On The Vorticity Of Unsteady MHD Free Convection Flow Through Porous Medium With Heat And Mass Transfer Past A Porous Vertical Moving Plate With Heat Source /Sink

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Abstract The paper is devoted to a study of a vorticity of the unsteady MHD free convection flow through porous medium with heat and mass transfer past a porous vertical moving plate with heat source /sink. The vorticity of the flow has been found for different values of Prandtl number (P_r) and Schimdt number (S_c). The results of various material parameters are discussed on the flow variables and are presented by tables and graphs.

Keywords: Vorticity MHD, Porous medium vertical plate, free convection flow, Heat transfer.

Notation

M= Magnetic Parameter

G_r = Grashof number

G_m = Modified Grashof number

P_r = Prandtl number

S_c = Schmidt number

D = concentration diffusivity

g = Acceleration due to gravity

B_0 = Magnetic induction

T_∞ = Temperature of fluid

t = time

C_p = Specific heat at constant pressure

k_0 = Porosity parameter

u^*, v^* = components of velocities along and perpendicular to the plate respectively n = Dimensionless exponential index

Greek symbols:

α = Porosity parameter

ρ = Fluid density

ν = Kinematic viscosity

ε = Scalar constant (<<1)

σ = Electric conductivity

β_f = Coefficient of volume expansion

β_c = Coefficient of concentration expansion.

1 Introduction

MHD plays an important role in power generation, space population, cure of diseases, control of thermonuclear reactor, boundary layer control in the field aerodynamics. In past few years, several simple flow problems associated with classical hydrodynamics have received new attention within the more general context of hydrodynamics. Convection in porous medium has applications in geothermal energy recovery, oil extraction and thermal energy storage. The effect of magnetic field on free convection flow is important in liquid metals and ionized gases. To study such applications, which are closely associated with magneto-chemistry, require a complete understanding of the equation of state and transfer properties such as diffusion, the shear stress, thermal conduction, electrical conduction, etc. Some of these properties will undoubtedly be influenced by the presence of external magnetic field and chemical reaction.

Soundalgekar [ ] obtained approximate solutions for the two-dimensional flow of an incompressible, viscous fluid past an infinite porous vertical plate with suction velocity normal to the plate, the difference between the temperature large causing free convection currents. Rapits [ ] studies mathematically the case of time-varying two-dimensional natural convective heat transfer of an incompressible, electrically-conducting viscous fluid through a highly porous medium bounded by an infinite vertical porous plate. Chamka [ ] studied unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Rapits et al. [] studied the effects of radiation in an optically thin gray gas flowing past a vertical infinite plate in the presence of a magnetic field. Cookey et al. [] have been investigated influence of viscous dissipation and radiation on unsteady MHD free-convection flow past in a porous medium with time-dependent suction.

Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction has been studied by Kim [12]. Das et al. [13] approached numerically the effect of mass transfer on unsteady flow past an accelerated vertical porous plate with suction.Ogulu and Prakash [14] analyzed the heat transfer to unsteady magnetohydrodynamic flow past an infinite vertical moving plate with variable suction. Sharma and Singh [17] reported the unsteady MHD free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation. Das and his associates [18] analyzed the mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. Das and his team [15] discussed the effect of induced magnetic field on MHD flow and heat transfer in a conducting elastico-viscous fluid past a continuously moving porous flat surface. In a separate paper, they [16] analyzed the effect of heat source and variable magnetic field on unsteady hydromagnetic flow of a viscous stratified fluid past a porous flat moving plate in the slip flow regime.

Mbeledogu et al. [] have been investigated unsteady MHD free convection flow of a compressible fluid past a moving vertical plate in the presence of radiative heat transfer. Singh and Gupta [ ] studied MHD free convective flow of viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime with mass transfer. Sharma et al. [ ] studied numerical steady of two dimensional MHD forward stagnation point flow in the presence of hall current.

