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Effect of Variable Viscosity, Radiation Absorption on Non-Darcy MHD Convective Heat and Mass Transfer Flow Past Stretching Sheet with Non-Uniform Heat Sources

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Abstract

We investigate the effect of variable viscosity on MHD non-Darcy flow and heat transfer over a continuous stretching sheet with electric field in present of Ohmic dissipation and non-uniform heat source/sink of heat. Highly non-linear momentum and heat transfer equations are solved numerically using fifth-order Runge-Kutta-Fehlberg method with shooting technique.

1. INTRODUCTION

It is worth mentioning that non-Darcial forced flow boundary layers from a very important group of flows, the solution of which is of great importance in many practical application such as in biomechemical problems. In filtration transpiration cooling and geothermal. Singh and Tewani [1] studied the effect of thermal stratification on non-Darcial free convection flow by using the Ergun model [2] to include the inertia effect. It is well known that there exists non-Darcial flow phenomena bodies inertia effect and solid-boudary viscous resistance. These non-Darcian effect include non-uniform porosity distribution and thermal dispersion. Tien and Hunt [3] analyzed these non-Darcian effect applicable for a boundary layer flow in porous beds.

The study of heart source/sink effect on heat transfer is very important in view of several physical problems. Aforementioned studies include only the effect of uniform heat source/sink on heat transfer. Abo-Eldahab and El-Aziz[4] have included the effect of non-uniform heat source with suction/blowing but confined to the case of viscous fluid only. Abel et al.[5] investigated on non-Newtonian boundary layer flow past a stretching sheet taking into account of non-uniform heat source and frictional heating. Abel and Mahesha[6] studied the magnetohydrodynamic boundary layer flow and heat transfer characterstic of a non-Newtonian viscoelastic fluid over a flat sheet with variable thermal conductivity in the presence of thermal radiation and non-uniform heat source. They have reported that the combined effect of variable thermal conductivity, radiation and non-uniform heat source have significant impact in controlling the rate of heat transfer in the boundary layer region.

The previous studies are based on the constant physical parameters of the fluid. For most realistic fluids, the viscosity shows a rather pronounced variation with temperature. It is known that the fluid viscosity changes with temperature shows a rather pronounced variation with temperature. It is known that the fluid viscosity changes with temperature [7]. Thus it is necessary to take into account the variation of viscosity with temperature in order to accurately predict the heat transfer rates. Ali[8] investigated the effect of variable viscosity on mixed convection heat transfer along a moving surface, Lai and Kulacki[9] analyzed the effects of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium. Pantokratoras [10,11] studied laminar free-convection over a vertical isothermal plate with uniform blowing of suction in water with variable physical properties. Later, Kafoussian and Williams[12] investigated on free forced convective boundary layer flow past a vertical isothermal flat considering temperature-dependent viscosity of the fluid. Recently, Pantokratoras [13] made a theoretical study to investigate the effect of variable viscosity on the classical Falkner-Skan flow with constant wall temperature and obtained results for well shear stress and the wall heat transfer for various values of ambient Prandtl numbers varying from 1 to 10,000.

In view of the above applications, we investigate the effect of variable viscosity on MHD non-Darcy flow and heat transfer over a continuous stretching sheet with electric field in present of Ohmic dissipation and non-uniform heat source/sink of heat. Highly non-linear momentum and heat transfer equations are solved numerically using fifth-order Runge-Kutta-Fehlberg method with shooting technique (Na,[14]).

