Variable Conductivity and Partial Slip on Flow over a Nonlinear Radiated Stretching Sheet

S. Sudhakar Reddy[1*]   N. Bhaskar Reddy[2]   K. Rananikanth[3]   M. Krishna Murthy[4]

1.S.V. College of Engineering, Department of Mathematics, Tirupati-517502, A.P., India

2.Department of Mathematics, Sri Venkateswara University, Tirupati 517502, A.P., India

3.Department of Mathematics, Dr. L.H.R.Govt. Degree College, Mylavaram,A.P. India

4.Department of Mathematics, Sri Venkateswara University, Tirupati 517502, A.P., India

Abstract

The heat and mass transfer effects on hydrodynamic slip flow past a non-linear stretching sheet is considered. For improving the thermal moment we also considered the thermal radiation and variable thermal conductivity. The condition of slip is one of the core concepts of the Navier – Stokes theory. The sheet is placed horizontally, so that buoyancy forces are less and negligible and the plate is stretching nonlinearly. The partial velocity slip may occur on the stretching boundary when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions. It has significant in biological and medicinal treatments such as controlling the heart attacks, improving the blood pumping, controlling high pressure etc. The thermal radiation is also significant in various treatments such as endoscopy, radiation treatment cancer, scanning and DNA checkup etc. The governing partial differential equations (PDEs) are transformed to ordinary differential equations (ODEs) by using similarity transformation and solutions are carried out with help of the Runge-Kutta based shooting technique. The effects of various material parameters on the velocity, temperature, concentration of the flow field as well as the skin-friction coefficient, heat and mass transfer rates are inspected in detail. It is found that the effects of slip parameter decelerates the fluid velocity, while raises the fluid temperature.

Keywords: Non-linear stretching sheet; variable conductivity; partial slip; thermal radiation.

Nomenclature:

κ^* is the temperature-dependent thermal conductivity	f is the stream velocity function
μ is the dynamic viscosity,	θ is the temperature
T_w, T_∞ is the fluid and ambient temperatures,	φ is the concentrations
C_w, C_∞ is the fluid and ambient concentrations	Pr is the Prandtl number
μ is the dynamic viscosity	λ is the velocity slip parameter
ρ is the fluid density,	f_w is the dimensionless blowing ($f_w < 0$)/suction parameter ($f_w > 0$)
σ is an electrical conductivity	Re_x is the Reynolds number.
q_r is the radiative flux in the boundary layer.	$f''(0)$ is the skin friction coefficient
$\lambda,$ is the velocity slip	$-\theta''(0)$ is the Nusselt number
S is the suction/injection parameter	$-\varphi''(0)$ is the Sherwood number.

INTRODUCTION

The stretching sheet has real life application in the fields of material processing, biological processing and manufacturing. In addition, there are extensive applications in numerous engineering procedures, such as stretching of plastic films, pumping the blood from various parts, polymer extrusion, glass fiber production, wire drawing, manufacturing of foods, and continuous casting. During the manufacture of these sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. The product with the preferred characteristics rigorously based on the stretching rate. Due to this Sakiadis [1] studied the constant velocity on flow over a solid moving continuous surface. The addition of this theory was deliberate Crane [2] by taking the stretched linearly surface with similarities. However, realistically broadening of elastic sheet may not essentially be linear. Further, the investigators studied the flow over stretching sheet by taking into various aspects of the flow controlling parameters such as continuous stretching sheet Carragher and Crane [3], exponentially continuous stretching sheet Magyari and Keller [4], Stagnation point flow Mahapatra and Gupta [5] and Pop et al. [6], vertical stretching sheet Ishak et al. [7] and Jat and Chaudhary [8]. However, Gupta and Gupta [9] claimed that, realistically, the stretching of a elastic sheet essentially may not be linear. This situation was attractively dealt with by Kumaran and Ramanaiah [10] in their work on boundary layer flow where, possibly for the first time, a common quadratic stretching surface was supposed. Ali [11] explored the thermal boundary layer flow for non-linear surface. Later on, the flow over a nonlinear stretching sheet was deliberated by Vajravelu [12] and Vajravelu and Cannon [13]. Akyildiz et al. [14] considered the velocity $u = cx^n$ at $y = 0$. By considering this

modification provided the solutions by Van Gorder and Vajravelu [15], one can usage this assumption for stretching velocity u = cxⁿ at y = 0 for any values, even non-integral of n > 1.

In the designing of radial diffusers and lunge bearing, the blowing and suction shows a dynamic role. Suction is applied to chemical processes to remove reactants from the body. Similarly, the injection is the extraction of fluid (blood, juices) through a permeable wall and is of great practical interest like polymer fiber coating, film cooling, and coating of wires. Blowing is used to add reactants, prevent corrosion or scaling and reduce the drag and cool the surface. The combination of suction or injection helps to control the body temperature, lungs cleaning and diffusion of concentration (water, ethylene glycol, nanofluids).The suction or injection and free convection on the magnetohydrodynamic flow past an accelerated vertical plate with uniform heat flux were studied by Kafousias and Raptis [16] and Raptis et al. [17]. Gupta and Gupta [18] examined the heat and ass transfer effects with suction/injection. Elbashbashy [19] investigated the heat transfer effects on a stretching surface with suction. Many processes in engineering occur at high temperatures and the full understanding of radiation effect on the rate of heat transfer is necessary in the design of equipment. Thermal radiation effects plays a vital role in governing heat transfer in polymer processing industry wherever the quality of the final manufactured goods depends on the heat regulatory factors to some amount. High temperature plasmas, liquid metal fluids, cooling of nuclear reactors, magnetohydrodynamic (MHD) accelerators, and power generation systems are some major applications of radiative heat transfer from a vertical wall to conductive gray fluids. View into this, the authors [20-25] studied the thermal radiation with various geometries. Hossain and Takhar [26] studied the finite difference approximation to analyze the influence of conduction-thermal radiation interaction on natural convection flow over an isothermal horizontal plate. Abdel-naby et al. [27] studied the thermal radiation on magnetohydrodynamic unsteady free convection over a vertical plate in the presence variable surface temperature. Palani and Kim [28] applied implicit finite difference scheme with Crank-Nicolson method to examine the prominence of Joule heating and viscous dissipation on magnetohydrodynamic flow over an inclined plate.

Partial velocity slip may happen on the stretching boundary when the fluid is particulate such as emulsions, suspensions, controlling the heart attacks, improving the blood pumping, controlling high pressure, foams and polymer solutions. In 1997, Thompson and Troian [29] showed that the slip velocity is interrelated to the slip length, the shear rate at the wall, and a critical shear rate at which the slip length diverges. Further, the fluids that exhibit boundary slip have significant scientific applications such as in the polishing of internal cavities and artificial heart valves. The effect of partial slip boundary condition on nanofluids past stretching surface with constant wall temperature has been considered by Noghrehabadi et al. [30] to extend the work done by Khan and Pop [31]. In addition, Nandeppanavar et al. [32] have tabulated the literature of the first-order slip. Recently, Raju and Sandeep [33] investigated the slip characteristics over a wedge and concluded that slip have tendency to control the heat transport phenomena. The diffusion is also having various applications for example: liquid metals, plasmas, salt water and electrolytes etc. Because of this significance the authors [34-39] investigated the flow characteristics of hydrodynamic with various flow controlling parameters such as (porosity, heat source or sink, non-uniform heat source or sink, partial slip and thermal radiation etc) and various geometries such as sheet, cone shrinking sheet, plate, peristalsis plate and vertical sheets. With this they concluded that the magnetic field have tendency to control the boundary layer.

With help of this background, in the present study, an attempt is made to model the mass transfer and variable thermal conductivity on the flow of a viscous fluid over a non-linearly stretching sheet with partial slip. The governing boundary layer conservation equations, which are parabolic in realistic nature. This are normalized into non-similar form and then solved numerically by Runge-Kutta fourth order scheme along with shooting technique. The effects of few pertinent parameters on the velocity, temperature, concentration as well as the skin friction coefficient, Nusselt and Sherwood numbers are computed and discussed in detail.

