Echo Cancelation Using Least Mean Square (LMS) Algorithm

Samira A. Mahdi
University of Babylon/College of Science/Physics Dept. Iraq/Babylon

Abstract
The aim of this work is to investigate methods for restoring signals that are corrupted by one or more echoes. Echo commonly occurs over communication channels such as telephone and ADSL lines. A reasonable model of the echo process at the time domain has been proposed. The echo removal system is modeled as an FIR filter with an unknown impulse response. We estimate the required impulse response by transmitting a known signal \( x[n] \) over the channel and observing the corresponding output \( y[n] \). We want a system that takes the echo signal \( y[n] \) as input, andoutputs the original signal \( x[n] \).

Keywords: Echo cancelation, FIR filter, Least Mean Square (LMS)

1. Introduction
Echo is an undesired signal impedance mismatch in the system, its occur over communication system channels such as telephone and ADSL lines. This problem was discussed in [1]. Two types of echo (Acoustic echo, hybrid echo) have been proposed and discussed in [2]. Many different algorithms have been implemented for removing the echo signal such as filter adaptation in AEC [3]. The most common one is the normalized least-mean squares (NLMS) algorithm [4] which has been shown to perform well for the AEC problem while at the same time having a rather low computational complexity. In addition, the other way that was adaptive an acoustic echo cancelation and Double-talk Detection problem has been investigated [5], as well as the applications of echo cancelation was discussed in [6]. Based on the two- channel fast recursive least-squares algorithm a complete implementation of stereophonic acoustic echo canceler is presented [7].

The goal of this research is to find methods for restoring signals that are distorted by one or more echo. The time domain model that represents echo process is

\[
y[n] = x[n] + a_1 x[n - n_d] + a_2 x[n - 2n_d] \ldots \ldots (1)
\]

Where:
\( n_d \) is the echo delay, \( a_1 \) and \( a_2 \) are the reflection coefficients.

2. Echo Cancelation
First of all, if all the parameters in (1) are known, then we can determine the inverse function analytically. Where \( G(z) = 1/H(z) \), \( H(z) = \frac{y(z)}{x(z)} \) which will help the echo cancellation processes. Assuming \( n_d = 1, a_1 = 2, a_2 = 1 \)

So,
\[
H(z) = \frac{y(z)}{x(z)} = 1 + 2z^{-1} + z^{-2}
\]

And the ROC: is \(|z| > 1\).

In case we do not know these parameters. Adaptive filters are therefore used to do echo cancellation.

3. Least Mean Square (LMS) Algorithm
The Least Mean Square (LMS) algorithm was first developed by Widrow and Hoff in 1959 through their studies of pattern recognition [8]. From there it has become one of the most widely used algorithms in adaptive filtering. The LMS algorithm is a type of adaptive filter known as stochastic gradient-based algorithms as it utilizes the gradient vector of the filter tap weights to converge on the optimal Wiener solution. It is well known and widely used due to its computational simplicity. This simplicity that has made it the benchmark against which all other adaptive filtering algorithms are judged. With each iteration of the LMS algorithm, the filter taps weights of the adaptive filter are updated according to the following formula.

\[
w(n+1) = w(n) + 2\mu e(n)x(n)
\]

Here \( x(n) \) is the input vector of time delayed input values, \( x(n) = [x(n) x(n-1) x(n-2) ... x(n-N+1)]^T \). The vector \( w(n) = [w0(n) w1(n) w2(n) ... wN-1(n)]^T \) represents the coefficients of the adaptive FIR filter tap weight vector at time \( n \). The parameter \( \mu \) is known as the step size parameter and is a small positive constant.

4. Results
4-1. One Reflection coefficient
The results using one reflection coefficient \( a_1=0. 5, \) LMS step size \( \mu= 5x10^{-7} \), and the delay parameter \( n_d=10 \) are shown in Fig.1. Where the desired echo-free signal on the top, and the echo signal is obviously shown in the
down of the figure. Whereas, the signal after passing through the adaptive filter is shown in Fig. 2, for different values of the echo delay parameter (nd=10, 100, and 1000). These values have been chosen according to reliability and the validity of the calculated mean error amount. Table 1 shows the value of the mean error which is decreased as nd decreased. Therefore, the echo delay is an important parameter that can control the efficiency of the adaptive filter. Note that, the other parameters (LMS step size and the value of the reflection coefficient $a_1$) were fixed.

$$\text{Figure 1. The echo free signal at the top and the echo signal on the down.}$$

$$\text{Figure 2. From the left, the output signal, error, square error at nd=10, nd=100 and nd=1000, LMS step size }= 5\times10^{-7}, \text{ and } a_1 = 0.5.$$ 

<table>
<thead>
<tr>
<th>Delay (nd)</th>
<th>LMS step size($\mu$)</th>
<th>$a_1$</th>
<th>Mean error</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$5 \times 10^{-7}$</td>
<td>0.5</td>
<td>0.784</td>
</tr>
<tr>
<td>100</td>
<td>$5 \times 10^{-7}$</td>
<td>0.5</td>
<td>0.795</td>
</tr>
<tr>
<td>1000</td>
<td>$5 \times 10^{-7}$</td>
<td>0.5</td>
<td>0.794</td>
</tr>
</tbody>
</table>

