Mathematical Model Describing Effect of S.Cerella on Paddy in the Environmental Ecosystem

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Abstract
In this paper we will establish a mathematical model with the help of second order differential equations by assuming that the rate of change of percentage germination loss with respect to grain infestation is proportional to germination loss. We have selected entomological data for our problem to be in the paper. The model is useful for better management of storage of grains and it will reduce the storage losses of grains under the consideration of various parameters like moisture, humidity etc.

Keywords: Germination loss, Infestation loss, Grains, Storage system etc.
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1. Introduction: In agriculture departments of various universities of World, where agricultural researches are going on, plentiful statistics are available to provide a basis for various analytical study. As a matter of fact, they are not of as much use as it ought to be because they are fragmentary and inconclusive. For this model we have gone through the research work of [1] Kapoor, J.N, [2] Pandey, V, [3] Prakash, A, [4] Singh R.A and Rizvi. S.M, [5] Singh V, [6] Arsmo M. O et al. Through mathematical models we give them systematic order to gain better insight into the problem for data has been collected, so that data may prove more useful for agricultural production. To have a clear and complete picture and the results of various problems consideration data are required for the formation of mathematical models. The storage condition which gives the validity and vigour of the seeds and various type of losses as a result of physiological and biochemical changes, differs considerably under the different storage system. In stores, grains absorb moisture because of humidity and get infected, resulting into development of insect population. Besides, these are perspiration or metabolic changes, depletion of foods reserves such as carbohydrates, proteins and oil, genetic changes and changes in membrane permeability. All these factors cause huge losses during storage. Therefore, it seems essential to study the effects of these factors and establish their relationship through modelling for better management and forecasting.

Due to high humidity harvested and processed paddy retains high moisture content, which predisposes grains for considerable storage losses every year. Paddy in storage is damaged by a dozen insects but only a few viz. angoumsis grain moth, Sitotroga Cerealella entomological etc are most harmful insects to damage grains. Pest status of an insect in rice storage period and local climatic conditions. A biotic environment like the grain moisture, humidity temperature, light and type of storage structure mentioned earlier and the storage losses due to insects. Losses may be quantitative and qualitative, i.e. reduced germination and contamination with mycotoxins. We have discussed some factors affecting the storage ecosystem, like atmosphere humidity and grain moisture content, temperature, light, season food etc.

We will establish a Mathematical model with the help of second order differential equations by assuming that the rate of change of percentage germination loss with respect to grain infestation is proportional to germination loss.

2. Formulation of the model
We conducted an experiments at Achrya Narendra Dev Agriculture University, crop research station, Faizabad U.P. we have selected twenty five grams of seed, each of 10 selected varieties, comprising of 5 grain types were kept in 100*25 mm size preterlised glass specimen tubes, were placed inside the desiciotor with some adult moth and performed the whole experiment under different environment conditions for different periods. We have extracted statistics from the experiments to develop mathematical models by considering two varieties viz. percentage grain infestation and percentage loss in germination.

Losses in grains weight and germination of rice varieties infested by S.Cerelella of 3, 6, and 9 months of duration.
## Table 1

<table>
<thead>
<tr>
<th>% Grain infestation</th>
<th>% Loss in germination</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 months</td>
<td>6 months</td>
</tr>
<tr>
<td>11.5</td>
<td>14.9</td>
</tr>
<tr>
<td>4.6</td>
<td>7.3</td>
</tr>
<tr>
<td>13.7</td>
<td>16.1</td>
</tr>
<tr>
<td>11.5</td>
<td>12.1</td>
</tr>
<tr>
<td>3.3</td>
<td>5.4</td>
</tr>
<tr>
<td>6.5</td>
<td>7.3</td>
</tr>
<tr>
<td>9.9</td>
<td>10.5</td>
</tr>
<tr>
<td>10.5</td>
<td>10.5</td>
</tr>
<tr>
<td>10.2</td>
<td>14.4</td>
</tr>
<tr>
<td>15.5</td>
<td>17.7</td>
</tr>
</tbody>
</table>

**Figure-1(a)**

% loss in germination with respect to % grain infestation

3 Months

**Figure-1(b)**

% loss in germination with respect to % grain infestation

6 Months
From figures, it evinces that percentage loss in germination value is behaving in oscillatory manner with respect to (kernel) percentage grain infestation. Therefore, it is postulated that percentage germination loss with respect to percentage grain infestation obeys a law where solution shows the oscillatory nature. Before trying any law to mould this situation we first idealise the experimental data to the extent that the idealised data is included most of the original data and some averaged data. The idealisation is required for the reason as follows:

To curve in figure is idealised to give a curve of frequency which shows the nature of oscillation. To obtain observation Percentage loss in germination $X_i$ are averaged at averaged kernel (grain infestation) for further use.

