

Study of the Potential of a Local Geoid Model for Extracting the Orthometric Heights from GPS Measurements in Topographic Works

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Abstract

Due to the rapid development and expansion of geodetic applications, the determination of orthometric heights in an accurate manner is considered as one of the most required conditions to carry out such projects. Obtaining orthometric heights using traditional methods of levelling is time and cost consuming. Hence, investigating other techniques that provide the same accuracy, as leveling methods, but requiring less time and cost is very gainful. Recently, satellite positioning techniques and their applications are being used increasingly in geodetic projects. So, it is interesting to study the efficiency of such techniques for obtaining orthometric heights. GPS methods provide highly accurate measurement of ellipsoidal heights. However, the conversion of ellipsoid heights to orthometric heights may be achieved using geoid models. The objective of this research is to study the efficiency of using a local geoid model as an alternative method to obtain orthometric heights from GPS measurements. This paper proposes a methodology to generate such local geoid models. Then, the results of using the generated local geoid model for Jeddah city in Saudi Arabia are presented. These results indicate that the difference among estimated undulations values from the local geoid model and undulations values calculated from leveling techniques ranges from 1.8 cm to -1.1 cm. with a maximum standard deviation of 56 mm. These results confirm that the creation of a local geoid model is an effective method that gives the required accuracy for topographic works.

Keywords: GPS/Levelling method, Local geoid model, Krigging method.

1. Introduction

With the advent of satellite-based technology, especially Global Positioning System (GPS), and its wide usage on different fields of surveying and geomatics, it becomes too easy to obtain three dimensional coordinates latitude, longitude, and ellipsoid height (h), relative to ellipsoid surface, with high accuracy. However, many of engineering projects depend upon the so-called orthometric height (H), relative to geoid surface represented by Mean Sea Level (M.S.L.). In order to transform ellipsoid heights into orthometric heights, we should accurately know the difference between both of these heights, which is called geoid undulation (N). Determination of the geoid undulation value isn't an easy task especially with insufficient available data along the study area. The commonly used procedures that can provide this geoid heights are categorized into three groups: Geometric, gravimetric geoid models, and Earth geopotential models (EGMs). On one hand, EGMs provide global geoid models of the Earth's gravity field based on satellite gravity mission [Förste et al., 2006]. On the other hand, geometric and gravimetric models can be developed for various areas, either locally or regionally. [Chen and Yang, 2001 - Seyed M. K. et al., 2016 - El Shoney et al., 2017].

The geometric method uses GPS/Levelling data without the need of gravity measurements in geoid determination. This method is suitable especially for relatively small areas with less variation in the mass density and distribution on the region of interest. While the gravimetric method is widely used throughout the world for the geoid determination because it is the most precise method. It consists of determining a geoid using gravity measurements. These two methods may be combined together to calculate this geoidal undulation and deflection by using available data obtained from both gravimetric and geometric approaches [Featherstone, 1998 - Erol and Celic, 2004 - Tripathi R.K. and Tripathi M., 2015 – El Shoney et al., 2017].

In this study, the geometric method is used for creating a local geoid model for the study area using GPS/leveling observations to decide to what extent we can replace levelling works with GPS observations to determine orthometric heights. To achieve this goal, a kriging interpolation method is used to generate geoid undulation for the region of interest. In addition, this interpolation method allows assessing the precision of such a geoid undulation model.

2. Local Geoid Model

Geoid models may be classified according to area they covered into global and local geoid models. Investigation of high accurate determination of these geoid models takes an important place in the recent studies of geodesy. A

large number of Global Geopotential Models (GGMs) have been released and developed in the last few years such as the Ohio State University (OSU-91A), The Earth Geopotential Models (EGM1996), and (EGM2008) which considered the most popular used one. Several studies have been performed to evaluate the precision of these geoid models in obtaining orthometric heights from ellipsoid heights. [Dawod et al., 2010 - Al-Krargy et al., 2014].

In relatively small areas, local geoid model may be used in place of GGMs. These local geoid models may be created using geometric or gravimetric approaches. The geometric approach is widely used due to the extensive use of GPS and leveling techniques. Many previous studies were interested in investigating these GPS/leveling methods for the creation of local geoid models as [Zhong, 1997 - Featherstone et al. 1998 - Erol and Celic, 2004 - Erol and Erol, 2013 - Seyed M. K. et al., 2016]. Also, the optimal combination of GPS and leveling observations along with the available geoid models were investigated by [Featherstone, 2000 - Soltanpour et al., 2006 - You, 2006].

The ellipsoid height (h) is accurately obtained from GPS measurements. The orthometric height (H) that is calculated according to the geoid is determined by geometric leveling. The undulation of geoid is defined as the distance between a point on the surface of the geoid and its projection on surface of the ellipsoid of reference according to the normal of the ellipsoid (n'). the deflection is defined as the angle between the normal to the ellipsoid (n') and the vertical to the geoid (n) as shown in figure (1). The relation among the ellipsoid height (h), the orthometric height (H) and the undulation (N) can be calculated using equation no. (1).

$$N = h - H \quad (1)$$

Where,

- h : ellipsoidal height,
- H : orthometric height, and
- N : geoid undulation.