MATHEMATICAL FORMULATION:

The graphical model of the problem with coordinate system was displayed in Fig.1. The system deals boundary layer hydrodynamic flow over a nonlinear stretching sheet. In most of the available literature the authors are considered the constant conductivity, whereas in my study we considered the variable conductivity. Let us assume that the speed at a point on the sheet is proportional to the power of its distance from the slit and the boundary layer approximations. In writing the following equations, it is assumed that the external electric field and the electric field due to the polarization of charges are negligible. The stretching velocity $U_w(x)$, the surface temperature and concentration are respectively $T_w(x)$ and $C_w(x)$.The sheet is supposed to be $U_w(x)$ vary non-linearly with the distance x from the leading edge, *i.e.* $U_w(x)=ax^m$, $T_w(x)= bx^s$ and $C_w(x)= cx^p$, where a and b are constants with $a > 0$ and $b > 0$. Under the usual conditions, the governing boundary layer equations of continuity, momentum, energy and concentration are
Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \vartheta\frac{\partial^2 u}{\partial x^2} \tag{2}$$

Energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho C_p}\left(\frac{\partial}{\partial y}\left(\kappa^*\frac{\partial T}{\partial y}\right)\right) - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

Species equation

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

Where u and v are the velocity components in the x and y directions, respectively, κ^*- the temperature-dependent thermal conductivity, μ - the dynamic viscosity, T_w and T_∞ - the fluid and ambient temperatures, respectively and C_w, C_∞ - the fluid and ambient concentrations, respectively. Further μ, ρ, σ and q_r are the dynamic viscosity, fluid density, electrical conductivity and radiative flux in the boundary layer.

The boundary conditions for the velocity, temperature and concentration fields are

$$y = 0 , \quad u = ax^m + L\frac{\partial u}{\partial y} , \quad v = V_w , \quad T = T_w(x) + bx^s , \quad C = C_w(x) + cx^p$$

$$y \to \infty , \quad u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \tag{5}$$

where

$$V_w(x) = -f_w\sqrt{\frac{\vartheta a(m+1)}{2}}x^{\frac{m-1}{2}}$$ Blowing/suction velocity. By using the Roseland approximation the radiative

heat flux q_r is given by

$$q_r = -\frac{4\sigma_s}{3k_e}\frac{\partial T^4}{\partial y} \tag{6}$$

Where σ_s is the Stephen Boltzmann constant and k_e - the mean absorption coefficient. It is noted that by using the Roseland approximation, the current examination is limited to optically thick fluids. If the temperature differences within the flow are sufficiently small, then the equation (6) can be linearized by expanding T^4 into the Taylor series about T_∞, which after neglecting higher order terms takes the form

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

Invoking the equations (6) and (7), the equation (3) can be modified as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho C_p}\frac{\partial}{\partial y}\left(\kappa^*\frac{\partial T}{\partial y}\right) + \frac{16\sigma_s T_\infty^3}{3\rho C_p k_e}\frac{1}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

The variable thermal conductivity is considered to vary linearly with temperature as given below

$$\kappa^* = \kappa(1 + \varepsilon\theta) \tag{9}$$

Where κ is thermal conductivity parameter and ε is a fluid parameter which depends on the nature of the fluid. To get similarity solutions of the equations (2), (4), (8) and (5) subject to the boundary conditions (7), we introduce the following similarity transformations.

$$\eta = \sqrt{\frac{(m-1)a}{2\vartheta}}x^{\frac{m-1}{2}}y, \quad v = -\sqrt{\frac{(m-1)\vartheta a}{2}}x^{\frac{m-1}{2}}\left(\frac{m-1}{m+1}\eta f'(\eta) + f(\eta)\right),$$

$$u = ax^m f'(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \quad \varphi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}, \operatorname{Pr} = \frac{\upsilon}{\alpha}, \operatorname{Re}_x = \frac{ax^{m-1}}{\vartheta},$$

$$Sc = \frac{\upsilon}{D}, Nr = \frac{4\sigma_s T_\infty^3}{kk_e}, \lambda = L\sqrt{\operatorname{Re}_x(1+m)},$$

(10)

Here f, θ and φ- the dimensionless stream function, temperature and concentrations, respectively, Pr - the Prandtl number, λ - the velocity slip parameter, f_w - the dimensionless blowing/suction parameter (blowing if $f_w< 0$ and suction if $f_w> 0$) and Re_x - the Reynolds number.

      Equation (2) gets satisfied automatically in view of the above similarity variables, while the remaining equations (3), (10) and (5) reduce to the following system of coupled, non-linear, dimensionless equations for momentum, energy and species,

$$f'''(\eta) + f(\eta)f''(\eta) - \frac{2m}{(m+1)}(f'(\eta))^2 = 0$$

(11)

$$\left(1+\varepsilon\theta+\frac{4}{3}R\right)\theta''(\eta) + \operatorname{Pr}\left(f(\eta)\theta'(\eta) - \frac{2s}{(s+1)}f'(\eta)\theta(\eta)\right) + \varepsilon(\theta'(\eta))^2 = 0$$

(12)

$$\varphi''(\eta) - Sc\left[\left(\frac{2p}{p+1}\right)f'(\eta)\varphi(\eta) - f(\eta)\varphi'(\eta)\right] = 0$$

(13)

The corresponding boundary conditions are

$$f(0) = f_w, \ f'(0) = 1 + \lambda f''(0), \ \theta(0) = 1, \ \varphi(0) = 1$$

$$f'(\infty) \rightarrow 0, \ \theta(\infty) \rightarrow 0, \ \varphi(\infty) \rightarrow 0$$

(14)

Here primes denote the differentiation with respect to η. λ and f_w are the velocity slip and suction/injection parameters respectively. The engineering design quantities of physical interest include the skin-friction coefficient, Nusselt number and Sherwood number. Knowing the velocity, the skin friction coefficient can be obtained, which in non-dimensional form is given by

$$C_f = \frac{-2\tau_{xy}(0)}{\rho U^2} \Rightarrow \operatorname{Re}_x^{1/2} C_f = f''(0)$$

(15)

Where $\tau_{xy} = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$

The rate of heat transfer coefficient can be attained, which in non- dimensional form, in terms of the Nusselt number, is given by

$$Nu_x = \frac{h(x)}{\alpha_\infty} \Rightarrow (\operatorname{Re}_x)^{-1/2} Nu_x = -\theta'(0)$$

(16)

Where $h(x) = \frac{-\alpha_\infty}{T_w - T_\infty}\left(\frac{\partial T}{\partial y}\right)_{y=0}$

\The rate of mass transfer coefficient can be achieved, which in non- dimensional form, in terms of the Sherwood number, is given by

$$Sh_x = \frac{Mu(x)}{D} \Rightarrow (\operatorname{Re}_x)^{-1/2} Sh_x = -\varphi'(0)$$

(17)

Where $Mu(x) = \dfrac{-D}{(C_w - C_\infty)} \left(\dfrac{\partial C}{\partial y} \right)_{y=0}$

SOLUTION OF THE PROBLEM:

The set of coupled non-linear governing ordinary differential equations (11) - (13) together with the boundary conditions (14) are explained numerically by using Runge-Kutta fourth order technique (R-K 4th) along with shooting method.

➤ First, we convert all the higher order non-linear differential equations (HDEs) into simultaneous linear differential equations (SDEs) of first order.

➤ These equations are transformed into an initial value problem (IVP), for this we need initial guesses or values. But no such values are given at the boundary. In order to choose appropriate finite values for η_∞, we apply shooting technique.

➤ To determine η_∞, we start with some primary guess value for some particular set of physical parameters. The selection of η_∞ is repeated until two successive values differ only by a specified significant digit. The last value of η_∞ is finally selected to be the suitable value of η_∞ for that specific set of parameters. The value of η_∞ may vary for another set of physical parameters. Once the finite value of η_∞ is determined then the integration is carried out.

$$G' = y_2, G'' = y_3, \theta' = y_5, \phi' = y7, \theta = y_4, \phi = y_6 \tag{18}$$

$$G''' = -y_1 y_3 + \left(2m/(m+1) y_2^{\,2} + M y_2 \right) \tag{19}$$

$$\theta'' = \left(\frac{1}{(1+\varepsilon\theta + (4/3)R)} \right) \left\{ \left(\Pr y_2 y_4 \frac{2m}{m+1} - \Pr y_1 y_5 \right) - \varepsilon y_5^{\,2} \right\} \tag{20}$$

$$\phi'' = Sc \left(\frac{2m}{m+1} y_2 y_6 - y_1 y_7 \right) \tag{21}$$

with boundary conditions are

$$y1 = f_w, y_2 = 1 + \lambda y_3, y_4 = 1, y_6 = 1, \text{ at } \eta \to 0$$
$$y_2 = 0, y_4 = 0, y_6 = 0 \text{ at } \eta \to \infty \tag{22}$$

We guess the values of $y_3(0), y_5(0), y_7(0)$ which are not given at the initial condition. The equations (18)-(22) are integrated by using Runge-Kutta fourth order method. The successive iterative step length is 0.01. Here η_{max} is η at ∞ and chosen large enough value so that the obtained solution shows larger η than η_{max}. For computation purpose we use MATLAB software ode45 solver. The accuracy of the guess values $y_3(0), y_5(0), y_7(0)$ is verified by comparing the calculated values of $y_2(0), y_4(0), y_6(0)$ at $\eta = \eta_{max}$ with their values given at $\eta = \eta_{max}$. Alternatively, we used Newton-Raphson method to find the accurate guess values. The obtained equations are integrated by using the Runge-Kutta method. The same procedure continued until the agreement between the calculated values and given condition at $\eta = \eta_{max}$.

