# MHD Heat Transfer Oscillatory Flow Jeffrey Fluid In An Inclined Channel Filled With Porous Medium

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#### Abstract

This paper investigate the effect of heat transfer on unsteady MHD and radiative oscillatory flow of a Jeffrey fluid in a horizontal channel filled with saturated porous medium and non-uniform walls temperature. Closed –form analytical solution are therefore constructed for the problem. The effects of various dimensionless parameters on the velocity and temperature profiles are considered and discussed detailed through graphs. It is observed that, the

velocity increases with increase in  $N, \alpha, \lambda_1$  and Gr. The velocity also decreases with increase H and Re. **Keywords**: MHD; Jeffrey fluid; Oscillatory flow; heat transfer.

#### INTRODUCTION

The effect of heat transfer on unsteady MHD oscillatory flow of fluid in horizontal media are encountered in a wide range of engineering and industrial applications such as molten iron flow, recovery extraction of crude oil, geothermal systems. Many chemical engineering processes like metallurgical and polymer extrusion processes involve cooling of a molten liquid being stretched in a cooling system; the fluid mechanical properties of penultimate product depend mainly on the cooling liquid used and the rate of stretching. Some polymers fluids like polyethylene oxide and polysobutylene solutions in a cetane, having better electromagnetic properties are normally used as cooling liquid as their flow can be regulated by external magnetic fields in order to improve the quality of the final product. Also, the radiative heat transfers is an important factor of thermodynamics of very high temperature systems such as electric furnaces, solar collectors, storage of nuclear wastes packed bed catalytic reactors, satellites, steel rolling, cryogenic engineering etc. The study of such flow under the influence of magnetic field and heat transfer has attracted the interest of many investigators and researchers.

Several simple flow problems associated with classical hydrodynamics have received new attraction within the more general context of magnetohydrodynmaics (MHD). Cogley et al.[1968] are discussed Differential approximation for radiative heat transfer in non-linear equations-grey gas heat equilibrium. Raptis et al. [1982] have analyzed hyderomagnetic free convection flow through a porous medium between two parallel plates. Moreau [1990] are discussed Magneto hydrodynamics studies in the technical fields. Anoter important field of application is electromagnetic propulsion. Basically, an electro-magnetic propulsion system consists of a power source(such as a nuclear reactor), plasma, and tube through which the plasma is accelerated by electromagnetic force. Unsteady MHD convection heat transfer past a semi-infinite vertical moving plate with variable section are studied by Kim[2000].Makinde et al. [2001] have studied MHD steady flow in a channel with slip at the permeable boundaries. Heat transfer to MHD oscillatory flow in a channel filled with porous medium are analyzed Makide et al.[2015] are steadied MHD flow of a Jeffery fluid with Newtonian heating .Kavith et al[2015] are analyzed per static transport of a Jeffery fluid in constant with Newtonian fluid in an inclined channel. Ali et al.[2016] are discussed analytic solution for oscillatory flow in a channel for Jeffery fluid .

#### MATHEMATICAL FORMULATION

Consider the flow of a conducting optimally thin fluid in a channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer as shown in Fig.1.It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. Take the Cartesian coordinates system (x,y) where ox lies along the center of the channel, y is the distance measured in the normal section .Then, assuming a Bossiness incompressible fluid model, the equations governing the motion are given as:

(1)



$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{v}{1+\lambda_1} \frac{\partial^2 u}{\partial y^2} - \frac{v}{(1+\lambda_1)k} u - \frac{\sigma_e B_0^2}{\rho} u + g\beta(T-T_0)\sin\alpha$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q}{\partial y}$$
(2)

$$u = 0, T = T_w,$$
 on  $y = 1$  (3)  
 $u = 0, T = T_0,$  on  $y = 0$  (4)

Following Cogley et al.[1], it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y} = 4\gamma^2 (T_0 - T) \tag{5}$$

Where  $\gamma$  is the mean radiation absorption co-efficient. The following dimensionless variable and parameters are introduced:

$$Re = \frac{Ua}{v}, \bar{x} = \frac{x}{a}, \bar{u} = \frac{u}{U}, \bar{y} = \frac{y}{a}, \theta = \frac{T - T_0}{T_w - T_0}, H^2 = \frac{a^2 \sigma_e B_0^2}{\rho v}, \bar{t} = \frac{tU}{a}$$
$$\overline{P} = \frac{aP}{\rho v U}, Da = \frac{K}{a^2}, Gr = \frac{g\beta(T_w - T_0)}{vU}, Pe = \frac{Ua\rho c_p}{k}, N^2 = \frac{4\alpha^2 a^2}{k}$$
(6)

Where U is the mean velocity. The dimensionless governing equations together with the appropriate boundary conditions, (neglecting the bars for clarity) can be written as

$$\operatorname{Re}\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2} - (\frac{s^2}{1+\lambda_1} + H^2)u + Gr\theta\sin\alpha$$
(7)

$$Pe\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial y^2} + N^2\theta$$
(8)

