A Note on "Solving Bimatrix Games with Fuzzy Payoffs by Introducing Nature as a Third Player"

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Abstract
In this note it is pointed out that in all the results proposed in the paper (M. Larabani, Solving bimatrix games with fuzzy payoffs by introducing Nature as a third player, Fuzzy Sets and Systems 160(2009) 657-666) a mathematical incorrect assumption is considered.
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1. Introduction

Mangasarian and Stone [2] proved that Nash equilibrium point (a pair of strategies where the objectives of both the players are fulfilled simultaneously) for bimatrix games (two person nonzero-sum games) can be obtained by solving a quadratic programming problem. On the same direction, Larbani [1] obtained a quadratic problem P1 to obtain $\alpha-$ maxmin Nash equilibrium point for bimatrix games with fuzzy payoffs by introducing Nature as a third player.

In Section 3 of this note, it is pointed out that Larbani [1] has considered a mathematical incorrect assumption to obtain the problem P1 as well as in all the results proposed in the paper [1].

Problem P1 [1, equation 11, pp. 662]

Maximize $\left(\sum_{i=1}^{m}\sum_{j=1}^{n} x_i\left(a(\beta)_{ij}+b(\delta)_{ij}\right)y_j-\lambda-\eta\right)$

Subject to

$\sum_{j=1}^{n}\left(a(\beta)_{ij}\right)y_j-\lambda\le 0,\ \ i=1,2,\ldots,m;$

$\sum_{i=1}^{m}\left(b(\delta)_{ij}\right)x_i-\eta\le 0,\ \ j=1,2,\ldots,n;$

$\sum_{i=1}^{m}x_i=1;$

$\sum_{j=1}^{n}y_j=1;$

$x_i\ge 0,\ i=1,2,\ldots,m;$

$y_j\ge 0,\ j=1,2,\ldots,n.$

where,

$a(\beta)_{ij}=\beta_{ij}\left(a_{ij}^{U_\alpha}-a_{ij}^{L_\alpha}\right)+a_{ij}^{L_\alpha}$,

$\left[a_{ij}^{L_\alpha},a_{ij}^{U_\alpha}\right]$: $\alpha-$ cut set of fuzzy number $\tilde{a}_{ij}$

β_{ij} : a real number number lying in the interval $[0,1]$

$b(\delta)_{ij}=\delta_{ij}\left(b_{ij}^{U_\alpha}-b_{ij}^{L_\alpha}\right)+b_{ij}^{L_\alpha}$,

$\left[b_{ij}^{L_\alpha},\ b_{ij}^{U_\alpha}\right]$: $\alpha-$ cut set of fuzzy number $\tilde{b}_{ij}$

δ_{ij} : a real number number lying in the interval $[0,1]$.

2. Mathematical formulation of bimatrix games with fuzzy payoffs

To point out the mathematical incorrect assumption considered by Larbani [1], it is necessary to explain the method, followed by Larbani [1], to obtain the mathematical formulation (P1) of bimatrix games with fuzzy payoffs. Therefore, in this section, the same is presented.

Let player 1 and player 2 have mixed strategies as $x_i,\ i=1,2,\ldots,m$ and $y_j,\ j=1,2,\ldots,n$ respectively. Let $\tilde{A}_{m\times n}=(\tilde{a}_{ij})_{m\times n}$ and $\tilde{B}_{m\times n}=(\tilde{b}_{ij})_{m\times n}$ be fuzzy payoff matrices of player 1 and player 2 respectively. Player 1 maximizes profit over rows of fuzzy matrix $\tilde{A}_{m\times n}=(\tilde{a}_{ij})_{m\times n}$ and player 2 maximizes profit over columns of fuzzy matrix $\tilde{B}_{m\times n}=(\tilde{b}_{ij})_{m\times n}$.

Therefore, the objective of player 1 is to

Maximize $\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_i\tilde{a}_{ij}y_j\right)$

Subject to

$\sum_{i=1}^{m}x_i=1,$

$x_i\geq 0,\ i=1,2,\ldots,m.$

and

the objective of player 2 is to

Maximize $\left(\sum_{j=1}^{n}\sum_{i=1}^{m}x_i\tilde{b}_{ij}y_j\right)$

Subject to

$\sum_{j=1}^{n}y_j=1,$

$y_j\geq 0,\ j=1,2,\ldots,n.$

It is obvious from Definition 2.2 [1, Section 2, pp. 660] that Larbani [1] has used the relation "$\tilde{a}\succeq\tilde{b}\Rightarrow a(\beta)\geq b(\beta)$ where $a(\beta)=\beta\left(a^{U_\alpha}-a^{L_\alpha}\right)+a^{L_\alpha}$ and $b(\beta)=\left(\beta\left(b^{U_\alpha}-b^{L_\alpha}\right)+b^{L_\alpha}\right)$" for comparing two fuzzy numbers. Using the same relation, the objective of player 1 is to

Maximize $\left(\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_ia_{ij}y_j\right)(\beta)\right)$

Subject to

$\sum_{i=1}^{m}x_i=1,$

$x_i\geq 0,\ i=1,2,\ldots,m.$

and the objective of player 2 is to

Maximize $\left(\left(\sum_{j=1}^{n}\sum_{i=1}^{m}x_i\left(b_{ij}\right)y_j\right)(\delta)\right)$

Subject to

$\sum_{j=1}^{n}y_j=1,$

$y_j\geq 0,\ j=1,2,\ldots,n.$

Larbani [1] assumed that Maximize $\left(\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_ia_{ij}y_j\right)(\beta)\right)$ is equivalent to Maximize $\left(\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_i\left(a_{ij}\left(\beta\right)_{ij}\right)y_j\right)\right)$ and to Maximize $\left(\left(\sum_{j=1}^{n}\sum_{i=1}^{m}x_i\left(b_{ij}\right)y_j\right)(\delta)\right)$ is equivalent to Maximize $\left(\left(\sum_{j=1}^{n}\sum_{i=1}^{m}x_i\left(b(\delta)_{ij}\right)y_j\right)\right)$.

