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Finite Time Stability of Linear Control Systems with Multiple Delays

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Abstracts

In this paper, we considered finite time stability of a class of linear control systems. By using suitable matrix measures and Coppel's inequality a bound for the solution of the linear control system with multiple delays in the state is determined. A sufficient delay dependent conditions for finite time stability of linear control system with delay are derived.

Key words: Linear systems, control systems, stability, time delay, Coppel's inequality

1. Introduction

The investigation of time delay systems has been carried out by many researchers over the years. It has been shown in several studies that time delays and external disturbances are unavoidable in many practical control systems such as hydraulic systems, long transmission lines, aircraft, manufacturing and process control systems, etc. and it is a source of instability (Zavarie, and Jamshidi, 1987).

The question of stability of linear and nonlinear control systems are of particular interest to researchers. In existing literature there are mainly two approaches in determining stability criteria, namely delay-independent criteria and delay-dependent criteria. Delay independent stability criteria provides conditions for stability for arbitrary large delays, while delay dependent conditions take into account the size of the maximum delays that can be tolerated by the system (Niculescu et al., 1997).

Classical stability concepts such as Lyapunov stability, BIBO stability, etc. deals with the behavior of a system within an infinite time interval. These concepts require that the state variable be bounded, but the values of the bounds are not prescribed. In practice one is not only interested in system stability, but also in bounds of system trajectories. A system could be stable but still completely useless if it possesses undesirable transient performance. Thus it may be useful to consider the stability of such systems with respect to certain subset of the state space which are defined a priori in a given problem. Besides that, it is of particular significance to consider the behavior of dynamical system only over a finite time interval. For this purpose, the concept of finite time stability and practical stability has been used.

The concept of finite-time stability was first introduced in the sixties (Dorato, 1961) . A system is said to be finite-time stable if, given a bound on the initial conditions and a specified time interval, its state does not exceed a certain bound during this time interval. The concept of finite time stability studies the behavior of the system within a finite interval, and requires the convergence of the solution in the specified finite time interval.

To verify the finite time stability of systems, several techniques has been introduced. (Amato et al. 2003) obtained necessary and sufficient conditions for the finite time stability and finite time boundedness of linear systems subject to exogenous disturbance by means of operator theory.

To solve the problem of finite time and practical stability of a class of linear continuous time delay system, Lyapunov like method have been used in (Debeljkovi et al., 2014; Debeljkovic et al., 2013).

Further in (Debeljkovic et al. 1997a) matrix measure approach is used to establish sufficient condition for the stability of linear dynamic systems over finite time interval. (Debeljkovi et al., 1997b; Kablar and Debeljkovic,

1998) used the Coppel's inequalities and matrix measures to investigate finite time stability of singular systems operating under perturbing forces.

Here we examine the problem of finite time stability for a class of linear multiple delay control system and presents sufficient conditions that enable system trajectories to stay within the a priori given sets.

2. Preliminary

Consider the following control system

$$\dot{x}(t) = A_0 x(t) + \sum_{i=1}^{k} A_i x(t - \tau_i) + Bu(t) \qquad (2.1a)$$

$$x(t) = \varphi(t) \qquad t \in [-\tau, 0] \qquad (2.1b)$$

Where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input control vector, $A_0, A_i \in R^{n \times n}$, $B \in R^m$ are constant system matrices, $0 < \tau_1, < \tau_2 <, \ldots < \tau_k$ are constant time delays and $\tau = \max\{\tau_1, \tau_2, \ldots \tau_k\}$

$\varphi \in C([-\tau, 0], R^n)$ is an admissible initial state and $C([-\tau, 0], R^n)$ is a Banach space of continuous function mapping the interval $[-\tau, 0]$ into R^n which converges uniformly. The system behavior is defined over the time interval $I = [0, T]$, where T is a positive number.

For the time invariant sets $S_{(\ )}$, used as bounds of the system trajectories are assumed to be bounded, open and connected. Let S_β be a given set of all allowable states of the system for $\forall t \in I$. Let S_α be the set of all initial states of the system such that $S_\alpha \subseteq S_\beta$ and S_γ denote the set of all allowable control actions. The sets S_α and S_β are connected and a priori known.

Let $\sigma_{\max}(.)$ be the largest singular value of matrix A and

$$\sigma_1 = \max_{1 \le i \le k}\{\sigma_{\max}(A_i)\}, \ \sigma = \max\{\sigma_{\max}(A_0), \sigma_1\} \quad \Rightarrow \|A_i\| < \sigma \ \forall i = 1,2,\ldots,k \qquad (2.2)$$

$$b = \sigma_{\max}(B) \qquad (2.3)$$

Before proceeding further, we will introduce the following definitions and theorems which will be used in the next section.

Matrix measures have been extensively studied in (Desoer, and Vidyasagar, 1975;Hu and Liu, 2004) and it is used to estimate upper bounds of matrix exponential. The following theorem relates an upper bound of a matrix exponentials to its matrix measures.

