Effect of Radiation and Chemical Reaction on Transient MHD Free Convective Flow over a Vertical Plate Through Porous Media

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Abstract:
This paper analyzes the Magneto hydrodynamic, Radiation and chemical reaction effects on unsteady flow, heat and mass transfer characteristics in a viscous, incompressible and electrically conduction fluid over a semi-infinite vertical porous plate through porous media. The porous plate is subjected to a transverse variable suction velocity. The transient, non-linear and coupled governing equations have been solved adopting a perturbative series expansion about a small parameter, ε. The effects of governing parameters on the flow variables are discussed graphically.

Keywords: Transient velocity, MHD, Chemical reaction, Radiation and Rarefaction Parameter.

Introduction:
Natural convection flow over vertical surfaces immersed in porous media has paramount importance because of its potential applications in soil physics, geohydrology, and filtration of solids from liquids, chemical engineering and biological systems. Study of fluid flow in porous medium is based upon the empirically determined Darcy’s law. Such flows are considered to be useful in diminishing the free convection, which would otherwise occur intensely on a vertical heated surface. In addition, recent developments in modern technology have intensified more interest of many researchers in studies of heat and mass transfer in fluids due to its wide applications in geothermal and oil reservoir engineering as well as other geophysical and astrophysical studies.

Cramer, K. R. and Pai, S. I. [1] taken transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible Muthucumaraswamy et al. [2] have studied the effect of homogenous chemical reaction of first order and free convection on the oscillating infinite vertical plate with variable temperature and mass diffusion. Sharma [3] investigate the effect of periodic heat and mass transfer on the unsteady free convection flow past a vertical flat plate in slip-flow regime when suction velocity oscillates in time. Chaudhary and Jha [4] studied the effects of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip-flow regime. Anjalidevi et al. [5] have examined the effect of chemical reaction on the flow in the presence of heat transfer and magnetic field. Muthucumaraswamy et al. [6] have investigated the effect of thermal radiation effects on flow past an impulsively started infinite vertical plate in the presence of first order chemical reaction.

Moreover, Al-Odat and Al-Azab [7] studied the influence of magnetic field on unsteady free convective heat and mass transfer flow along an impulsively started semi-infinite vertical plate taking into account a homogeneous chemical reaction of first order. The effect of radiation on the heat and fluid flow over an unsteady stretching surface has been analyzed by El-Aziz [8]. Singh et. al. [9] studied the heat transfer over stretching surface in porous media with transverse magnetic field. Singh et. al. [10] and [11] also investigated MHD oblique stagnation-point flow towards a stretching sheet with heat transfer for steady and unsteady cases. Elbashbeshy et. al. [12] investigated the effects of thermal radiation and magnetic field on unsteady boundary layer mixed convection flow and heat transfer problem from a vertical porous stretching surface. Ahmed Sahin studied influence of chemical reaction on transient MHD free Convective flow over a vertical plate. Recently, The chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification have been presented by Kandasamy et al.[13]. The opposing buoyancy effects on simultaneous heat and mass transfer by natural convection in a fluid saturated porous medium investigated by Angirasa et al.[14]. Ahmed [15] investigates the effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate. Ahmed Sahin [16] studied the Magneto hydrodynamic and chemical reaction effects on unsteady flow, heat and mass transfer characteristics in a viscous, incompressible and electrically conduction fluid over a semi-infinite vertical porous plate in a slip-flow regime.
The objective of the present study is to investigate the effect of various parameters like chemical Reaction parameter, thermal Grashof number, mass Grashof number, rarefaction parameter, magnetic field parameter, radiation parameter, suction parameter on convective heat transfer along an inclined plate embedded in porous medium. The governing non-linear partial differential equations are first transformed into a dimensionless form and thus resulting non-similar set of equations has been solved using the perturbation technique. Results are presented graphically and discussed quantitatively for parameter values of practical interest from physical point of view.