Therefore, the objective of player 1 is to

Maximize $\left(\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_i\left(a(\beta)_{ij}\right)y_j\right)\right)$

Subject to

$\sum_{i=1}^{m}x_i=1,$

$x_i\geq 0,\ i=1,2,\ldots,m.$

and the objective of player 2 is to

Maximize$\left(\left(\sum_{j=1}^{n}\sum_{i=1}^{m}x_i\left(b(\delta)_{ij}\right)y_j\right)\right)$

Subject to

$\sum_{j=1}^{n}y_j=1,$

$y_j\geq 0,\ \ j=1,2,\ldots,n.$

According to Mangasarin and Stone [2], the point $\left(x_i^0,\ y_j^0\right)$ will be Nash equilibrium point (a pair of strategies $x_i^0,\ y_j^0$ where the objectives of both the players are fulfilled simultaneously) if there exist real numbers $\lambda^0,\ \eta^0$ such that $x_i^0,\ y_j^0,\lambda^0,\eta^0$ satisfy the following conditions:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left(x_i^0\left(a(\beta)_{ij}\right)y_j^0\right)-\lambda^0=0;$$

$$\sum_{j=1}^{n}\sum_{i=1}^{m}\left(x_i^0\left(b(\delta)_{ij}\right)y_j^0\right)-\eta^0=0;$$

$$\sum_{j=1}^{n}\left(\left(a(\beta)_{ij}\right)y_j^0\right)-\lambda^0\leq 0,\ \ i=1,2,\ldots,m;$$

$$\sum_{i=1}^{m}\left(\left(b(\delta)_{ij}\right)x_i^0\right)-\eta^0\leq 0,\quad j=1,2,\ldots,n;$$

$$\sum_{i=1}^{m}x_i^0=1;$$

$$\sum_{j=1}^{n}y_j^0=1;$$

$$x_i^0\geq 0,\ \ i=1,2,\ldots,m;$$

$$y_j^0\geq 0,\ \ j=1,2,\ldots,n.$$

Also, according to Mangasarin and Stone [2], the values of $x_i^0,\ y_j^0,\lambda^0,\eta^0$ which will satisfy the above conditions will be optimal solution of the quadratic programming problem P1.

3. Mathematical incorrect assumption

It is obvious from Section 2 that to obtain the mathematical formulation i.e. problem P1, Larbani [1] has assumed that to Maximize $\left(\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_ia_{ij}y_j\right)(\beta)\right)$ is equivalent to Maximize $\left(\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_i\left(a_{ij}(\beta)_{ij}\right)y_j\right)\right)$ and to Maximize $\left(\left(\sum_{j=1}^{n}\sum_{i=1}^{m}x_i\left(b_{ij}\right)y_j\right)(\delta)\right)$ is equivalent to Maximize $\left(\left(\sum_{j=1}^{n}\sum_{i=1}^{m}x_i\left(b(\delta)_{ij}\right)y_j\right)\right)$.

However,

$$\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_i\left(a_{ij}\right)y_j\right)(\beta)=\beta\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_ia_{ij}^{U_\alpha}y_j-\sum_{i=1}^{m}\sum_{j=1}^{n}x_ia_{ij}^{L_\alpha}y_j\right)+\sum_{i=1}^{m}\sum_{j=1}^{n}x_ia_{ij}^{L_\alpha}y_j$$

and

$$\sum_{i=1}^{m}\sum_{j=1}^{n}x_i\left(a(\beta)_{ij}\right)y_j=\sum_{i=1}^{m}\sum_{j=1}^{n}x_i\left(\beta_{ij}\left(a_{ij}^{U_\alpha}-a_{ij}^{L_\alpha}\right)+a_{ij}^{L_\alpha}\right)y_j$$

$$\Rightarrow\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_i\left(a_{ij}\right)y_j\right)(\beta)\neq\sum_{i=1}^{m}\sum_{j=1}^{n}x_i\left(a(\beta)_{ij}\right)y_j$$

$$\Rightarrow\text{Maximize}\left(\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_i\left(a_{ij}\right)y_j\right)(\beta)\right)\neq\text{Maximize}\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_i\left(a(\beta)_{ij}\right)y_j\right)$$

This clearly indicates that to Maximize $\left(\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_i\left(a_{ij}\right)y_j\right)(\beta)\right)$ is not equivalent to Maximize

$\left(\left(\sum_{i=1}^{m}\sum_{j=1}^{n} x_i \left(a_{ij}(\beta)\right) y_j\right)\right)$.

Similarly, it can be proved that to Maximize $\left(\left(\sum_{j=1}^{n}\sum_{i=1}^{m} x_i \left(b_{ij}\right) y_j\right)(\delta)\right)$ is not equivalent to Maximize $\left(\left(\sum_{j=1}^{n}\sum_{i=1}^{m} x_i \left(b(\delta)_{ij}\right) y_j\right)\right)$.

However, Larbani [1], has considered the above mentioned mathematical incorrect assumption for obtaining the mathematical formulation i.e. problem P1 as well as in all the results.

Therefore, the mathematical formulation i.e. problem P1 as well as all the results proposed by Larbani [1] are not valid.

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