Nuclear Shell Model Calculations on Positive and Negative Parity States in upper $0f_{7/2}$- Shell Nuclei

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Abstract
The excitation energies for both the positive and negative parities of (^{90}Sr,^{90}Y,^{92}Nb and ^{92}Zr) isotopes have been calculated by employing modified surface delta interaction. A comparison between our results and the available experimental data to theoretical test for shell model description in isotopes above. It was seen that the obtained theoretical results are in agreement with the experimental data for each of the all isotopes under study.
Keywords: Excitation energy , Modified surface delta interaction ,Shell model.

Theory:
The underlying idea in the shell model, is that the nucleons outside the core (the valence nucleons) specify most of the properties of the nucleus. Thus that the model space is restricted to include only the degrees of freedom relevant for the valence nucleons, and the solution is found by a large-scale diagonalization within this model space , if a restricted space (model space) is to be used, the effects of the configurations left out of the model space has to be included in an effective interaction [1].The effective interaction is a key ingredient for the success of the nuclear shell model ,we can describe various nuclear properties accurately and systematically ,which helps us to understand nuclear structure. In this study ,we using modified surface delta interaction(MSDI) are an interaction between individual nucleons which exist near the Fermi surface. The primary principle behind of the MSDI is that only the nucleons on the surface interact with each other, while those within the nuclear interior are inert outside the surface[2] .The interaction are also not important since the probability of having a nucleon outside of the mean nuclear radius rapidly approaches zero. Thus, it is logical to restrict the residual interaction to the nuclear surface, and define the MSDI as [3,4,5] :-

$$V(\vec{r}_{12}) = -2F.(R_0\ u_0)^{-4}.\delta(r_1 - R_0).(r_2 - R_0).\delta(Cosw_{12} - 1) + B'(\tau(1).\tau(2)) + C \quad (1)$$

where R_0 is the nuclear radius, u_0 is radial wave function (here the radial wave functions are approximated to be same at the nuclear surface) and w_{12} is angle between the position vectors of the nucleons,Eq. (1) can be rewritten in terms of spherical harmonics as:-

$$V(\vec{r}_{12}) = -V_0 \sum_{lm} \frac{\delta(r_1 - R)}{r_1} Y^*_{lm}(w_1).\frac{\delta(r_2 - R)}{r_2} Y_{lm}(w_2) + B'(\tau(1).\tau(2)) + C \quad (2)$$

Here V_0 is the strength for (proton –proton , neutron-neutron and proton –neutron) interactions among the active nucleons, B and C are parameters which are constants in coordinate space.This is a reasonable interaction which has been used in various mass regions[6].In order to overcome some systematic discrepancies between the experimental and the(MSDI) predictions of the level energies and the spacing of (T =0 and T = 1) centroids of upon as a linear combination of the Heisenberg and Wigner terms .

3.Results and Discussions :
In this study, the selection model space of radioactive(^{90}Sr,^{90}Y,^{92}Nb and ^{92}Zr)isotopes between the N = 38 and N=40 shell closures provide the longest chain of semi-magic nuclei accessible to nuclear structure studies .

3.1 Energy Levels of { ^{90}Sr(Strontium) and ^{90}Y(yttrium) }nuclei :-
In order to estimate the energy levels of ^{90}Sr and ^{90}Y nuclei , we have performed shell model calculations by using MSDI interaction , after choice of ^{88}Sr as an inert core(semi doubly- magic nucleus), we choose a suitable model space to the valance nucleons which distributed over the single particle-orbits for ^{90}Sr nucleus was ($2d_{5/2}$,$1g_{7/2}$,$2d_{3/2}$,$3s_{1/2}$and $1h_{11/2}$) model space , as well as the ($1p_{1/2}$, $1g_{9/2}$) for proton and ($2d_{5/2}$,$3s_{1/2}$) for neutron were as model space in ^{90}Y nucleus .

