Rainfall-runoff modelling of a watershed

Pankaj Kumar   Devendra Kumar

GBPUA & T Pantnagar (US Nagar) ,India

Abstract

In this study an adaptive neuro-fuzzy inference system was used for rainfall-runoff modelling for the Nagwan watershed in the Hazaribagh District of Jharkhand, India. Different combinations of rainfall and runoff were considered as the inputs to the model, and runoff of the current day was considered as the output. Input space partitioning for model structure identification was done by grid partitioning. A hybrid learning algorithm consisting of back-propagation and least-squares estimation was used to train the model for runoff estimation. The optimal learning parameters were determined by trial and error using gaussian membership functions. Root mean square error and correlation coefficient were used for selecting the best performing model. Model with one input and 91 gauss membership function outperformed and used for runoff prediction.

Keywords: Rainfall, runoff, modelling, ANFIS

Introduction

The hydrologic behavior of rainfall-runoff process is very complicated phenomenon which is controlled by large number of climatic and physiographic factors that vary with both the time and space. The relationship between rainfall and resulting runoff is quite complex and is influenced by factors relating the topography and climate.

In recent years, artificial neural network (ANN), fuzzy logic, genetic algorithm and chaos theory have been widely applied in the sphere of hydrology and water resource. ANN have been recently accepted as an efficient alternative tool for modeling of complex hydrologic systems and widely used for prediction. Some specific applications of ANN to hydrology include modeling rainfall-runoff process (Sajikumar *et. al.*, 1999). Fuzzy logic method was first developed to explain the human thinking and decision system by Zadeh (1965). Several studies have been carried out using fuzzy logic in hydrology and water resources planning (Mahabir *et al.* 2003; Chang *et al.,* 2002).

Adaptive neuro-fuzzy inference system (ANFIS) which is integration of neural networks and fuzzy logic has the potential to capture the benefits of both these fields in a single framework. ANFIS utilizes linguistic information from the fuzzy logic as well learning capability of an ANN. Adaptive neuro fuzzy inference system (ANFIS) is a fuzzy mapping algorithm that is based on Tagaki-Sugeno-Kang (TSK) fuzzy inference system (Jang *et al*., 1997; Loukas, 2001). ANFIS used for many application such as, database management, system design and planning/forecasting of the water resources (Nayak *et. al.,* 2004; Firat *et. al*., 2009 and Wang *et. al* 2009).

Study area

The Nagwan watershed is located at Upper Siwane river of Damodar-Barakar basin in the Hazaribagh District of Jharkhand, India, and lies between 85.25^0 to 85.43^0 E longitudes and 23.99^0 to 24.12^0 N latitude. The location and topographic map of Nagwan watershed is shown in Figure 3.1. The catchment is rectangular in shape with an area of 92.46 sq km and length-width (L/W) ratio as 2.7. The maximum and minimum elevation in the catchment is 640 m and 550 m respectively above mean sea level. The catchment has very undulating and irregular slope varying from 1 to 25%. The climate of watershed is sub-tropical with three distinct seasons viz. winter (October to February), summer (March to May) and monsoon (June to September). The average annual
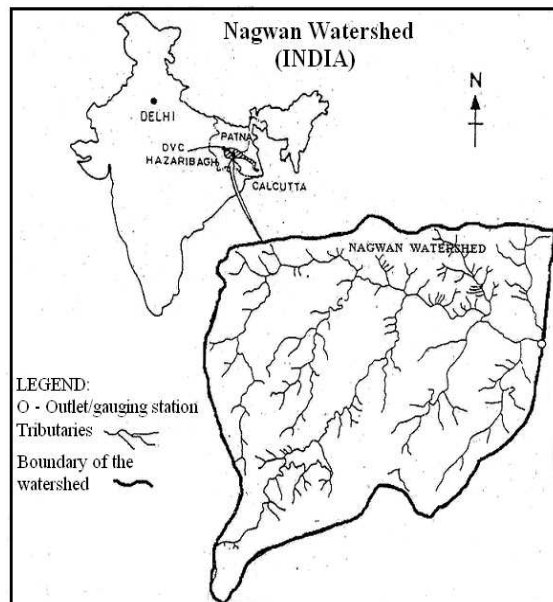