RESULTS AND DISCUSSION:

In order to get a physical insight into the problem, the possessions of governing physical parameters on the velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ as well as the skin friction coefficient $\mathrm{Re}_x^{1/2} C_f$, Nusselt number $(\mathrm{Re}_x)^{-1/2} Nu_x$ and Sherwood number $(\mathrm{Re}_x)^{-1/2} Sh_x$ are calculated and offered in figures 2-22.

The influence of the suction/injection parameter on the velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ is depicted in Figs. 2-4. A negative value of f_w indicates injection and positive values of f_w deals the case of suction. It is observed that growing values of injection parameter ($f_w > 0$), fluid velocity is decrease, whereas the suction parameter ascends ($f_w < 0$), the fluid speed accelerates. i.e. suction causes to decrease the velocity of the fluid in the boundary layer region. This influence decreases the wall shear stress. Growth in suction causes advanced thinning of the boundary layer. The physical clarification for such a behavior is as in case of suction the heated fluid is pushed towards the wall where the buoyancy forces can perform to retard the fluid. Fig.3 explains the effect of f_w on the temperature. The fluid temperature decreases with

increasing injection parameter (f_w>0) near the boundary layer. The thermal boundary thickness decreases with a raise in the suction parameter (f_w<0). Here it is noticed that the suction parameter reaches a maximum point at η = 4 and then the temperature decreases. The physical explanation for such an activities is that the fluid is carried nearer to the surface and reduces the thermal boundary layer thickness in case of suction.Fig.4 indicates the effect of suction/injection parameter on the species concentration. It is seen that the fluid concentration decreases for an increase inf_w. Figs. 5 and 6 describe the behaviors of the longitudinal $f'(\eta)$ and $\phi(\eta)$ for dissimilar values of power law stretching parameter m and it is noted that rise in m results in a decline of longitudinal velocity. It is noticed that the concentration increases with an increase in the stretching parameter and converges smoothly (Fig. 6).

The effect of s (power law variation of temperature) on the temperature is shown in Fig. 7. As s increases, the temperature of the flow decreases. Fig.8 depicts the influence of the power-law variation of concentration on the species of the flow field. The concentration $\phi(\eta)$ of the flow decreases for an increase in the power law parameter (p). The effect of the slip parameter on the velocity is presented in Fig.9. As the slip parameter increases the velocity in the existence of slip at the fluid-solid interface and declines monotonically to zero far away from the solid surface. In case of no-slip (i.e.,δ =0), the fluid velocity neighboring to the solid surface and is equal to the velocity of the stretching, then $f'(0)$= 1 which is obviously satisfied in this figure. Figs. 10 and 11 designates that an increase in the velocity slip tends to thermal rise and consequently increases in the species concentration (Fig.11). The influence of the radiation parameter Nr on the temperature field is illustrated in Figs.12. The Nr being reciprocal of Stephan-Boltzmann constant is the measure of comparative importance of thermal radiation transfer to the conduction heat transfer. Thus larger values of Nr sound dominance of thermal radiation over conduction. Consequently, larger values of Nr are revealing the larger amount of radiative heat energy being dispensed into the system, causing rise in $\theta(\eta)$.

Fig. 13 portrays the effect of the variable thermal conductivity on the temperature $\theta(\eta)$. It is evident that $\kappa^* > \kappa$ i.e. the thermal conductivity is higher when ε >0, and increase in parameter ε results in increase of thermal conductivity. This effect is seen as increase in the temperature profile with the increase in ε parameter. Figs.14-16 represents the graphs on interested physical quantities like Skin friction $\mathrm{Re}_x^{1/2} C_f$, Nusselt number $\left(\mathrm{Re}_x\right)^{-1/2} Nu_x$ and Sherwood number $\left(\mathrm{Re}_x\right)^{-1/2} Sh_x$. The shear stress profiles for various values of injection parameter with different power-law stretching parameter. It is clear that, an increase in either injection or stretching parameter increases the skin friction coefficient $\mathrm{Re}_x^{1/2} C_f$ (Fig.14).The rate of heat transfer $\left(\mathrm{Re}_x\right)^{-1/2} Nu_x$ profiles for different values of variable thermal conductivity parameterκ^* against radiation parameter Nr is illustrated in Fig. 15. It is evident that, the heat transfer rate $\left(\mathrm{Re}_x\right)^{-1/2} Nu_x$ decreases gradually. For ε = 0, 0.1 and 1 the graph is curvy whereas ε increases, we can see the linearized line graph. The radiation parameter increases, the heat transfer rate $\left(\mathrm{Re}_x\right)^{-1/2} Nu_x$ decreases.Fig.16 shows the graph for Nusselt number $\left(\mathrm{Re}_x\right)^{-1/2} Nu_x$ versus Prandtl number for different values of radiation parameter. Increase in the Prandtl number increases the heat transfer rate $\left(\mathrm{Re}_x\right)^{-1/2} Nu_x$. There exists a minimum at the point η~ 0.2, after that the profiles are increasing before the minima the trend is reversed. And clearly, it is seen that increase Nr decreases the Nusselt number $\left(\mathrm{Re}_x\right)^{-1/2} Nu_x$ after the minimum point and a reverse phenomenon is observed before η~ 0.2.Fig.17 projects rate of mass transfer profiles $\left(\mathrm{Re}_x\right)^{-1/2} Sh_x$ against power-law concentration parameter (p) for various Sc values. It is observed that mass transfer rate $\left(\mathrm{Re}_x\right)^{-1/2} Sh_x$ increases for increasing p values but decreases for increasing Sc values.

CONCLUSIONS

The condition of slip is one of the core concepts of the Navier – Stokes theory. The sheet is placed horizontally, so that buoyancy forces are less and negligible and the plate is stretching nonlinearly. The partial velocity slip may occur on the stretching boundary when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions. The set of governing equations and the boundary condition are reduced to ordinary differential equations (ODEs) with appropriate boundary conditions using similarity transformations. The resultant dimensionless equations are solved numerically by shooting technique with Runge-Kutta scheme. A

comparison with previously published work is performed and the results are found to be in good agreement. Based on the obtained results, the following conclusions may be drawn:

- Increase in suction/injection parameter decreases the velocity, temperature and concentration as well.
- Temperature and concentration of the fluid increase as the power-law stretching parameter increases.
- Velocity decelerates for acceleration in stretching parameter, while the skin friction accelerates for increasing m.
- Ascending magnetic parameter descends the motion of the fluid but ascends the shear stress.
- Increase in the velocity slip parameter decreases the velocity, but rises the temperature as well as concentration. Skin-friction rises for increase in f_w or m.
- Temperature rises for an increase in the radiation parameter or varying thermal conductivity parameter or velocity slip parameter. But decreases for increasing values of Pr or s.
- The species concentration decreases with an increase in the values of Schmidt number or p. Mass transfer rate increases with an increase in Sc or p.
- The rate of heat transfer decreases for an increase in ε or Nr while it decreases with increasing values of Pr.