2. FORMULATION

We consider two-dimensional incompressible, electrically conducting fluid flow over a vertical non-linear stretching sheet embedded in non-darcy porous medium with the plane y=0 of a co-ordinate system. The fluid properties are constant, except for the fluid viscosity μ which is assumed to vary as an inverse linear function of temperature T, in the form (Lai and Kulacki [9]):

$$\frac{1}{\mu} = \frac{1}{\mu_{\infty}} [1 + \gamma_0 (T - T_{\infty})]$$
(1)

$$\frac{1}{\mu_{\infty}}\alpha(T-T_{\infty}) \tag{2}$$

where $\alpha = \frac{\gamma_0}{\mu_{\infty}}$ and $T_r = T_{\infty} - \frac{1}{\gamma_0}$

in which α and T_r are constants, and their respective values are based on the liquid's thermal charecteristic, i.e., g_0 . Generally, $\alpha > 0$, $\alpha < 0$ represent for liquids and gases respectively. θ r is a constant which is defined by

 $\theta_r = \frac{T_r - T}{T_w - T} = -\frac{1}{\gamma(T_w - T)}$ and primes denote differentiation with respect to η . It is worth

mentioning here that for $\gamma \gamma \rightarrow 0$ *i.e* $\mu = \mu$ (Constant) then $\theta_r \rightarrow .$ It is also important to note that θ_r is negative for liquids and positive for gases. T is free stream temperature. The flow region is exposed under uniform transverse magnetic fields Bo=(0,0,Bo) and uniform electric field $\overline{E} = (0,0,-E_0)$ (see fig.1).Since such imposition of electric and magnetic fields stabilizes the boundary layer flow. It is assumed that the flow is generated by stretching of an elastic boundary sheet from a slit by imposing two equal and opposite forces in such a way that velocity of the boundary sheet is of linear order of the flow direction. We know from Maxwell's equation that $\nabla .\overline{B} = 0$ and $\nabla x\overline{E} = 0$. When magnetic field is not so strong then electric field and magnetic field obeys Ohm's law $\overline{J} = \sigma(\overline{E} + \overline{q}x\overline{B})$, where \overline{J} is the coule current. The viscous dissipation and velocity of the fluid far away from the plate are assumed to be negligible. We assumed that magnetic Reynolds number of the fluid is small so that induced magnetic field and Hall effect may be neglected. We take into account of magnetic field effects as well as electric field in momentum. Under these assumptions, the governing boundary layer equations for momentum, energy and diffusion for mixed convection under Boussinesq's approximation are

Conservation of mass:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

Conservation of momentum:

$$\frac{1}{\delta^{2}}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) = \frac{1}{\delta\rho_{m}}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \rho_{m}g_{0}\beta_{\tau}\left(T-T_{\infty}\right) + \rho_{m}g_{0}\beta_{c}\left(C-C_{\infty}\right) - \left(\frac{v}{k}\right)u - \left(\frac{C_{b}}{\sqrt{k}}\right)u^{2} + \frac{\sigma}{\rho}\left(E_{0}B_{0}-B_{0}^{2}u\right)$$
(4)

Conservation of energy:

$$\rho_m C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_f \frac{\partial^2 T}{\partial y^2} + q''' + Q_1' (C - C_\infty) + \mu \left(\frac{\partial u}{\partial y} \right)^2 - \frac{\partial (q_R)}{\partial y} + \frac{D_m K_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} + \sigma (u B_0 - E_0)^2$$
(5)

Mass diffusion of Species equation:

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - k_0 (C - C_\infty) + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2}$$
(6)
Poundary conditions are:

Boundary conditions are:

(13)

$$u = U_w(x) = bx, v = 0, T = T_w = T + A_0 \left(\frac{x}{l}\right)^2$$

$$C = C_w = C + A_1 \left(\frac{x}{l}\right)^2 \qquad \text{at } y = 0$$

$$u = 0, T \to T, C \to C \qquad \text{as } y \to$$
(7)

Where u and v are the velocity components along x and y directions respectively, T_w , are C_w the temperature and concentration at the wall, T and C are the temperature and concentration far away from the stretching sheet, Dm is the mass diffusivity, Tm is mean temperature, K_T is the thermal diffusion ratio, ko is the chemical reaction coefficient.

To solve the governing boundary layer equations (*)-(*), we define following similarity transformations(18,19):

$$u = bx, v = -(\sqrt{bv})f(\eta), \eta = \sqrt{\frac{b}{v}}y, \theta = \frac{T-T}{T_w - T}, \phi = \frac{C-C}{C_w - C}$$

$$\tag{8}$$

The non-dimensional heat/sink, q''' (20,21) is modelled as

$$q''' = \frac{k_f u_w(x)}{v x} (A^{\bullet} (T_w - T_{\infty}) u + B^* (T - T_{\infty}))$$
(9)

Where A^{\bullet} and B^{\bullet} are the coefficient of space and temperature dependent heat source/sink respectively. Here we make a note that the case $A^{\bullet} > 0$, $B^{\bullet} > 0$ corresponds to internal heat generation and that $A^{\bullet} < 0$, $B^{\bullet} < 0$ corresponds to internal heat absorption.

Substitution of equation(8) and (9) into the governing equations(3)-(6) and using the above relations we finally obtain a system of non-linear ordinary differential equations with appropriate boundary conditions

$$\frac{f'''}{\delta} + (1 - \frac{\theta}{\theta_r})\frac{ff''}{\delta^2} + (\frac{1}{\theta_r - \theta})\frac{\theta'f''}{\delta} + (1 - \frac{\theta}{\theta_r})M^2(E_1 - f') = (1 - \frac{\theta}{\theta_r})(\frac{f'^2}{\delta^2} + F^*f'^2 - G(\theta + N\phi) + D^{-1}f' = 0$$

$$\theta'' - (1 - \frac{\theta}{\theta_r})\Pr((2f'\theta - f\theta') - M^2Ec(E_1 - f')^2) = -Ec\Pr(f'')^2 - (11)$$

$$(41f' + B1\theta) - O1\phi$$

$$\phi'' + Sx(f\phi' - 2f'\phi) - \gamma\phi = -ScSr\theta''$$
⁽¹²⁾

The boundary conditions (7) becomes

$$f(0) = 0, f'(0) = 1, \theta(0) = 1, \phi(0) = 1$$
 on $\eta = 0$
 $f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0$ as $\eta \to \infty$
Where

$$D^{-1} = \frac{v}{bk} (Inverse \ Darcy \ parameter), \ M = \sqrt{\frac{\sigma}{\rho b}} B_0 (HartmannNumber)$$

$$E_1 = \frac{E_0}{B_0 bx} (Local \ inertial coefficient) \ G = \frac{\beta_T g(T_w - T)}{b^2 x} (Buoyancy \ or \ Mixed \ convection \ parameter)$$

$$Pr = \frac{\rho v C_p}{k_f} (Ambient \ Prndtl \ Number), \ Pr = (1 - \frac{\theta}{\theta_r})^{-1} \ Pr \ (Prandtl \ number)$$

$$Ec = \frac{b^2 l^2}{A_0 C_p} (Ec \ ker \ tNumber) , \ Sr = \frac{D_m K_T (C_w - C)}{T_m (T_w - T)} (Soret \ Parameter)$$

$$Du = \frac{D_m K_T (T_w - T_\infty)}{C_s (C_w - C_\infty)} (Du four \ parameter)$$
$$Q1 = \frac{Q_1' (C_w - C_\infty)}{D_w (T_w - T_\infty)} (Radiation \ absorption \ parameter)$$

The local Nusselt number which are defined as

$$Nu_x = \frac{xq_w}{k_f(T_w - T)} \tag{14}$$

Where q_w is the heat transfer from the sheet is given by

$$q_w = -k_f \left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{15}$$

Using the non-dimensional variables(8), we get from equations(14) and (15) as

$$\frac{Nu_x}{\operatorname{Re}_x^{1/2}} = -\theta'(0) \tag{16}$$

The physical quantity of interest is the local Sherwood number which are defined as

$$Sh_x = \frac{xq_m}{D_m(C_w - C)} \tag{17}$$

Where q_m is the mass transfer which is defined by

$$q_m = -D_m \left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{18}$$

Using the non-dimensional variables(8) and (18) we get from equations(17) as

$$\frac{Sh_x}{\operatorname{Re}_x^{1/2}} = -\phi'(0) \tag{19}$$

where

$$\operatorname{Re}_{x} = \frac{xU_{w}(x)}{v}$$
 is the local Reynolds

3. THE METHOD OF SOLUTION

The Galerkin finite element method has been implemented to obtain numerical solutions of coupled non-linear equations (10) to (12) of third-order in f and second order in h,θ , ϕ under boundary conditions (13). This technique is extremely efficient and allows robust solutions of complex coupled, nonlinear multiple degree differential equation systems. The fundamental steps comprising the method are

1]Discretization of the domain into elements

2] Derivation of element equations

3] Assembly of Element Equations

4] Imposition of boundary conditions

5] Solution of assembled equations

4. DISCUSSION

In this analysis, we investigate the combined influence of thermal radiation, radiation absorption sorest and Dufour effects on convective heat and mass transfer flow of a viscous chemically reacting electrical conducting fluid through a porous medium past stretching sheet in the presence of non-uniform heat source. The non-linear ,coupled equations governing the flow, heat and mass transfer have been solved by using Galerkin finite element analysis with quadratic approximation functions.

Figs.2a-2c predicts the behaviour of velocity, temperature and concentration with Eckert number Ec. It can be seen from figs.2a&2b that an increase in Eckert number reduces the velocity and temperature. This is due to the fact that the thermal energy is absorbed in the fluid on account of friction heating. The concentration enhances in the solutal boundary layer with increasing Ec. It can be attributed to the fact that an increase in Ec increases the thickness of the solutal boundary layer(fig.2c).





Figs.3a-3c represent the behaviour of f', θ and C with the local inertial coefficient,E1.It can be seen from fig.3a that an increase in the inertial coefficient E1 \leq 1.0, results in a reduction in the velocity while for higher E1 \geq 1.5, we notice an increment in u in the entire flow region. The temperature reduces with increase in E1 \leq 0.75 and enhances with E1 \geq 1.0 in the boundary layer(fig.3b).From fig.9c we find that the concentration experiences an enhancement with increase in E1. This may be attributed to the fact that higher the inertial coefficient E1 larger the thickness of the solutal boundary layer.

Figs.4a-4b show the variation of the velocity, temperature and concentration with strength of the heat source. The velocity and temperature reduces with increase in A1>,B1>0 ,owing to the absorption of energy in the boundary layer while an increase in A1,B1<0,leadts to an increase in f' and θ . This is due to the fact that the energy is generated in the boundary layer. From fig.4c we find that the concentration enhances in the boundary layer with increase in both the cases A1,B1>0&A1,B1<0.





Figs.5a-5b predicts the behaviour of f', θ and C with different values of the Fochheimir's number F. From fig.5a we find that the velocity reduces with increase in F≤6 and for higher F≥10,This may be attributed to the fact the thickness of the momentum boundary layer reduces with F≤6 and increases with higher F≥10.From fig.5b we find that the thickness of the thermal boundary layer reduces with F≤4 and increase s with F≥6,which shows that the temperature reduces with F≤4 and for higher F≥6,we notice an increment in the temperature in the thermal boundary layer.

Figs.6a-6c shows the variation of f', θ and C with different values of Soret parameter Sr and Dufour parameter Du. From fig.6a we find that increasing Sr \leq 1.0(or decreasing Dufour parameter Du \geq 0.6) smaller the velocity in the boundary layer and higher Sr \geq 1.5(or Du \leq 0.04) larger the velocity in the boundary layer. From fig.6b,we find that the temperature reduces with increasing Sr \leq 1.5(or decreasing Du \geq 0.04) and for higher Sr=2.0(or Du=0.03),we notice an enhancement in the temperature in the thermal boundary layer.Increasing the Soret parameter Sr(or decreasing Du) decreases the concentration in the solutal boundary layer(fig.6c).





Figs. 7a – 7c represent a variation of f', θ and C with radiation absorption parameter (Q₁). The velocity depreciate with increase in Q₁. From fig.7b, we find the temperature reduces with Q₁. From fig.7c we notice an enhancement in concentration in the boundary layer.