Mathematical Analysis:

Consider the unsteady two dimensional MHD free convective flow of a viscous incompressible, electrically conducting and radiating fluid in an optically thin environment past an infinite heated vertical porous plate embedded in a porous medium in presence of thermal and concentration buoyancy effects. Let the x-axis be taken in vertically upward direction along the plate and y-axis is normal to the plate. It is assumed that there exist a homogeneous magnetic field is applied in the direction perpendicular to the plate. The viscous dissipation and the Joule heating effects are assumed to be negligible in the energy equation. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible [22]. Also it is assumed that there is no applied voltage, so that the electric field is absent. The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species, which are present, and hence the Soret and Dufour effects are negligible and the temperature in the fluid flowing is governed by the energy concentration equation involving radiative heat temperature. Under the above assumptions as well as Boussinesq’s approximation, the equations of conservation of mass, momentum, energy and concentration governing the free convection boundary layer flow over a vertical porous plate in porous medium can be expressed as:

\[
\frac{\partial v^1}{\partial y^1} = 0
\]

\[
\frac{\partial u^1}{\partial t^1} - v_0 \left(1 + \varepsilon A e^{i\omega t}\right) \frac{\partial u^1}{\partial y^1} = g \beta \left(T^1 - T_{\infty}^1\right) + g \beta \left(C^1 - C_{\infty}^1\right) + \nu \frac{\partial^2 u^1}{\partial y^2} - \frac{\rho \beta^2}{\rho} \frac{u^1}{k^1} - v^1 \frac{u^1}{k^1}
\]

\[
\rho C_p \left[\frac{\partial T^1}{\partial t^1} - v_0 \left(1 + \varepsilon A e^{i\omega t}\right) \frac{\partial T^1}{\partial y^1}\right] = k \frac{\partial^2 T^1}{\partial y^2} - \frac{\partial q_i}{\partial y^1}
\]

\[
\frac{\partial C^1}{\partial t^1} - v_0 \left(1 + \varepsilon A e^{i\omega t}\right) \frac{\partial C^1}{\partial y^1} = D \frac{\partial^2 C^1}{\partial y^2} - K^1 \left(C^1 - C_{\infty}^1\right)
\]

Where \(g, T^1, C^1, B_0, D, \sigma, k, C_p, \nu, \beta, \beta_i, q^1\) and \(K\) are acceleration due to gravity, fluid temperature, species concentration, magnetic field, chemical molecular diffusivity, electrical conductivity, thermal conductivity, specific heat constant pressure, kinematic viscosity, density, coefficient of volume expansion for heat transfer, volumetric coefficient of expansion with species concentration, radiative heat flux and chemical reaction parameter respectively. The corresponding boundary conditions of the problem are

\[
u^1 = L \left(\frac{\partial u^1}{\partial y^1}\right), T^1 = T_w^1 + \left(T_{\infty}^1 - T_w^1\right) e^{i\omega t}
\]

\[
C^1 = C_w^1 + \left(C_{\infty}^1 - C_w^1\right) e^{i\omega t} \text{ at } y^1 = 0
\]

\[
u^1 \to 0, T^1 \to T_{\infty}^1, C^1 \to C_{\infty}^1 \text{ at } y^1 \to \infty
\]
Where \( T^w \) and \( T^\infty \) is the temperature at the wall and infinity, \( C^w \) and \( C^\infty \) is the species concentration at the wall and at infinity respectively.

By using Rosseland approximation the radiative heat flux \( q^1_r \) is given by

\[
q^1_r = -\frac{4\sigma_\gamma}{3k} \frac{\partial T^4}{\partial y}
\]

Where \( \sigma_\gamma \) is the Stefan Boltzmann constant and \( k^\gamma \) is the mean absorption coefficient.

By expanding \( T^4_w \) in to the Taylor series about \( T^4_i \) which after neglecting higher order terms takes the form

\[
T^4_w \approx 4T^4_i T^4_w - 3T^4_i .
\]

From the equation of continuity (1), it is clear that the suction velocity at the plate is either a constant or a function of time only. Hence, the suction velocity normal to the plate is assumed to be in the form

\[
v^1 = v_0 (1 + \epsilon A e^{i \omega \epsilon t})
\] 

