Effect of Wiggler Magnetic Field on Second Harmonic Generation in Quantum Plasma

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Abstract
Second harmonic generation due to linearly polarized laser pulse propagating through quantum plasma immersed in a transverse wiggler magnetic field is studied using the quantum hydrodynamic (QHD) model. The effects associated with the Fermi pressure, the Bohm potential and the electron spin have been taken into account. Wiggler magnetic field plays both a dynamic role in producing the traverse harmonic current as well as kinematical role in ensuring phase-matching. The quantum dispersive effects also contribute to the intensity of second harmonics.

Keywords: Quantum plasma, Harmonic generation, Phase matching

1. Introduction
The study of electron-wave interaction in the presence of background plasma has attracted a lot of interest over the past few decades. The physical phenomenon of interaction of a high-intensity laser radiation with plasma leads to a number of relativistic and nonlinear effects such as self modulation, self-focusing, Raman scattering, and harmonic generation [1-2]. The generation of harmonic radiation is an important area of laser-plasma interaction for its value in many applications [3-10]. Harmonic generation offers an alternative source for short wavelength generation and important tool for diagnostics of nonlinear media. With the development of high intensity short pulse lasers, the electron motion becomes highly nonlinear, giving rise to nonlinearity rather than the anharmonicity of the bounded electron oscillation in atoms and molecules. Theory for coherent emission in the direction of propagation of laser beam, referred to as relativistic harmonic generation, has been established [1,11]. It pronounces that because of the mismatch between the phase velocities of the laser pulse and the generated harmonics under the collective response of the plasma, the conversion efficiency should be low unless a means for phase-matching [12] is applied. Matsumoto and Tanaka [13] have presented analysis of quasi phase-matched second-harmonic generation in the reflection by backward propagating interaction and showed that bistability appears in the generated second-harmonic power if the amount of phase mismatch is suitably chosen. Meyer and Zhu [14] claimed to have observed the second relativistic harmonic generated under the condition of beam filamentation. However, such second harmonic light has been later identified by several groups [15,16] to be associated with the transverse-density depression derived by laser self-channeling or filamentation, as is evident from its broad angular width caused by the plasma density gradient [14]. To increase the efficiency of the harmonic generation process which is significantly affected due to the phase mismatch between the fundamental and generated harmonics radiation several schemes have been proposed to make harmonic generation a resonant one. Singh et. al. [17] has shown that a density ripple in a plasma could be properly employed for resonant second harmonic generation. Nitikant and Sharma [18,19] found that wiggler magnetic field plays both a dynamic role in producing the transverse harmonic current and a kinematical one in ensuring phase-matching. Parashar and Pandey [20] proposed the employing of a density ripple to compensate for the momentum mismatch between the pump and second- harmonic wave in plasma and semiconductor respectively. Shkolnikov et al. [21] demonstrated the feasibility of optimal quasi phase matching for higher-order harmonic generation in gases and plasmas with modulated density. Rax and Fisch [22] studied phase modulated relativistic third harmonic generation employing resonant density modulation in plasma. Agrawal et al. [23] have studied resonant second harmonic generation of a millimeter wave in a plasma filled waveguide in the presence of a helical magnetic wiggler. Weissman et. al. [24] have studied second harmonic generation in Bragg-resonant quasi-phase-matched periodically segmented waveguides. Ding et al. [25] have developed a theory for quasi-phase-matched backward second and third harmonic generation in a periodically doped semiconductor.

All the above work has been done for classical plasma. Classical plasma physics has mainly focused on regimes of high temperatures and low densities, in which quantum mechanical effects play no role. Plasma where the density is quite high and the de–Broglie thermal wavelength associated with the charged particle i.e., \( \lambda_B = \frac{h}{(2\pi mk_B T)^{1/2}} \) approaches the electron Fermi wavelength \( \lambda_{Fe} \) and exceeds the electron Debye radius \( \lambda_D \).