The energy levels spectra of ^{90}Sr nucleus are presented in Table. (1) .It can seen that the agreement is good for the states(0.831, 1.655 ,1.892 and 4.240} MeV with our predicted theoretical results . The experimental states {2.674, 3.146 ,3.449 and 4.947}MeV were uncertain in the spins and parities such as {0^+, 5^- , (2^+,3,4^+) and 2^+ } are predicted at our calculations by {0^+ , 5^+, (1^+ , 3^+) and 2^+}.The theoretical levels such as{2.486,3.533 and 4.129 }were satisfactory agreement with experimental data{2.497, 3.594and 4.073}MeV ,which were specific spins{(2,3),(3^-,4^+)and (3^-,4^+)}. We predict spins and parities for experimental levels such as { 2.570, 3.032, 3.383, 3.394, (3.845 to 4.019) , 4.148,(4.335 to 4.919) ,(5.142 to 5.431) and (5.557 to 5.827)} MeV. On the other hand the theoretical levels as {5.641 and (6.027 to 7.118) } MeV, were undeterminate the energies ,spins and parties experimentally .

While : Table .(2)is showing the comparison of our calculations using the mentioned effective interaction for the energy levels spectrum of positive and negative parities at ^{90}Y nucleus are in better agreement with the experimental values [7] .The theoretical levels such{0.116 to 1.052} MeV, were excellent corresponds with experimental data. The level 1.298 MeV ,was uncertain experimentally$(5,6,7)^+$,it was predicted theoretically by 6^+ .Finally the levels{ 1.189 and 2.021 } MeV have been predicated by our results at spins and parities{4^+ and 4^+} respectively were undeterminate experimentally.

3.2 Energy Levels of (^{92}Nb(Niobium) and ^{92} Zr(Zirconium)) Nuclei :

The nucleus ^{90}Zr is taken as an inert core for ^{92}Nb and ^{92}Zr nuclei. The calculated energy levels of the^{92}Nb by using the model space ($1g_{9/2}$) for proton and ($2d_{5/2}$) for neutron are compared with the experimental data [8] , as shown in Table .(3) .The agreement for the excitation levels were excellent of this nucleus as{0.135,0.357, 0.285 , 0.594 and 0.657 }MeV ,were uncertain of parity experimentally .

While at ^{92}Zr nucleus, the model space were ($2d_{5/2}$,$1g_{7/2}$,$2d_{3/2}$,$3s_{1/2}$and $1h_{11/2}$) . The positive and negative parity of the calculated energy levels and experimental results [8]of low-lying states presented in Table.(4) . The comparison was obtained in acceptable agreement, for this nucleus in both, positive and negative states. There is uncertainty in the spins of some energy levels experimentally such{2.182 and 5.115}MeV and also uncertainty in spins and parities of experimental levels as {3.325, 3.379, 4.183, 4.380,4.606, 4.894, 5.310, 5.490, 5.581, 5.680, and 6.240} MeV. The levels {4.670 and 5.012}MeV ,were undeterminate the spins experimentally. We obtain in our calculations of ^{92}Zr nucleus on some energy levels were undeterminate the energies , spin and parties in experimental data .

Conclusions

In this present work , we predicted that the agreement between the calculated and experimental excitation energies of each nuclei which under to study are good .It were showed at the deviations from the experimental values were small values because model space increase for these nuclei and also in the framework of shell model calculations of energy levels were determined of levels undetermined experimentally .This investigation increases the theoretical knowledge of all isotopes with respect to energy levels. Its concluded that more experimental data were required to fully investigation the level structure of these nuclei.

Table (1): The comparison of the experimental excitation energies[7] with shell model predictions for ^{90}Sr nucleus

Theoretical Results		Experimental Results	
J^{π}	**E(MeV) MSDI Interaction**	**J^{π}**	**E(MeV)**
0_1^+	0	0^+	0
2^+	1.229	2^+	0.831
4^+	1. 558	4^+	1.655
2^+	2.120	2^+	1.892
3^+	2.486	2,3	2.497
3^+	2.857	——	2.570
0^+	2.981	(0^+)	2.674
4^+	3.026	——	3.032
5^+	3.184	(5^-)	3.146
2^+	3.338	——	3.383
7^-	3.380	——	3.394
$1^+,3^+$	3.461	$(2^+,3,4^+)$	3.449
$4^+,6^-,8^-$	3.533	$3^-,4^+$	3.594
6^+	3.738	——	3.845
2^+	3.983	——	3.954
4^-	4.012	——	4.019
$1^+,3^-,5^-$	4.129	$3^-,4^+$	4.073
0^+	4.206	——	4.148
2^+	4.282	2^+	4.240
5^-	4.327	——	4.335
1^+	4.493	——	4.493
6^-	4.564	——	4.580
4^+	4.922	——	4.919
2^+	5.096	(2^+)	4.947
3^+	5.161	——	5.142
7^-	5.173	——	5.239
5^-	5.458	——	5.426
6^-	5.540	——	5.431
4^+	5.641	——	——
6^-	5.685	——	5.557
9^-	5.777	——	5.600
8^+	5.787	——	5.623
2^+	5.793	——	5.785
10^+	5.823	——	5.822
0^+	5.883	——	5.827
2^-	6.027	——	——
4^-	6.069	——	
7^-	6.099	——	——
$3^+,5^-$	6.136	——	——
5^-	6.139	——	——
3^-	6.167	——	——
$2^-,4^-,6^-,8^+$	6.208	——	——
0^+	6.967	——	——
4^+	7.089	——	——
6^+	7.091	——	——
2^+	7.118	——	——

