

Superposed Degenerate Parametric Oscillator with Coherent Light

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Abstract
In this paper, we have studied the statistical and squeezing properties of the light produced by superposed light beams generating by a pair of degenerate parametric oscillator whose cavity mode is driven by a coherent light and is coupled to a vacuum reservoir via a single port mirror. We obtain c-number Langevin equation associated with the normal ordering using the pertinent master equation. Employing the solution of the c-number Langevin equation and the density operator, we have determined the mean and the variance photon number, quadrature variances and squeezing spectrum of the superposed light beams.

1.Introduction

The quantum properties of light are largely determined by the state of the light modes. The well known quantum states of light are the number state, the coherent state, the chaotic state and the squeezed state[1-4]. Number state, coherent state and chaotic state have sub-poissonian, poissonian and super-poissonian photon statistics, respectively. The squeezing properties of a single-light mode are described by two Hermitian operators satisfying the uncertainty relation $\Delta a_{+}\Delta a_{-} \geq 1$. Squeezed light has potential applications in the detection of weak signals and in low-noise communications[1,2,4] and a light can be generated by quantum optical processes such as parametric oscillation [1-10], second harmonic generation[1-4,6,11].

Parametric oscillator is an important source of squeezed light. In this device a pump photon interacts with a nonlinear crystal inside a cavity is down-converted into two highly correlated photons. If these photons have the same frequencies the device is called degenerate parametric oscillator, otherwise it is nondegenerate parametric oscillator. Different authors have studied the quantum and statistical properties of the signal mode light produced by degenerate parametric oscillator[5-7,9,10].

In this paper, we seek to investigated the statistical and squeezing properties of superposed light beams generated by a pair of degenerate parametric oscillator whose cavity mode is driven by coherent light and coupled to a vacuum reservoir via a single port-mirror. We carry out our analysis employing the solution of the c-number Langevin equation and the density operator for the superposition of light beams.

2. The Density Operator

Suppose $\hat{\rho}'(\hat{a}_1^{+}, \hat{a}_1)$ is the density operator for signal light beams. Expanding the density operator in the normal order, we have

$$\hat{\rho}'(t) = \sum_{lm} C\, lm\hat{a}1 + l\hat{a}1m \tag{1}$$

Moreover, applying the completeness relation for coherent state

$$I = \int d2\alpha 1/\pi/\alpha 1\rangle\langle\alpha 1/, \tag{2}$$

and the relations

$$/\alpha 1\rangle\langle\alpha 1/\hat{a}m = (\alpha 1 + \partial/\partial\alpha 1*)m\, /\alpha 1\rangle\langle\alpha 1/, \tag{3}$$

$$/\alpha 1\rangle\langle\alpha 1/\hat{a} + l = /\alpha 1\rangle\langle\alpha 1/\hat{a}1 * l. \tag{4}$$

Eq. (1) can be rewrite as

$$\hat{\rho}'(t) = \frac{1}{\pi}\int d2\,\alpha 1 \sum_{lm} C\, lm\alpha 1 * l(\alpha 1 + \partial/\partial\alpha 1*)m\, /\alpha 1\rangle\langle\alpha 1/, \tag{5}$$

Or this equation can be rewritten as

$$\hat{\rho}'(t) = \int d2\,\alpha 1 Q1(\alpha 1 *, \alpha 1 + \partial/\partial\alpha 1*, t)\, \hat{D}(\alpha 1)\hat{\rho}0\hat{D} + (\alpha 1), \tag{6}$$

Where $\hat{\rho}_0 = |0\rangle\langle 0|$ is the density operator for a vacuum state at initial time and

$$Q1(\alpha 1 *, \alpha 1 + \partial/\partial\alpha 1*, t) = \frac{1}{\pi}\sum_{lm} C\, lm\alpha 1 * l(\alpha 1 + \partial/\partial\alpha 1*, t)m\, . \tag{7}$$

Considering the density operator for the beam at initial time in state other than vacuum. On the basis of Eq. (6), the density operator for the superposition of two light beams can be written as

$$\hat{\rho}(t) = \int d2\,\alpha 2 Q2(\alpha 2 + \partial/\partial\alpha 2*, t)\, \hat{D}(\alpha 2)\hat{\rho}0\hat{D} + (\alpha 2), \tag{8}$$

in which

$$Q2(\alpha 2 *, \alpha 2 + \partial/\partial\alpha 2*, t) = \frac{1}{\pi}\sum_{sr} C\, sr\alpha 2 * s(\alpha 2 + \partial/\partial\alpha 2*, t)r \tag{9}$$