Table : 1			
Values of Nusselt Number (Nu), Sherwood Number (Sh) at $\eta = 0$			
Parameter		Nu(0)	Sh(0)
Ec	0.1	0.215633	0.779862
	0.3	0.284921	0.740745
	0.5	0.357043	0.658202
	0.7	0.4342	0.592133
Q ₁	0.5	0.215633	0.779862
	1.0	0.24462	0.775792
	1.5	0.222115	0.774368
	2.5	0.213317	0.782196
Sr / Du	2.0/0.03	0.215633	0.779862
	1.5/0.04	0.266347	0.762302
	1.0.0.06	0.259989	0.747716
	0.6/0.1	0.258979	0.748826
E ₁	0.5	0.215633	0.779862
	1.0	0.266714	0.746374
	1.5	0.261702	0.716851
	2.5	0.252456	0.636257
F	2	0.216472	0.782598
	4	0.265755	0.748743
	6	0.26348	0.739312
	10	0.263132	0.737875
A_1, B_1	0.1/0.1	0.215633	0.779862
	0.3/0.3	0.336757	0.69196
	-0.1/-0.1	0.13654	0.85415
	-0.3/-0.3	0.0618939	0.922482

The rate of heat transfer(Nusselt number) at the wall $\eta=0$ is evaluated for different values of the governing parameters. An increase in the radiation absorption parameter Q1 \leq 1.0or inertial parameter E1 \leq 0.75,enhances Nu at wall. For still higher Q1 \geq 1.505 E1 \geq 1.0,we notice a reduction in Nu at the wall. Increasing the Soret parameter Sr (or Dufour parameter Du) enhances the Nusselt number at the wall. The variation of Nu with non-uniform heat source parameter A1,B1,we find that the rate of heat transfer enhances with increase in the heat generating source case(A1,B>0) and reduces in the heat absorption source(A1,B1<0).

The rate of mass transfer (Sherwood number) at the wall $\eta=0$ is evaluated for different parametric variations. It is found that the rate of mass transfer increases with increase in E1 fixing the other parameters. Higher the Lorentz force or Radiative heat flux or dissipative heat lesser the Sherwood number at the wall. An increase in Q1 \leq 1.5 or F \leq 4, results in a reduction in Sh at the wall and for higher Q1 \geq 2.0 or F \geq 4, we notice a reduction in Sh. The rate of mass transfer reduces with the strength of the heat generating source and increases with that of heat absorption source.

CONCLUSIONS

The non-linear ,coupled equations governing the flow, heat and mass transfer have solved by using Runge Kutta-Felhberg method. The velocity, temperature and concentration have been represented graphically for different parametric variations. The conclusions of this analysis are

- Higher the Lorentz force /lesser the permeability of the porous medium smaller the velocity ,Sherwood number and larger the temperature,concentration,skin friction and Nusselt number at the wall.
- Higher the thermal radiation//higher the dissipation reduces the velocity,larger the concentration. The temperature reduces with Ec. The Nusselt number enhances Ec on the wall. Sherwood number reduces with Ec.
- The velocity,temperature,skin friction reduces ,the concentration ,Nusselt number and Sherwood number enhances in the degenerating chemical reaction case while in generating chemical reaction case ,a reversed effect is noticed.
- The velocity ,temperature ,skin friction reduces while thye concentration,Nusselt number and Sherwood number increases with increase in inertia coefficient E1.
- An increase in A1,B1>0,reduces thy velocity,temperature,skin friction,Sherwood number and enhances the concentration,Nusselt number on the wall,while an increase in A1,B1<0,a reversed effect is noticed.</p>
- The velocity, concentration reduces and thy temperature enhances with increasing Q1.Thr Nusselt number enhances and the Sherwood number reduces with Q1.
- Increasing Soret parameter (or decreasing Du)reduces the velocity, temperature and concentration.

- An increase in Forchheimer parameter F reduces the velocity and temperature and enhances the Nusselt number.
- Lesser the thermal diffusivity larger the velocity, temperature and concentration in the flow region.

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