We now introduce the following non-dimensional quantities into the equations (1) to (5)

\[
y = \frac{v_0 y}{v}, u = \frac{u^1}{v_0}, \tau = \frac{t}{v^3/v_0^2}, \omega = 4\omega^1 v/v_0^2, \nu = \nu/\rho, Pr = \frac{\mu C^\rho}{k}
\]

\[
\theta = \frac{T^1_i - T^1_w}{T^1_w - T^1_\infty}, \phi = \frac{C^1_i - C^1_w}{C^1_w - C^1_\infty}, Gr = \frac{v g \beta (T^1_i - T^1_w)}{v^3}, \quad Gc = \frac{v g \beta (C^1_i - C^1_w)}{v^3}
\]

\[
M = \frac{\sigma f^2 |v|}{\rho v_0^2}, Sc = \frac{\nu}{D}, h = \frac{v_0 L}{v}, K_r = \frac{k^1 v_0^2}{v^2}, \kappa = \frac{k^1 v_0^2}{v^2}, R = \frac{kk^1 v_0^2}{4\sigma T^4_i v^2}.
\] 

The governing equations (2) to (4) can be rewritten in the non-dimensional form as follows

\[
\frac{1}{4} \frac{\partial u}{\partial t} - \left(1 + \epsilon A e^{i \omega \epsilon t}\right) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr \theta + Gc \phi - (M + \kappa) u
\] 

\[
\frac{1}{4} \frac{\partial \theta}{\partial t} - \left(1 + \epsilon A e^{i \omega \epsilon t}\right) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \left(1 + \frac{4}{3R}\right) \frac{\partial^2 \theta}{\partial y^2}
\]

\[
\frac{1}{4} \frac{\partial \phi}{\partial t} - \left(1 + \epsilon A e^{i \omega \epsilon t}\right) \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - K_r \phi
\]

The transformed boundary conditions are

\[
u = h \frac{\partial u}{\partial y}, \quad \theta = 1 + \epsilon A e^{i \omega \epsilon t} \quad \phi = 1 + \epsilon A e^{i \omega \epsilon t} \quad \text{at} \quad y = 0\]

\[
u \to 0, \theta \to 0, \phi \to 0 \quad \text{at} \quad y \to \infty
\]

**Solution of the Problem:**

The equations (8) to (10) are coupled, non-linear partial differential equations and these cannot be solved in closed form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. So this can be done, when the amplitude of oscillations (\( \epsilon << 1 \)) is very small, we can assume the solutions of flow velocity \( u \), temperature field \( \theta \) and concentration \( \phi \) in the neighborhood of the plate as:

\[
f(y, t) = f_1(y) + \epsilon A e^{i \omega \epsilon t} f_2(y) + \ldots.
\] 

Where \( f \) stands for \( u, \theta \) or \( \phi \)
\[
\theta_0^{11} + \Pr \left( 1 + \frac{4}{3R} \right) \theta_0^1 = 0 \quad (13)
\]

\[
\theta_1^{11} + \Pr \left( 1 + \frac{4}{3R} \right) \theta_1^1 - i \omega \Pr \left( 1 + \frac{4}{3R} \right) \theta_1^1/4 = -2A \Pr \left( 1 + \frac{4}{3R} \right) \theta_0^1 \quad (14)
\]

\[
u_0^{11} + u_0^1 - (M + \kappa) u_0 = -Gr \theta_0^1 - Gc \phi_0^1 \quad (15)
\]

\[
u_1^{11} + u_1^1 - (M + \kappa + i \omega/4) u_1 = -Gr \theta_1^1 \quad (16)
\]

\[
\phi_0^{11} + Sc \phi_0^1 - K_r Sc \phi_0 = 0 \quad (17)
\]

\[
\phi_1^{11} + Sc \phi_1^1 - Sc \left( K_r + i \omega/4 \right) \phi_1 = ASc \phi_0^1 \quad (18)
\]

where prime denotes differentiation with respect to \( y \). The corresponding boundary conditions are

\[
u_0 = h \left( \frac{\partial u_0}{\partial y} \right), \quad u_1 = h \left( \frac{\partial u_1}{\partial y} \right), \quad \theta_0 = 1
\]

\[
\theta_1 = 1, \phi_0 = 1, \phi_1 = 1 \text{ at } y = 0
\]

\[
u_0 \rightarrow 0, u_1 \rightarrow 0, \theta_0 \rightarrow 0, \theta_1 \rightarrow 0, \phi_0 \rightarrow 0, \phi_1 \rightarrow 0 \text{ at } y \rightarrow \infty \quad (19)
\]