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is the Q- function corresponding to the second light beam. Now inserting Eq.(6) into Eq.(8), we have

$$\hat{\rho}(t) = \int d2\,\alpha1\alpha2Q1(\alpha1*,\alpha1+\partial/\partial\alpha1*,t)Q2\alpha2\,Q2(\alpha2*,\alpha2+\partial/\partial\alpha2*,t)\hat{D}(\alpha2)\,\hat{D}(\alpha1)/0\rangle\langle0/\hat{D}+(\alpha1)\hat{D}+(\alpha2), \quad (10)$$

so that applying the relation

$$\hat{D}(\alpha2)\,\hat{D}(\alpha1)/0\rangle = exp[\tfrac{1}{2}\,(\alpha2\alpha1*-\alpha2*\alpha1]/\alpha2+\alpha1\rangle, \quad (11)$$

$$\langle0/\hat{D}+(\alpha1)\hat{D}+(\alpha2) = exp[\tfrac{1}{2}\,(\alpha2*\alpha1-\alpha2\alpha1*]\langle\alpha1+\alpha2\,/, \quad (12)$$

We find

$$\hat{\rho}(\mathrm{t}) = \int d2\,\alpha1d2\alpha2Q1(\alpha1*,\alpha1+\partial/\partial\alpha1*,\mathrm{t})\,Q2(\alpha2*,\alpha2+\partial/\partial\alpha2*,\mathrm{t})|\alpha2+\alpha1\rangle\langle\alpha1+\alpha2| \quad (13)$$

3. Photon Statistics

We seek to calculate the mean and variance of the photon number for the superposition of a pair of light beams employing the density operator and the Q-function. We now proceed to obtain the Q-function. The Q-function can be expressed as

$$Q\,(\alpha*,\alpha,t) = \frac{1}{\pi}\int d\,2\omega.\frac{1}{\pi}\,\emptyset a\,(\omega*,\omega,t)exp(\omega*\alpha-\omega\alpha*), \quad (14)$$

Where

$$\emptyset a\,(\omega*,\omega,t) = Tr\,(\hat{\rho}(0)exp(-\omega*\hat{a}(t))\,exp(\omega\hat{a}+(t)) \quad (15)$$

is the antinormally order characteristic function defined in the Heisenberg picture. Applying the Baker-Housedorf identity

$$e^{\hat{A}}e^{\hat{B}} = e^{\hat{A}}e^{\hat{B}}e^{[\hat{A},\hat{B}]}, \quad (16)$$

the characteristic function can be expressed in terms of c-number variables associated with the normal ordering is written in the form

$$\emptyset a\,(\omega*,\omega,t) = exp[-a\omega*\omega+\tfrac{1}{2}b(\omega*2+\omega2)+c(\omega-\omega*)], \quad (17)$$

in which

$$a = 1-\frac{\varepsilon1}{2\lambda+}(1-e-\lambda+t)+\frac{\varepsilon1}{2\lambda-}(1-e-\lambda-t), \quad (18)$$

$$b = -\frac{\varepsilon1}{2\lambda+}(1-e-\lambda+t)-\frac{\varepsilon1}{2\lambda-}(1-e-\lambda-t), \quad (19)$$

$$c = \frac{2\varepsilon2}{\lambda+}(1-e-\tfrac{1}{2}\lambda+t), \quad (20)$$

where ε_1 and ε_2, real and constant, are proportional to the amplitudes of the pump mode and the driving mode, respectively. $\lambda\pm = \kappa\pm2\varepsilon1$, where κ is the cavity damping constant and $\hat{a}$ is the annihilation operator for the cavity mode.

Now using Eq.(17) into Eq.(14), we have

$$Q(\alpha*,\alpha,t) = \frac{1}{\pi2}\int d\,2\omega exp[-a\omega*\omega+\tfrac{1}{2}b(\omega*2+\omega2)-\omega(\alpha*-c)+\omega*(\alpha-c)]. \quad (21)$$

Thus on performing the integration employing the relation

$$\int\frac{d2}{\pi}z\,exp(-az*z+bz+cz*+Az2+Bz*2) = [\frac{1}{a2-4AB}]1/2e^{(abc}+Ac2+Bb2)/a2-4AB, a>0, \quad (22)$$

$$Q(\alpha*,\alpha,t) = \frac{\sqrt{u2-v2}}{\pi}\,e^{-c\eta}e^{-u\alpha*\alpha}+\eta(\alpha*+\alpha)+\tfrac{1}{2}v\,(\alpha*2+\alpha2), \quad (23)$$

Where

$$u = \frac{\mathrm{a}}{\mathrm{a}2-\mathrm{b}2}, \quad (24)$$

$$v = \frac{b}{\mathrm{a}2-\mathrm{b}2}, \quad (25)$$

$$\eta = c(u-v). \quad (26)$$

This represents the Q-function for the light produced by a degenerate parametric oscillator whose cavity mode is driven by coherent light and coupled to a vacuum reservoir.