Fig 1. Location and topographic map of Nagwan watershed

rainfall in the watershed is about 1137 mm. About 90% of the rainfall occurs due to southeast monsoon during the period of 1st June to 30th September. The daily mean temperature of the watershed ranges from 3^0C to 40^0C. The mean monthly relative humidity varies from a minimum of 40% in the month of April to a maximum of 85% in the month of July.

Materials and methods

Adaptive neuro-fuzzy inference systems (ANFIS)

Adaptive Neuro Fuzzy Inference System (ANFIS) is a fuzzy mapping algorithm that is based on Tagaki-Sugeno-Kang (TSK) fuzzy inference system (Jang *et al.,* 1997 and Loukas, 2001).ANFIS is integration of neural networks and fuzzy logic and have the potential to capture the benefits of both these fields in a single framework. ANFIS utilizes linguistic information from the fuzzy logic as well learning capability of an ANN for automatic fuzzy if-then rule generation and parameter optimization.

A conceptual ANFIS consists of five components: inputs and output database, a Fuzzy system generator, a Fuzzy Inference System (FIS), and an Adaptive Neural Network. The Sugeno- type Fuzzy Inference System, (Takagi and Sugeno, 1985) which is the combination of a FIS and an Adaptive Neural Network, was used in this study for rainfall-runoff modeling. The optimization method used is hybrid learning algorithms.

For a first-order Sugeno model, a common rule set with two fuzzy if-then rules is as follows:

Rule 1: If x_1 is A_1 and x_2 is B_1, then $f_1 = a_1 x_1 + b_1 x_2 + c_1$.

Rule 2: If x_1 is A_2 and x_2 is B_2, then $f_2 = a_2 x_1 + b_2 x_2 + c_2$.

where, x_1 and x_2 are the crisp inputs to the node and A_1, B_1, A_2, B_2 are fuzzy sets, a_i, b_i and c_i $(i = 1, 2)$ are the coefficients of the first-order polynomial linear functions. Structure of a two-input first-order Sugeno fuzzy model with two rules is shown in Figure 1 It is possible to assign a different weight to each rule based on the structure of the system, where, weights w_1 and w_2 are assigned to rules 1 and 2 respectively.

and f = weighted average

The ANFIS consists of five layers (Jang, 1993), shown in Figure 3.

The five layers of model are as follows:

Layer1: Each node output in this layer is fuzzified by membership grade of a fuzzy set corresponding to each input.

$$O_{1,i} = \mu_{Ai}(x_1) \qquad\qquad i = 1, 2$$

or

$$O_{1,i} = \mu_{Bi-2}(x_2) \qquad\qquad i = 3, 4 \qquad\qquad \text{... (1.1)}$$