$$y \to \infty , \quad u \to 0, \quad T \to T_\infty, \quad C \to C_\infty$$

$$y = 0 , \quad u = \alpha x^m + L \frac{\partial u}{\partial y} , \quad v = V_w , \quad T = T_w(x) + bx^s , \quad C = C_w(x) + cx^p$$
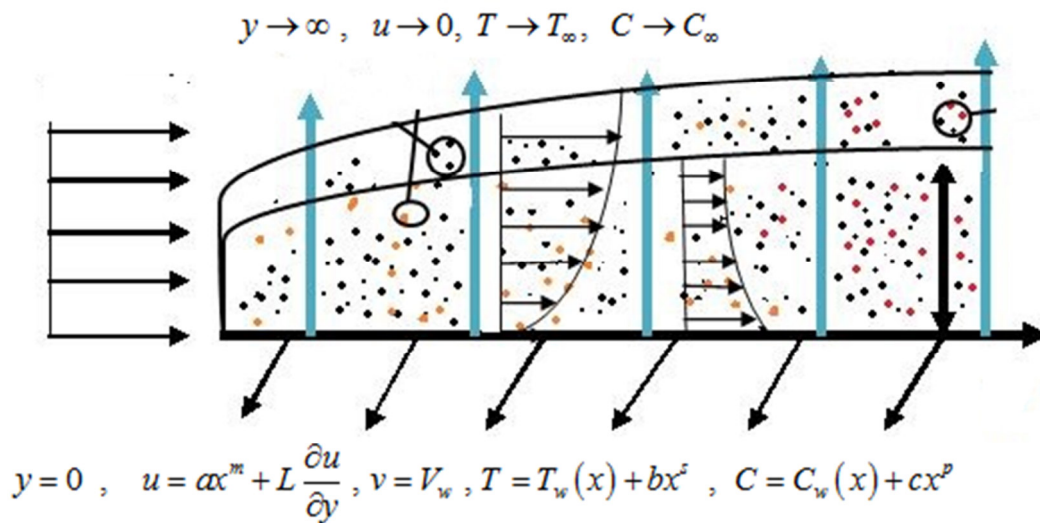
Fig.1: Physical model of the flow
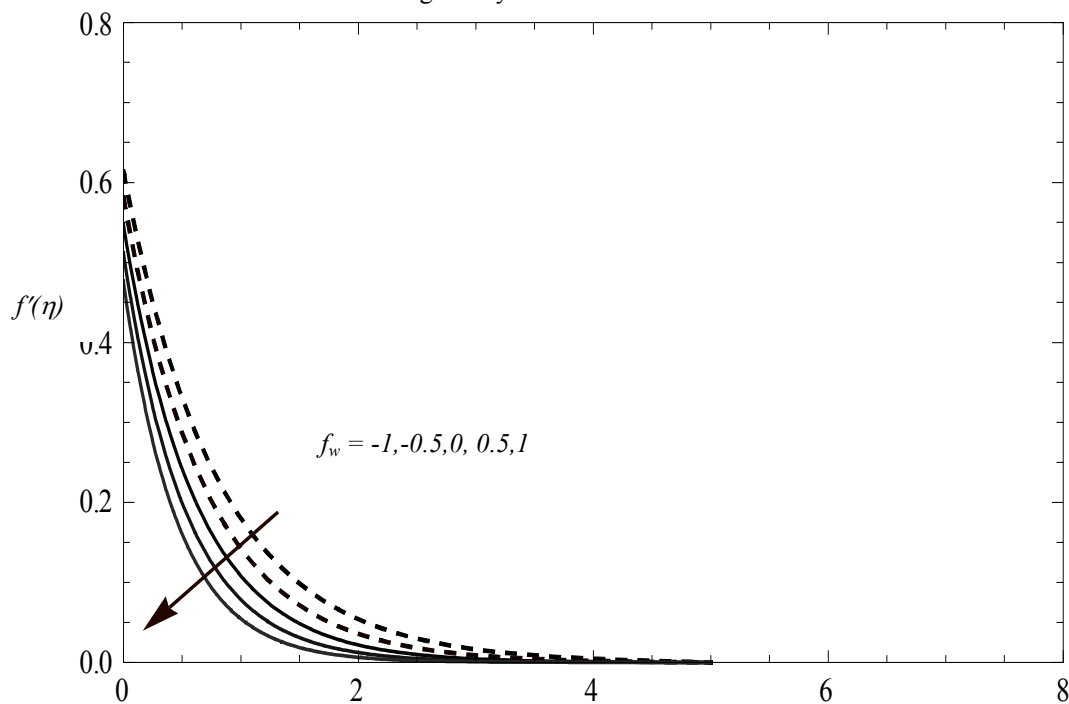
Fig.2: Effect of suction/injection on the velocity
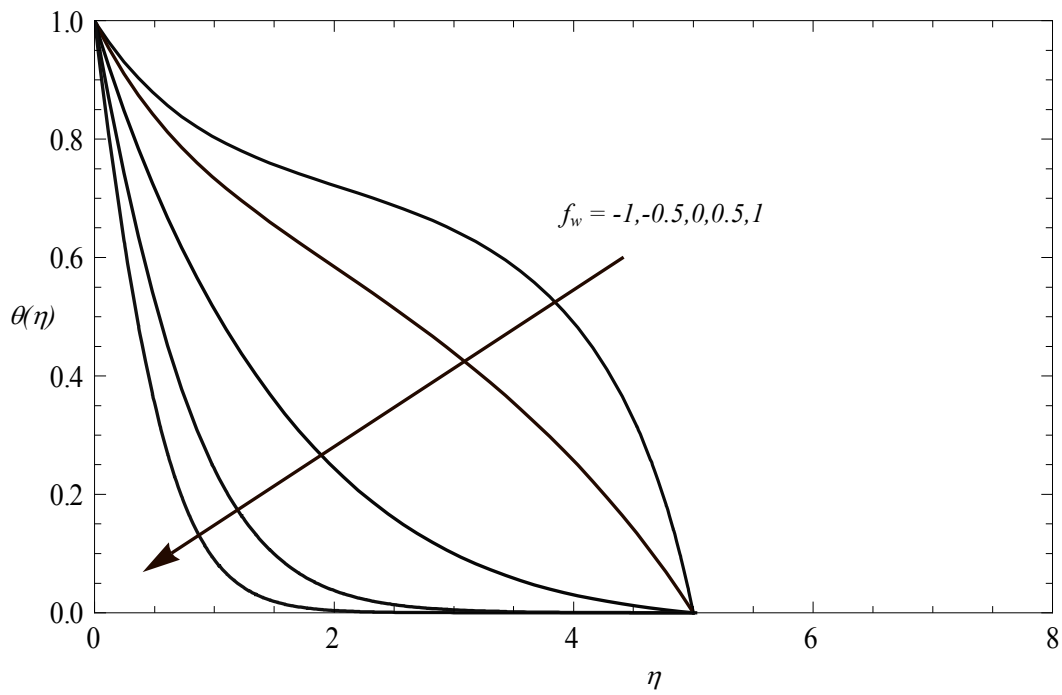
Fig.3: Effect of suction/injection on the temperature
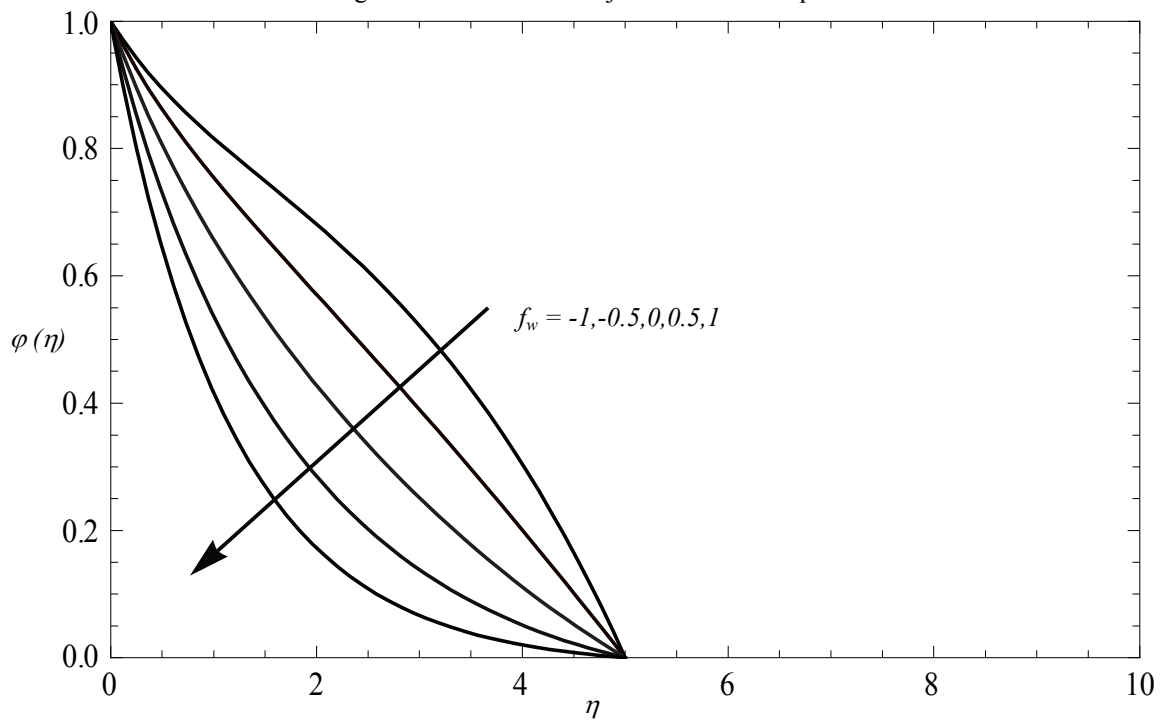
Fig.4**:** Effect of suction/injection on the species concentration
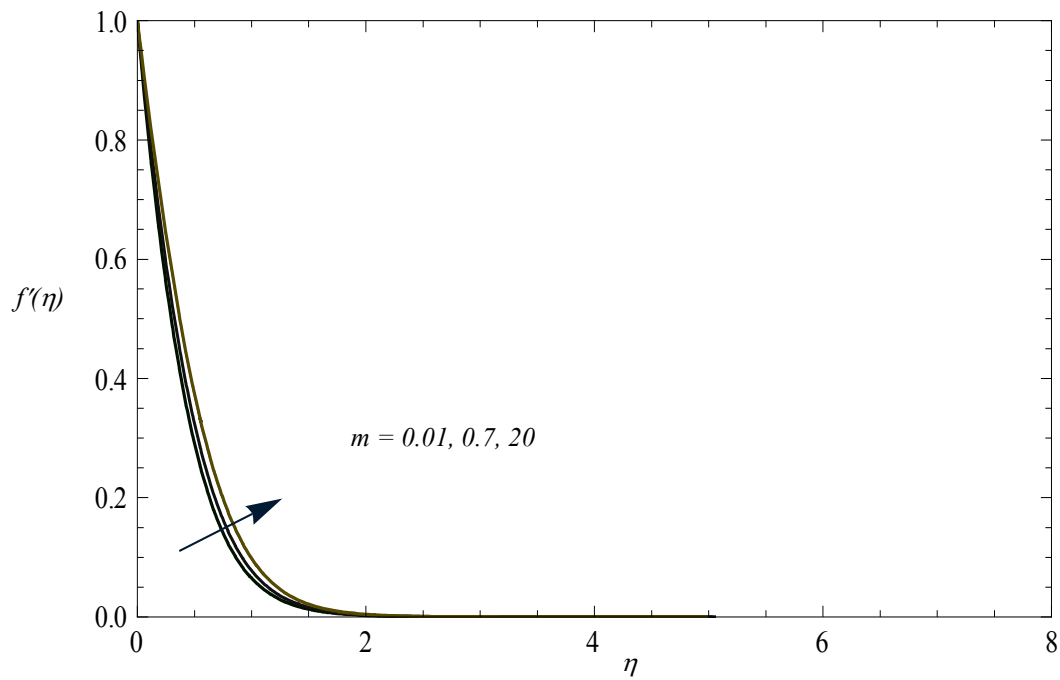
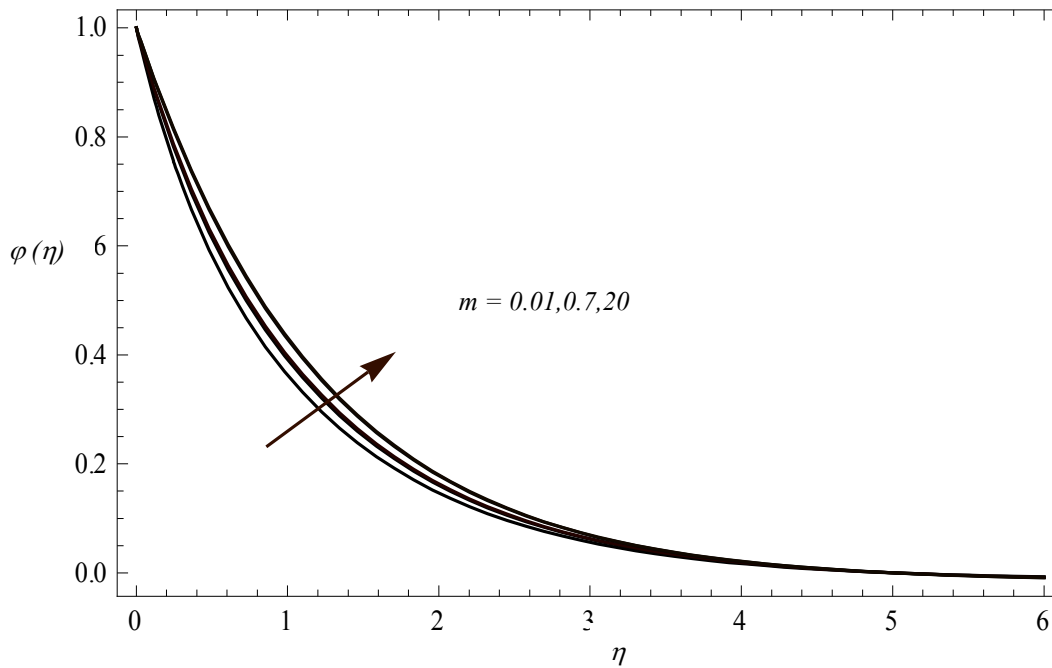Fig.5:Effect of power-law stretching parameter on the temperature