3·1. The Mean of Photon Number

Using the density operator, the mean photon number can be expressed as

$$\bar{n} = \langle\hat{a}+(t)\,\hat{a}(t)\rangle = Tr(\hat{\rho}(t)\,\hat{a}+\hat{a}), \quad (27)$$

Introducing Eq. (13), we see that

$$\bar{n} = \int d\,2\alpha1d2\alpha2\;Q1\left(\alpha1*,\alpha1+\frac{\partial}{\partial\alpha1*},t\right)Q2\left(\alpha2*,\alpha2+\frac{\partial}{\partial\alpha2*},t\right)[\alpha1*\alpha1+\alpha2*\alpha2+\alpha1*\alpha2+\alpha2*\alpha1], \quad (28)$$

Eq. (28) can be put in the form

$$\bar{n} = \langle\hat{a}1+(t)\,\hat{a}1(t)\rangle+\langle\hat{a}2+(t)\,\hat{a}2(t)\rangle+\langle\hat{a}1+(t)\rangle\langle\hat{a}2(t)\rangle+\langle\hat{a}1(t)\rangle\langle\hat{a}2+(t)\rangle,$$

(29)

in which

$$\langle \hat{a}i + \hat{a}i\rangle = \int d\,2\alpha i Qi(\alpha i *, \alpha i\ +\ \partial/\partial\alpha i *, t)\ \alpha i * \alpha i, \quad (30)$$
$$\langle \hat{a}i\rangle = \int d\,2\alpha i Qi(\alpha i *, \alpha i\ +\ \partial/\partial\alpha i *, t)\alpha i, \quad (31)$$

with i = 1, 2 corresponds to the first and second light beams.

Replacing α^* by α_i^* and α by $\alpha_i+\partial/\partial\alpha_i^*$ in Eq. (23) we can make a relation between the function

$$Qi(\alpha i *, \alpha i\ +\ \partial/\partial\alpha i *, t) = Qi(\alpha i *, \alpha i, t)\ exp[(-u\alpha i *\ + \eta\ +\ v\alpha i)\ \partial/\partial\alpha i * + \tfrac{v}{2}\ \partial 2/\partial\alpha i * 2]. \quad (32)$$

Now on account of Eq. (32), we see that

$$\langle \hat{a}i + \hat{a}i\rangle = \int d\,2\alpha i\ Qi(\alpha i *, \alpha i, t)\ exp[(-u\alpha i *\ + \eta\ +\ v\alpha i)\ \partial/\partial\alpha i * + \tfrac{v}{2}\ \partial 2/\partial\alpha i * 2]\ \alpha i * \alpha i, \quad (33)$$

With the aid of the eigenvalue equation [12,13]

$$\hat{A}\, f(x)\ = a\, f(x), \quad (34)$$

that satisfies

$$e^{\widehat{A}\, f(x)}\ = e^{a}\ f(x), \quad (35)$$

then we get

$$\langle \hat{a}i + \hat{a}i\rangle = \int d\,2\alpha i\ Qi(\alpha i *, \alpha i, t)\ \frac{\partial}{\partial\lambda}\exp(-u\alpha i *\ + \eta\ +\ v\alpha i)\lambda\alpha i\ + \tfrac{1}{2}\ v\lambda 2\alpha i 2\ + \lambda\alpha i * \alpha i\ /\lambda = 0 \quad .$$
(36)

Furthermore carrying out differentiate with respect to λ and setting λ = 0, we get

$$\langle \hat{a}i + \hat{a}i\rangle = \int d\,2\alpha i\ Qi(\alpha i *, \alpha i, t)[(1-u)\alpha i * \alpha i\ +\ \eta\alpha i\ +\ v\ \alpha i 2]. \quad (37)$$

On account of Eq. (23), we see that

$$\langle \hat{a}i + \hat{a}i\rangle = \sqrt{u2 - v2}\ e^{-c\eta}\ \frac{1}{\pi}\int d\,2\alpha i\ exp[-u\alpha i * \alpha i\ + m\ \alpha i *\ + n\alpha i\ + \tfrac{1}{2}v(\alpha i * 2\ +\ \alpha i 2]\ [(1-u)\ \alpha i * \alpha i\ + \eta\ \alpha i\ + v\alpha i 2]/m = n = \eta \quad ,$$
(38)

This equation can be rewritten in the form

$$\langle \hat{a}i + \hat{a}i\rangle = \sqrt{u2 - v2}\ e^{-c\eta}[(1-u)\frac{\partial 2}{\partial m\partial n} + \frac{\partial}{\partial n} + v\frac{\partial 2}{\partial n 2}]$$
$$\times \frac{1}{\pi}\int d\,2\alpha i\ exp[-u\alpha i * \alpha i\ + m\ \alpha i *\ + n\alpha i\ + \tfrac{1}{2}v(\alpha i * 2\ +\ \alpha i 2]/m = n = \eta\ . \quad (39)$$

With the help of Eq. (24), (25) and (26), we can write as

$$\langle \hat{a}i + \hat{a}i\rangle\ = c2 + a\ - 1. \quad (40)$$

Following the same procedure, we easily find

$$\langle \hat{a}i\rangle\ = c. \quad (41)$$

Employing Eq. (18) and (20), we have

$$\bar{n} = 2[\frac{2(2\varepsilon 2)2}{\lambda+2}\ (1 - e^{-\frac{\lambda+t}{2}}) - \frac{\varepsilon 1}{2\lambda+}(1 - e^{-\lambda}{}_{+}t) + \frac{\varepsilon 1}{2\lambda+}(1 - e^{-\lambda}{}_{-}t)]. \quad (42)$$

At steady state, we have

$$\bar{n}_{ss} = \frac{(4\varepsilon 2)2}{(\kappa+2\varepsilon 1)2} + \frac{(2\varepsilon 1)2}{\kappa 2-(2\varepsilon 1)2}\ . \quad (43)$$

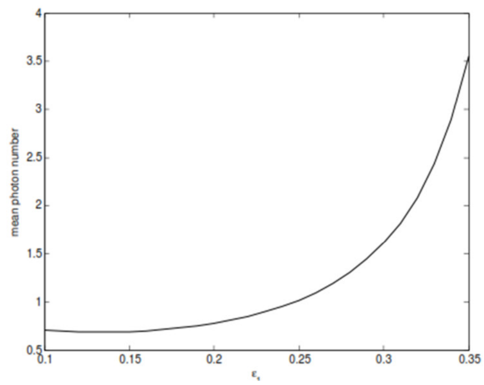

Figure 1: A plot of the mean photon number for the superposed light beams [Eq. (43)] versus ε_1 for ε_2= 0.2 and κ = 0.8.

The result in Eq. (43) shows the mean photon number at steady state for the superposed light beams is not a sum of the mean photon numbers of the constituent light beams. Figure.1 represents that the amplitude proportional to the pump mode increases the mean photon number for superposed beams.

For the case, in the absence of coherent light where ε_2 =0, Eq. (43) reduces to

$$\bar{n}_{ss} = \frac{(2\varepsilon 1)2}{\kappa 2-(2\varepsilon 1)2}\ . \quad (44)$$

This equation shows that the mean photon number of the superposition of a pair of light beams is twice the mean photon numbers of a degenerate parametric oscillator.

3.3 The Variance of Photon Number

The variance of a superposed light beams expressed by

$$(\Delta n)2 = \langle \hat{n}2 \rangle - \langle \hat{n} \rangle 2, \tag{45}$$

can be written as

$$(\Delta n)2 = \langle \hat{a} + 2\hat{a}2 \rangle + 2\bar{n} - \bar{n}2. \tag{46}$$

Using Eq. (13), we have

$$\langle \hat{a} + 2\hat{a}2 \rangle = \int d\, 2\alpha 1 d2\alpha 2\; Q1(\alpha 1 *, \alpha 1 + \partial/\partial\alpha 1 *, t)\; Q2(\alpha 2 *, \alpha 2 + \partial/\partial\alpha 2 *, t)/\alpha 12 + \alpha 22 + 2\alpha 1\alpha 2/2, \tag{47}$$