Where, x_1 and x_2 are the inputs to node i ($i = 1, 2$ for x_1 and $i = 3, 4$ for x_2) and x_1 (or x_2) is the input to the ith node and A_i (or B_{i-2}) is a fuzzy label.
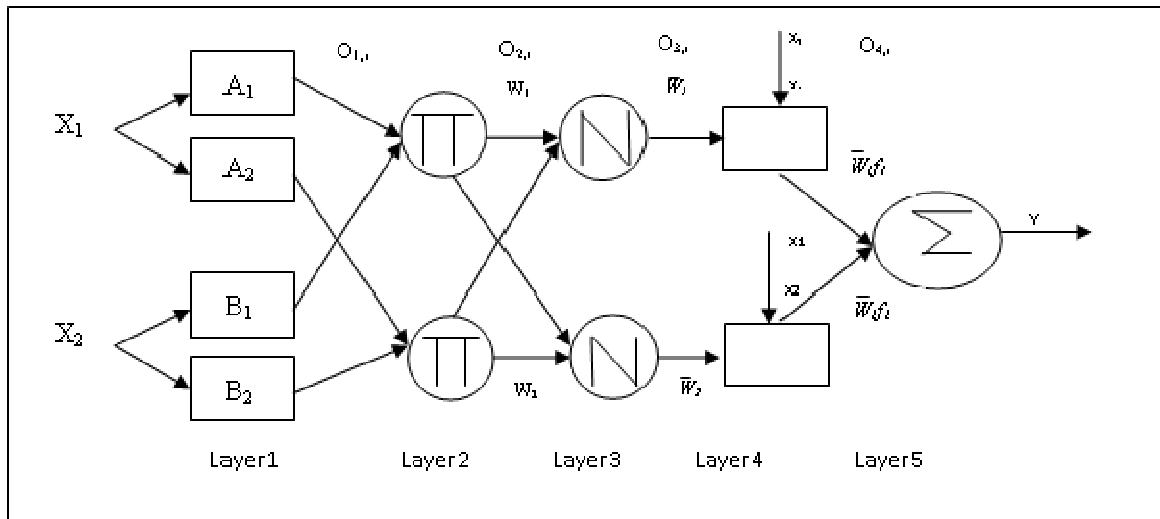
Fig 2. ANFIS architecture

Layer 2: Each node output in this layer represents the firing strength of a rule, which performs fuzzy, AND operation. Each node in this layer, labeled Π, is a stable node which multiplies incoming signals and sends the product out.

$$O_{2,i} = W_i = \mu_{Ai}(x_1)\ \mu_{Bi}(x_2) \qquad i = 1, 2$$

... (1.2)

Layer 3: Each node output in this layer is the normalized value of layer 2, i.e., the normalized firing strengths.

$$O_{3,i} = \overline{W_i} = \frac{w_i}{w_1 + w_2} \qquad i = 1, 2$$

… (1.3)

Layer 4: Each node output in this layer is the normalized value of each fuzzy rule. The nodes in this layer are adaptive .Here $\overline{W_i}$ is the output of layer 3, and $\{a_i, b_i, c_i\}$ are the parameter set. Parameters of this layer are referred to as consequence or output parameters.

$$O_{4i} = \overline{W_i} f_i = \overline{W_i}(a_i x_1 + b_i x_2 + c_i) \qquad i = 1, 2$$

…(1.4)

Layer 5: The node output in this layer is the overall output of the system, which is the summation of all coming signals.

$$Y = \sum_1^2 \overline{W_i} f_i = \frac{\sum_1^2 W_i f_i}{\sum_1^2 W_i}$$

$$\ldots (1.5)$$

In this way the input vector was fed through the network layer by layer. The two major phases for implementing the ANFIS for applications are the structure identification phase and the parameter identification phase. The structure identification phase involves finding a suitable number of fuzzy rules and fuzzy sets and a proper partition feature space. The parameter identification phase involves the adjustment of the premise and consequence parameters of the system.

Optimizing the values of the adaptive parameters is of vital importance for the performance of the adaptive system. Jang et al. (1997) developed a hybrid learning algorithm for ANFIS to approximate the precise value of the model parameters. The hybrid algorithm, which is a combination of gradient descent and the least-squares method, consists of two alternating phases: (1) in the backward pass, the error signals recursively propagated backwards and the premise parameters are updated by gradient descent , and (2) least squares method finds a proper set of consequent parameters (Jang *et al*., 1997). In premise parameters set for a given fixed values, the overall output can be expressed as a linear combination of the consequent parameters.

$$AX = B \qquad \ldots (1.6)$$

Where, *X* is an unknown vector whose elements are the consequent parameters. A least squares estimator of X, namely X^*, is chosen to minimize the squared error $\|AX - B\|^2$. Sequential formulas are employed to compute the least squares estimator of *X*. For given fixed values of premise parameters, the estimated consequent parameters are known to be globally optimal.