Table (2) :The comparison of the experimental excitation energies[7] with shell model predictions for ^{90}Y nucleus

Theoretical Results		Experimental Results	
J^π	E(MeV) MSDI Interaction	J^π	E(MeV)
2^-	0	2^-	0
3^-	0.116	3^-	0.202
7^+	0.691	7^+	0.681
2^+	0.768	2^+	0.776
5^+	0.956	5^+	1.046
3^+	0.976	3^+	0.953
0^-	1.008	0^-	1.211
1^-	1.052	1^-	1.371
4^+	1.192	—	1.189
6^+	1.253	$(5,6,7)^+$	1.298
5^+	1.941	$5^+,6^+$	1.962
4^+	2.088	—	2.021

Table (3): The comparison of the experimental excitation energies[8] with shell model predictions for ^{92}Nb nucleus

Theoretical Results		Experimental Results	
J^π	E(MeV) MSDI .Interaction	J^π	E(MeV)
7^+	0	7^+	0
2^+	0.134	$(2)^+$	0.135
5^+	0.305	$(5)^+$	0.357
3^+	0.329	$(3)^+$	0.285
4^+	0.594	$(4)^+$	0.480
6^+	0.657	$(6)^+$	0.501

Table (4) : The comparison of the experimental excitation energies[8] with shell model predictions for ^{92}Zr nucleus

Theoretical Results		Experimental Results	
J^π	E(MeV) (MSDI Interaction)	J^π	E(MeV)
0^+	0	0^+	0
2^+	1.213	2^+	0.934
4^+	1.508	4^+	1.495
2^+	2.218	$(2)^+$	2.182
3^+	2.645	—	2.666
4^+	2.873	4^+	2.864
6^+	2.952	6^+	2.957
3^+	2.958	3	3.039
0^+	3.027	—	—
2^+	3.227	2^+	3.262
5^+	3.283	$(^+)$	3.325
$1^+,3^+,5^-$	3.322	5^-	3.345
4^+	3.381	—	—
2^+	3.417	—	—
7^-	3.460	(7^-)	3.379
$1^+,3^-,5^+$	3.482	1	3.667
$4^-,6^-,8^-$	3.610	$3^+,4^+,5^+$	3.675
0^+	4.190	$(^+)$	4.183
2^+	4.230	$2^+,3^+$	4.213
4^+	4.323	(4^+)	4.380
3^+	4.527	—	4.504
5^-	4.560	(5^-)	4.606
1^+	4.687	$^+$	4.670
6^+	4.815	—	4.813
2^+	4.892	$(^+)$	4.894
9-	5.078	-	5.012
4^+	5.111	$(4)^+$	5.115
7^-	5.129	—	5.197
6^+	5.242	—	5.215
2^+	5.314	$(2^+,3^+)$	5.310
4^+	5.358	—	5.358
3^+	5.364	—	—
0^+	5.396	(0^+)	5.490
5^-	5.409	—	—
3^-	5.455	—	—
$2^+,4^+,6^-,8^-$	5.492	(2^+)	5.581
2^+	5.537	—	—
7^-	5.561	—	—
5^-	5.607	—	—
$4^-,6^-$	5.652	(4^+)	5.680
3^+	5.909	—	—
10^+	5.943	—	—
6^+	6.004	—	6.125
4^+	6.025	(4^+)	6.240
2^+	6.227	—	—
0^+	6.539	—	—

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