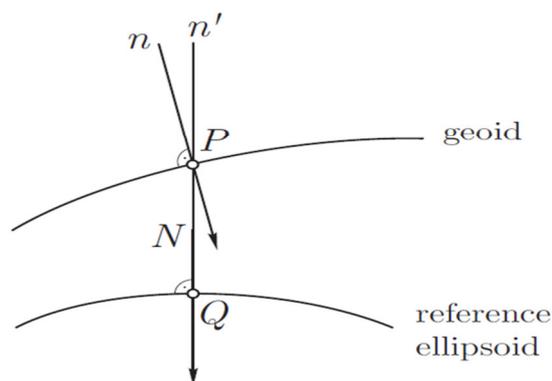


Figure 1. Relation between ellipsoidal height, orthometric height and geoid undulation (geoid height) [Hofmann-Wellenhof and Moritz, 2006]

The geoid heights at any other GPS measurement points can be calculated using analytical or graphical interpolation methods depending on the known geoid heights of some surrounding reference points. These interpolation surface modeling methods refers to as the process of generating a surface through the study area with available data set and it is aimed at formulating an object in a grid system, in which each grid cell contains an estimate of the object that is representative for that particular location. [Elshouny and Yakoub, 2015].

There are several important factors that affect the accuracy of GPS/ Leveling method for creation local geoid model. These factors are; [Erol and Çelik, 2004]

1. The distribution and the number of GPS/ Leveling reference points. These points must be distributed homogeneously into the coverage area of the model. Also, they have to be chosen according to the expected changes of the interpolated geoid surface.
2. The accuracy of the ellipsoidal heights (h) (derived from GPS measurements) and the orthometric heights (H) (derived from levelling measurements).
3. Characteristic of the mass distribution in the area of interest.
4. Used the appropriate interpolation method while modelling the geoid.

Regarding this last point, there are different interpolation algorithms where each of them may provide different results when interpreting data. Some of the interpolation algorithms that may be used for generating local geoid models are: Polynomial Regression, Inverse Distance Weighting to Power (IDW), Natural Neighbor, Triangulation linear interpolation, and Kriging [El- Hallaq, 2012 - Elshouny and Yakoub, 2015].

The Kriging method, compared to other interpolation techniques, provide an estimation of the error during the interpolation process because it has the advantage of integrating geo-statistical constraints in the process of

interpolation based upon spatial variance. Kriging interpolation methods have proven to be useful and popular in many fields as well as geodesy. The kriging interpolation weights the surrounding measured values to derive a prediction for unmeasured locations [Oliver and Webster, 1990]. However, in kriging, the weights are based not only on the distance between the measured points and the prediction location but also on the overall spatial arrangement of the measured points. Kriging assumes that the distance or direction between sample points reflects a spatial correlation that can be used to explain the variation in the surface [Longley et.al, 2010]. To use the spatial arrangement in weights assignment, the spatial autocorrelation must be quantified through empirical semi-variograms.

The constructed semi-variogram of the data is used to weight nearby sample points when interpolating, also it provides a means for users to understand and model the directions (e.g., north–south, east–west) trends of their data. The semi-variogram can have one of the following models: circular, spherical, exponential, Gaussian, and linear. There are two kriging methods: ordinary and universal. The ordinary kriging, the most common method, assumes that the constant mean is unknown, while the universal kriging assumes that there is an overriding trend in the data and this trend is modeled by a polynomial. Kriging is multi-steps process including: exploratory statistical analysis of the data, variogram modeling, creating the surface, and (optionally) exploring a variance surface. Kriging uses the following equation no. (2) [Qulin Tan and Xiao Xu, 2014]:

$$Z(s_0) = \sum_{i=1}^N \lambda_i Z(s_i) \quad (2)$$

Where,

$Z(S_i)$ = the i -th position of the measured value,

λ_i = the i -th position measurement values of the unknown weight,

S_0 = the predicated position, and

N = the number of measurements.

Kriging is a very flexible gridding method where by the default parameters may be accepted to produce an accurate of the source data; alternatively, Kriging can be custom-fit to a data set by specifying an appropriate variogram model. Kriging can be either an exact or a smoothing interpolator depending on the user specified parameter [Elshouny and Yakoub, 2015].

3. Numerical Case Study

An early study for the geoid modeling in the Kingdom of Saudi Arabia (KSA) was conducted by Algarni [1997]. He employed the GPS points and the orthometric heights to derive the geometrical geoid model using a least-squares fitting model. Currently, there are two existing geometrical geoid models, the first is the KSA geoid model by Ngiboglu [2008], derived from the GPS points co-located with the basic levelling network established by the Arabian–American Oil Company (ARAMCO). The variation in the density of points distribution is noticeable in the western region where the point measurements are rare, compared to the eastern region where there is a highly coverage particularly over the oil field areas. The second geoid model is produced by the Saudi Ministry of Municipality and Rural Affairs (MOMRA) as described by Alrajhi et al. [2009]. The MOMRA model is combined with a first-order levelling network, high precision GPS measurements attached to the International Terrestrial Reference Frame (ITRF2000), and EGM08 [Pavlis et al., 2012]. The least-squares collocation was utilized for fitting the model with the GPS-levelling data and EGM08 in an iterative way [see Alothman et al. 2011, 2012, 2013].

In this paper, the study area of this research is located in the middle part of Jeddah city on the west coast of the Kingdom of Saudi Arabia, in the middle of the eastern shore of the Red sea. The selected study area located in the middle part of Jeddah city and with an approximate total area of (100) kilometers square as represented in figure (2).