Chamka A.J. [ ] have investigated MHD free convection from a vertical plate embedded in a thermally stratified porous medium with Hall effects. Senapati et al. [ ] studied the effect of chemical reaction on MHD unsteady free convection flow through a porous medium bounded by a linearly accelerated vertical infinite plate.

There has been a renewed interest in studying magneto-hydrodynamics (MHD) flow and heat transfer in porous to effect of magnetic fields on the boundary layer flow control and the performance of many system using electrically conducting fluids. Rapits et al. [ ] analyzed hydromagnetic free convection flow through a porous medium between two parallel plates. Gribben [ ] presented the boundary layer flow over a semi-infinte plate with an aligned magnetic field in the presence of pressure gradient. He obtained solutions for large and small magnetic Prandtl number using the method of matched asymptotic expansion. Gregantopoulos et al. [ ] studied two dimensional unsteady free convection and mass transfer flow of an incompressible viscous dissipative and electrically conducting fluid past an infinite vertical porous plate.

The object of the present paper is to study the vorticity of unsteady MHD free convection flow through porous medium with heat and mass transfer past a porous vertical moving plate with heat source/sink.

2 Formulation of the Problem

We consider the two-dimensional unsteady free convective heat and mass transfer flow of a laminar, incompressible, conducting fluid past a semi-infinite vertical porous moving plate with heat source/sink in the presence of uniform magnetic field applied normal to the direction of flow. Let the components of velocity along with x^* and y^* axis should be $u*$, v^* and which chosen in the upward direction along the plate and normal to the plate respectively. Under these conditions the governing equation can be written in a Cartesian frame of reference as follows:

2.1Continuity Equation

$$\frac{\partial v^*}{\partial y^*} = 0 \qquad \text{....(1.1)}$$

2.1.1 Momentum Equation

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_f (T - T_\infty) + g\beta_c (C - C_\infty) - \upsilon \frac{u^*}{k_0} - \frac{\sigma}{\rho} B_0^2 u^* \qquad .(1.2)$$

2.1.2Energy Equation

$$\frac{\partial T}{\partial t^*} + v^* \frac{\partial T}{\partial y^*} = \frac{k}{\rho c_p} . \frac{\partial^2 T}{\partial y^{*2}} \qquad .\ (1.3)$$

2.1.3Diffusion Equation

$$\frac{\partial C}{\partial t^*} + v^* \frac{\partial C}{\partial y^*} = D \frac{\partial^2 T}{\partial y^{*2}} \qquad .\ (1.4)$$

Where x^* and y^* are dimensional distances and perpendicular to the plate, respectively, u^* and v^* the components of dimensional velocities along the x^* and y^* directions, respectively, K_0 is porosity parameter , ρ is density, σ is the electrical conductivity of the fluid, T_∞ is the temperature of the fluid in the free stream, C_∞ is the concentration at infinite, k is thermal conductivity, β_f is the coefficient of volume expansion, β_c is the coefficient of concentration expansion, B_0 is the magnetic induction.

3 Method of Solution: The equation of continuity (1.1) gives

$$v^* = -v_0^* = \text{constant} \qquad \text{…. (1.5)}$$

Where $v^* > 0$ corresponds to steady suction velocity at the surface. In the view of equation (1.5), the equations (1.2), (1.3) and (1.4) can be written as

$$\frac{\partial u^*}{\partial t^*} - v_0^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_f (T - T_\infty) + g\beta_c (C - C_\infty) - \upsilon \frac{u^*}{k_0} - \frac{\sigma}{\rho} B_0^2 u^* \qquad (1.6)$$

$$\frac{\partial T}{\partial t^*} - v^* \frac{\partial T}{\partial y^*} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^{*2}} \qquad \text{…. (1.7)}$$

$$\frac{\partial C}{\partial t^*} - v^* \frac{\partial C}{\partial y^*} = D\frac{\partial^2 C}{\partial y^{*2}} \qquad \text{… (1.8)}$$

The corresponding boundary conditions are;

$$t = 0, u^* = v_0^*(1 + \varepsilon e^{-nt}), \frac{dT}{\partial y} = -\frac{q}{k}, \frac{\partial C}{\partial y} = \frac{m}{D} \text{ at y = 0}$$

$$t = 0, u^* = \rightarrow 0, T = T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \qquad \text{… (1.9)}$$

