Computer Engineering and Intelligent Systems www.iiste.org
ISSN 2222-1719 (Paper) ISSN 2222-2863 (Online)
Vol 3, No.1, 2012

An Algorithm for Generating New Mandelbrot and Julia Sets

R. P. Pant[1] R. K. Bisht[1*]

1. Department of Mathematics, D. S. B. Campus, Kumaun University, Nainital-263002, India

* E-mail of the corresponding author: ravindra.bisht@yahoo.com

Abstract

The present paper is motivated from the paper of John R. Tippetts (Tippetts 1992) in which he gave an algorithm to generate an interesting Mandelbrot set. We not only generate Julia sets using Tippetts algorithm (Tippetts 1992), but also generate some new Julia and Mandelbrot sets by slightly modifying the Tippetts algorithm. This approach yields a new class of algorithms to produce new and alluring fractals with virtually infinite complexity.

Keywords: Mandelbrot set, Julia set, recursion formula, algorithm

1. Introduction

The study of complex iterated polynomials has been a very active field of research activity during the last two decades. The study of Julia sets and Mandelbrot sets derived from polynomials is an area of abiding interest in the field of fractal geometry (Peitgen *et al* 2006).

The well-known algorithm for generating Mandelbrot set from the recursion formula;

$Z \rightarrow Z^2 + C$ (where $Z = x + iy$ and $C = a + ib$), requires the following steps (Peitgen *et al* 2006):

[1] $x_{new} = x^2 - y^2 + a$
[2] $y_{new} = 2xy + b$
[3] $x = x_{new}$
[4] $y = y_{new}$
[5] return to step (1).

For Mandelbrot set, a point *(a, b)* belongs to the set if, after repeated iteration, x and y remain finite (*x* and *y* being initially zero). In 1992, John R. Tippetts gave a simple algorithm for generating an interesting Mandelbrot set by slightly modifying the above algorithm. He replaced y_{new} and x_{new} by *y* and *x* respectively (Tippetts 1992) and got the following algorithm:

[1] $x = x^2 - y^2 + a$
[2] $y = 2xy + b$
[3] return to step (1).

Then, simply by putting value of x from step (1) to step (2), Tippetts got an explicit equation for the new *y*-value:

$$y_{new} = 2x^2 y - 2y^3 + 2ay + b,$$

which produces a new class of fractals having virtually infinite complexity.

In this paper we generate a new Julia set using the Tippetts algorithm (Tippetts 1992). We also generate some new Mandelbrot and Julia sets by slightly modifying Tippetts algorithm. Like the Tippetts algorithm, the modified algorithm is also very interesting.

2. Main Results

The main steps required to display the Julia sets for the recursion $Z \rightarrow Z^2 + C$ (where $Z = x + iy$ and $C = a + ib$), consists the following statements (Peitgen *et al* 2006):

(1) $x_{new} = x^2 – y^2 + a$
(2) $y_{new} = 2xy + b$
(3) $x = x_{new}$
(4) $y = y_{new}$
(5) return to step (1).

For fixed C, a point x, y belongs to the set if, after repeated iteration, steps (1) and (2) remain finite. Following Tippetts (Tippetts 1992) we can replace y_{new} by simply y. Further, if we also replace x_{new} by x so that the steps (3) and (4) can be discarded. Now only three statements remain:

(1) $x = x^2 – y^2 + a$

(2) $y = 2xy + b$

(3) return to step (1).

Now, simply by putting value of x from step (1) to step (2), we get an explicit equation for the new y-value which is same as (Tippetts 1992):

$$y_{new} = 2x^2 y – 2y^3 + 2ay + b.$$

The algorithm formed by the above statements is equivalent to the following:

(1) $x_{new} = x^2 – y^2 + a$
(2) $y_{new} = 2x^2 y – 2y^3 + 2ay + b$
(3) $x = x_{new}$
(4) $y = y_{new}$
(5) return to step (1).