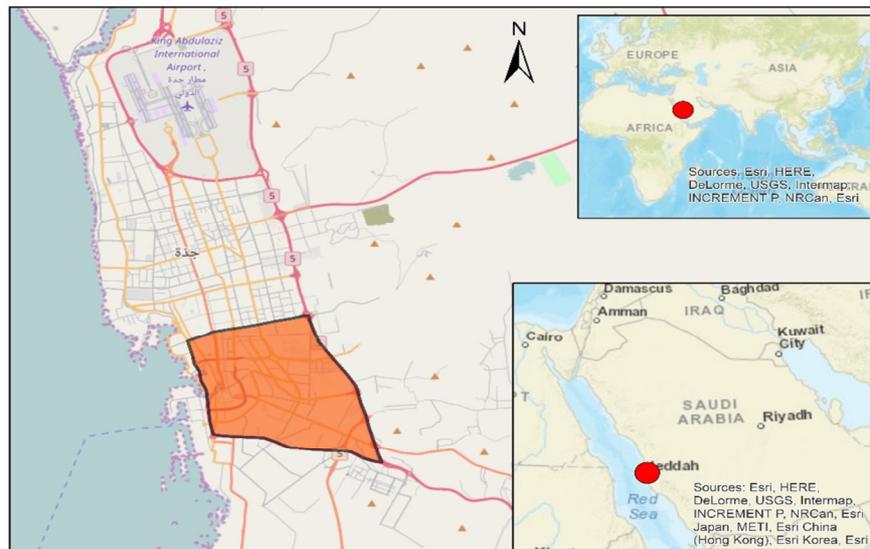


Figure 2. The selected study area

3.1. Designing the network of reference points

Available data used in the study consists of (19) reference points of second order class established by Jeddah Municipality. These points have been selected from all available points after exclusion of missing and bad condition points as a result of field reconnaissance stage for all available existing points in the study area. These points are defined by their accurate geographic coordinates latitude (Φ), longitude (λ), in addition to their ellipsoid heights (h). Also, orthometric heights (H) of these available points were determined according to the second order class levelling – standards and specifications. The distribution of these (19) reference points along the study area is represented in figure (3).

Before starting the densification of the existing reference points, it was important to study their distribution to determine locations where densification is required. The newly established reference points aim to improve the geometry of the network that will be used for generating local geoid model of the study area. Three newly reference points were added to the existing (19) reference points as shown in figure (3). These reference points were observed using GPS observations in order to determine their accurate geographic coordinates and ellipsoid heights. Also, as detailed in section 3.2, the new reference points were observed using geometric leveling process to obtain their orthometric height.

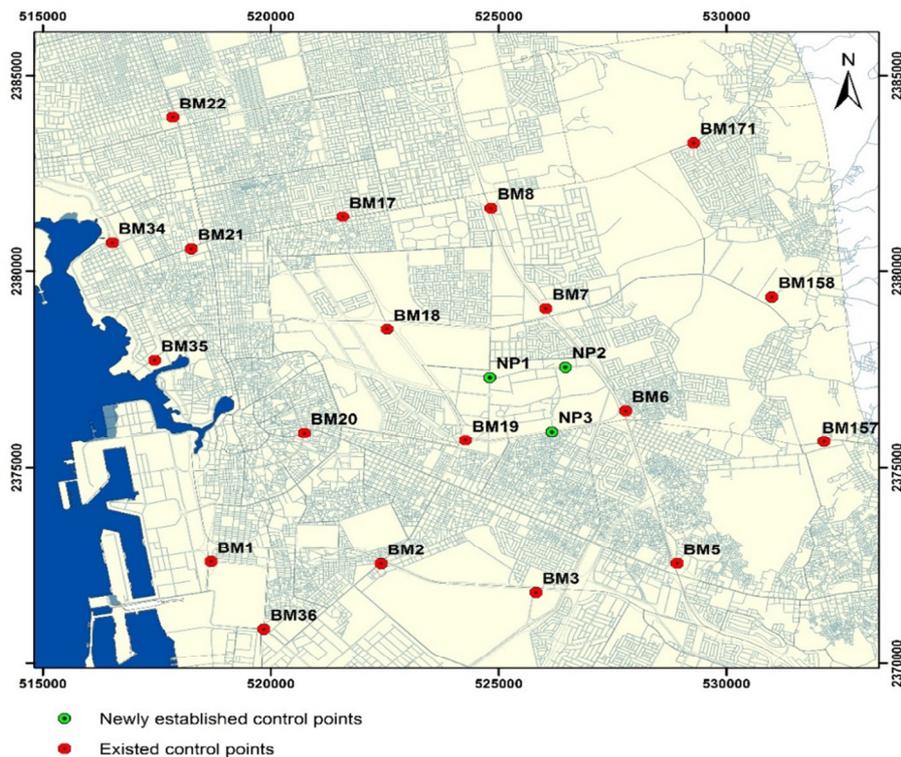


Figure 3. The distribution of existed and newly established reference points along the study area

3.2. Levelling Observations

A closed leveling network was performed in order to check the quality of orthometric heights of existing reference points and also to calculate the heights of the newly established reference points. This network is composed by six reference points, three of them were already established with known elevation while the other three points are newly established where the elevations needs to be calculated. This circuit were deigned assuming that only 2 of these 3 known reference points have known elevations in order to check the quality of the existing points used in the leveling network. Two ways leveling (forward and backward) between these points were observed with the reading sequences "backward – forward – forward – backward" and then "forward – backward – backward – forward" to have redundant measurements to eliminate blunder errors and to enable performing least-squares adjustment technique for calculating unknown elevations. The design of this network consists of 18 routes between reference points with about (47) km length, are shown in figure (4).