Fig. 3 shows the output signal, error, and square error for several different amounts of the reflection coefficient ($a_1 = 0.5, 2, \text{ and } 6$) while the others parameters were fixed LMS step size $= 5\times10^{-7}$, and nd $=100$. The results are illustrated in Table 2. Where the mean error values fluctuate as the value of $a_1$ was increased.

$$\text{Figure 3. From the left, the output signal, error, square error at } a_1=0.5, a_1=2 \text{ and } a_1=6, \text{ respectively } \text{LMS step size }= 5\times10^{-7}, \text{ and } \text{nd }=100.$$
Table 2. The variation of Mean error with the value of $a_1$

<table>
<thead>
<tr>
<th>$a_1$</th>
<th>LMS step size ($\mu$)</th>
<th>Delay ($n_d$)</th>
<th>Mean error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$5 \times 10^{-7}$</td>
<td>100</td>
<td>0.795</td>
</tr>
<tr>
<td>2.0</td>
<td>$5 \times 10^{-7}$</td>
<td>100</td>
<td>0.797</td>
</tr>
<tr>
<td>6.0</td>
<td>$5 \times 10^{-7}$</td>
<td>100</td>
<td>0.785</td>
</tr>
</tbody>
</table>

Fig. 4 and Table 3 are utilized the effect of the LMS step size ($\mu$) of the mean error value under constant values of the reflection coefficient $a_1=0.5$ and echo delay $n_d=100$. The results show that the mean error decreased as $\mu$ is increased, however the degradation in the mean value is limited at a certain amount of $\mu=0.5$ the mean value reach about Inf.

Figure 4. From the left, the output signal, error, square error at LMS step size $\mu = 5 \times 10^{-5}$, $\mu = 5 \times 10^{-3}$, and $\mu = 0.5$, $a_1=0.5$, and $n_d=100$.

Table 3. The variation of Mean error with the value of $\mu$

<table>
<thead>
<tr>
<th>LMS step size ($\mu$)</th>
<th>$a_1$</th>
<th>Delay ($n_d$)</th>
<th>Mean error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times 10^{-7}$</td>
<td>0.5</td>
<td>100</td>
<td>0.795</td>
</tr>
<tr>
<td>$5 \times 10^{-5}$</td>
<td>0.5</td>
<td>100</td>
<td>0.579</td>
</tr>
<tr>
<td>$5 \times 10^{-3}$</td>
<td>0.5</td>
<td>100</td>
<td>0.527</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>100</td>
<td>Inf</td>
</tr>
</tbody>
</table>

4-2. Two Reflection coefficient

In this section we will discuss the effect of using two reflection coefficient and how would change the value of the mean error. Fig. 5 illustrates the error, square error at LMS step size $\mu =5 \times 10^{-7}$, $5 \times 10^{-5}$, $5 \times 10^{-3}$, and $\mu = 0.5$ respectively when $a_1= a_2=0.5$, and $n_d=100$. Table 4 shows the amount of mean error reach about $2 \times 10^0$, while it reached about 0.576 at the same conditions that was shown in Table 3. In this case, the addition of 2nd reflection coefficient in the Eq. 1 make the situation is harder to investigate the typical values of $\mu$.

Figure 5. From the left, the output signal, error, square error at LMS step size $\mu = 5 \times 10^{-7}$, $\mu = 5 \times 10^{-5}$, $5 \times 10^{-3}$, and $\mu = 0.5$ respectively $a_1= a_2=0.5$, and $n_d=100$.

Table 4. The variation of Mean error with the value of $\mu$

<table>
<thead>
<tr>
<th>LMS step size ($\mu$)</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>Delay ($n_d$)</th>
<th>Mean error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times 10^{-7}$</td>
<td>0.5</td>
<td>0.5</td>
<td>100</td>
<td>0.787</td>
</tr>
<tr>
<td>$5 \times 10^{-5}$</td>
<td>0.5</td>
<td>0.5</td>
<td>100</td>
<td>0.633</td>
</tr>
<tr>
<td>$5 \times 10^{-3}$</td>
<td>0.5</td>
<td>0.5</td>
<td>100</td>
<td>$2.433 \times 10^3$</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>100</td>
<td>Inf</td>
</tr>
</tbody>
</table>
To evaluate the effect of the 2nd reflection coefficient we introduce three different values of $a_2$, and as shown in Fig. 6 and Table 5 under constant values of $\text{LMS step size} = 5 \times 10^{-5}$, $a_1=0.5$, and $n_d=100$. The amount of mean error starts to increase as $a_2$ increase, therefore adding another reflection coefficient to adaptive filter increase the amount of error.

Finally, the effect of the echo delay $n_d$ has been studied under a constant value of the other parameters, LMS step size $\mu = 5 \times 10^{-5}$, and $a_1=a_2=0.5$. As shown in Table 6 and Fig. 7 from the left $n_d=10$, 100, 500, 1000, respectively. The results show that the mean error value increases as echo delay increase when the 2nd reflection parameter is used.

5. Conclusion:
1- The most significant variable is the step size of adapter filter, because it controls the influence of the updating factor. Selection of a suitable value for $\mu$ is imperative to the performance of the LMS algorithm, if the value is too small the time of adaptive filter takes to converge to the optimal solution will be too long; if $\mu$ is too large the adaptive filter becomes unstable and its output diverges.
2- The second important parameter is the echo delay. A long delay required a large impulse response length, so the computational expense becomes very high.
3- The mean error value increasing as the value of reflection coefficient increase.

References