The corresponding Julia set for the above algorithm, contains fractal boundary. Julia set of the above algorithm for $C= -1.0+0i$, is the boundary between the inner capture zone and the outer regions of fast escape is shown in Figure1. The Julia set and its zoomed-in view for $C=-0.73+0.40i$ are shown in Figures 2 and 3.
In the next section, we generate some new fractals containing both Julia set and Mandelbrot set, by slightly modifying the algorithm:

(1) $x = x^2 – y^2 + a$
(2) $y = 2xy + b$
(3) return to step (1).

In the above algorithm by putting value of y from step (2) to step (1), we get an explicit equation for the new *x*-value:

$$x_{new} = x^2 - 4x^2y^2 + 4bxy - b^2 + a.$$

The newly formed algorithm is equivalent to the following statements:

(1) $x_{new} = x^2 - 4x^2y^2 + 4bxy - b^2 + a$
(2) $y_{new} = 2xy + b$
(3) $x = x_{new}$
(4) $y = y_{new}$
(5) return to step (1).

For Mandelbrot set, a point *(a, b)* belongs to the set if, after repeated iteration, *x* and *y* remain finite (*x* and *y* being initially zero) (Tippetts 1992). The corresponding Mandelbrot set for the quartic function x_{new} along with y_{new} is shown in Figure 4 and represents the boundary between the inner capture zone (black) and the outer regions of fast escape (yellow). Other zoomed-in views for Mandelbrot set [M-Set] are shown in Figures 6 and 7.

Again, Julia set for fixed $C= -0.1+0i$, a point *(x, y)* belongs to the set if, after repeated iteration, steps 1 and 2 remain finite. The corresponding Julia set for the quartic function x_{new} along with y_{new} is shown in Figure 7 and represents the boundary between the inner capture zone and the outer regions of fast escape. Some other Julia sets for different C-values are shown in Figures 8 and 9.

3. Conclusion:

Many interesting fractals can be generated from the modified algorithm for the recursion formula $Z \rightarrow Z^2 + C$. This approach opens up scope for obtaining new algorithms with more complex iterated functions having fascinating fractal boundaries.