Theorem 2.1: (Desoer, and Vidyasagar, 1975; Hu and Mitsui, 2012) For any matrix $A \in R^{n \times n}$ the estimate

$$\|\exp(A(t))\| \le \exp(\mu(A)(t))$$

holds.

Theorem 2.2: The matrix norm or Lozinskii logarithm norm of a $n \times n$ matrix A is

$$\mu(A) = \lim_{h \to 0} \frac{|I + hA| - I}{h}$$

Where $\|(.)\|$ is any matrix norm compatible with some vector norm $|x|_{(.)}$. The matrix measure define in theorem 2.2 has three variants depending on the norm utilized in the definition.

It is assumed that the usual smoothness condition is satisfied by system (2.1) so that there will be no difficulty with the question of existence, uniqueness and continuity of solutions with respect to initial data.

Before stating our results, we introduce the concept of finite-time stability for time-delay system (2.1). This concept can be formalized through the following definition.

Definition 2.1: Time delayed control system is finite time stable with respect to $\{S_\alpha, S_\beta, T, \|(.)\|, \mu(A_0) \neq 0\}$, $\alpha < \beta$ if and only if :

$$\varphi(t) \in S_\alpha,\ \forall t \in [-\tau, 0] \text{ and } u(t) \in S_\gamma, \forall t \in T$$

implies

$$x(t : t_0, x_0) \in S_\beta, \forall t \in [0, T]$$

See (Debeljkovi, et al., 2001; Lazarevic, and Debeljkovi, 2005)

3. Main result

Definition: System (2.1) satisfying the initial condition is finite time stable w.r.t. $\{t_0, T, \alpha, \beta, \gamma, \tau\}$, $\alpha < \beta$ if and only if:

$$\|\varphi(t)\| < \alpha, \|u(t)\| < \gamma,\ \forall t \in T \tag{3.1}$$

implies

$$\|x(t)\| < \beta, \forall t \in T \tag{3.2}$$

Theorem: System (2.1) with initial condition is said to be finite time stable with respect to $\{\alpha, \beta, \gamma, \tau, T, \mu(A_0) \neq 0\}$, if the following condition is satisfied:

$$e^{\mu(A_0)t} < \frac{\beta / \alpha}{1 + \mu^{-1}(A_0)(k\sigma + b\gamma)\left(1 - e^{-\mu(A_0)\tau}\right)},\ \forall t \in [0, T] \tag{3.3}$$

Where $\|(.)\|$ denotes the Euclidean norm.

Proof: Using variation of parameter, the solution of system (2.1) can be expressed in terms of matrix exponential as

$$x(t) = e^{A_0(t)}\varphi(t) + \int_{-\tau}^{0} e^{A_0(t)}\left(\sum_{i=1}^{k} A_i(t)x(s-\tau_i) + Bu(s)\right)ds \qquad (3.4)$$

Evaluating the norm of both sides of equation (3.4) and using theorem 2.1 yields

$$\|x(t)\| \le e^{\mu(A_0)t}\|\varphi(0)\| + \int_{-\tau}^{0} e^{\mu(A_0)(t-s)}\left(\sum_{i=1}^{k}\|A_i\|\|x(s-\tau_i)\| + \|B\|\|u(s)\|\right)ds$$

Applying conditions (2.2) and (2.3) we obtain

$$\|x(t)\| \le e^{\mu(A_0)t}\|\varphi(0)\| + \int_{-\tau}^{0} e^{\mu(A_0)(t-s)}\left(\sum_{i=1}^{k}\sigma\|x(s-\tau_i)\| + b\|u(s)\|\right)ds$$

Using condition (3.1)

$$\|x(t)\| \le e^{\mu(A_0)t}\|\varphi(0)\| + \int_{-\tau}^{0} e^{\mu(A_0)(t-s)}\left(k\sigma\|x(s)\| + b\gamma\right)ds$$

$$\le e^{\mu(A_0)t}\|\varphi(0)\|\left(1 + (k\sigma + b\gamma)\int_{-\tau}^{0} e^{-\mu(A_0)s}\,ds\right)$$

This yields

$$\|x(t)\| \le \alpha e^{\mu(A_0)t}\left(1 + \mu^{-1}(A_0)(k\sigma + b\gamma)\left(1 - e^{-\mu(A_0)\tau}\right)\right)$$

Finally applying the basic condition of the main theorem, equation (3.3) on the preceding inequality, it yields:

$$\|x(t)\| < \beta,\ \forall t \in [0, T]$$

which completes the proof.

Conclusions

Matrices has been used to provide bounds on systems trajectories. In this paper using suitable matrix measures and Coppel's inequality, a non-Lyapunov stability criteria for the solution of linear control system with multiple delays in the state is determined. A sufficient delay dependent conditions for finite time stability of linear control system with delay are derived, which extends some basic results.

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