<table>
<thead>
<tr>
<th>Kernel infestation</th>
<th>Germination percent loss</th>
<th>Kernel infestation</th>
<th>Germination percent loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>75.9</td>
<td>20.6</td>
<td>16</td>
<td>21</td>
</tr>
<tr>
<td>7.4</td>
<td>22.0</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>17.2</td>
<td>26.9</td>
<td>17</td>
<td>27</td>
</tr>
<tr>
<td>18.1</td>
<td>24.1</td>
<td>19</td>
<td>28</td>
</tr>
<tr>
<td>6.1</td>
<td>22.0</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>7.2</td>
<td>11.6</td>
<td>7</td>
<td>17</td>
</tr>
<tr>
<td>10.1</td>
<td>14.8</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>11.4</td>
<td>14.4</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>14.1</td>
<td>20.7</td>
<td>14</td>
<td>21</td>
</tr>
<tr>
<td>18.9</td>
<td>26.8</td>
<td>19</td>
<td>27</td>
</tr>
</tbody>
</table>

Also we have omitted one data of germination losses $X_i$ at grain infestation $Y_i$. Simply because they are not behaving according to the general trends of the other data, may be due to some other variables presented at the time of recording the data but they are not present here. Other observations of the table are as the same five in table 2. According to the follow up of the idealised data and corresponding curve.

It is postulated that the rate of percentage germination loss ‘$Z$’ with respect to grain infestation ‘$I$’ is proportional to the Z i.e.

$$\frac{d^2Z}{dI^2} \propto Z$$

Or,

$$\frac{d^2Z}{dI^2} = -\omega^2 Z \quad \text{(1)}$$

Where $\omega^2$ is constant of proportionality.

2.1 Discussion and solution:
Solution of equation (1) is given by
\[ Z = C_1 \cos \omega I + C_2 \sin \omega I \tag{2} \]

Where \( C_1 \) and \( C_2 \) are arbitrary constant.

An equivalent expression for the solution of equation (1) is

\[ Z = A \sin (\omega I + \phi) \tag{3} \]

Where \( \sin (\omega I + \phi) = \sin \omega I \cos \phi + \cos \omega I \sin \phi \tag{4} \)

Using equation (4) in equation (3) we arrive at

\[ Z = A \sin \omega I \cos \phi + A \cos \omega I \sin \phi \tag{5} \]

In view of equations (3) and (5) we get,

\[ C_1 = A \sin \phi \tag{6} \]
\[ C_2 = A \cos \phi \tag{7} \]

Which evidently provide us

\[ A^2 = C_1^2 + C_2^2 \quad \text{and} \quad \phi = \tan^{-1} \left( \frac{C_1}{C_2} \right) \]

Where \( A \) is called amplitude of oscillation and the oscillation is \( (\omega I + \phi) \) Phase angle is \( \phi \)

If we start from maximum point i.e. \( \phi = \frac{\pi}{2} \) in equation (5) reduces to

\[ Z = A \cos \omega I \tag{8} \]

Where \( w \) is the circular frequency and equation (6) and (7) reduces to

\[ C_1 = A, C_2 = 0 \]

\( \omega \) is also expressed in terms of period as

\[ \omega = \frac{2\pi}{T} \tag{9} \]

Where \( T \) is the period of oscillation or distance between two points.

Using equation (8) and (9), we get,

\[ p = A \cos \left( \frac{2\pi}{T} \right) \tag{10} \]

We claim that equation (10) in the model representing the data given in table. To justify the claim we proceed to test the model.

Equation (10) is expected to generate data represented by fig (2). It is evident that equation (10) represents a wave equation where the displacement curve is represented with respect to a line of symmetry which we term as a "line of best fit" of the data table, so fitting a line \( y = mx + c \), to the data we obtain the equation.

\[ y = 0.90x + 11.5 \tag{11} \]

the inclination of line is 0.90 which gives remarkable effect i.e. germination value of grains is directly depend upon percent grain infestation.

Let, \( u = x - 10, v = y - 20 \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>u</th>
<th>v</th>
<th>uv</th>
<th>u²</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>21</td>
<td>-6</td>
<td>1</td>
<td>-6</td>
<td>36</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>-3</td>
<td>-10</td>
<td>30</td>
<td>9</td>
</tr>
<tr>
<td>17</td>
<td>27</td>
<td>7</td>
<td>7</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td>19</td>
<td>28</td>
<td>9</td>
<td>8</td>
<td>72</td>
<td>81</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>-4</td>
<td>-8</td>
<td>32</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>17</td>
<td>-3</td>
<td>-3</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>0</td>
<td>-5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>1</td>
<td>-10</td>
<td>-10</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>21</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

\[ \sum u = 16 \quad \sum v = 19 \quad \sum uv = 198 \quad \sum u^2 = 209 \]
\[
x = 10 + \left( \frac{16}{9} \right) = 11.7
\]
\[
y = 20 + \left( \frac{19}{9} \right) = 22.1
\]
\[
b_{yx} = \frac{\left( 9 \times 198 \right) - \left( 16 \times 19 \right)}{\left( 9 \times 209 \right) - \left( 16 \right)^2}
\]
\[
b_{yx} = 0.90
\]

Hence regression line is given by
\[
y = 22.1 = 0.90(x - 11.7)
\]
\[
y = 0.9x + 11.4
\]

3. Conclusion

We conclude from this model that Infestation by S. Cerealella inflicted considerable damage to impact significant varieties in germination between different storage periods. As insect infestation increased progressively with the storage period, germination decreased following more or less similar trend. Insect infestation is dependent upon the number of insect present in the store. Also increase in population of insect depends upon the environmental factors i.e. environmental ecosystem. This mathematical model lays emphasis on the role insect infestation during storage of grains, in reducing germination which proves its efficiency in itself.

4. References: