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Radiation and Mass Transfer Effects on MHD Viscous Flow Past an Impulsively Started Vertical Plate through a Porous Medium

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Abstract

In this paper we have analyzed the effects of Radiation and mass transfer on MHD viscous flow. The flow is assumed to be past an impulsively vertical plate in porous medium. The governing equations of the flow are solved numerically using finite difference scheme. Finally the influence of various physical parameters involved in the equations of velocity, temperature and concentration are discussed through graphs. More over through this numerical study, we observed that velocity as well as temperature profiles decreases with an increase in Radiation parameter. Increase in Magnetic and permeability parameters decreases and increases the velocity respectively. **Keywords:** MHD, Radiation, Viscous dissipation, Vertical plate, Finite difference scheme.

NOMENCLATURE

С	-	dimensionless concentration, [-]
C	-	concentration, $[molm^{-3}]$
C_{w}^{\prime}	-	species concentration at the plate, $[molm^{-3}]$
$\mathcal{C}_{\infty}^{'}$	-	species concentration far away from the plate, $[molm^{-3}]$
c_p	-	specific heat at constant pressure, [Jkg ⁻¹ K ⁻¹]
Ď	-	chemical diffusivity, [m ² s ⁻¹]
Ec	-	Eckert number, [-]
$g^{'}$	-	acceleration due to gravity, [ms ⁻²]
Gc	-	solutal Grashof number, [-]
Gr	-	Grashot number, [-]
B_0	-	applied magnetic field, [-]
k	-	permeability parameter, [-]
κ	-	thermal conductivity, [Wm ⁻¹ K ⁻¹]
Pr	-	Prandtl number, [-]
p'	-	pressure, [kgm ⁻¹ s ⁻²]
Sc	-	Schmidt number, [-]
T,	-	dimensionless fluid temperature, [-]
T	-	fluid temperature, [K]
T'_{∞}	-	temperature of the fluid far away from the plate, [K]
t .	-	dimensionless time, [-]
ť	-	the time, [s]
u	-	dimensionless velocity of the fluid at the x' - direction, [-]
u'	-	velocity of the fluid at the x' - direction, [ms ⁻¹]
ν΄	-	velocity of the fluid at the y' - direction, [-]
v_0	-	suction velocity, [ms ⁻¹]
<i>x</i> ′	-	co-ordinate axis along the plate, [-]
y'	-	co-ordinate axis normal to the plate, [-]
Greek symbols		
β	-	coefficient of thermal expansion, [K ⁻¹]
eta^*	-	coefficient of concentration expansion, [(molm ⁻³) ⁻¹]
v	-	kinematic viscosity, [m ² s ⁻¹]
ρ	-	fluid density, [kgm ⁻³]

σ^{*}	-	Stefan – Boltzman constant, [Wm ⁻² K ⁻⁴]		
ω	-	dimensionless frequency of vibration of the fluid, [-]		
ω΄	-	frequency of vibration of the fluid, [rads ⁻¹]		
Subscript				
W	-	conditions at the wall		
∞	-	conditions in the free stream		

INTRODUCTION

There has been a renewed interest in studying magneto hydrodynamic (MHD) flow and heat transfer in porous and non-porous media due to the effect of magnetic fields on the performance of many systems using electrically conducting fluids. In addition, that this type of flow applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors and geothermal energy extractions. Saravana et al. (1) studied the mass transfer effects on MHD viscous flow past an impulsively started infinite vertical plate with constant mass flux. Gireesh Kumar and Ramakrishna (2) discussed the MHD flow of viscous fluid past an impulsively moving isothermal moving vertical plate through porous medium with chemical reaction. Noushima et al (3) investigated hydro magnetic free convective Rivin-Erickson flow through a porous medium with variable permeability.

At high operating temperatures, radiation effect can be quite significant. Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are example of such engineering areas. Ramana et al. (4) analyzed the mass transfer and radiation effects of unsteady MHD free convective fluid flow embedded in porous medium with heat generation/absorption. Rajput and Kumar (5) analyzed the radiation effects on MHD flow past an impulsively started vertical plate with variable heat and mass transfer. Suneetha et al. (6) studied the thermal radiation effects on MHD free convection flow past an impulsively started vertical plate with variable surface temperature and concentration. Ibrahim et al. (7) discussed the effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with heat source and suction. Reddy and Srihari (8) investigated the numerical solution of unsteady flow of a radiating and chemically reacting fluid with time-dependent suction.

Viscous mechanical dissipation effects are very important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. Mohammed Ibrahim and Bhaskar Reddy (9) studied the radiation and mass transfer effects on MHD free convection flow along a stretching surface with viscous dissipation and heat generation. Ramachandra Prasad and Bhaskar Reddy (10) discussed the radiation and mass transfer effects on an unsteady MHD convection flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with viscous dissipation. Hemanth Poonia and Chaudhary (11) analyzed the MHD free convection and mass transfer flow over an infinite vertical porous plate with viscous dissipation. Gregantopoulos et al. (12) investigated two-dimensional unsteady free convection and mass transfer flow of an incompressible viscous dissipative and electrically conducting fluid past an infinite vertical porous plate.

However the interaction of radiation with mass transfer of an electrically conducting and diffusing fluid impulsively started infinite vertical plate has received little attention. Hence an attempt is made to investigate the radiation effects on an unsteady visco-elastic second order Rivin-Erickson fluid past an impulsively started infinite vertical plate in the presence of foreign mass on taking into account of viscous dissipation. The governing equations are transformed by using similarity transformation and the resultant dimensionless equations are solved numerically using the finite difference method. The effects of various governing parameters on the velocity, temperature and concentration are shown in figures and analyzed in detail.

FORMULATION OF THE PROBLEM:

Consider the flow of a viscous incompressible visco-elastic second order Rivin-Erickson radiating fluid past an impulsively started infinite vertical plate through porous medium. The x'-axis is taken along the plate in the vertically upward direction and the y'-axis is chosen normal to the plate. Initially the temperature of the plate and the fluid is T'_{∞} , and the species concentration at the plate C'_w and in the fluid throughout C'_{∞} are assumed to be the same. At time t' > 0, the plate temperature is changed to T'_w causing convection currents to flow near the plate and mass is supplied at a constant rate to the plate and the plate starts moving upward due to impulsive motion, gaining a velocity of U_0 . A uniform magnetic field of intensity H_0 is applied in the y'-direction. Therefore the velocity and the magnetic field are given by $\bar{q} = (u, 0, 0)$ and $\bar{H} = (0, H_0, 0)$. The flow being slightly conducting the magnetic Reynolds number is much less than unity and hence the induced magnetic field can be neglected in comparison with the applied magnetic field in the absence of any input electric field, the flow is

governed by the following equations:

Continuity equation

$$\frac{\partial v'}{\partial y'} = 0$$
(1)
Momentum equation

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_{\infty}) + g\beta^{*}(C' - C'_{\infty}) + v\frac{\partial^{2}u'}{\partial y'^{2}} + K_{0}^{*}\frac{\partial^{3}u'}{\partial y'^{2}\partial t'} - \frac{\sigma\mu_{e}^{2}H_{0}^{2}}{\rho}u' - \frac{vu'}{k'} \quad (2)$$
Energy equation

$$\rho C_{p}\frac{\partial T'}{\partial t'} = \kappa \frac{\partial^{2}T'}{\partial y'^{2}} - \frac{\partial q_{r}}{\partial y'} + \mu \left(\frac{\partial u'}{\partial y'}\right)^{2} \quad (3)$$
Diffusion equation

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^{2}C'}{\partial y'^{2}} \quad (4)$$

The initial and boundary conditions are:

$$\begin{array}{cccc} t' \leq 0: & u' = 0, \ T' = \ T'_{\infty}, \ C' = \ C'_{\infty} \ for all \ y' \\ t' > 0: u' = \ U_0, \ T' = \ T'_{w}, \ C' = \ C'_{w} \ at \ y' = 0 \\ & u' \to 0, \ T' \to \ T'_{\infty}, \ C' \to \ C'_{\infty} \ as \ y' \to \infty \end{array} \right\}$$
(5)

Where u' is the velocity of the fluid along the plate in the x'-direction, t' is the time, g is the acceleration due to gravity, β is the coefficient of volume expansion, β^* is the coefficient of thermal expansion with concentration, T' is the temperature of the fluid near the plate, T'_{∞} is the temperature of the fluid, are the plate, T'_{∞} is the temperature of the fluid, C' is the species concentration in the fluid near the plate, C'_{w} is the concentration of the fluid, C'_{∞} is the species concentration in the fluid near the plate, v is the kinematic viscosity, K_0^* is the coefficient of kinematic visco-elastic parameter, σ is the electrical conductivity of the fluid, μ_e is the magnetic permeability, H_0 is the strength of applied magnetic field, ρ is the density of the fluid, D is the molecular diffusivity, U_0 is the velocity of the plate, k' is the permeability parameter.

By using the Rosseland approximation, the radiative heat flux q_r is given by

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T'}{\partial y'} \tag{6}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. It should be noted that by using the Rosseland approximation the present analysis is limited to optically thick fluids. Assuming that the differences in temperature within flow are such that T'^{4} can be expressed as a linear combination of the temperature, we expand T'^{4} in a Taylor's series about T'_{2} as follows

$$T^{\prime A} = T^{\prime A}_{\infty} + 4T^{\prime 3}_{\infty}(T^{\prime} - T^{\prime}_{\infty}) + 6T^{\prime 2}_{\infty}(T^{\prime} - T^{\prime}_{\infty})^{2} + \dots$$

and neglecting higher order terms beyond the first degree in $(T^{\prime} - T^{\prime}_{\infty})$ we get
$$T^{\prime A} \cong 4T^{\prime A}_{\infty}T^{\prime} - 3T^{\prime A}_{\infty}$$
(7)

From equations (6), (7) we obtain

$$\frac{\partial q_r}{\partial y'} = -\frac{16\sigma^* T_{\infty}^{\prime 4}}{3k^*} \frac{\partial^2 T'}{\partial y'^2}$$
(8)

From (3) and (8) we have

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma^* T_{\infty}^{\prime 4}}{3k^*} \frac{\partial^2 T'}{\partial y'^2} + \mu \left(\frac{\partial u'}{\partial y'}\right)^2 \tag{9}$$

On introducing the following dimensionless variables and parameters

$$u = \frac{u'}{U_0}, t = \frac{t'U_0^2}{v}, y = \frac{y'U_0}{v}, \theta = \frac{T' - T'_{\infty}}{T'_w - T'_{\infty}}, C = \frac{C' - C'_{\infty}}{(j''v/DU_0)}$$

$$Gr = \frac{vg\beta(T'_w - T'_{\infty})}{U_0^3}, Gc = \frac{vg\beta^*(j''v/DU_0)}{U_0^3}, \lambda = \frac{K_0^*U_0^2}{v^2}, \Pr = \frac{v\rho C_p}{\kappa}$$

$$M = \frac{\sigma\mu_e^2 H_0^2 v}{\rho U_0^2}, Ec = \frac{\mu U_0}{v\rho C_p(T'_w - T'_{\infty})}, Sc = \frac{v}{D}, N = \frac{k_e k}{4\sigma_s T_s^{-3}}, k = \frac{k'U_0^2}{\rho}$$
(10)

In terms of the above dimensionless quantities, Equations (2), (4) and (9) reduces to

$$\frac{\partial u}{\partial t} = Gr\theta + Gc C + \frac{\partial^2 u}{\partial y^2} + \lambda \frac{\partial^3 u}{\partial y^2 \partial t} - \left(M + \frac{1}{k}\right)u \tag{11}$$

$$\frac{\partial\theta}{\partial t} = \frac{1}{Pr} \left(1 + \frac{4}{3N} \right) \frac{\partial^2 \theta}{\partial y^2} + Ec \left(\frac{\partial u}{\partial y} \right)^2$$
(12)

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} \tag{13}$$

With the following initial and boundary conditions:

$$t \le 0: u = 0, \quad T = 0, \quad C = 0 \quad forall \ y \\ t > 0: u = 1, \quad \theta = 1, \quad C = 1 \quad at \quad y = 0 \\ u \to 0, T \to 0, \quad C \to 0 \quad as \quad y \to \infty$$
 (14)

NUMERICAL TECHNIQUE:

Equations (11) - (13) are coupled non-linear partial differential equations, and are to be solved by using the initial and boundary conditions (14). However, exact solution is not possible for this set of equations and hence we solve these equations by finite-difference method. The equivalent finite difference scheme of equations for (11) - (13) is as follows:

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = Gr\theta_{i,j} + Gc.C_{i,j} + \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{(\Delta y)^2} + \lambda \left(\frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1} - u_{i+1,j} + 2u_{i,j} - u_{i-1,j}}{\Delta t.(\Delta y)^2}\right) - \left(M + \frac{1}{k}\right)u_{i,j}$$
(15)

$$\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} = \frac{1}{Pr} \left(1 + \frac{4}{3N} \right) \left(\frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta y)^2} \right) + Ec \left(\frac{u_{i+1,j} - u_{i,j}}{\Delta y} \right)^2$$
(16)

$$\frac{C_{i,j+1} - C_{i,j}}{\Delta t} = \frac{1}{Sc} \frac{C_{i+1,j} - 2C_{i,j} + C_{i-1,j}}{(\Delta y)^2}$$
(17)

Where index i refers to y and j to time t, and during computation $\Delta y = 0.1$ and $\Delta t = 0.001$. From the initial conditions in (14), we have the following equivalent:

 $u(i,0) = 0, \theta(i,0) = 0, C(i,0) = 0$ for all *i* The boundary conditions from (14) are expressed in finite- difference form as follows: $u(0,j) = 1, \theta(0,j) = 1, C(0,j) = 1$ for all *j* $u(i_{max}, j) = 0, \theta(i_{max}, j) = 0, C(i_{max}, j) = 0$ for all *j* Here i_{max} was taken as ∞