4. Generation of Julia Sets using Tippetts algorithm:

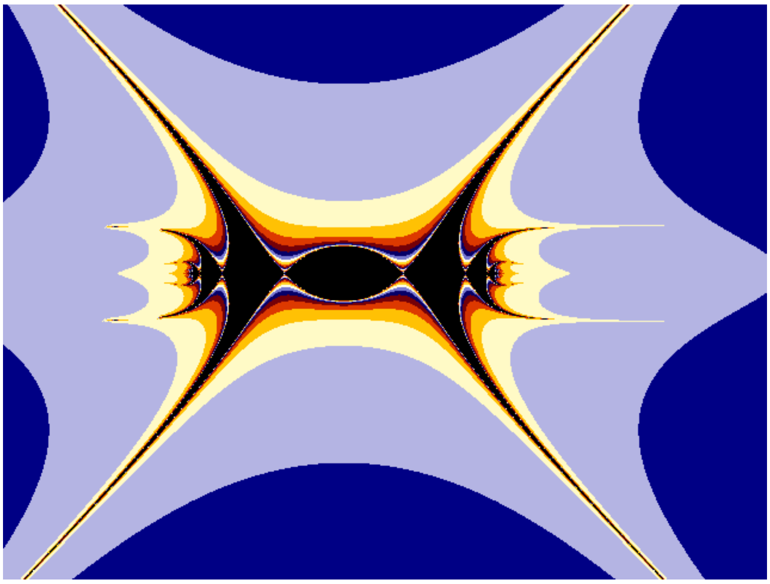

Figure1 Julia set for *C* = *(-1.0, 0)*

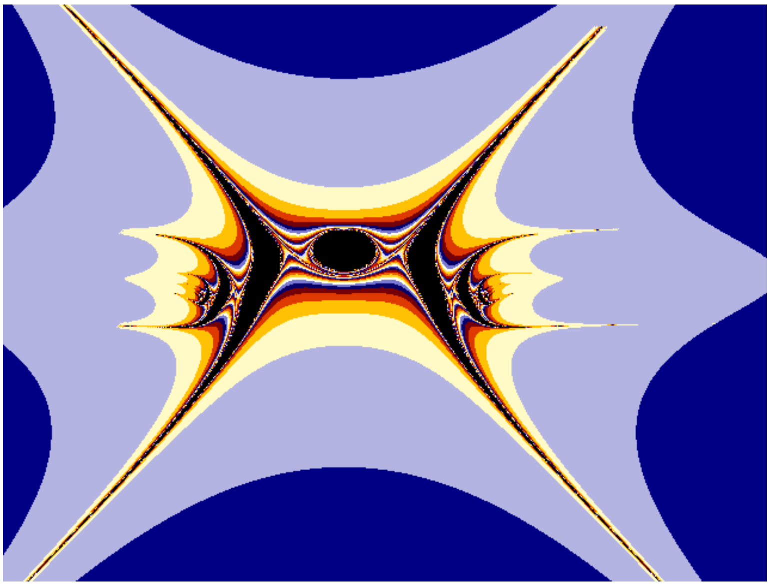

Figure 2 Julia set for *C* = *(-0.73+0.40i)*

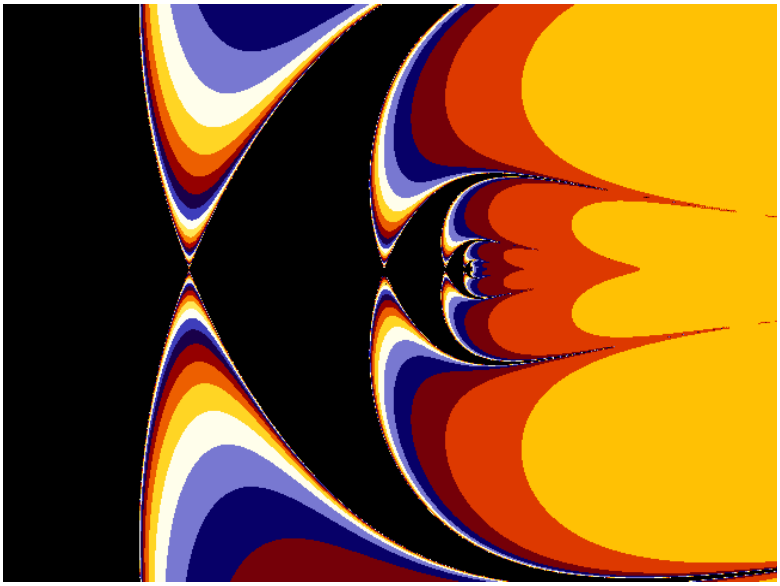

Figure 3 Zoomed in-view for C = *(-0.73+0.40i)*

5. Generation of Mandelbrot Sets using modified algorithm:

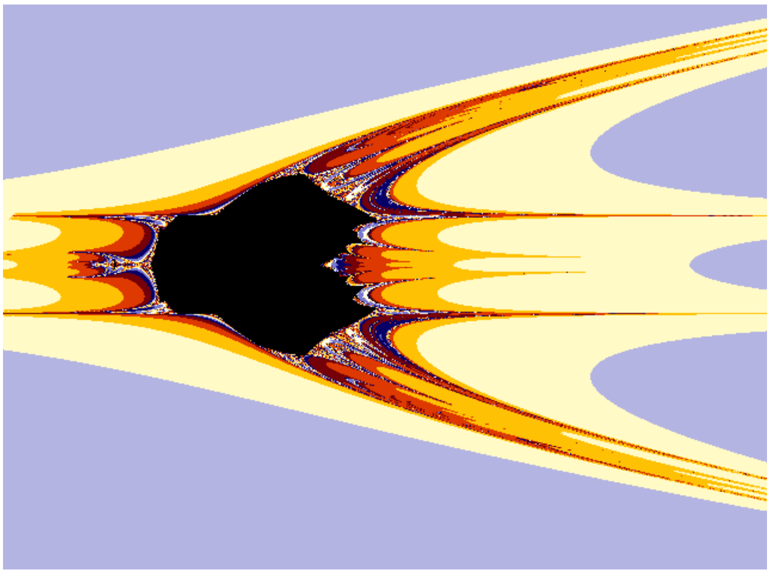

Figure 4 Mandelbrot set for x-new

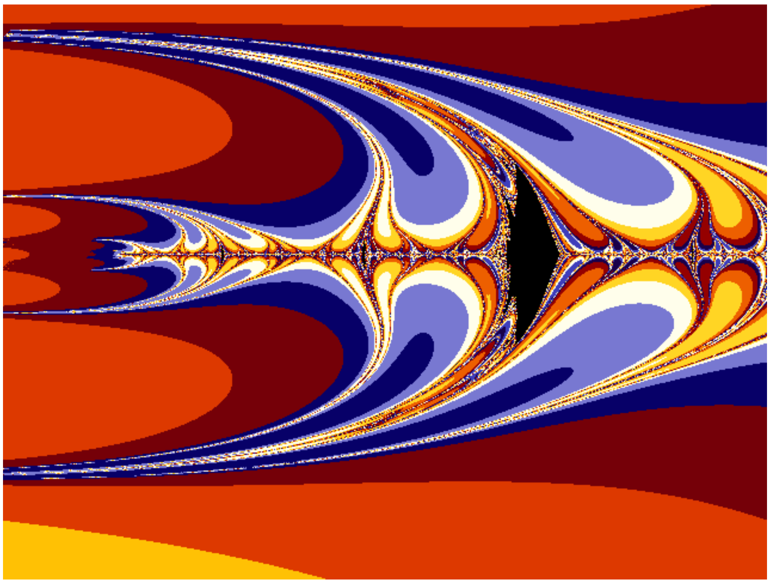

Figure 5 Zoomed in view for M. Set.

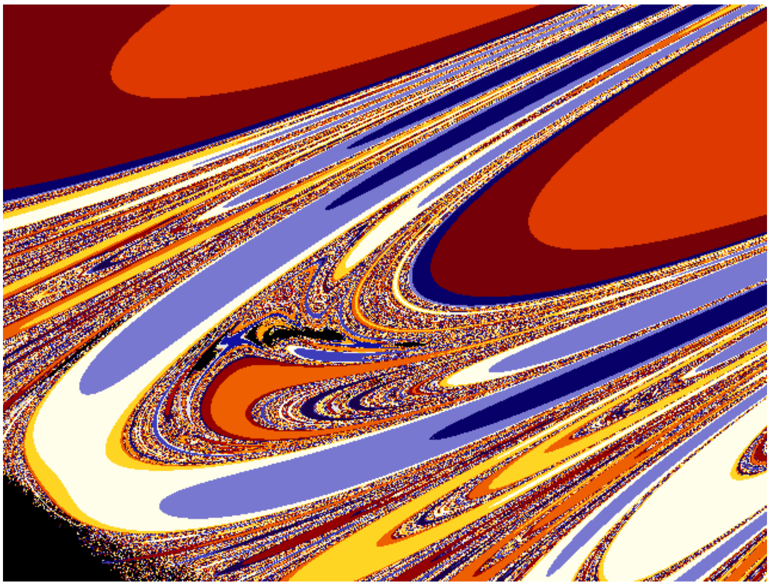

Figure 6 Zoomed in view for M. Set.