Introducing the following non- dimensionless variables and parameters,

$$u = \frac{u^*}{v_0}, y = \frac{v_0 y^*}{\nu}, P_r = \frac{\mu c_p}{k}, S_c = \frac{\nu}{D}, t = \frac{v_0^2 t^*}{\nu}$$

$$Q = \frac{(T - T_\infty) v_0 k}{q\nu}, \phi = \frac{(C - C_\infty) v_0 D}{m\nu}$$

$$M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}\ , \ P_r = \frac{\mu c_p}{k}\ , S_c = \frac{\nu}{D}\ , \alpha = \frac{v_0^2 k_0}{\nu^2}\ , G_r = \frac{g\beta_f q \nu^2}{v_0^4 k}\ , G_m = \frac{g\beta_c m \nu^2}{v_0^4 D}$$

Hence, using the above non-dimensional quantities, the equations (1.6), (1.7), and (1.8) after dropping the star (*) can be written as

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - \left\{ \frac{\partial u}{\partial t} + u(\frac{1}{\alpha} + M \right\} = -(G_r \theta + G_m \phi) \qquad \text{… (1.11)}$$

$$\frac{\partial^2 \theta}{\partial y^2} + P_r \frac{\partial \theta}{\partial y} = P_r \frac{\partial \theta}{\partial t} \qquad \text{… (1.12)}$$

$$\frac{\partial^2 \phi}{\partial y^2} + S_c \frac{\partial \phi}{\partial y} = S_c \frac{\partial \phi}{\partial t} \qquad \text{… (1.13)}$$

And the corresponding dimensionless boundary conditions are:

$$u = 1 + \varepsilon e^{-nt}, \theta = -1, \phi = -1 \text{ at } y = 0$$

$$u \rightarrow 0, \theta = 0, \phi = 0 \text{ as } y \rightarrow \infty \qquad \text{… (1.14)}$$

The equation (1.11) is solved using series method discussion. Now, let us substitute

$$u(y,t) = u_0(y) + \varepsilon u_1(y)e^{-nt} \quad \ldots (1.15)$$

Substituting equation (1.15) into equation (1.11)

$$u_0'' + (\frac{1}{\alpha} + M)u_0' = -(G_r\theta_0 + G_m\phi_0) \quad \ldots (1.16)$$

$$u_1'' + u_1' - (\frac{1}{\alpha} + M - n)u_0 = -(G_r\theta_1 - G_m\phi_1) \quad \ldots (1.17)$$

With corresponding boundary conditions are;

$$u_0 = u_1 = 0, \theta_0 = 0, \phi_0 = 0 \text{ at } y = 0$$

$$u_0 = u_1 = 1, \theta_0 = 0, \phi_0 = 0 \text{ at } y \to \infty \quad \ldots (1.18)$$

The solutions of equations (1.16) and (1.17) with satisfying boundary conditions (1.18) are given by

$$u_0(y) = (1 + R_2G_r + R_3G_m)e^{-R_1 y} - R_2G_re^{-P_r y} - R_3G_me^{-S_c y} \quad \ldots (1.19)$$

And $u_1(y) = e^{-R_4 y}$ … (1.20)

Where, $R_1 = \left[\dfrac{1 + \{1 + 4(\frac{1}{\alpha} + M)\}^{1/2}}{2}\right]$, $R_2 = \left[\dfrac{P_r^2 - P_r - \left(\dfrac{1}{\alpha} + M\right)}{P_r}\right]$

$$R_3 = \left[\frac{S_c^2 - S_c - \left(\dfrac{1}{\alpha} + M\right)}{S_c}\right], R_4 = \left[\frac{1 + \{1 + 4(\frac{1}{\alpha} + M - n)\}^{1/2}}{2}\right]$$