To achieve standards of second order leveling, a Leica Sprinter 150 M digital level, with 0.6 mm precision, was used during observations (figure 5a). According to the second order standards the maximum sight distance is 70 meters and the differences between the backward and forward sight distance should never exceed 5 meters per setup to avoid columniation errors. Also, the maximum allowable value of mis-closure between the two leveling ways (backward and forward) is $\pm 8\sqrt{L}$ in mm where L is the length of the leveling (only one way) in kilometers [Adm and Bossler, 1984]. After this, the final orthometric height differences of each route was calculated by the mean of the two leveling ways values as shown in table (1).

A Least-squares technique was then performed to obtain the optimal solutions of orthometric heights for the 4 unknown reference points using equation no. (3) (on of them as check point and the three others as new points). The final orthometric heights of all reference points along the study area are included also in table (1).

(3)

$$X = \begin{bmatrix} bm\ 12 \\ bm\ 03 \\ bm\ 06 \\ f\ 174 \end{bmatrix} = (A^T \cdot W \cdot A)^{-1} \cdot A^T \cdot W \cdot L$$

where:

- X : is elevation unknown values,
- A, A^T : is design matrix, and its transpose matrix,
- W : weight matrix, and
- L : observations matrix

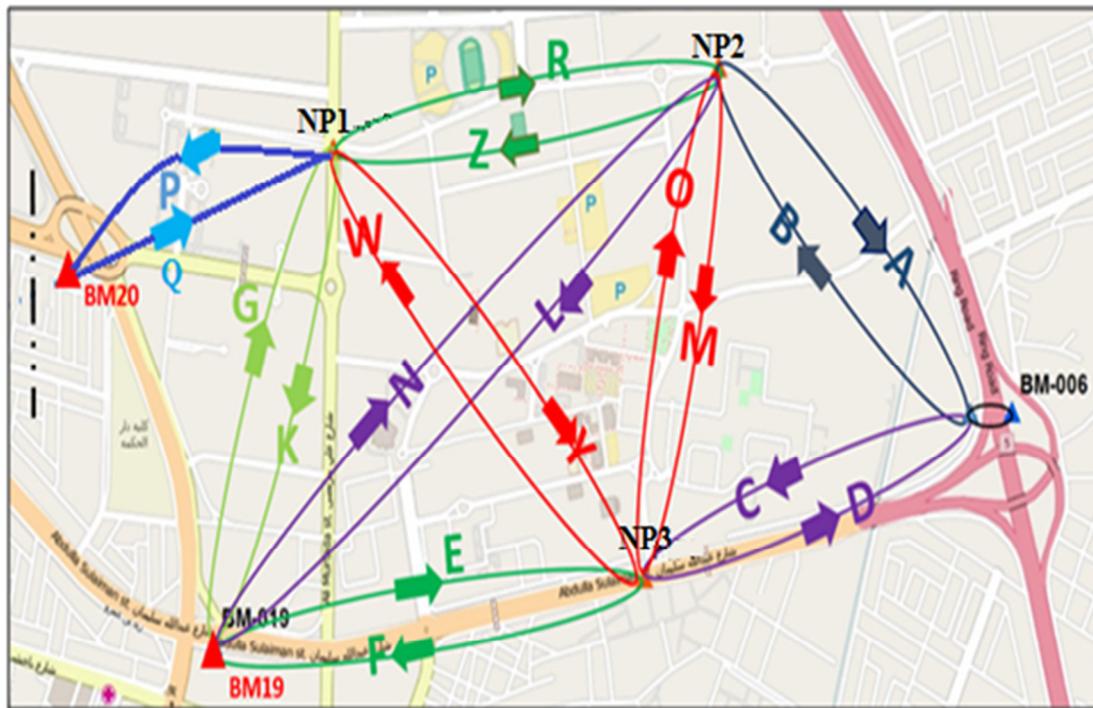


Figure 4. Leveling circuit design and its routes between reference points

3.3. GPS Observations

A Differential GPS mission was carried out for all the 22 reference points used along the study area to calculate their precise three dimensional coordinates including latitude, longitude, and ellipsoid heights. The GPS network was designed as an over-constrained network with two fixed points with known coordinates. The observations were performed using (10) successive sessions, as shown in figure (6), with five Leica Viva GNSS instruments (figure 5b). Two instruments were installed as bases and the three other ones as rovers. All GPS observations were in static mode with duration of a about one hour for each session. Dilutions of precision, number of available satellites and best observations period were taken into consideration during observation sessions.

Table (1): All leveling routes data and final obtained orthometric heights.

Direction	Route ID	Route Points	Each Route Length (m)	Total Route Length (m)	Height Differences (m)	Mis-closure (m)	Allowable Mis-closure	Correction Values	Final orthometric heights
Go	A	NP2–BM6	1975.855	3980.55	6.6199	0.0029	0.019	0.0014	6.6185
Return	B	BM6–NP2	2004.695		-6.6170			0.0015	-6.6185
Go	C	NP3–BM6	1895.87	3828.162	10.5681	0.0148	0.019	0.0075	10.5608
Return	D	BM6– NP3	1932.292		-10.5533			0.0073	-10.5608
Go	E	BM19– NP3	1957.09	3920.492	11.2787	-0.0115	0.019	0.0058	11.2811
Return	F	NP3–BM19	1963.402		-11.2869			0.0058	-11.2811
Go	G	BM19– NP1	2018.952	4100.372	4.7110	0.019	0.020	0.0094	4.7016
Return	K	NP1–BM19	2081.42		-4.692			0.0096	-4.7016
Go	W	NP3– NP1	2392.95	4764.84	-6.609	0.0099	0.021	0.005	-6.6049
Return	Y	NP1– NP3	2371.89		6.600			0.0049	6.6049
Go	M	NP2– NP3	1836.61	3700.32	-3.9359	0.0142	0.019	0.0071	0-3.943
Return	O	NP3– NP2	1863.71		3.9501			0.0071	3.9430
Go	R	NP1– NP2	1716.397	3417.787	10.5393	0.0005	0.018	0.0025	10.53905
Return	Z	NP2– NP1	1701.390		-10.5388			0.0025	-10.53905
Go	N	BM19– NP2	3811.450	7327.53	15.2274	-0.0126	0.027	0.0066	15.2340
Return	L	NP2–BM19	3516.08		-15.240			0.0060	0-15.234
Go	P	NP1–BM20	5920.06	11865.76	-21.135	-0.016	0.027	0.0079	-21.127
Return	Q	BM20– NP1	5945.70		21.119			0.0081	21.127