6. Generation of Julia Sets using modified algorithm:

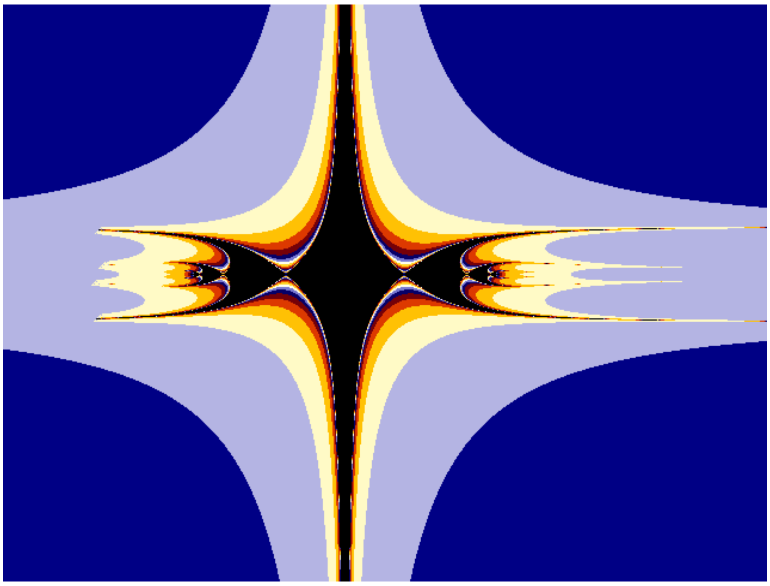

Figure 7 Julia set for, *(C=-0.1+0 i)*

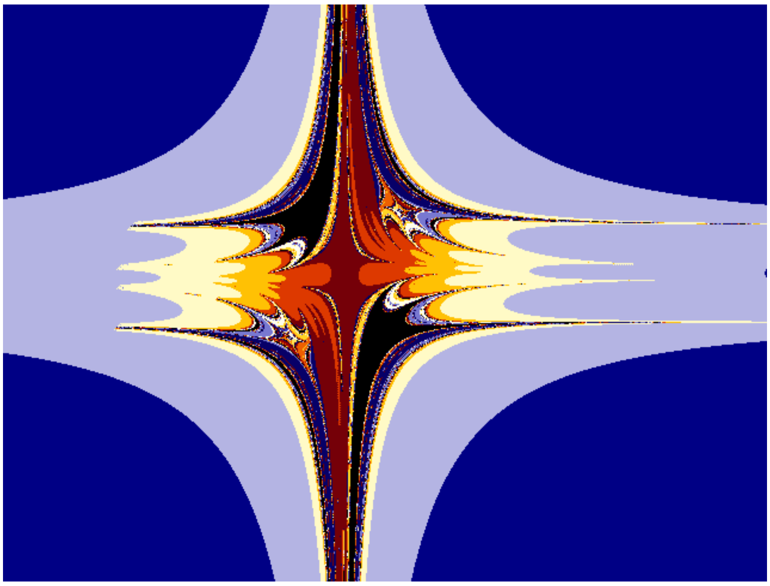

Figure 8 Julia set for, *C=(0.66- 0.12 i*)

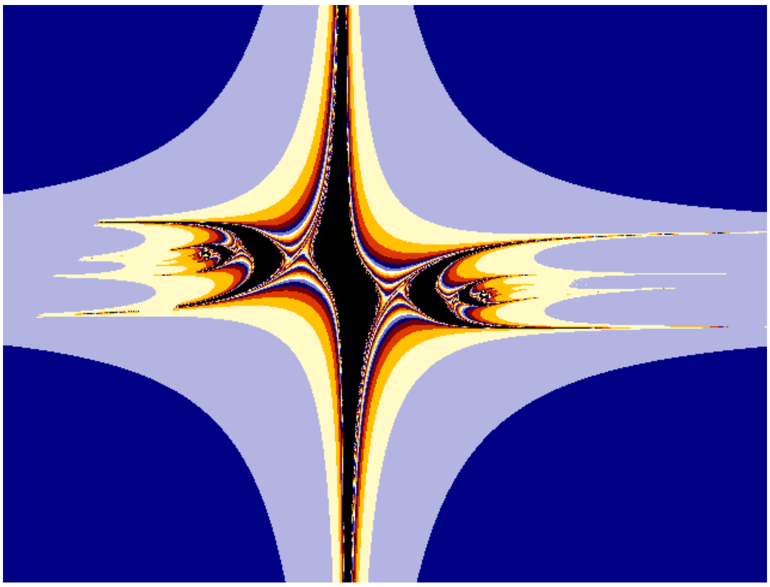

Figure 9 Julia set for, *(C=-0.73 +0.40 i)*

References

[1] Tippetts, John R. (1992), "A Simple Algorithm Giving an Interesting Mandelbrot Set", *The visual computer* **8**, Springer-Verlag , 200-201.

[2] Mandelbrot B. B. (1980), **"**Fractal Aspects of the Iteration of $z \to \boldsymbol{\lambda}\, z\,(1 - z)$ for Complex $\boldsymbol{\lambda}$ and z", *Ann of NY Acad Sci* **357**, 249-259.

[3] Peitgen H.O., Jurgens H. & Saupe D. (2006), **"**Chaos and Fractals", New Frontier of Science, (2^{nd} edition), MR2031217.