51
Consider a linearly polarized laser beam with electric field, $\vec{E}_y = \hat{y} E_o \exp(k_o z - \omega_o t)$. The interaction dynamics is governed by the following set of QHD equations.

$$\frac{\partial \vec{v}}{\partial t} = -\frac{e}{m} \left[ \vec{E} + \frac{1}{c} (\vec{v} \times \vec{B}) \right] - \frac{\hbar^2}{2m^2} \vec{v} \frac{\partial^2 \hat{n}}{\partial \vec{r}^2} + \frac{\hbar^2}{m \hbar} \vec{v} (\frac{1}{\sqrt{n}} \frac{\partial^2 \sqrt{n}}{\partial \vec{r}^2}) - \frac{2\mu_B}{m \hbar} \vec{S} \cdot \vec{B},$$

$$\left( \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \vec{S} = \left( \frac{2\mu_B}{\hbar} \right) \left( \vec{B} \times \vec{S} \right),$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \vec{v}) = 0.$$
where, $\vec{v}$ is the velocity, $\hbar$ is the Planck’s constant divided by $2\pi$, $v_F$ is the Fermi velocity and $\vec{S}$ is the spin angular momentum with $|S_o| = \hbar/2$ and $\mu = (-g/2)\mu_B$, with $g = 2.0023193$ and $\mu_B = e\hbar/2m$ being the Bohr magneton. The third term on the right-hand side of eq. (2) denotes the Fermi electron pressure. The fourth term is the quantum Bohm force and is due to the quantum corrections in the density fluctuation. The last term is the spin contribution to the momentum. The above equations are applicable even when different spin states are well represented by a macroscopic average. The wave equation for the current source is.

$$\left( \frac{\partial^2}{c^2 \partial t^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \vec{E}(\vec{r}, t) = \frac{4\pi}{c^2} \frac{\partial \vec{J}}{\partial t}. \quad (5)$$

where, $\vec{J}$ is the current density.

Perturbative expansion of the set of QHD equations governing the electron plasma dynamics for the first order of the electromagnetic field gives

$$\frac{\partial \vec{v}^{(1)}}{\partial t} = -\frac{e}{m} \vec{E}^{(1)} - \frac{v_F^2}{n_o} \vec{\nabla} n^{(1)} + \frac{\hbar^2}{4m^2n_o} \vec{\nabla} (\vec{v}^2 n^{(1)}) - \frac{2\mu_B}{m\hbar} \vec{S}_o \cdot (\vec{\nabla} \vec{B}^{(1)}), \quad (6)$$

$$\frac{\partial \vec{S}^{(1)}}{\partial t} = \frac{2\mu_B}{\hbar} (\vec{B}^{(1)} \times \vec{S}_o), \quad (7)$$

$$\frac{\partial n^{(1)}}{\partial t} + (\eta_{\vec{v}} \vec{\nabla} \vec{v}^{(1)} + \vec{\nabla} n_o \vec{v}^{(1)}) = 0. \quad (8)$$

From the above equations, it is evident that wiggle field effects the motion of electron both in longitudinal as well as in the transverse direction. Due to the above oscillations the first order perturbations in electron density are written as,

$$\vec{v}_x^{(1)} = v_{1x}^{(1)} E_o \exp(i(k_o z - \omega_o t)). \quad (9)$$

$$\vec{v}_y^{(1)} = v_{1y}^{(1)} E_o \exp(i(k_o z - \omega_o t)) \exp(i(k_w z - \omega_w t)). \quad (10)$$

$$\vec{v}_z^{(1)} = v_{1z}^{(1)} E_o \exp(i(k_w z - \omega_w t)) \exp(i(k_o z - \omega_o t)). \quad (11)$$

where,

$$\Omega_{q} = v_F^2 + \frac{\hbar^2 k_w^2}{4m^2}, \quad v_{1x}^{(1)} = \frac{\Omega_{q} k_o \eta_{1x}^{(1)}}{n_o \omega_o} + \frac{\mu_B S_o k_o}{m\hbar \omega_o}, \quad v_{1y}^{(1)} = \left[ \frac{e}{m} \frac{\Omega_{q} k_o \eta_{1y}^{(1)}}{n_o \omega_o} \right],$$

$$v_{1z}^{(1)} = \left[ \frac{k_o \Omega_{q} \eta_{1z}^{(1)}}{n_o \omega_o} \right] = \left[ \frac{i\omega_o S_o \mu_B k_o}{m\hbar \omega_o} \right].$$