Leica Geo-Office software (LGO) was used to process the collected GPS data with a 95% confidence level. During the processing, the ambiguity was first solved. Also, errors such as antenna phase center and ionospheric and tropospheric delays were corrected. Precise ephemeris was used instead of the Broadcasted one to increase the accuracy of the adjusted coordinates of reference points. After LGO processing and adjustment processes, the final coordinates of all points with their horizontal precision are calculated. Table 2 contains the calculated coordinates and ellipsoid heights obtained from GPS measurements with their corresponding horizontal and vertical precision, the orthometric height obtained from levelling process, and also the undulation value of each reference point.



(a) Leica sprinter 150 m
 (b) Leica viva GS 15
Figure 5. Level and GPS instruments used in the observation processes

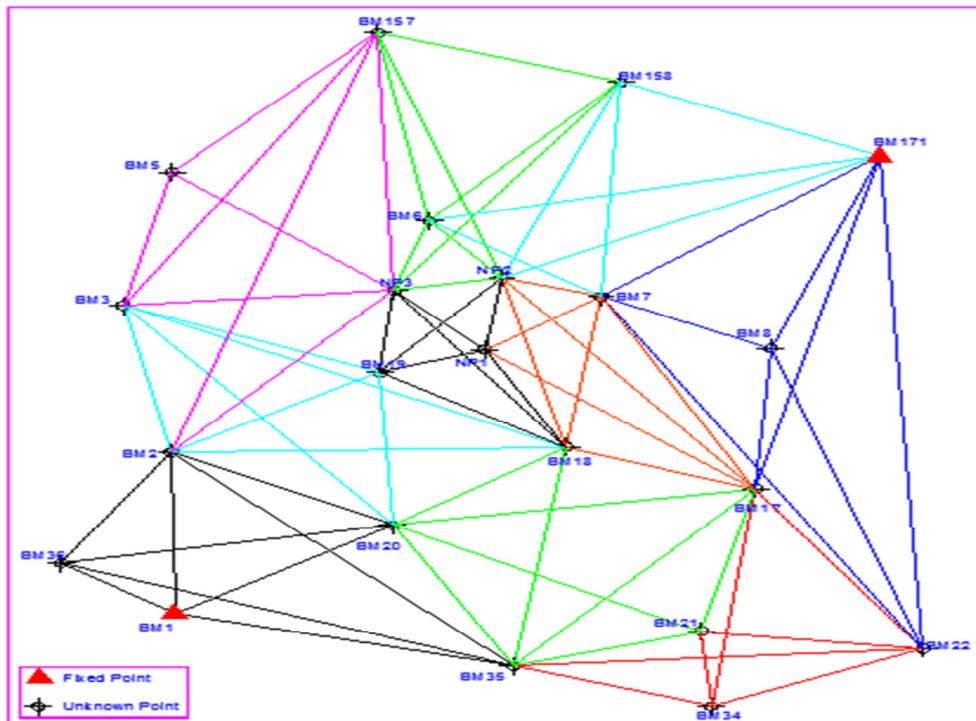


Figure 6. GPS over-constrained network with observation sessions

Table (2): Reference points coordinates with HZ and VL precision of GPS measurements, orthometric heights and geoid undulations.

Point ID	Northing	Easting	h Ellipsoid	GPS measurements Precision		H Geoid	Geoid Undulation N
				HZ. (meter)	VL. (meter)		
NP1	2377290.997	524804.891	35.164	0.017	0.035	30.400	4.764
NP2	2377552.250	526468.150	45.755	0.011	0.019	40.939	4.816
NP3	2375923.788	526169.910	41.773	0.009	0.016	36.995	4.778
BM1	2372598.804	518686.339	6.607	Fixed	Fixed	1.960	4.647
BM2	2372547.126	522417.938	18.756	0.019	0.017	14.108	4.648
BM3	2371814.606	525822.383	34.849	0.011	0.018	30.173	4.676
BM5	2372552.563	528913.032	54.861	0.009	0.016	50.050	4.811
BM6	2376453.755	527787.790	52.311	0.01	0.02	47.557	4.754
BM7	2379054.010	526035.305	44.061	0.009	0.019	39.235	4.826
BM8	2381621.695	524830.129	41.114	0.017	0.026	36.237	4.877
BM17	2381395.720	521579.438	19.532	0.011	0.02	14.795	4.737
BM18	2378523.168	522548.554	23.562	0.014	0.048	18.814	4.748
BM19	2375706.652	524274.147	30.460	0.015	0.024	25.705	4.755
BM20	2375892.499	520735.530	13.958	0.02	0.047	9.271	4.687
BM21	2380567.375	518256.019	10.915	0.019	0.031	6.179	4.736
BM22	2383928.094	517849.380	15.952	0.017	0.036	11.236	4.716
BM34	2380722.221	516520.999	6.356	0.011	0.025	1.555	4.801
BM36	2370880.532	519845.662	7.781	0.015	0.018	3.244	4.537
BM35	2377724.088	517451.287	6.398	0.011	0.017	1.646	4.752
BM157	2375672.962	532141.434	91.859	0.01	0.027	87.042	4.817
BM158	2379348.322	530995.611	76.367	0.017	0.019	71.535	4.832
BM171	2383279.085	529279.511	89.131	Fixed	Fixed	84.108	5.023