Material and methods

The daily rainfall and runoff data of monsoon period (June to September) for the period 1993-1999 were used to describe daily time series and development of models. Daily rainfall and runoff data for the year of 1993 to 1999 were used for the training (calibration) of the developed model whereas daily data for the year of 2000 to 2002 were used for verification (testing) of the models.

Different combinations of rainfall and runoff were considered as the inputs to the model, and runoff of the current day was considered as the output. Input space partitioning for model structure identification was done by grid partition. Hybrid learning algorithm was used to train the models for runoff prediction. The optimal learning parameters were determined by trial and error (Kim *et al*., 2002) for gaussian membership function. In order to choose better model among developed models root mean square error and correlation coefficient was computed (Nayak *et al*., 2005). Different combinations of the runoff and rainfall were used to construct the appropriate input structure in the runoff forecasting model.

Result and Discussions

The study revealed that the highest value of correlation coefficient and least value of root mean square error were obtained for model with one input as current day rainfall and output as current day runoff. There were vague results for increasing number of inputs (Previous day's rainfall as well as previous day's runoff). It implies that runoff mainly depends upon rainfall of current day. It is due to small area of watershed and varying slope condition. Among triangular, bell shaped, trapezoidal and gaussian membership function, the gaussian membership function was found most suitable for this study. By increasing membership function number, best fit model was found for 91 gauss membership functions. Input space partitioning for model structure identification was done by grid partition because of only one input. Quantitative performance indices such as root mean square error and correlation coefficient for model are 0.964 and 1.087. In case of testing period root mean square error and correlation coefficient are 0.867 and 1.390 respectively.
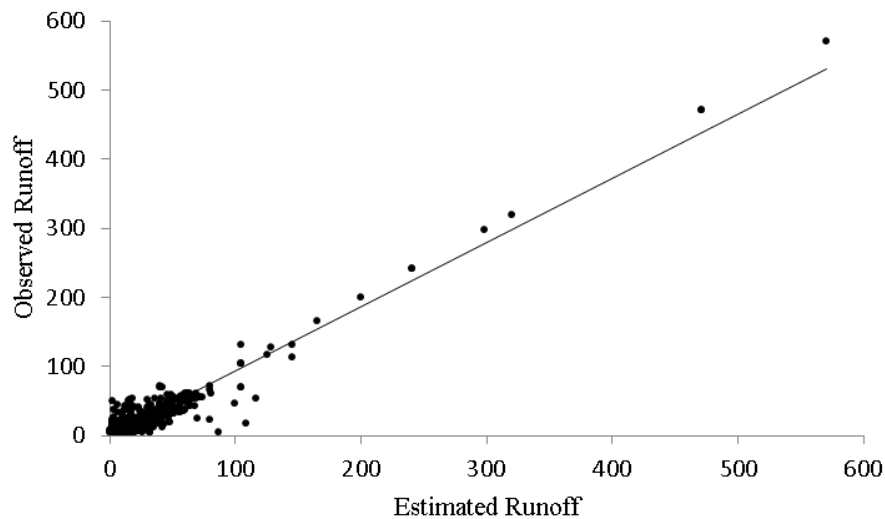
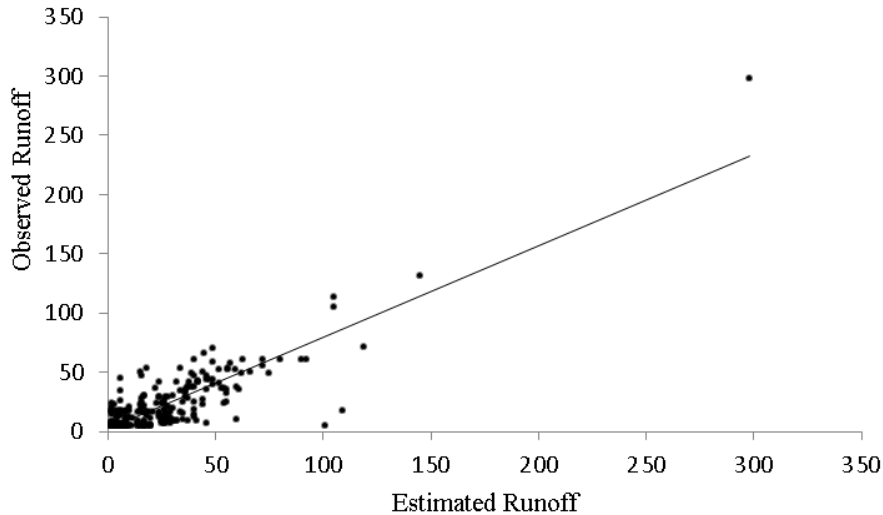Fig 3. Observed and estimated runoff during training period