From the above equations, it is evident that wiggle field effects the motion of electron both in longitudinal as well as in the transverse direction. Due to the above oscillations the first order perturbations in electron density are,

$$n_x^{(1)} = n_{1x}^{(1)} E_o \exp(i(k_o z - \omega_o t)). \quad (12)$$

$$n_y^{(1)} = n_{1y}^{(1)} E_o \exp(i(k_w z - \omega_w t)) \exp(i(k_o z - \omega_o t)). \quad (13)$$

$$n_z^{(1)} = n_{1z}^{(1)} E_o \exp(i(k_w z - \omega_w t)) \exp(i(k_o z - \omega_o t)). \quad (14)$$

where the quantities $\eta_x^{(1)}, \eta_y^{(1)}, \eta_z^{(1)}$ and $\eta_w^{(1)}$ are obtained using continuity eq. (8),

$$\eta_{1x}^{(1)} = \frac{\mu_B S_o n_o k_o^2}{m\hbar \omega_o^2}, \quad \eta_{1y}^{(1)} = \frac{2S_o n_o (k_o + k_w) k_w B_{ow} \mu_B^2}{i\hbar^2 \omega_o^2}, \quad \eta_{1z}^{(1)} = \frac{2S_o n_o (k_o + k_w) k_w B_{ow} \mu_B^2}{i\hbar^2 \omega_o^2}.$$
Due to the presence of the magnetic field, electrons attain a spin angular moment. The dynamics of spin angular magnetic moment leads to dispersion. The components of first order perturbed spin angular momentum are obtained using eq. (3) as,

\[
\tilde{S}^{(1)}[\exp] = \left[ S^{(1)}_{ix} \exp i(k_o z - \omega_o t),\right.
\]

\[
\tilde{S}^{(1)}_{ix} = S^{(1)}_{ix} E_o \exp i(k_o z - \omega_o t),
\]

\[
\tilde{S}^{(1)}_{ix} = S^{(1)}_{ix} E_o \exp i(k_o z - \omega_o t),
\]

where,

\[
S^{(1)}_{ix} = -\frac{2S_o B_o v_o^2}{\alpha_o h^2}, \quad S^{(1)}_{iy} = \frac{\mu_B S_o}{i\hbar \omega_o}, \quad \text{and} \quad S^{(1)}_{iz} = \frac{i\mu_B S_o}{\hbar \omega_o}.
\]

The perturbed density and spin motion of electrons due to oscillatory velocities generate oscillating current. The current density is the sum of conventional source current \( \left\langle J_c = -nev \right\rangle \) and the spin current due to spin magnetic moment \( \left\langle J_s = \frac{2\mu_B}{h} \mathbf{v} \times \mathbf{S} \right\rangle \), whose components are,

\[
J^{(1)}_{ix} = J^{(1)}_{ix} + J^{(1)}_{ix} = J^{(1)}_{ix} + J^{(1)}_{ix} \exp i(k_o z) E_o \exp i(k_o z - \omega_o t),
\]

\[
J^{(1)}_{iy} = J^{(1)}_{iy} + J^{(1)}_{iy} = J^{(1)}_{iy} + J^{(1)}_{iy} \exp i(k_o z) E_o \exp i(k_o z - \omega_o t),
\]

\[
J^{(1)}_{iz} = J^{(1)}_{iz} + J^{(1)}_{iz} = J^{(1)}_{iz} + J^{(1)}_{iz} \exp i(k_o z) E_o \exp i(k_o z - \omega_o t).
\]

Eqs. (18) - (20) contain the collective effects of the laser and magnetic fields on the plasma electrons. The first term in eqs. (18-20) arises due to the action of the radiation field on plasma electrons while the second term denotes the force of wiggler field, under the influence of electron spin and other quantum effects. First order velocity beats with wiggler magnetic field at \( (2\omega_o, 2\kappa_o + \kappa_w) \) to produce ponderomotive force, \( F^2 = (2\omega_o, 2\kappa_o + \kappa_w) \). The plasma electrons acquire oscillatory velocity at \( (2\omega_o, 2\kappa_o + \kappa_w) \) due to the force \( F^2 \), whose components are,

\[
v^{(2)}_x = \frac{v^{(2)}_x \exp i(k_o z) E_o^2 \exp 2i(k_o z - \omega_o t),}{2i\omega_o},
\]