The boundary conditions are  

$$u = 0, \theta = 1, \dots, v = 1$$
(0)

$$y = 0, \theta = 0 \qquad \qquad y = 0 \tag{9}$$

$$u = 0, 0 = 0, \quad \text{on} \quad y = 0$$
 (10)

(16)

$$s^2 = \frac{1}{Da}$$

Where

## Method of Solution: In order to solve equat

In order to solve equations (7)-(10) for purely oscillatory flow,

Let 
$$-\frac{\partial P}{\partial x} = \lambda e^{i\omega t}, u(y,t) = u_0(y)e^{i\omega t}, \theta(y,t) = \theta_0(y)e^{i\omega t}$$
(11)

Where  $\lambda$  is a constant and  $\omega$  is the frequency of the oscillation. Substituting the above expressions in equation (11) into equations (7) to (10), we obtain :

$$\frac{d^2 u_0}{dy^2} - m_2^2 u_0 = -\lambda (1+\lambda_1) - (1+\lambda_1) Gr \theta_0 \sin \alpha$$
(12)

$$\frac{d\theta_0}{dy^2} + m_1^2 \theta_0 = 0 \tag{13}$$

With

$$u_0 = 0, \theta_0 = 1,$$
 on  $y = 1$  (14)

$$u_0 = 0, \theta_0 = 0, \qquad \text{on } y = 0 \tag{15}$$

Where  $m_1 = \sqrt{N^2 - i\omega Pe}$  and  $m_2 = \sqrt{s^2 + H^2(1 + \lambda_1) + (1 + \lambda_1)i\omega Re}$  equations (12) to (15) are solved and the solution for fluid velocity and temperature are given as follows:

$$u(y,t) = \begin{cases} \frac{\lambda(1+\lambda_{1})}{m_{2}^{2}} [1-\cosh m_{2}y] + \frac{\lambda(1+\lambda_{1})}{m_{2}^{2}} \left[\frac{\sinh m_{2}y}{\sinh m_{2}}\right] [\cosh m_{2}-1] + \\ \frac{(1+\lambda_{1})Gr}{m_{1}^{2}+m_{2}^{2}} \left[\frac{\sin m_{1}y}{\sin m_{1}} - \frac{\sin m_{2}y}{\sin m_{2}}\right] \sin \alpha \end{cases} e^{i\omega t}$$

$$\theta(y,t) = \frac{\sin m_1 y}{\sin m_1} e^{i\omega t}$$
<sup>(17)</sup>

The shear stress at the upper wall of the channel is given by

$$\tau = -\mu \frac{\partial u}{\partial y}$$

$$= \begin{cases} \frac{(1+\lambda_{1})Gr}{m_{1}^{2}+m_{2}^{2}} \left[ \frac{m_{1}\cosh(m_{1}y)}{\sin m_{1}} - \frac{m_{2}\cosh(m_{2}y)}{\sinh m_{2}} \right] \sin \alpha + \frac{\lambda(1+\lambda_{1})}{m_{2}} \left[ \frac{\cosh m_{2}y}{\sinh m_{2}} \right] [\cosh m_{2}-1] \\ + \frac{\lambda(1+\lambda_{1})}{m_{2}} [\sinh m_{2}y] \end{cases} e^{i\omega t}$$
(18)

### **RESULT AND DISCUSSION**

In this paper, MHD heat transfer oscillatory flow Jeffery fluid in an inclined channel filled with porous medium are investigated and the results are discuss for various physical parameters Peclet number Pe, Grashof number Gr, Reynolds number Re ,time variable t, frequency of Oscillation  $\omega$ , porous medium shape factor s, Hartmann number H, Radiation parameter N, angle of inclination  $\alpha$  and Jeffrey parameter  $\lambda_1$ . We have taken the real part of the results obtained in equation (16)-(18) and made use of the following parameter values

 $Pe = 0.71, Gr = 1, Re = 1, \lambda = 1, t = 0, \omega = 1, s = 1, H = 1, N = 1, \alpha = \frac{\pi}{4}$  and  $\lambda_1 = 1$ . These values kept as Common in entire study except for varied values as displayed in figures 2-8.

The variation of velocity u with y is calculated for different values of N or  $\alpha$  or  $\lambda_1$  or Gr and shown in Figures 1,2,3 and 4. It is seen that velocity increases with increasing N or  $\alpha$  or  $\lambda_1$  or Gr.

The variation of velocity u with y is calculated for different values of H or Re and shown in Figures 5 and 6. It is seen that velocity decreases with increasing H or Re.

The variation of Temperature  $\theta$  with y is calculated for different values of N and shown in Figure 8. It is seen that Temperature increases with increasing N.





Figure 4: Velocity profile for different values of  $\lambda_1$ 



Figure 3:Velocity profile for different values of  $\alpha$ 



Figure 5: Velocity profile for different values of Gr



Figure 4: Velocity profile for different values of H



Figure 4:Tempurture profile for different values of N

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Figure 4: Velocity profile for different values of Re