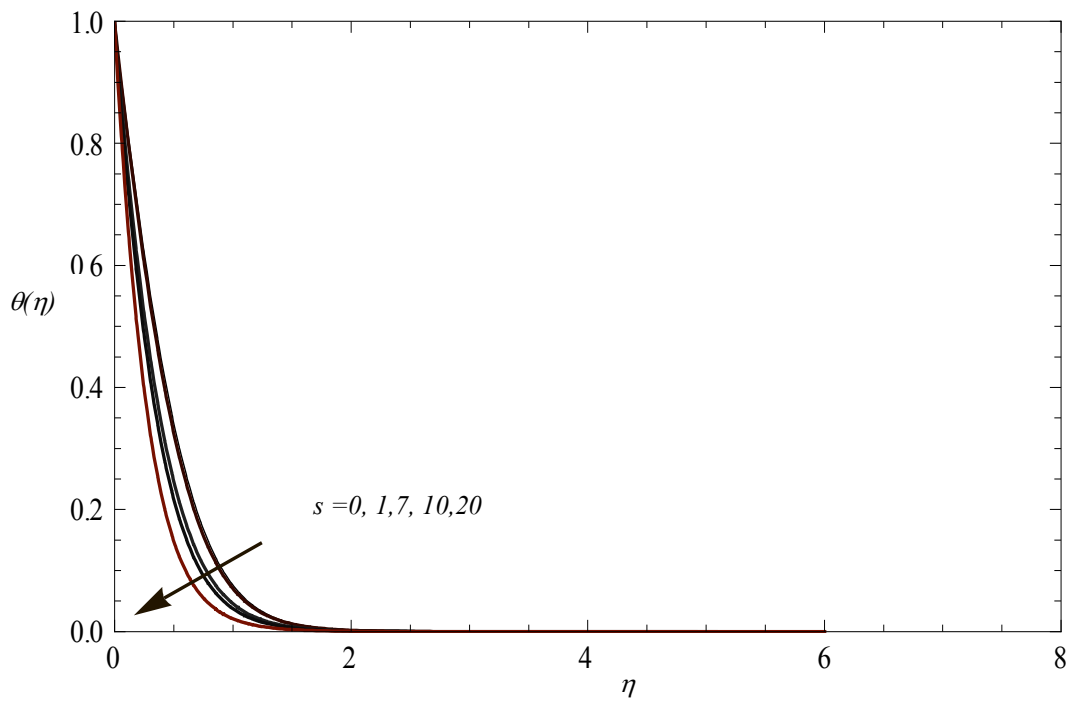Fig.6:Effect of power-law stretching parameter on the concentration
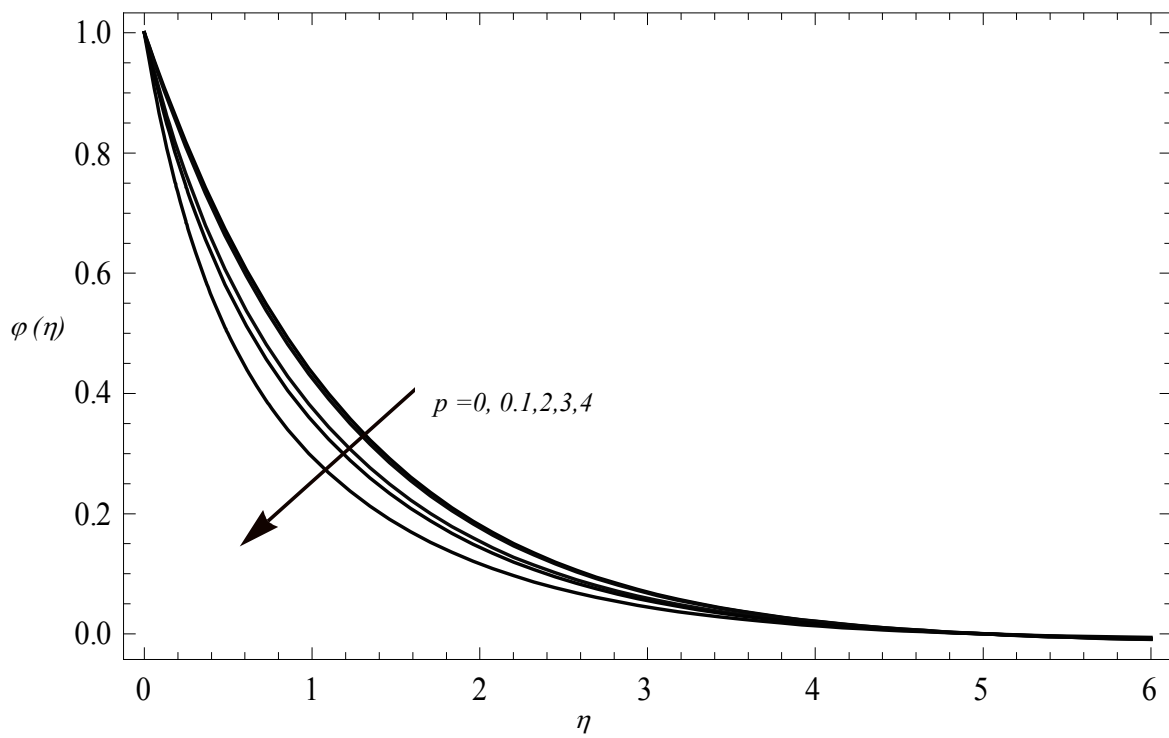
Fig.7: Temperature profiles for various s