\[
v^{(2)}_{y} = \frac{v^{(2)}_{y} \exp i(k_o z) E_o^2 \exp 2i(k_o z - \omega_o t),}{4i\omega_o},
\]

\[
v^{(2)}_{z} = \frac{v^{(2)}_{z} \exp i(k_o z) E_o^2 \exp 2i(k_o z - \omega_o t),}{2i\omega_o mc},
\]

where,

\[
v^{(2)}_{1x} = \frac{\omega_{w} v^{(2)}_x}{2i\omega_o}, \quad v^{(2)}_{1y} = \frac{\omega_{w} v^{(2)}_y}{4i\omega_o} + \frac{\mu_B B_o E_o S^{(1)}_{1y}}{i\hbar \omega_o}, \quad v^{(2)}_{1z} = \frac{\omega_{w} v^{(2)}_z}{2i\omega_o mc},
\]

and \( v^{(2)}_{2z} = \frac{\omega_{w} v^{(2)}_z}{2i\omega_o mc} \).
\[ n_x^{(2)} = \left[ \eta_x^{(2)} + \eta_x^{(2)} \exp i(k_o z) \right] E_o^2 \exp i(k_o z - \omega_o t). \]  
(24)

\[ n_y^{(2)} = \left[ \eta_y^{(2)} + \eta_y^{(2)} \exp i(k_o z) \right] E_o^2 \exp i(k_o z - \omega_o t). \]  
(25)

\[ n_z^{(2)} = \left[ \eta_z^{(2)} + \eta_z^{(2)} \exp i(k_o z) \right] E_o^2 \exp i(k_o z - \omega_o t). \]  
(26)

\[ S_x^{(2)} = S_{1x}^{(2)} \exp i(k_o z) E_o^2 \exp i(k_o z - \omega_o t). \]  
(27)

\[ S_y^{(2)} = S_{1y}^{(2)} + S_{2y}^{(2)} \exp i(k_o z) E_o^2 \exp i(k_o z - \omega_o t). \]  
(28)

\[ S_z^{(2)} = S_{1z}^{(2)} + S_{2z}^{(2)} \exp i(k_o z) E_o^2 \exp i(k_o z - \omega_o t). \]  
(29)

where,

\[ \eta_{1x}^{(2)} = \frac{n_o \left( 2 k_o + k_o \right) v_{1x}^{(2)}}{2 \omega_o}, \quad \eta_{1y}^{(2)} = \frac{k_o v_{1y}^{(2)} \eta_{1y}}{\omega_o}, \quad \eta_{1z}^{(2)} = \frac{n_o \left( 2 k_o + k_o \right) v_{1z}^{(2)}}{2 \omega_o} \]

\[ \eta_{2x}^{(2)} = \frac{n_o v_{1y}^{(2)} + \eta_{1y}^{(2)} + \eta_{2y}^{(2)} v_{1y}^{(2)}}{2 \omega_o}, \quad \eta_{2y}^{(2)} = \frac{n_o k_o v_{1y}^{(2)}}{2 \omega_o}, \quad \eta_{2z}^{(2)} = \frac{n_o \left( 2 k_o + k_o \right) v_{1z}^{(2)}}{2 \omega_o} \]

\[ S_{1x}^{(2)} = \frac{\left( k_o + k_o \right) v_{1y}^{(2)} S_{1y}^{(2)} - \mu_B B_{ow} S_{1z}^{(2)}}{2 \omega_o}, \quad S_{1y}^{(2)} = \frac{k_o v_{1y}^{(2)} S_{1y}^{(2)}}{2 \omega_o} + \frac{\mu_B S_{1y}^{(2)}}{2i \omega_o}, \quad S_{1z}^{(2)} = \frac{k_o v_{1z}^{(2)} S_{1z}^{(2)}}{2 \omega_o} \]

\[ S_{2x}^{(2)} = \frac{-\mu_B S_{1x}^{(2)}}{2i \omega_o}, \quad \text{and} \quad S_{2z}^{(2)} = -\frac{k_o v_{1z}^{(2)} S_{1z}^{(2)}}{2 \omega_o} \]