or this equation can be rewritten as

$$\langle \hat{a}^{+2}\hat{a}^2 \rangle = \langle \hat{a}_1{}^{+2}\hat{a}_1{}^2 \rangle + 4\langle \hat{a}_1{}^{+}\hat{a}_1 \rangle\langle \hat{a}_2{}^{+}\hat{a}_2 \rangle + 2\langle \hat{a}_1{}^{+2}\hat{a}_1 \rangle\langle \hat{a}_2 \rangle + 2\langle \hat{a}_1{}^{+}\hat{a}_1{}^2 \rangle\langle \hat{a}_2{}^{+} \rangle + \langle \hat{a}_2{}^{+2}\hat{a}_2{}^2 \rangle$$
$$+ 2\langle \hat{a}_2{}^{+2}\hat{a}_2 \rangle\langle \hat{a}_1 \rangle + 2\langle \hat{a}_2{}^{+}\hat{a}_2{}^2 \rangle\langle \hat{a}_1{}^{+} \rangle + \langle \hat{a}_1{}^{+2} \rangle\langle \hat{a}_2{}^2 \rangle + \langle \hat{a}_2{}^{+2} \rangle\langle \hat{a}_1{}^2 \rangle, \tag{48}$$

Then the variance is found to be

$$\Delta n^2 = \left(\frac{32\varepsilon 2}{\lambda +}\right)^2 (1 - e^{-\frac{1}{2}\lambda^{+t}})^2 \left[1 - \frac{2\varepsilon 1}{\lambda +}(1 - e^{-\lambda^{+t}})\right] + 2\left[\left(\frac{\varepsilon 1}{\lambda +}\right)^2 (1 - e^{\lambda + t})^2 + \left(\frac{\varepsilon 1}{\lambda -}\right)^2 (1 - e^{-\lambda^{-t}})^2\right]$$
$$2\left[\frac{\varepsilon 1}{\lambda +}(1 - e^{\lambda + t}) + \frac{\varepsilon 1}{\lambda -}(1 - e^{-\lambda^{-t}})\right]. \tag{49}$$

Eq. (58) represents the photon number variance for the superposition of a pair of degenerate parametric oscillator whose cavity driven by coherent light. At steady state, the variance of the superposed light beams can be written in the form

$$(\Delta n)^2_{ss} = 2\bar{n} - \frac{64\varepsilon 22\varepsilon 1}{(k + 2\varepsilon 1)3} + \frac{4\varepsilon 12(k2 + 4\varepsilon 12)}{(k2 - 4\varepsilon 12)2}. \tag{50}$$

We see from Fig. 2 that the photon statistics of the light produced by system under consideration has super-Poissonian photon statistics.

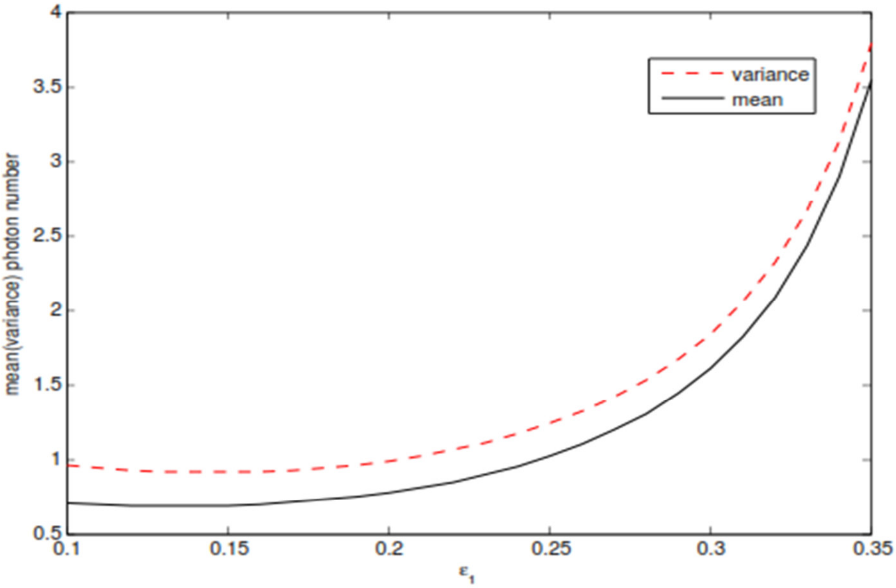

Figure 2: Plots the mean (solid black curve [Eq. (43)]) and variance (red dashed curve[Eq. (58)]) of photon number versus ε_1 for ε_2=0.2 and κ = 0.8.