Fig 4. Observed and estimated runoff during testing period

Conclusions

The present study discusses the application and usefulness of adaptive neuro fuzzy inference system based modelling approach for estimation of runoff. The visual observation based on the graphical comparison between observed and predicted values and the qualitative performance assessment of the model indicates that ANFIS can be used effectively for hydrological rainfall runoff modelling. The ANFIS model is flexible and has options of incorporating the fuzzy nature of the real-world system.

REFERENCES

[1] Chang ,F.J.; Chang, L.C., Huang, H.L.(2002).″ Real time recurrent neural network for streamflow forecasting". Hydrol Process , 16:2577–88

[2]   Firat, M.,Turan, M. and Yurdusev, M.A. (2009). ″Comparative analysis of fuzzy inference systems for water consumption time series prediction". Journal of Hydrology ,374 :235–241.

[3]   Jang, J.-S. R. (1993) ANFIS: "adaptive network-based fuzzy inference   systems". IEEE Trans. Syst., Man Cybern. 23: 665–685.

[4] Jang, J.S.R.; Sun, C.T. and Mizutani, E.(1997). "Neuro-Fuzzy and Soft Computing, A Computational Approach to Learning and Machine Intelligence". Prentice Hall, NJ,USA ISBN 0-13-261066-3.

[5]   Kim, B.; Park, J.H.; Kim, B.S.(2002). "Fuzzy logic model of Langmuir probe discharge data". Comput Chem,26(6):573–581.

[6] Loukas, Y.L .(2001). "Adaptive neuro-fuzzy inference system: an instant and architecture-free predictor for improved QSAR studies". J Med Chem 44(17):2772–2783.

[7] Mahabir, C.; Hicks, F.E. and   Robinson F. A.(2003). "Application of fuzzy logic to forecast seasonal runoff". Hydrological Process, 17:3749-3762.

[8]   Nayak, P.C., Sudheer, K.P., Rangan, D.M. and Ramasastri, K.S. (2004).   "A neuro-fuzzy computing technique for modelling hydrological time series". J. Hydrology, 291 : 52-66.

[9]   Sajikumar, N. and Thandaveswara, B.S. (1999). "A non-linear rainfall-runoff model using artificial neural networks". J. Hydrol., 216: 32-55.

[10]   Takagi, T., and M. Sugeno.(1985). "Fuzzy identification of systems and its application to modeling and control". IEEE Transactions on Systems, Man, and Cybernetics,15: 116–132.

[11]   Wang,W.C.; Chau, K.W.; Cheng,C.T. and Lin,Q.(2009). "A comparison of performance of several artificial intelligence methods for forecasting monthly discharge time series." Journal of hydrology,374: 294-306

[12]   Zadeh, L.A. (1965). "Fuzzy Sets." Information and Control, (8): 338-353.