First order velocity betas with first order density perturbation and spin angular momentum perturbation to produce second harmonic currents at \((2 \omega_o, 2 k_o + \sqrt{k_o})\). The second order current densities are,

\[ J_{x}^{(2)} = \left[ J_{1x}^{(2)} + J_{2x}^{(2)} \right] \exp i(k_o z) E_o^2 \exp i(k_o z - \omega_o t). \]  
(30)

\[ J_{y}^{(2)} = \left[ J_{1y}^{(2)} + J_{2y}^{(2)} \right] \exp i(k_o z) E_o^2 \exp i(k_o z - \omega_o t). \]  
(31)

\[ J_{z}^{(2)} = \left[ J_{1z}^{(2)} + J_{2z}^{(2)} \right] \exp i(k_o z) E_o^2 \exp i(k_o z - \omega_o t). \]  
(32)

where,

\[ J_{1x}^{(2)} = - \frac{4i \mu_B S_{1x} \eta_{1x}^{(2)}}{\hbar}, \quad J_{2x}^{(2)} = \frac{-2 \mu_B}{\hbar} \left[ (2 k_o + k_o) (S_o \eta_{2x}^{(2)} + n_o S_{1x}^{(2)} + \eta_{1x}^{(2)} S_{1x}^{(2)}) \right] \]

\[ J_{1y}^{(2)} = - \frac{2 \mu_B}{\hbar} \left[ 2i k_o (S_o \eta_{1y}^{(2)} + n_o S_{1y}^{(2)} + \eta_{1y}^{(2)} S_{1y}^{(2)}) \right] \]

\[ J_{2y}^{(2)} = - \frac{2 \mu_B}{\hbar} \left[ 2i k_o (S_o \eta_{2y}^{(2)} + n_o S_{2y}^{(2)} + \eta_{2y}^{(2)} S_{1y}^{(2)}) \right] \]

\[ J_{1z}^{(2)} = - \frac{2 \mu_B}{\hbar} \left[ 2i k_o (S_o \eta_{1z}^{(2)} + n_o S_{1z}^{(2)} + \eta_{1z}^{(2)} S_{1z}^{(2)}) \right] \]

\[ J_{2z}^{(2)} = - \frac{2 \mu_B}{\hbar} \left[ 2i \left( k_o + k_o \right) (S_o \eta_{2z}^{(2)} + n_o S_{2z}^{(2)} + \eta_{1z}^{(2)} S_{1z}^{(2)}) \right] \]

Thus, the total second order nonlinear current (using eqs. (30-32)) is,

\[ J_{NL}^{2} = \left( J_{x}^{(2)} + J_{y}^{(2)} + J_{z}^{(2)} \right) E_o^2 \exp i(2 k_o + k_o) z - 2 \omega_o t. \]  
(33)

There also exists a self-consistent second harmonic field, \( \bar{E}_{2 \omega_o} = E_o \exp i((2 k_o + k_o) z - 2 \omega_o t) \) due to which the linear current density is,

\[ J_{2 \omega_o}^{L} = - \frac{n_o e^2 \bar{E}_{2 \omega_o}}{2i \omega_o}. \]  
(34)
3. Second harmonic field

The wave equation governing the generation of second harmonic is given by,

\[ \left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \hat{E}_{2\omega} = \frac{4\pi}{c^2} \frac{\partial}{\partial t} \left( J_{2\omega}^{NL} + \tilde{J}_{2\omega}^{ll} \right) \hat{E}_{\omega}^2 \exp(i(2k_x + k_y)z - 2\omega_f t). \]

On simplifying the above equation, we get the normalized amplitude

\[ \frac{E_2}{E_\omega} = \frac{8\pi \omega_\omega (J^{(2)}_{2x} + J^{(2)}_{2y} + J^{(2)}_{2z})}{c^2 (2k_x + k_y)^2 - 4\omega_\omega^2 + \omega_\omega^2 \left( \frac{a_m c \omega_\omega}{e} \right)^2} \]  

(35)

where, \( a_m = \left( \frac{eE_\omega}{m \omega_\omega} \right) \).