Hence, the velocity is given by

$$u(y,t) = (1 + R_2G_r + R_3G_m)e^{-R_1 y} - R_2G_re^{-P_r y} - R_3G_me^{-S_c y} + \varepsilon e^{-R_4 y}e^{-nt} \quad \ldots (1.21)$$

The vorticity of the flow will be;

$$\zeta = -R_1(1 + R_2G_r + R_3G_m)e^{-R_1 y} + P_rR_2G_re^{-P_r y} + S_cR_3G_me^{-S_c y} - \varepsilon R_4e^{-R_4 y}e^{-nt} \quad \ldots (1.22)$$

4 Results and discussions

Table-1 Value of Vorticity at G_r = 5, M = 1.0, G_m = 2, α = 1.0, n = 0.1, E = 0.2, t = 0

y → P_r ↓ S_c ↓	0	1	2	3	4	5
0.7 0.4	36.881	-1.53	-3.793	-2.568	-1.559	-0.938
0.9 0.4	21.570	-2.820	-2.252	-1.680	-1.221	-1.131
0.7 0.6	13.688	-1.933	-1.721	-1.413	-1.115	-1.001

Table-2 Value of Vorticity at G_r = 5, M =1.0, G_m = 2, α = 1.0, n = 0.1, E = 0.2, t = 6

y → P_r ↓ S_c ↓	0	1	2	3d	4	5
0.7 0.4	36.222	-1.477	-4.477	-3.216	-2.396	-1.968
0.9 0.4	20.629	-2.211	-2.210	-1.231	-1.131	-1.000
0.7 0.6	12.329	-1.510	-1.532	-1.112	-1.013	-0.937

Analysis1 (a)

At y = 0, i.e., at x-axis the value of vorticity is maximum and it drops instantaneously to a negative value as we move away the axis (Fig. 1). It drops continuously till y = 2 and then begins to increase and it may be predicted that at high values of y, it may become zero and so flow may become irrotational. It is obvious that between y = 0 and y =1 the flow will also be irrotational.

If P_r or S_c are further increased the maximum value of vorticity decreases and the rate of decrease of vorticity also decreases. When time is increased the same effect is observed. But in all these cases the first region of zero vorticity will shift towards x-axis, while the farther region will move still farther. Thus the change in vorticity of MHD free convection and mass transfer flow through porous medium induced by the motion of a plate moving with velocity decreasing exponentially with time depends on Grashof number (P_r) and Schmidt number (S_c).

5 Conclusion

It is concluded that the vorticity decreases with increases Prandtl number or Schmidt number. Also the vorticity decreases with increases time (t).

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Analysis1 (a)