The solutions of the equations (13) to (18) under the boundary the conditions (19) are

\[
\theta_0 (y) = e^{-Pr \left( 1 + \frac{4}{3R} \right) y}, \quad (20)
\]

\[
\phi_0 (y) = e^{-ny}, \quad (21)
\]

\[
\theta_1 (y) = B_1 e^{-\lambda y} + B_2 e^{-Pr \left( 1 + \frac{4}{3R} \right) y}, \quad (22)
\]

\[
u_0 (y) = B_3 e^{-ny} - B_4 e^{-Pr \left( 1 + \frac{4}{3R} \right) y} - B_5 e^{-ny}, \quad (22)
\]

\[
\phi_1 (y) = B_6 e^{-ny} - B_7 e^{-ny}, \quad (23)
\]

\[
u_1 (y) = B_{10} e^{-ny} - B_4 e^{-ny} - B_{11} e^{-ny} + B_{12} e^{-ny} - B_{13} e^{-Pr \left( 1 + \frac{4}{3R} \right) y} - B_{14} e^{-ny} \quad (24)
\]

Results and Discussion:

The effects of governing parameters like magnetic field, chemical reaction, rarefaction parameter, suction parameter, thermal Grashof number as well as mass Grashof number porosity parameter and radiation parameter on the transient velocity have been presented in the respective Figures 1 to 20 for both the cases of air and water and in presence of foreign species \( Sc = 0.22 \), \( \omega = \frac{\pi}{2} \). Here we restricted our discussion to the aiding of favourable case only, for fluids with Prandtl number \( Pr = 0.71 \) which represent air at 20\(^\circ\)C at 1 atmosphere and for fluids \( Pr = 7 \) which represent water. The value of thermal Grashof number \( Gr \) is taken to be positive, which corresponds to the cooling of the plate. The diffusing chemical species of most common interest in air has Schmidt number (Sc) and is taken for Hydrogen (Sc = 0.22), Oxygen (Sc = 0.66), and Carbon dioxide (Sc = 0.94).

Figures (1) and (2) shows the effect of Grashof number on transient velocity for \( Pr = 0.71 \) and \( Pr = 7 \). Here Chemical reaction parameter (\( Kr=0.1 \)), Magnetic field parameter (\( M=1 \)),Radiation parameter (\( R=3 \)),Suction parameter (\( A=0.5 \)),Refraction parameter (\( h = 0.2 \)),Porosity parameter (\( \kappa=0.1 \)),Frequency
parameter ($\omega=30$), Mass Grashof number ($G_c=5$) on the velocity profiles. We observe that the velocity gradient at the surface increase with the increase of Grashof number.

Figures (3) and (4) represents the effect of porosity parameter on transient velocity for $Pr = 0.71$ and $Pr = 7$. Here Chemical reaction parameter ($Kr=0.1$), Magnetic field parameter ($M=1$), Radiation parameter ($R=2$), Suction parameter ($A=0.5$), Refraction parameter ($h = 0.2$), Frequency parameter ($\omega=30$), Mass Grashof number ($G_c=5$), Thermal Grashof number ($Gr=5$) on the velocity profiles. We observe that the velocity gradient at the surface increase with the decrease of porosity Parameter.

Figures (5) and (6) shows the effect of Radiation parameter on transient velocity for $Pr = 0.71$ and $Pr =7$. Here Chemical reaction parameter ($Kr=0.1$), Magnetic field parameter ($M=1$), Suction parameter ($A=0.5$), Refraction parameter ($h = 0.2$), Porosity parameter ($\kappa=0.1$), Frequency parameter ($\omega=30$), Mass Grashof number ($G_c=5$) on the velocity profiles. We observe that the velocity gradient at the surface increase with the decrease of radiation parameter but it is reversed in case of water.