3.4. Creating Local Geoid Model for the Study Area

The main objective of this research is to assess the quality of obtained orthometric heights from GPS measurements using a local geoid model as an alternative method of geometric leveling techniques. After obtaining ellipsoid heights from GPS measurements, and orthometric heights from leveling technique for reference points along the study area, we will compute geoid undulation values for these points using equation no.(1) as shown in table (2).

In order to calculate the value of geoid undulation in the whole study area based on the reference points, a local geoid model is created using the interpolation kriging interpolation method.

Two different local geoid models were created and investigated along the study area (Figure 7). The first model covers only a part of the study area which contains the newly established or densified reference points. The second model covers the whole study area and contains all available reference points except the newly established ones. The purpose of creating these two different models is to study and compare the effect of densification (increasing) of reference points along the study area on the precision of orthometric height extracted from these local geoid models. The characteristics of these two models are shown in table (3). Reference points along each model area are divided into two groups. The first group was used in the creation of the geoid model, while the second group of these points was used as check points to investigate the obtained precision of each geoid models. Note that one of these check points (NP3) is mutual to both models to be compared.

Table (3): The characteristics of the two generated geoid models

Created model	Number of the used reference points		Model Covered area (km ²)	Distances between reference points statistics		
	for creating the model	for testing the model		Minimum distance (km)	Maximum distance (km)	Average of distances (km)
First local geoid model	6 points	2 points	15.74	1.516	5.633	3.329
Second local geoid model	17 points	3 points	140.14	2.073	6.986	4.271

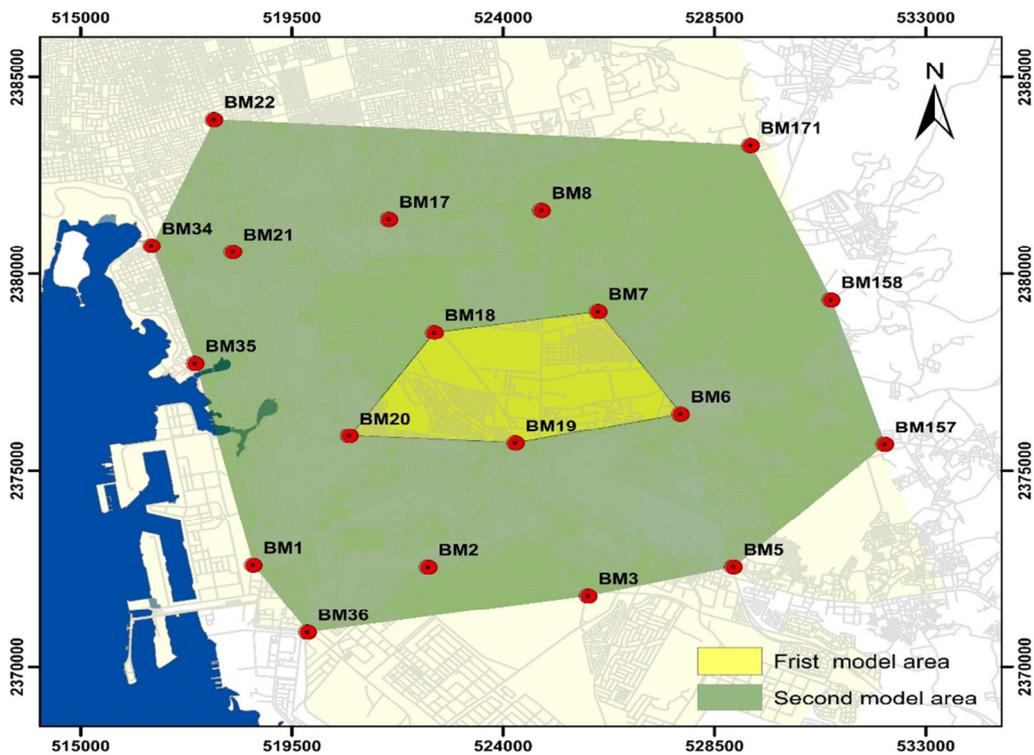


Figure 7. Areas of the two local geoid model for the study area.

4. Results and Discussion

The results obtained from the two local geoid models are presented and discussed in the following sections.

4.1. First Local Geoid Model

Figure (8) shows geoid undulation values of the first local geoid mode. Table (4) represents the assessment of orthometric heights of the two check points used for this model. The results indicate that difference between geoid undulations values estimated from the created local geoid model and obtained from GPS/leveling measurements ranges between (1.1) cm and (-0.7) cm. In addition, the standard deviation map that results from the kriging interpolation of the first local geoid model shows a maximum of 42 mm (Figure 9).