The second harmonic power density can be written as

\[ \bar{P}_2 = c/8\pi (\hat{E}_2^* \times \hat{H}_2) = \frac{(2k_x + k_y)c^2}{16\omega_\omega \pi} |E_\omega|^2 \]  

(36)

and the fundamental power density can be written as,

\[ \bar{P}_\omega = c/8\pi (\hat{E}_\omega^* \times \hat{H}_\omega) = \frac{k_\omega c^2}{8\omega_\omega \pi} |E_\omega|^2. \]

(37)

The ratio of second harmonic power density to that of the fundamental power density gives the efficiency of second harmonic generation as,

\[ \frac{P_2}{P_\omega} = \left( \frac{2(2k_x + k_y) (mn_e \omega_\omega c^2 \omega_\omega^2)}{k_\omega \omega_\omega^2} \right) \left( \frac{8\pi \omega_\omega (J^{(2)}_{2x} + J^{(2)}_{2y} + J^{(2)}_{2z})}{c^2 (2k_x + k_y)^2 - 4\omega_\omega^2 + \omega_\omega^2 \left( \frac{a_m c \omega_\omega}{e} \right)^2} \right)^2. \]

(38)

In figure 1, the power efficiency \( \langle P_2 / P_\omega \rangle \) (in %) has been plotted as a function of normalized wiggler frequency \( \omega_{\text{wiggler}} / \omega_\omega \), for different values of plasma density. The figure shows that for a constant plasma density, the harmonic grows with an increase in the wiggler field. The maximum efficiency is attained at about \( \omega_{\text{wiggler}} / \omega_\omega \approx 0.76 \). Maximum efficiency appears at higher densities with increase in the wiggler field. The cut off value for the harmonic generation and the saturation value for magnetic field also increase with plasma density. The strong magnetic field and quantum effects both contribute to increase in the cut-off value of second harmonic generation.

The figure 2 is plotted for the power efficiency \( \langle P_2 / P_\omega \rangle \) (in %) of second harmonic generation in magnetized quantum plasma as function of magnetic field strength \( \omega_\omega / \omega_\omega \), where \( \omega_\omega = \epsilon \omega / mc \) (B = \( \hat{y} \)B is static magnetic field), for the different values of normalized electron density. It is seen that the power efficiency variation is completely different in the phase-mismatched condition (in absence of wiggler). In the Phase mismatch condition, the second harmonic efficiency reduces with the increase in magnetic field and plasma density.

Figure 3 shows the variation of power efficiency \( \langle P_2 / P_\omega \rangle \) (in %) of second harmonic generation as a function of normalized electric field parameter \( a_\omega \) at different densities. The resonant power efficiency increases sharply for lower values of intensity, however at higher values it saturates.

In addition, we can conclude that the efficiency for phase matched is distinguished from efficiency of phase mismatched. The power efficiency of second harmonic generation in phase mismatched condition is always below the phase matched one. It is worth mentioning that in low density plasma we need a super strong magnetic field to get maximum power efficiency of harmonic generation whereas in quantum plasma which is highly dense, the excitation of efficient harmonics becomes easy by applying lesser magnetic field strength. The quantum diffraction also enhance the harmonic generation. A balance between the plasma density and applied field is required to obtain optimum efficiency.

References
Figure 1: Variation of power efficiency \( \left(\frac{|P_2|}{P_o}\right) \) (in %) for phase-matched second harmonic with normalized wiggler frequency \( \left(\frac{\omega_{ow}}{\omega_o}\right) \) for \( n_o = 10^{28} \text{ cm}^3 \), \( \alpha_o = 0.271 \) and different values of normalized electron density.

Figure 2: Variation of power efficiency \( \left(\frac{|P_2|}{P_o}\right) \) (in %) for phase-mismatched second harmonic with respect to the normalized wiggler frequency \( \left(\frac{\omega_{ow}}{\omega_o}\right) \) for \( n_o = 10^{28} \text{ cm}^3 \), \( \alpha_o = 0.271 \) and different values of normalized electron density.
Figure 3: Variation of power efficiency \( \left( \frac{P_2}{P_o} \right) \) (in \%) for phase-matched second harmonic with respect to the laser intensity \( a_o \) for \( \omega_{ow} = 2.82 \times 10^{18} \) Hz, \( n_o = 10^{28} \) \( cm^3 \) and different values of normalized electron density.