Figures (7) and (8) represents the effect of Chemical reaction parameter on transient velocity for $Pr = 0.71$ and $Pr =7$. Magnetic field parameter ($M=1$), Radiation parameter ($R=2$), Suction parameter ($A=0.5$), Refraction parameter ($h = 0.2$), Frequency parameter ($\omega=30$), Mass Grashof number ($G_c=5$), Thermal Grashof number ($Gr=5$) on the velocity profiles. We observe that the velocity high at the surface with the decrease of Chemical reaction parameter.

Figures (9) and (10) shows the effect of Magnetic field parameter on transient velocity for $Pr = 0.71$ and $Pr =7$. Here Chemical reaction parameter ($Kr=0.1$), Radiation parameter ($R=2$), Suction parameter ($A=0.5$), Refraction parameter ($h = 0.2$), Porosity parameter ($\kappa=0.1$), Frequency parameter ($\omega=30$), Mass Grashof number ($G_c=5$) on the velocity profiles. We observe that the velocity gradient at the surface increases with the decrease of Magnetic parameter.

Figures (11) and (12) shows the effect of rarefaction parameter on transient velocity for $Pr = 0.71$ and $Pr =7$. Here Chemical reaction parameter ($Kr=0.1$), Magnetic field parameter ($M=1$), Radiation parameter ($R=3$), Suction parameter ($A=0.5$), Refraction parameter ($h = 0.2$), Porosity parameter ($\kappa=0.1$), Frequency parameter ($\omega=30$), Mass Grashof number ($G_c=5$) on the velocity profiles. We observe that the velocity gradient at the surface decrease with the increase of rarefaction parameter. From figure (13) it is observed that effect of suction parameter on velocity. Here velocity profiles decreases decreases in water and flow is together. From figures (14) and (15) we observe the same results as in figures (1) and (2).

Figure (16) shows the effect of porosity parameter on temperature profiles for $Pr = 0.71$ and $Pr =7$. Here Chemical reaction parameter ($Kr=0.1$), Magnetic field parameter ($M=1$), Radiation parameter ($R=3$), Suction parameter ($A=0.5$), Refraction parameter ($h = 0.2$), Porosity parameter ($\kappa=0.1$), Frequency parameter ($\omega=30$), Mass Grashof number ($G_c=5$) on the velocity profiles.

Figure (17) shows the dimensionless concentration profiles ($\phi$) for chemical reaction ($K$), Schmidt number and suction parameter ($A$) against frequency parameter. A decrease in concentration with increasing $K$ as well as $Sc$ is observed from this figure, while the concentration rises with the increase of suction parameter. This is because as the suction rate is increased, more warm fluid is taken away from the boundary layer. Also, it is noted that the concentration boundary layer becomes thin as the Schmidt number as well as chemical reaction parameter increases.
Fig (1): Effect of Grashof number on transient velocity for Pr=0.71.

Fig (2): Effect of Grashof number on transient velocity for Pr=7.

Fig (3): Effect of porosity parameter on transient velocity for Pr=0.71.

Fig (4): Effect of porosity parameter on transient velocity for Pr=7.

Fig (5): Effect of radiation parameter on transient velocity for Pr=0.71.

Fig (6): Effect of radiation parameter on transient velocity for Pr=7.
Fig (7): Effect of Chemical Reaction parameter on transient velocity for Pr=7

Fig (8): Effect of Chemical Reaction parameter on transient velocity for Pr=0.71

Fig (9): Effect of magnetic parameter on transient velocity for Pr=7

Fig (10): Effect of magnetic parameter on transient velocity for Pr=0.71

Fig (11): Effect of rarefaction parameter on transient velocity for Pr=0.71

Fig (12): Effect of rarefaction parameter on transient velocity for Pr=7
Fig (13): Effect of suction parameter on transient velocity for $Pr=0.71$ and $Pr=7$.

Fig (14): Effect of mass Grashof number on transient velocity for $Pr=7$.

Fig (15): Effect of mass Grashof number on transient velocity for $Pr=0.71$.

Fig (16): Temperature profiles against $y$.

Fig (17): Concentration profile $\phi$ against $y$ here

<table>
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<th>A</th>
<th>R</th>
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<tr>
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<td>10</td>
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When $\omega=20$, $\omega t = \pi/2$, $\epsilon = .05$, $Gr=Gc=5$

References:


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