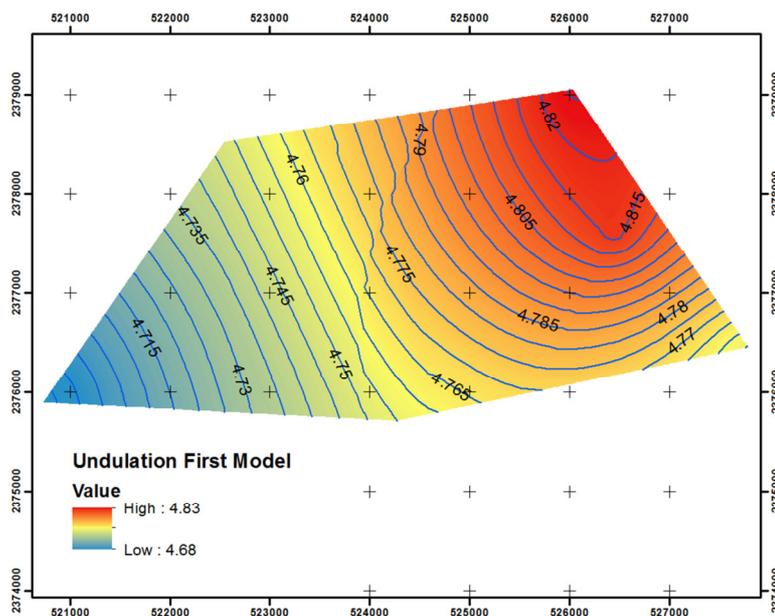


Figure 8. Undulations value of the first local geoid model in meter

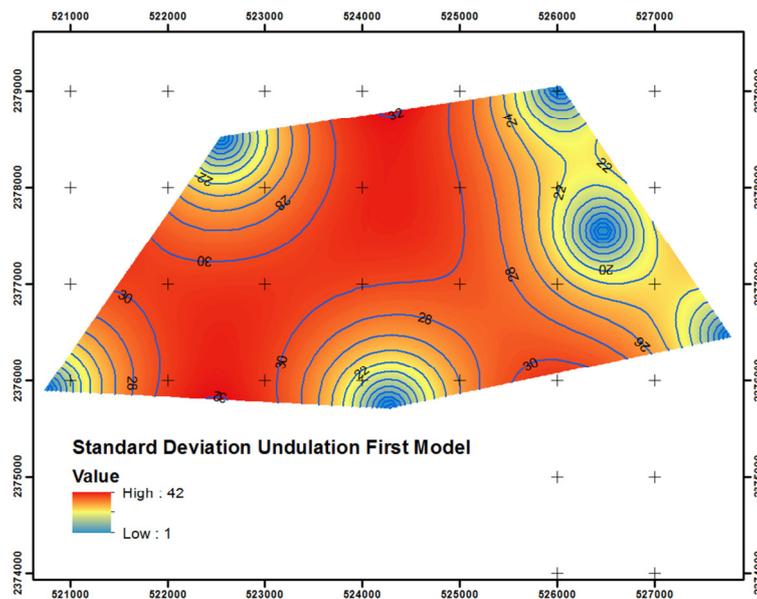


Figure 9. Standard deviation values of the undulation that corresponds to the first local geoid model in millimeter.

Table (4): First geoid model test points data.

Test point ID	GPS	Leveling	GPS/leveling	local Geoid model		Geoid Undulation difference (m)
	Ellipsoid height (m)	Geoid height (m)	Geoid undulation (m)	Geoid undulation (m)	Orthometric height (m)	
NP3	41.773	36.995	4.778	4.771	37.002	-0.007
NP1	35.164	30.400	4.7642	4.775	30.389	0.011

4.2. Second Geoid Model Results

Geoid undulation values of the second geoid model created for the whole study area are shown in figure (10). The results of the three check points used for evaluating the quality of this model are shown in table (5). The results indicate that differences of estimated and obtained geoid undulations values are ranges between (1.8) cm and (-1.1) cm. Also, the maximum standard deviation value for the second local geoid model is 56 mm (Figure 11).

