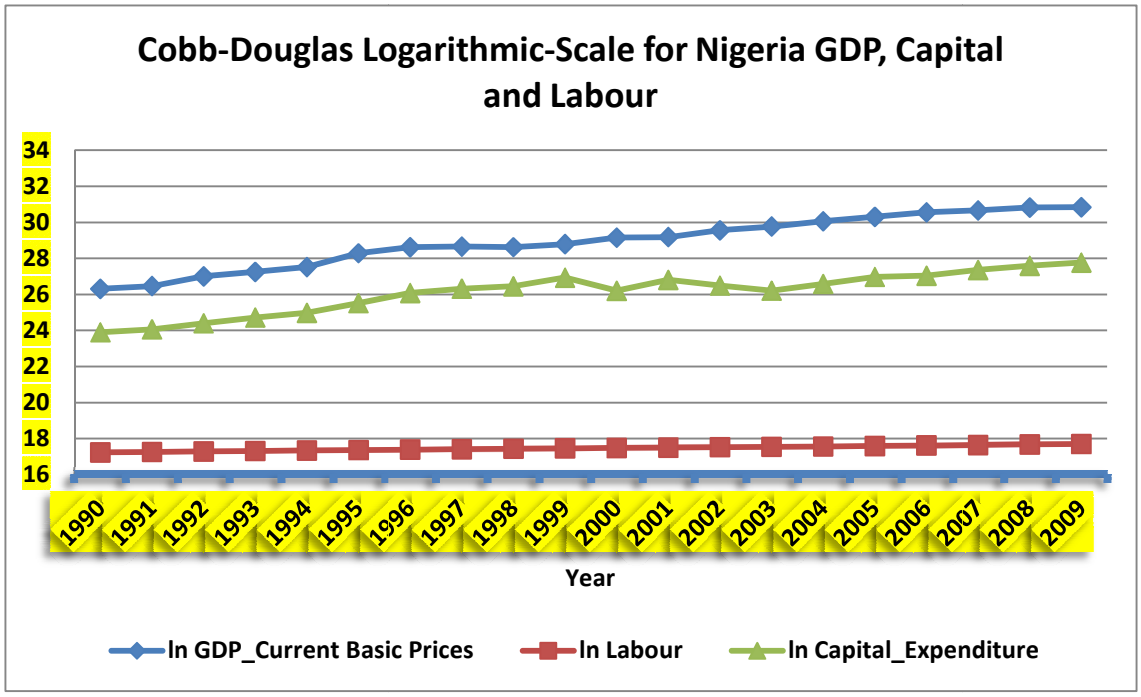

Figure 1

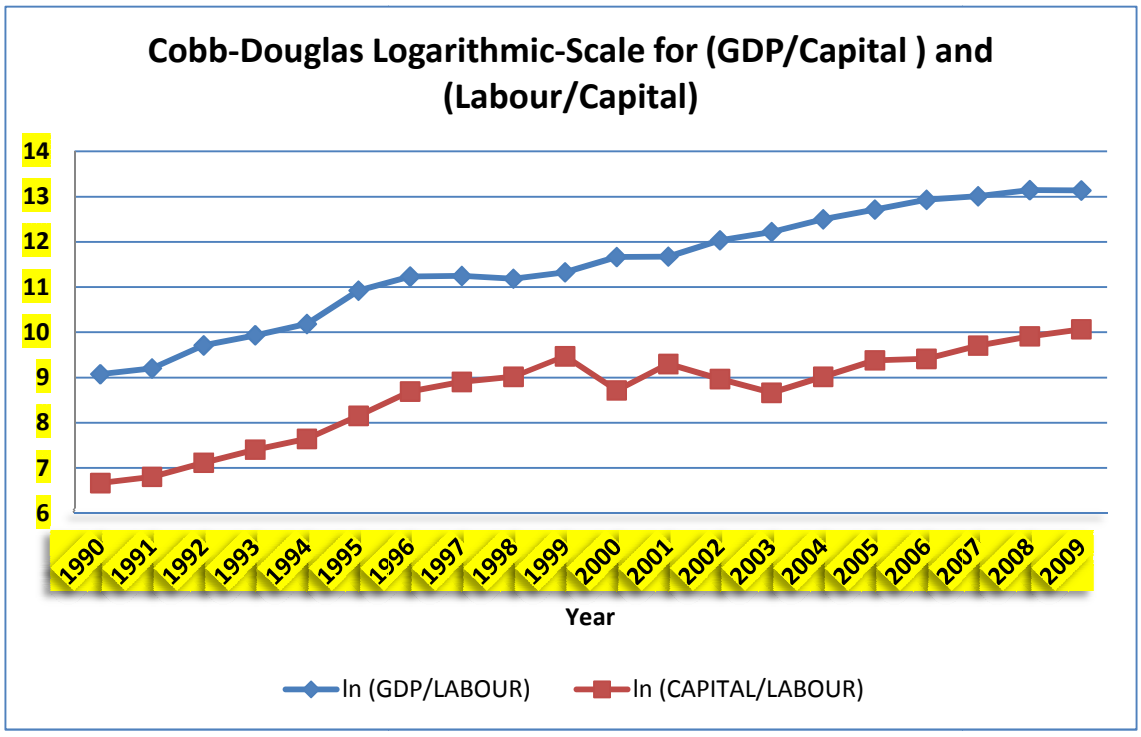

Figure 2

Figures 1 and 2 above show upward movements for the log transformation of all the variables under consideration.