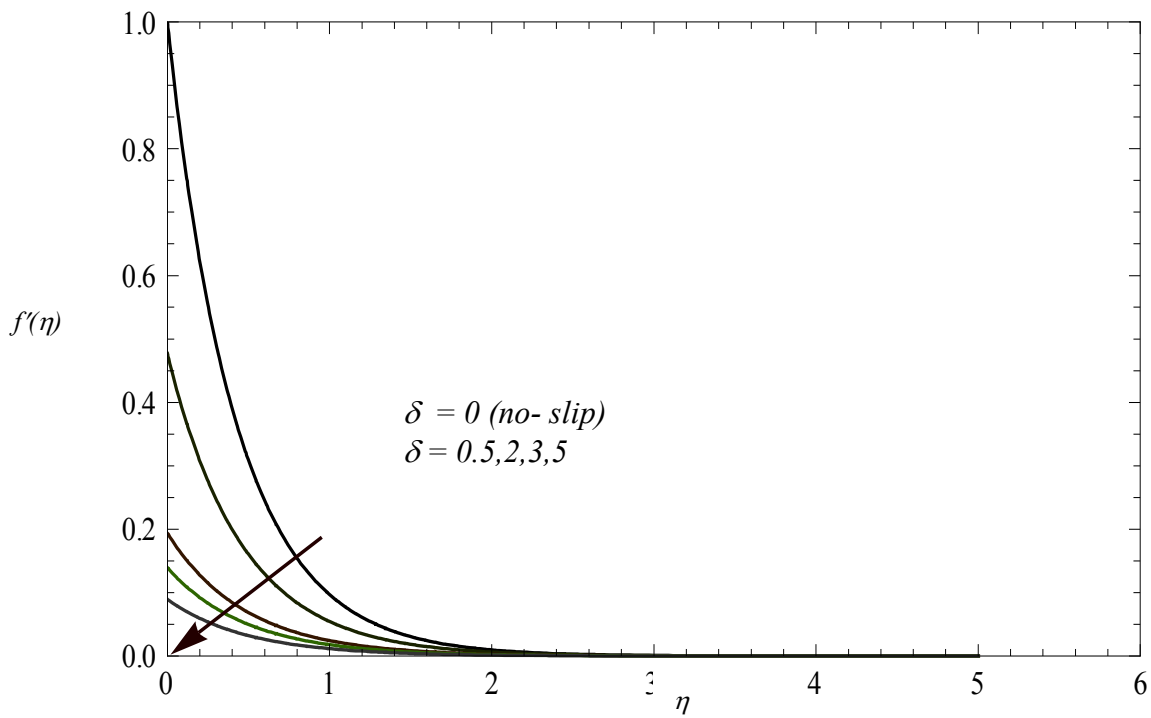
Fig.8 Concentration profiles for different *p*

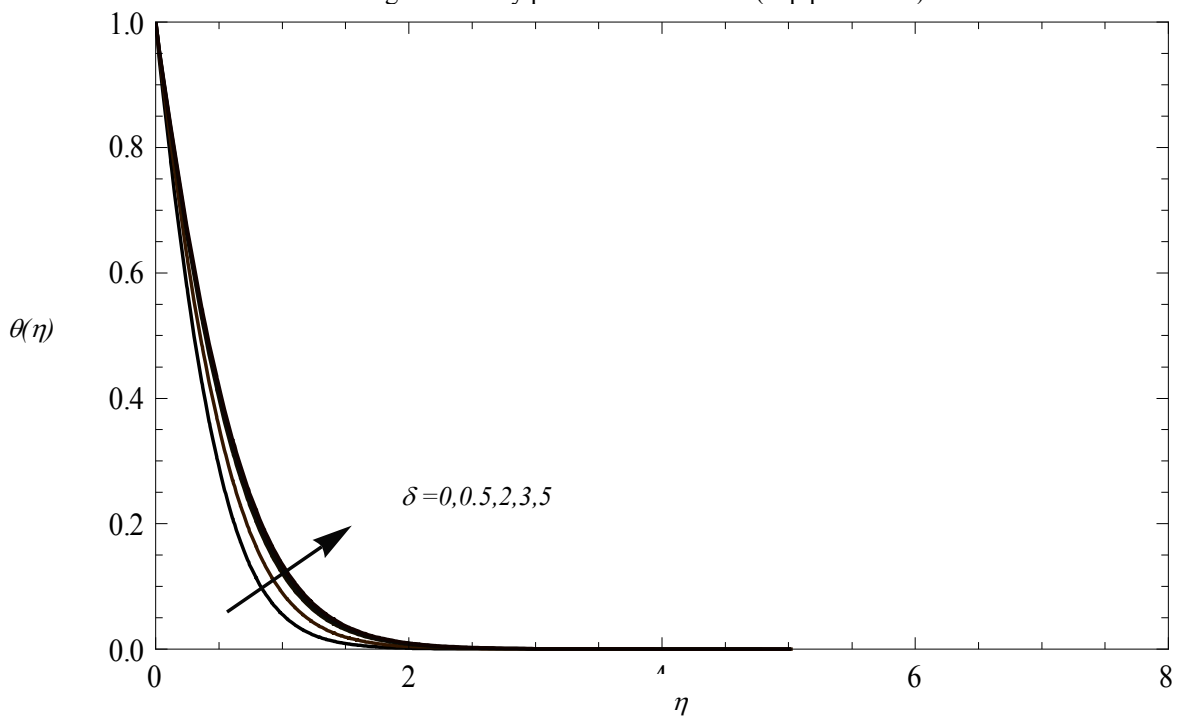Fig.9: Velocity profiles for various δ(slip parameter)
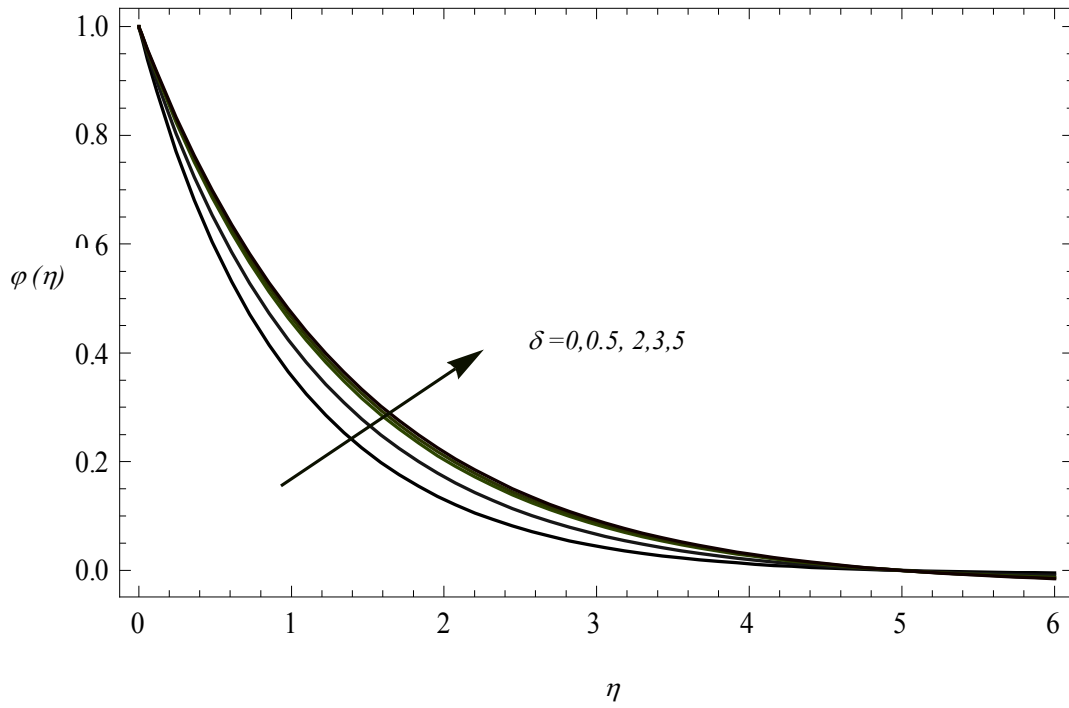

