Neti-Neti-Onomasiological Onerousness And E Pluribus Unum La Escallonia Adventitious Ad Infinitum Model

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**ABSTRACT:** In knowledge, there are logical contradictions and a logical incompatibilities, divergential affirmation and dialectic transformation. Brahmn is not the universe, nor sky, nor senses, nor mind. By Brahman we understand the one agency that stimulates the human being in to action. Good. And Anti Brahman is a part of Brahman and this is the agency of stimulus response of individuals based on their Rajas and Tamás equilibrium or proportion of disequilibrium (Kind attention drawn to the author’s articles on the subject matter. And we have to recapitulate the bank example given wherein the conservativeness generalisationality do not abnegate or revoke applicability to the individual systems which are conservatism or which might be unconservational,like in self transformational, system mutational, syllogistically disintegrational, configurationally changeable. Anti Brahman stimulates one to do bad things. All individual records of commission and omission of Gratification or deprivation to others are assumed to be recorded in Neuron DNA (in space). We will not repeat this explanation again and holds good for the entire monograph. Separate Papers are written (Free will and Destiny, Natures general Ledger, A Model for God etc..) and bad. This depends on freewill and destiny. We have already defined freewill as the in discretionary usage of power be it financial embezzlement, pecuniary misappropriation or fiscal defalcation. The other person might have been desensitized for such actions and might have been taught by reference groups (See Merton) to take things in his “stride”. But our General Ledger of all actionable and actions accounts.

The General Ledger is the General Theory Of all actions of all individual human beings. It is also assumed that the reactionary potentiality depends on the genes (See Neurobiology by John Smith). This is a very important aspect not to be overlooked. Because Schrodinger’s equation is applicable to general system of the Universe, it does not mean its inapplicability to individual systems and the conservativeness and /or non conservativeness notwithstanding. In consideration to the fact that we have already built a model encompassing Brahman-Anti Brahman, Rajas-Tamás, we shall not include this in the model. Instead we take “Neti Neti concept” by its representation in the Schrodinger’s form (See Penrose on Schrödinger’s cat) and stimulus-response as the further constituents to tie up with the cosmic thread. One sentence about association need be said. Einstein said that matter is the one that gives rise to space time world lines. In our case human beings with their pen chance, predilections, proclivities, and propensities “svabhava” are responsible for the world lines of space time. The characteristics we have already stated depend on Rajas-Tamás-Sattva combination of the individual (all the three qualities are in the individual and in disequilibrium state, albeit we are not sure about the conservativeness of the states holistically. This we have codified in the General Ledger which is the General Theory of actions in the Universe. So we have the structure of space and time is determined by “Svabhava”. Like they say: “That area is not OK”. This is true of all characteristics, inertia, static, dynamic (Rajas, Tamás, Sattva means those words). These are most salient and perennial concepts that
are to be remembered in the read of monograph. This is ‘neti/ neti’ process, repeating which one becomes thoughtless. In jnana the entire world is illusory like a dream; only Brahma is real- ‘Brahma satya jagat mithya.’ The world is neither unreal nor real. This means that reality is equivalent to unreality. Here let us go by the usual definition of “reality” and “unreality”’. Unreality is pathological. Many philosophers like Constantine and Lester have said, it is schizophrenics who are suffering from the Truth. we shall marginalize the proposition in so fact as the study of monograph is concerned. This implies that Unreality is a part of reality. Further we assume that Objective reality and subjective experience are necessary to realize the Truth. Lest probably a courtesan would have become “enlightened” or a Professor had attained “Nirvikalpa Samadhi”. The world is mere projection of self. Jnana requires strong discriminating ability. Krishna says, “It is more difficult to realize the formless God than One with form”. We reiterate that formlessness is nothing but more than mere loss of style and substance and is well within space time. This is “subjective experience “most recruitment agency calls for. You can see without staring, and can hear without cupping your ears. Dealings with fellow people, bonhomie, and joie de vivre are the ones that could tackle a tricky situation involving a belligerent, cantankerous and recalcitrant person. Here the importance of zero is to be emphasized.” Experience “is nothing but the “experience” so assiduously and avidly asked by recruiter agencies while they recruit people. When they say that whether you know the proficiency in anagrammatism like addition, subtraction, multiplication, division And may be differentiation and integration, which a probationer does, but something more, a knack to deal with people, know them, adjustable temperament, which comes by subjective experience. Both “Objective Reality” and “Subjective Experience” are necessary for the determination of Truth. And Schrödinger and Wagner have said that Truth is always elusive. That we state in Model form. It is not “emptiness”, but “Nothing” which is zero. And this zero is the cancellation of two opposite terms or particle-antiparticle duality, and the zyphera that is written with so much of gusto and aplomb in every equation.