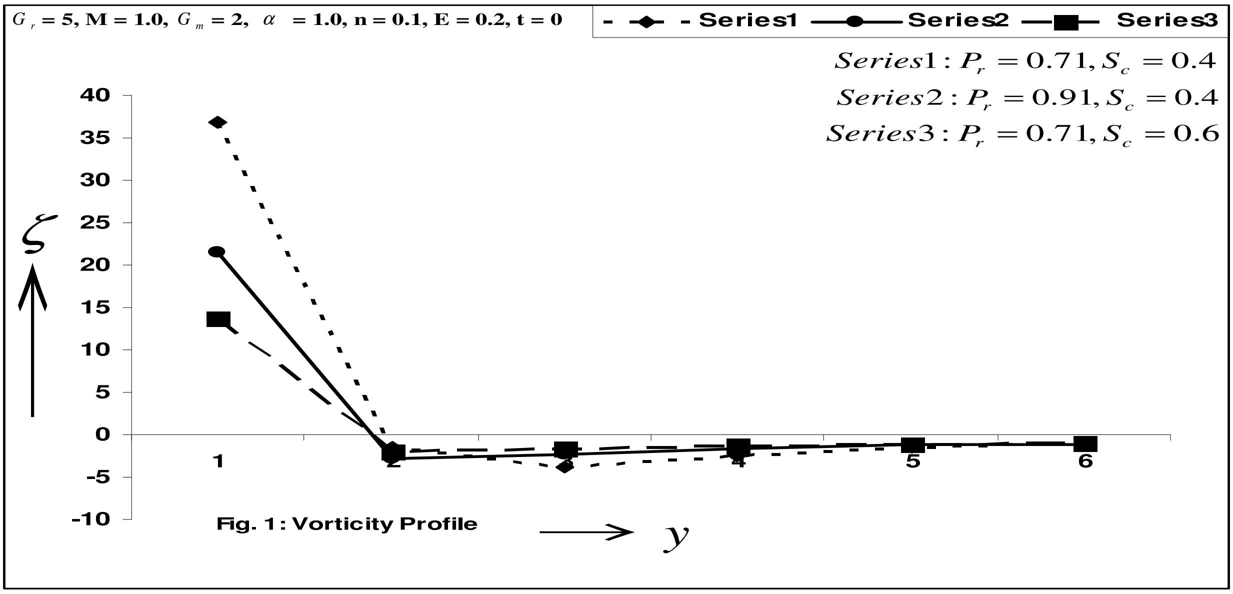

Fig. 1: Vorticity Profile

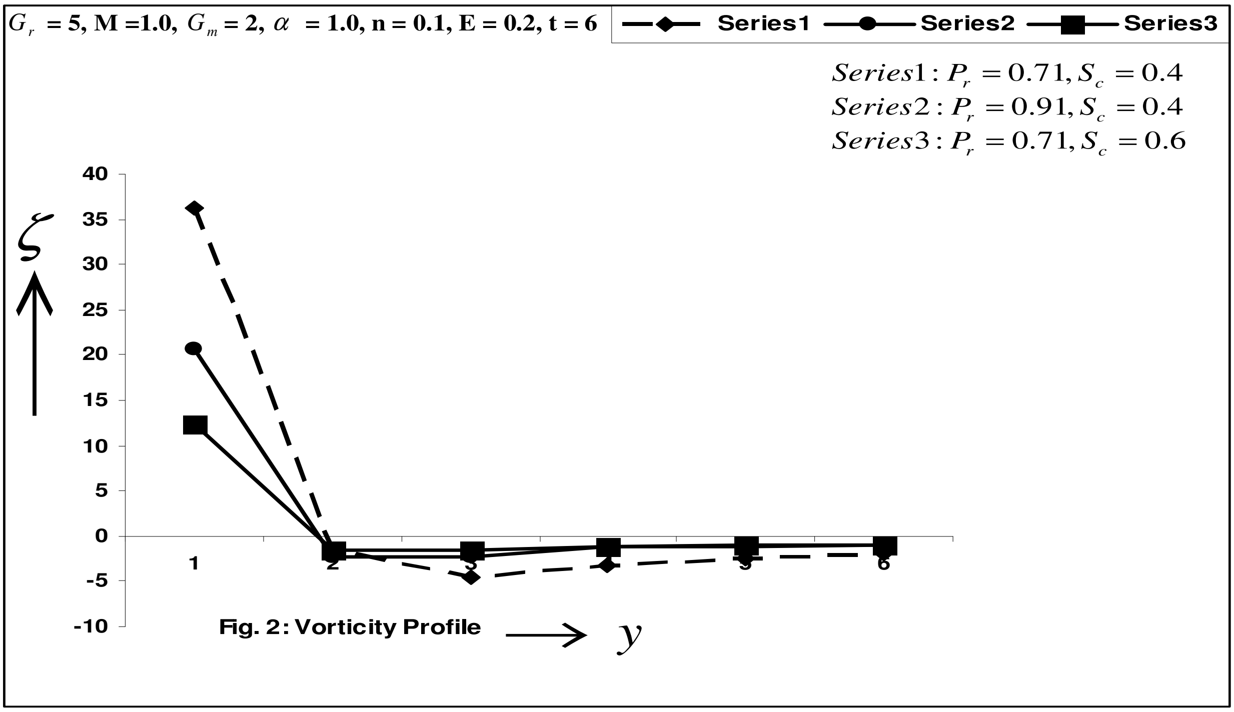

Fig. 2: Vorticity Profile

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