Fig.10:Temperature profiles for various δ

Fig.11:Concentration profiles for different δ
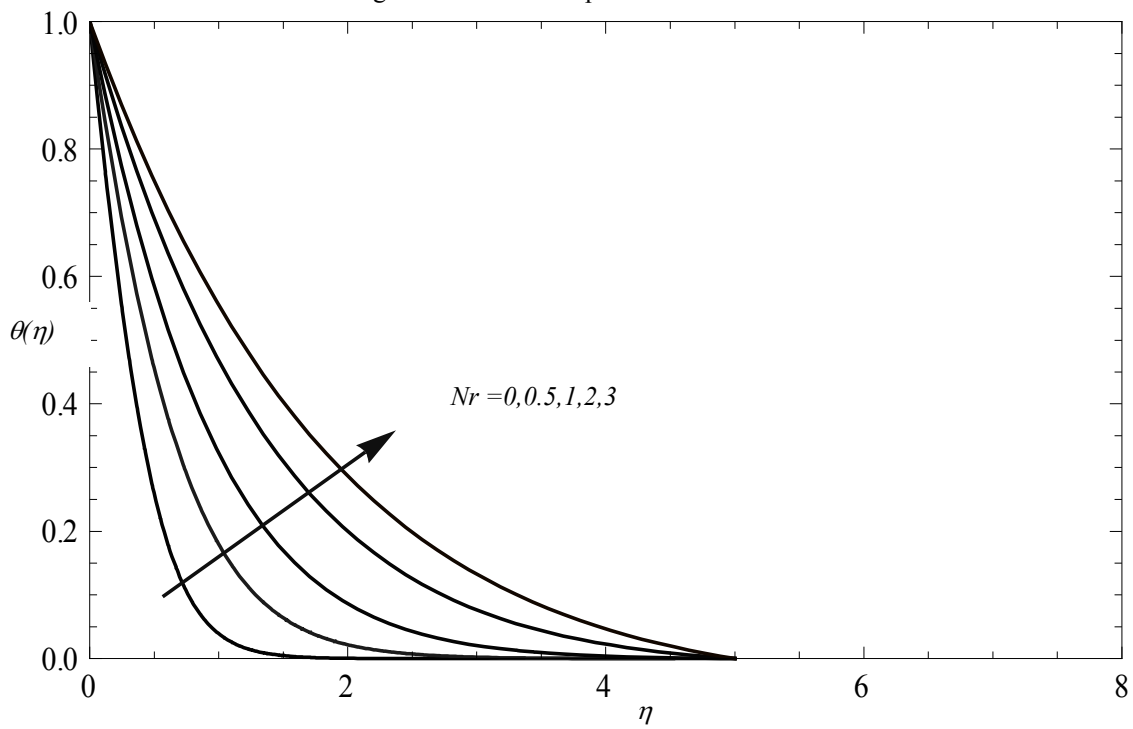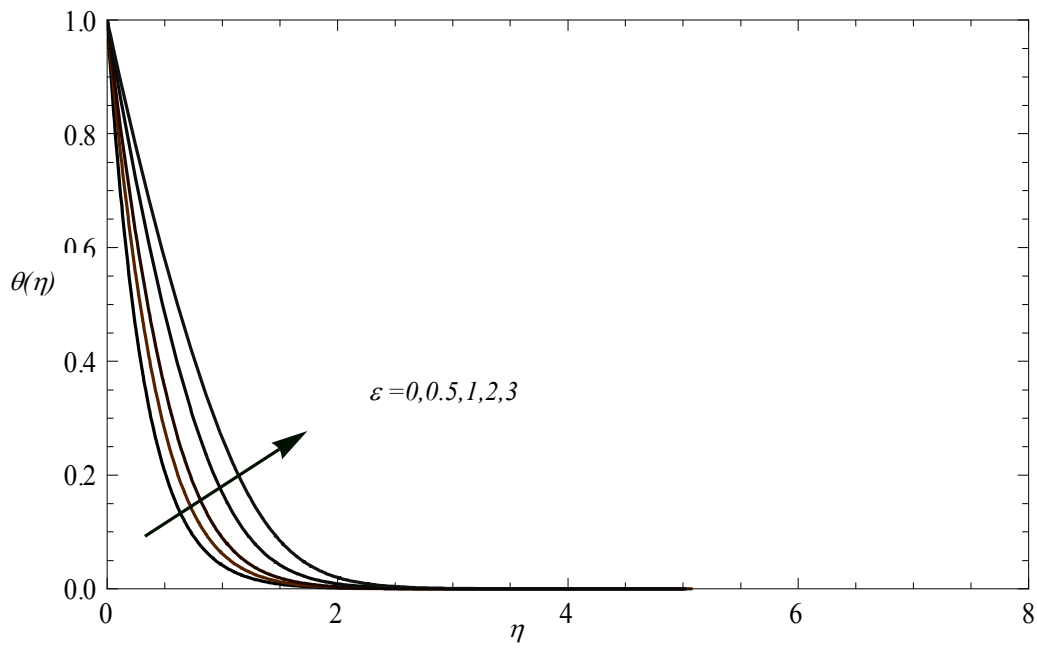