4. Quadrature Squeezing

In this section we seek to analyze the quadrature variance and squeezing spectrum for the superposed light beam produced by a pair of degenerate parametric oscillator whose cavity mode driven by coherent light. We define the quadrature operator for the superposition of two light beams as

$$\hat{a}_{\pm} = \sqrt{\pm 1}\,(\hat{a}^{+} \pm \hat{a}). \tag{51}$$

With a modified commutation relation for two modes

$$[\hat{a}, \hat{a}^{\dagger}] = 2. \tag{52}$$

Using Eq. (60) and (61), the superposed light beams operators satisfy the commutation relation

$$[\hat{a}_{+}, \hat{a}_{-}] = 4i. \tag{53}$$

The quadrature variance for the superposition of two modes can be expressed as

$$\Delta a_{\pm}{}^2 = 2 + 2\langle \hat{a}^{+}\hat{a} \rangle \pm \langle \hat{a}^2 \rangle \pm \langle \hat{a}^{+2} \rangle \mp \langle \hat{a} \rangle^2 \mp \langle \hat{a}^{+} \rangle^2 - 2\langle \hat{a}^{+} \rangle\langle \hat{a} \rangle. \tag{54}$$

Using Eqs. (41), (42) and (55) and their conjugates, we find

$$\Delta a_{\pm}^{2} = 2 + 4(a - 1) \pm 4b. \quad (55)$$

In view of Eqs. (18) and (19), we write as

$$\Delta a_{\pm}^{2} = 2 \mp \frac{4\varepsilon 1}{\lambda \pm}(1 - e^{-\lambda_{\pm} t}). \quad (56)$$

Eq. (65) represents the quadrature variance for the system under consideration. The result of the superposed light beams is in a squeezed state and the squeezing occurs in the plus quadrature. We see that the driving coherent light has no effect on the quadrature variance of the superposed light beams. Moreover, the quadrature variances for the superposition are the sum of the quadrature variances of the separate light beams. At steady state, we have

$$\Delta a_{\pm}^{2} = 2 \mp \frac{4\varepsilon 1}{\kappa \pm 2\varepsilon 1}. \quad (57)$$

At threshold $\kappa = 2\varepsilon_1$, we see that

$$\Delta a_{+}^{2} = 1. \quad (58)$$

The quadrature squeezing for a signal beams relative to the corresponding coherent light can be expressed as

$$S_{+} = \frac{\Delta c + 2 - \Delta a + 2}{\Delta c + 2}, \quad (59)$$

where $\Delta c_{+}^{2} = 2$ is the quadrature variance for the two modes coherent state. Taking into account the quadrature variance for two coherent states and Eq. (66), the quadrature squeezing for the superposition of a pair of degenerate parametric oscillator with driving coherent lights turns out to be

$$S_{+} = \frac{2\varepsilon 1}{\kappa \pm 2\varepsilon 1}. \quad (60)$$

The quadrature squeezing for the superposed light beam is the average of a separate light beams 50% below coherent-state level.

4.1. Squeezing Spectrum

The squeezing spectrum of a signal mode light with central frequency ω_0 by

$$S_{\pm}^{out}(\omega) = 2 + Re\int_0^{\infty} d\tau \langle \hat{a}_{\pm}^{out}(t), \hat{a}_{\pm}^{out}(t+\tau)\rangle_{ss} e^{i\omega\prime\tau}, \quad (61)$$

Using the relation for output and cavity modes

$$\alpha_{\pm}^{out}(t) = \sqrt{\kappa}\, \alpha_{\pm}(t), \quad (62)$$

$$\hat{a}(t+\tau) = \sqrt{\pm 1}\, \hat{a}^{+}(t+\tau) \pm \hat{a}(t+\tau)\,). \quad (63)$$

Applying Eqs. (71) and (72), and along with the relation $\langle u, v\rangle = \langle uv\rangle - \langle u\rangle\langle v\rangle$, we get

$$S_{\pm}^{out}(\omega) = 2 + \kappa Re\int_0^{\infty} d\tau[\langle \pm\hat{a}^{+}(t)\, \hat{a}^{+}(t+\tau)\rangle_{ss} + \langle \hat{a}^{+}(t)\, \hat{a}(t+\tau)\rangle_{ss} + \langle \hat{a}(t)\, \hat{a}^{+}(t+\tau)\rangle_{ss}$$
$$\pm \langle \hat{a}(t)\, \hat{a}(t+\tau)\rangle_{ss}]e^{i\omega\prime\tau}. \quad (64)$$

The two-time correlation function that appears in the squeezing spectrum is expressible in the Schrödinger picture as

$$\langle \hat{a}^{+}(t)\, \hat{a}^{+}(t+\tau)\rangle = Tr[\hat{\rho}(t)\, \hat{a}^{+}(0)\, \hat{a}^{+}(\tau)]. \quad (65)$$