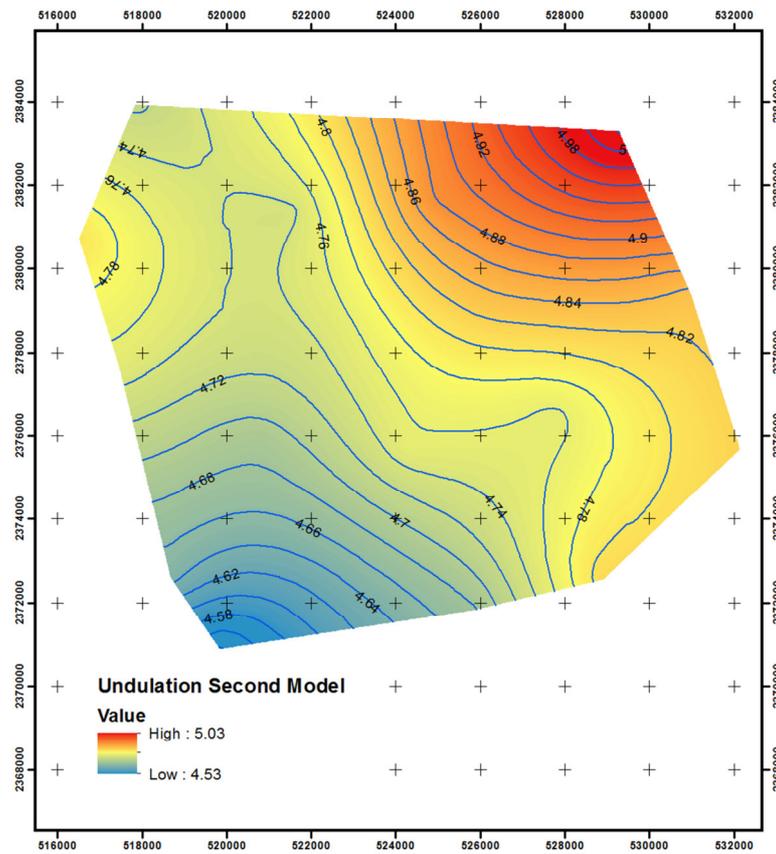


Figure 10. Undulations value of the second local geoid model in meter

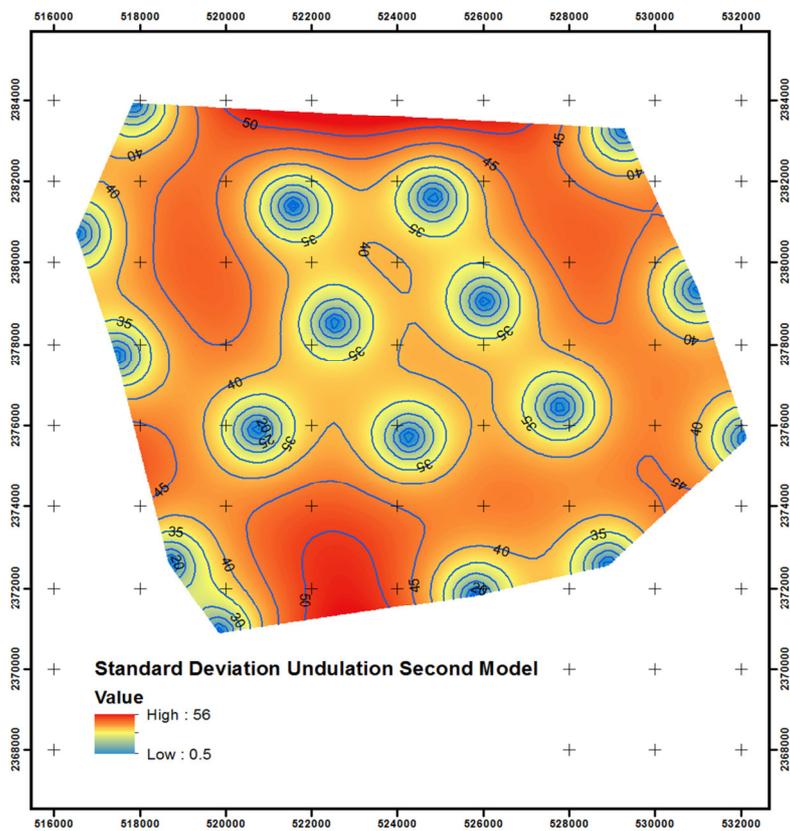


Figure 11. Standard deviation values of the undulation that corresponds to the second local geoid model in millimeter.

Table (5): Second geoid model test points data.

Test point ID	GPS	Leveling	GPS/leveling	local Geoid model		Geoid Undulation difference (m)
	Ellipsoid height (m)	Geoid height (m)	Geoid undulation (m)	Geoid undulation (m)	Orthometric height (m)	
NP3	41.773	36.995	4.778	4.7675	37.0055	-0.0105
BM2	18.756	14.108	4.648	4.637	14.119	-0.011
BM21	10.915	6.179	4.736	4.754	6.161	0.018

4.3. Discussion

From comparing the results corresponding to the two generated models, we notice that the first model gives higher precision than the second model. This is mainly due to the more densified points and short separated distances between reference points of the first model. More specifically, the check point (NP3), which existed in both models, shows that the calculated geoid undulation difference is 0.69 cm and 1.05 cm from the first model and second model respectively. Also, the standard deviation maps show that first model (with a maximum standard deviation value of 42 mm) is more precise than the second model (with a maximum standard deviation value of 56 mm). However, it is very important to mention that the two models give an acceptable standard deviation for orthometric heights in surveying works that corresponds to 10 cm (2σ for 95% confidence interval).

5. Conclusion and Recommendations

Orthometric height is a very important information that is crucial for several geodetic applications. However, the classical methods for obtaining this kind of height (such geometric leveling) are very time and money consuming. The conversion of the ellipsoid heights obtained from GPS measurements into orthometric heights becomes recently an important alternative thanks to the rapid growth of GNSS solutions. The aim of this paper is to study the efficiency and the potential of using local geoid model method for extracting orthometric heights from ellipsoid heights.

GPS/Leveling methods are the simplest and popular methods used in the creation of local geoid models. Precise leveling and differential GPS should be used to determine orthometric and ellipsoid height of reference points with a high precision to increase the accuracy of the obtained heights from the appropriate interpolation method. The Kriging method is one of the best interpolation methods that allows predicting the value of undulation while characterizing the corresponding the precision of predications.

Based on the analysis of the obtained results, creating a local geoid model is an effective method for obtaining orthometric height using GPS observations instead of levelling works. This can reduce the field-time of the survey and, as a consequence, saving costs of this field works. However, to achieve the required precision, the study area has to be covered with sufficient and well distributed reference points. The number and location of these reference points depend mainly on the topography of the study area and the required precision of the local geoid model to be generated.

Finally, establishing a strong cooperation among local surveying organizations is highly recommended in order to create local geoid models. This kind of collaboration will facilitate sharing all available data and avoiding repetitive surveying works.

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