Fig.12: Influence of radiation on the temperature

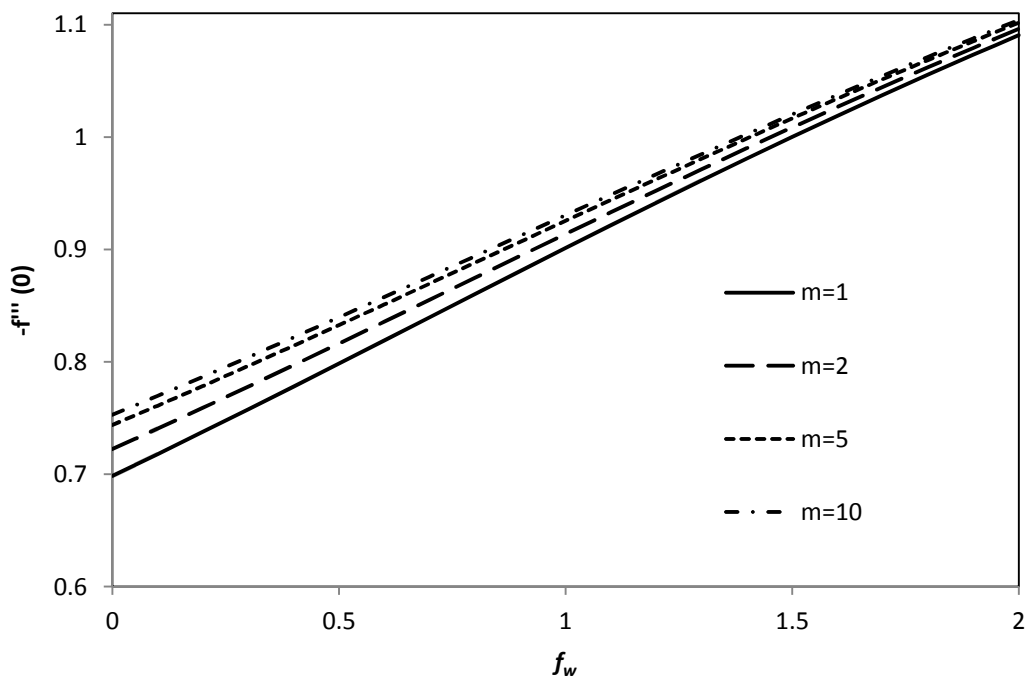Fig.13:Influence of variable thermal conductivity on the temperature

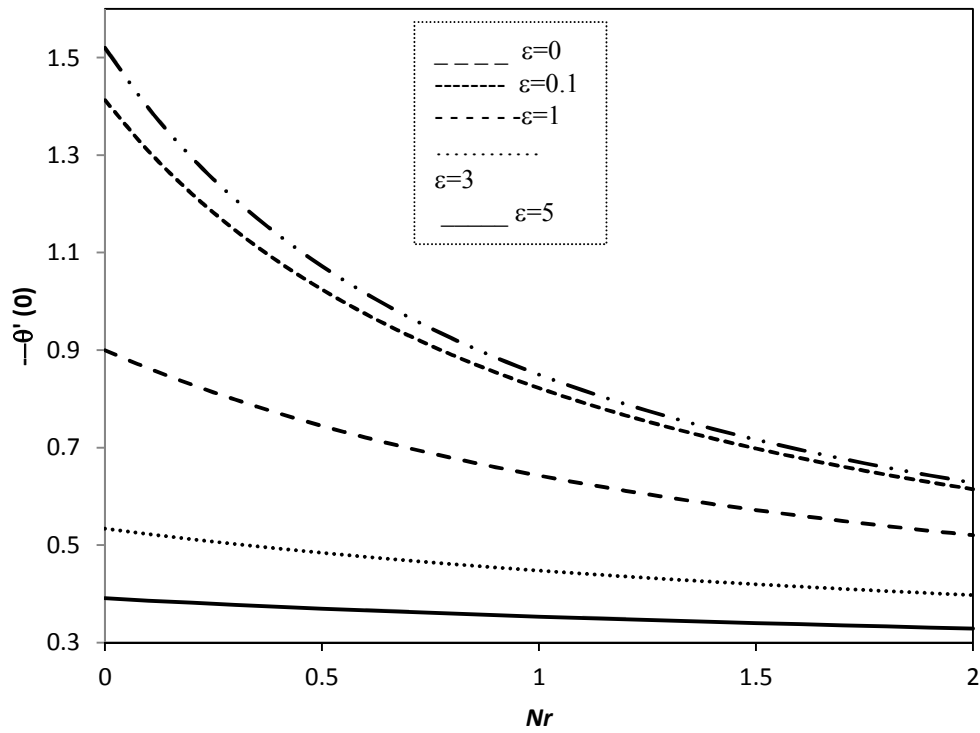Fig. 14:Shear stress versus f_w for different values of m
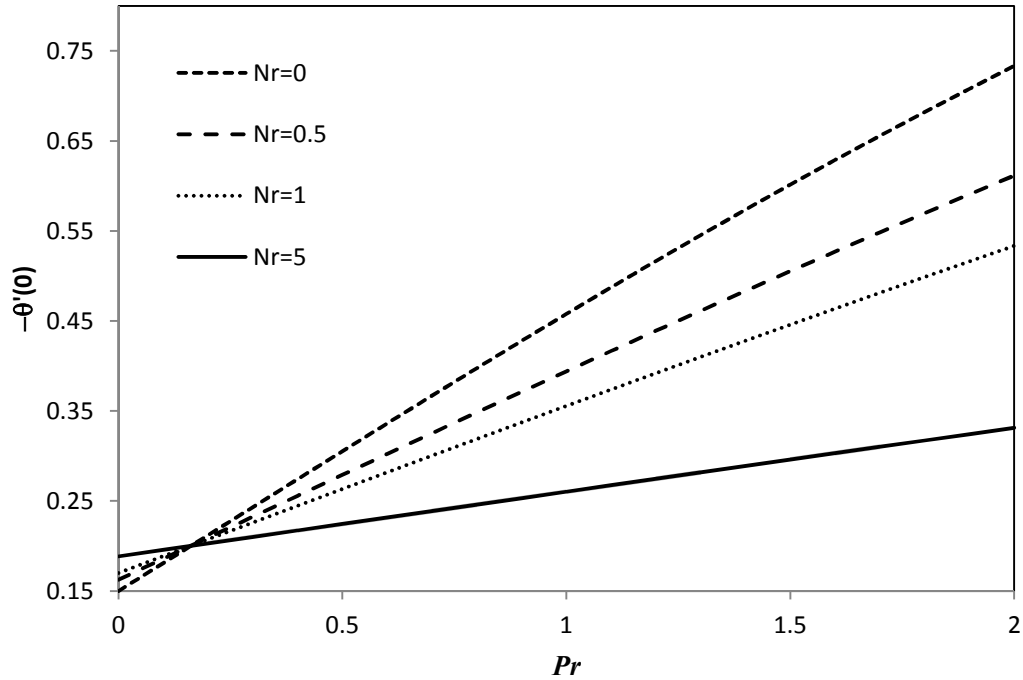
Fig. 15: Heat transfer rate against Nr for different ε

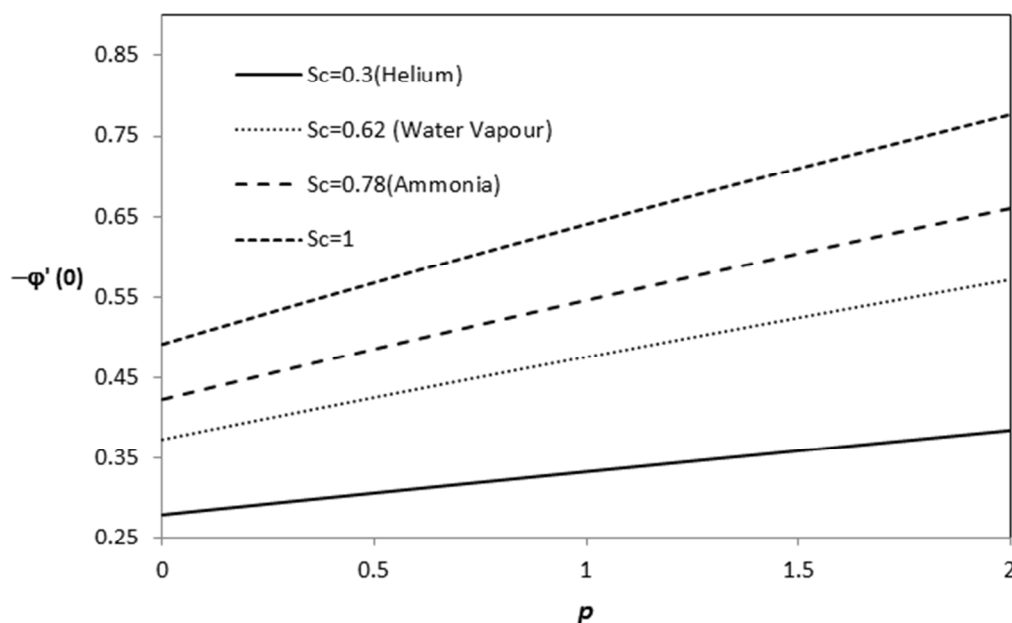Fig.16: Heat transfer rate against Pr for different Nr

Fig.17: Mass transfer rate against *p* for different *Sc*

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