With the aid of Eq. (13) into Eq. (74), leads to

$$\langle \hat{a}^{+}(t)\, \hat{a}^{+}(t+\tau)\rangle = \int d^2\alpha_1 d^2\alpha_2\, Q_1(\alpha_1^{*}, \alpha_1 + \partial/\partial\alpha_1 *, t)\, Q_2(\alpha_2^{*}, \alpha_2 + \partial/\partial\alpha_2 *, t)$$
$$\times(\alpha_1^{*} + \alpha_2^{*})Tr[|\alpha_2 + \alpha_1\rangle\langle \alpha_1 + \alpha_2|\hat{a}^{+}(\tau)]. \quad (66)$$

We note that

$$Tr[\hat{\rho}(0)\, \hat{a}^{+}(\tau)] = Tr[\hat{\rho}(\tau)\, \hat{a}^{+}(0)], \quad (67)$$

in which $\hat{\rho}(0) = |\alpha_2 + \alpha_1\rangle\langle \alpha_1 + \alpha_2|$. Now replacing $(\alpha_i^{*}, \alpha_i, t)$ by $((\boldsymbol{\lambda}_i^{*}, \boldsymbol{\lambda}_i, \tau)$ in Eq. (13), the density operator $\hat{\rho}(\tau)$ can be written as

$$\hat{\rho}(\tau) = \int d^2\boldsymbol{\lambda}_1 d^2\boldsymbol{\lambda}_2\, Q_1(\lambda_1^{*}, \lambda_1 + \partial/\partial\boldsymbol{\lambda}_1 *, \tau)\, Q_2(\lambda_2^{*}, \lambda_2 + \partial/\partial\boldsymbol{\lambda}_2 *, \tau)|\boldsymbol{\lambda}_2 + \boldsymbol{\lambda}_1\rangle\langle \boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2|. \quad (68)$$

On account of Eq. (76), along with Eq. (77), we get

$$Tr[\hat{\rho}(\tau)\, \hat{a}_i^{+}(0)] = \langle \hat{a}_i^{+}(\tau)\rangle, \quad (69)$$

Where

$$\langle \hat{a}_i^{+}\rangle = \int d^2\boldsymbol{\lambda}_i\, Q_i(\lambda_1^{*}, \lambda_1 + \partial/\partial\lambda_1 *, \tau)\, \lambda_i^{*}. \quad (70)$$

Using Eqs. (75) and (31), we can write

$$\langle \hat{a}^{+}(t)\hat{a}^{+}(t+\tau)\rangle = \int d^2\alpha_1 d^2\alpha_2\, Q_1(\alpha_1^{*}, \alpha_1 + \partial/\partial\alpha_1 *, t)\, Q_2(\alpha_2^{*}, \alpha_2 + \partial/\partial\alpha_2 *, t)$$
$$\times(\alpha_1^{*} + \alpha_2^{*})Tr[\langle \hat{a}_1^{+}(\tau)\rangle + \langle \hat{a}_2^{+}(\tau)\rangle]. \quad (71)$$

For the component light beams initially to be

$$\langle \hat{a}_1^{+}(\tau)\rangle + \langle \hat{a}_2^{+}(\tau)\rangle = A_{+}(\tau)\,(\alpha_1^{*} + \alpha_2^{*}) + A_{-}(\tau)\,(\alpha_1 + \alpha_2) + 2c(\tau), \quad (72)$$

in which

$$A_{\pm}(t) = \frac{1}{2}(e^{-\frac{1}{2}\lambda_{+} t} \pm e^{-\frac{1}{2}\lambda_{-} t}). \quad (73)$$

Combining Eq. (80) and (81), we have

$\langle \hat{a}^{+}(t)\hat{a}^{+}(t+\tau)\rangle = \int d^2\alpha_1 d^2\alpha_2\, Q_1(\alpha_1^*,\alpha_1+\partial/\partial\alpha_1*, t)\, Q_2(\alpha_2^*,\alpha_2+\partial/\partial\alpha_2*, t)[2c(\tau)(\alpha_1^*+\alpha_2^*)$
$A_+(\tau)(\alpha_1^{*2}+2\alpha_1^*\alpha_2^*+\alpha_2^{*2})+A_-(\tau)(\alpha_1^*\alpha_1+\alpha_1^*\alpha_2+\alpha_2^*\alpha_1+\alpha_2^*\alpha_2)]$. (74)