The values of 'C, θ and u' are known at all grid points at t = 0 from the initial conditions. The values of 'C, θ and u' at time level 'j+1' using the known values at previous time level 'j' are calculated as follows. The finite difference equation (17) at every internal nodal point on a particular j- level constitutes a tri-diagonal system of equations. Such a system of equations is solved by using finite difference scheme. Thus, the values of 'C' are known at every nodal point at (j+1)th time level. Similarly the values of ' θ ' are calculated from equation (16). Using the values of 'C' and ' θ ' at (j+1)th time level in equation (15), the values of 'u' at (j+1)th time level are found in similar manner. This process is continued to obtain the solution till desired time 't'. Thus the values of 'C, θ and u' are known, at all grid points in the rectangular region at the desired time level. During computation Δt was chosen as 0.001.

To judge the accuracy of the convergence and stability of finite difference scheme, the same program was run with different values of Δt i.e., $\Delta t = 0.0009, 0.0001$ and no significant change was observed. Hence, we conclude that the finite-difference scheme is stable and convergent.

RESULTS AND DISCUSSION:

The formulation of the problem that accounts for the effects of radiation and viscous dissipation on the flow of an

incompressible viscous fluid along an infinite vertical plate in the presence of transverse magnetic field through porous medium was accomplished in the preceding sections. The governing equations of the flow field were solved by using finite difference method. In order to get a physical insight of the problem, the physical quantities are computed numerically for different values of the governing parameters viz., Grashof number Gr, solutal Grashof number Gc, Prandtl number Pr, Schmidt number *Sc*, the radiation parameter N, Eckert number Ec, Hartmann number (magnetic parameter) M and permeability parameter k.

The effect of radiation parameter N on the transient velocity (u) and temperature (θ) variations are depicted in Figs. 1(a) and 1(b). The radiation parameter N (i.e., Stark number) defines the relative contribution of conduction heat transfer to thermal radiation transfer. As N increases, considerable reduction is observed in both velocity and temperature profiles.









Figs. 2(a) and 2(b) illustrate the behavior of the velocity (u) and temperature (θ) for different values of the Prandtl number Pr. The numerical result shows that the effect of increasing values of Prandtl number results in a decreasing velocity. From Fig. 2(b), it is observed that an increase in the Prandtl number results in a decreasing temperature.

The effects of viscous dissipative heat (Ec) on the transient velocity (u) as well as temperature (θ) have been plotted in Figs. 3(a) and 3(b). It is noticed that an increase in viscous dissipative heat leads to increase in both the velocity as well as the temperature.



Fig. 2(a): Velocity profiles for different values of Prandtl number 'Pr' when M = 1, Sc = 0.24, k = 0.01, Gr = 5, Gc = 5, Ec = 0.02, N = 1



Fig. 2(b): Temperature profiles for different values of Prandtl number 'Pr' when M = 1, Sc = 0.24, k = 0.01, Gr = 5, Gc = 5, Ec = 0.02, N = 1

















Fig. 5(a): Velocity profiles for different values of Schmidt number 'Sc' when M = 1, Pr = 0.71, k = 0.01, Gr = 5, Gc = 5, Ec = 0.02, N = 1







Fig. 7: Velocity profiles for different values of permeability parameter 'N' when M = 1, Sc = 0.24, Pr = 0.71, Gr = 5, Gc = 5, Ec = 0.02, N = 1

The velocity profiles for different values of the Grashof number Gr are described in Fig. 4(a). It is observed that an increase in Gr leads to rise in the values of velocity. For the case of different values of the solutal Grashof number Gc, the velocity profiles are shown in the Fig. 4(b). It is observed that an increase in Gc leads to a rise in the values of velocity.

Figs. 5(a) and 5(b) illustrate the influence of the Schmidt number Sc on the transient velocity and concentration. As Sc increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reduction in the velocity and concentration profiles is

accompanied by simultaneous reductions in velocity and cencentration boundary layer. These behaviors are clear from Figs. 5(a) and 5(b). The effect of different values of Hartmann number (magnetic parameter M) for velocity profile is shown in Fig.6. It is observed that an increase in M leads to decrease in velocity. The effect of different values of porosity parameter k for velocity profile is shown in Fig. 7. It is observed that an increase in k leads to increase in velocity.

CONCLUSIONS:

We summarize below the following results of physical interest on the velocity, temperature and concentration distributions of the flow field.

- 1. The velocity decreases with the increase of the radiation parameter.
- 2. A growing magnetic parameter or Prandtl number or Schmidt number parameter retards the velocity of the flow field at all points.
- 3. The effect of increasing Grashof number or modified Grashof number or permeability parameter or Eckert number is to accelerate velocity of the flow field at all points.
- 4. A growing Prandtl number decreases temperature of the flow field at all points
- 5. The growing Schmidt number decreases the concentration of the flow field at all points.

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