With the help of Eqs. (40), (41) and (55) and their conjugates, and Eq. (75), at steady state we get

$$\langle \hat{a}^{+}(t)\hat{a}^{+}(t+\tau)\rangle_{ss} = -\frac{\varepsilon 1}{\lambda+}e^{-\frac{1}{2}\lambda_+\tau} - \frac{\varepsilon 1}{\lambda-}e^{-\frac{1}{2}\lambda_-\tau} + \left(\frac{4\varepsilon 2}{\lambda+}\right)^2, \quad (75)$$

$$\langle \hat{a}^{+}(t)\hat{a}(t+\tau)\rangle_{ss} = -\frac{\varepsilon 1}{\lambda+}e^{-\frac{1}{2}\lambda_+\tau} + \frac{\varepsilon 1}{\lambda-}e^{-\frac{1}{2}\lambda_-\tau} + \left(\frac{4\varepsilon 2}{\lambda+}\right)^2, \quad (76)$$

Using Eqs. (84) and (85) and their conjugates, Eq. (70), can be written as

$$S_{\pm}^{out}(\omega) = 2 \mp 2\kappa Re\int_0^\infty \left[\frac{2\varepsilon 1}{\lambda\pm}\left(e^{-\frac{1}{2}\lambda_\pm\tau} + \left(\frac{4\varepsilon 2}{\lambda+}\right)^2(-1\mp 1)\right)\right] e^{i\omega'\tau}\, d\tau. \quad (77)$$

Based on the relation,

$$\int_0^\infty e^{-ax}dx = \frac{1}{a}, \qquad a>0, \quad (78)$$

we carry out the integration and taking the real part we get

$$S_{\pm}^{out}(\omega) = 2 \mp \frac{2\kappa\varepsilon 1}{\left(\frac{\kappa}{2}\pm\varepsilon 1\right)2+\omega'2}. \quad (79)$$

The squeezing spectrum of the output signal mode is found to be

$$S_{\pm}^{out}(\omega) = \left[2 \mp \frac{2\kappa\varepsilon 1}{\left(\frac{\kappa}{2}\pm\varepsilon 1\right)2+\omega'2}\right], \quad (80)$$

At threshold at $\kappa = 2\varepsilon_1$, we see that

$$S_{+}^{out}(\omega) = \frac{\kappa 2+2\omega'2}{\kappa 2+\omega'2}, \quad (81)$$

$$S_{-}^{out}(\omega) = \frac{\kappa 2+2\omega'2}{\omega'2}, \quad (82)$$

The plots in Figure 3 shows that the maximum squeezing occurs for any value of ε_1 at $\omega'= 0$ and the squeezing spectrum increases when $|\omega'|$ increases. We note that the effect of the parameter ε_1 is to increase the degree of squeezing. From Eq. (89) the driving coherent light ε_2 has also no effect on the squeezing spectrum of the output mode. The superposed squeezing spectrum are the average of separate beams.

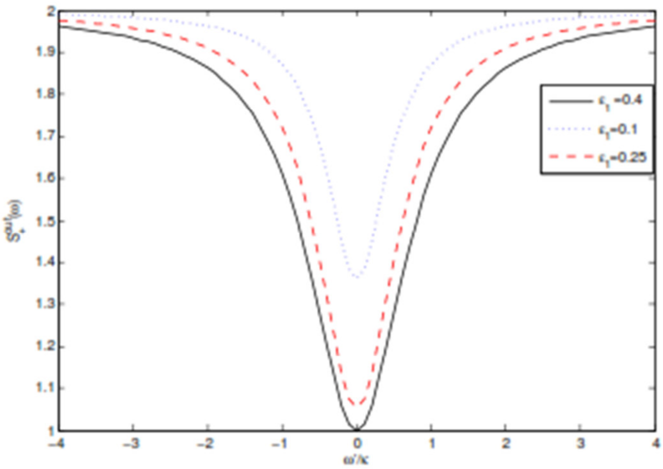

Figure 3: Plots the plus squeezing spectrum Eq. (89) versus ω/κ for ε_1=0.1, ε_1= 0.25, ε_1= 0.4 and κ = 0.8.

5. Conclusion

We have considered the superposition of a pair of degenerate parametric oscillator with the cavity mode driven by coherent light. We have determined the density operator of the superposed light beams with the aid of the resulting density operator and the Q-function. We have calculated the mean and variance of the photon number as well as the quadrature variance and squeezing spectrum of the superposed light beams. The result shows that the mean photon number of the superposed light beams is not the sum of the separate light beams. This is due to the driving coherent light increases the mean photon number. The results also show that the quadrature variance and squeezing spectrum of the superposed light beams are the sum of the two beams. The quadrature squeezing of the superposed light beams are the average of the two beams. As the parameter ε_1 increase, the quadrature squeezing increases. Moreover, the maximum quadrature squeezing and squeezing spectrum for superposed light beams is 50% below coherent state.

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