An equation equivalent to zero means that all the constituents cancel out each other in some form. We shall for our purpose choose one permutation and combination. It is to be mentioned in this connection that waking state is one of the three states of dream state and dreamless state and space time is only projection on the screen of consciousness (which we have defined as collection of all knowledge – individuals-his peer group references, belief systems etc., including visual representations, with anagrammatic mind acting on its to fabricate concoct realistic, consistent, surrealistic, unreal dreams and actual “thinking”. Like a movie being watched. Mind doth act upon it and weaves a web like a spider. Lastly we state that ‘association’ is the part and parcel of space and time and once taught. The “association” fails to leave you. Even eminent scientists fall prey for this defending their theories till the end instead of applying their minds to the twists their own theories have taken. This statement is made in a dispassionate manner of witness consciousness and not participatory consciousness. When over thinking about association takes place in the spatio temporal actualization of events it becomes pathological. Be it love, murder, mayhem, we have various psychiatric cases. And we want to “renormalize them”. So there is reality, surrealism, unreality, inconsistencies, contradictions in the waking state, dream state-note you experience in dream state also- and dreamless state.)

PREFACE:

Section 1

Since the beginning of the human race, there has been a tug of war between the corporeal depth and sonorous continuum, locus of essence of expression and sense, propositional subsistence and corporeality, perceptual field; form background, theme potentialities, transitive states and substantive subsystems, determinate orientation, heights of cognition, evaluative integralnational collective cultural pamauntacy, generalized strain of rationalized consistency, essential cognitive orientation and choice variables, cathartic evaluative impressionistic mechanisms of individual minds, and componential clustering’s principal frontier of diurnal dynamics, principal concept of determinate apriori, and differential apostoeity, aphorism, anecdote, syndromes, internal differentiation, comparative variability, structural morphology and normative aspect of credibility of both religion and science. Many riveting riposte and recrimination and dialectic deliberations, polemical conversations and argumentative confabulations have taken place as to which is superior or not. But the best of minds have taken the spirit of the Eastern Science and have adapted to the materialistic science which has helped the science and progressed the frontiers, with generalization manifestations and personalized significations. Krishna
in Bhagavad Geeta explicitly states that he who is carried away by the alliterations, epistemological engendered propositions, phenomenological transcendental constitutional ties, and metaphorical constitutive conditionals, radical cleavage of the disjunctive syllogisms, non modalized root forms, apotheosis of willful injustices and ignominies, dissententious resentient and dejurational resentient, apodictic knowledge of successful effectuality, extrinsic predications and non consummative abstractions would never be able to understand Geetha in its wide ranging manifestations and he shall be consigned to dustbins of history despite knowing it y heart. In recent days Penrose has been in the forefront in the study of Consciousness and without the inclusion and occlusion of Quantum neuropsychology, has been able to use some of the concepts to the scientific advantage. Walker albeit little ambiguous, in His Physics of Consciousness mentions certain equations of consciousness one may not completely agree with albeit Quantum Mechanics has become a subjective science, and the necessarily compulsion of Quantum Computation has lead many a people in to forays in to eastern mysticisms. Fritzaf Capra is another name that has done much collaborative work in the field of consciousness and Buddhism

And western physics. William James one of the respected exponents of Consciousness states: There is no science of the soul without a metaphysical basis to it and without spiritual remedies at its disposal. Penrose uses Schrödinger’s equation in the context of superposition of the cat and there have been criticisms over that especially by Hawking. We have in a series of papers (Theory of hidden variable: perception is not reality; A model for destiny and free will, and God does not put signature-a Brahman and Anti Brahman model for god) have tried to provide a scientific interpretation of our own age old predications and postulation alcovishness which every Indian knows but unfortunately fails to consummate either in practice or in theory due to irresolute concern of the mundane musings and day to day drooling. If perception is not reality what is it? What you see is not what you see, what you do not see is not what you do not see, what you do see is what you do see and what you do not see is what you see. Quintessentially some variables have to be added to perception to know the truth. For the physicist and the instrument are governed by the same laws as is dictated by the universe to the system under investigation. Infact recently, on a discourse on Paramahamsa Yogananda, author found Von Neumann grudgingly acceded that some factor is to be added to perception and this is consciousness. Unless you know that there is a cricket match how do you explain the rush at the stadium? You go on and on without any end. In this monograph we apply the concepts of both time dependent and time independent equations of Schrodinger, which define the state of perception of the Universe defined by the “collapse of the wave function” and apply to the fundamental concept of Brahman namely Neti Neti, meaning neither this nor that. We only hope that his shall be a small contribution to further distanced invigoration and ramified resonations in the field. Counter actualisation of consciousness is also ridden with condensation of singularities and we write to state that the evolutionary concept of integrative structure of system of moral value orientation patterns on the relational and transcendental level is relevant to the institutionalization of the widest commonalty spread. Lord does not need either gratification or mortification. A dispassionate view is one which God shows and a sensitive passionate observer would come to know what reality of “Grand design” is all about. All it takes is to keep the eyes and ears open.

I am grateful to professor B.S.Kiranagi and Professor C.S.Bagewadi for their invaluable suggestions on the subject matter in question

Consciousness is in its original nature quiet, pure, and above the dualism of subject and object./But here appears a principle of particularization., and with the rise of wind of action the waves are agitated over the tranquil surface of mind . It is now differentiated and evolved in to various levels. --D.T.Suzuki

There is an incessant multiplication of the inexhaustible one and the Unification of the indefinitely many. Such are the beginnings and endings of the worlds and of the individual beings, expanded from a point without position or dimensions and a “Now” without date or duration.--Ananada Coomaraswamy

Consciousness is Maya (Please see model on Hidden variable Theory)

-Schroedinger
Neti, Neti principle we write in mathematical form as follows (See Schrodinger’s cat in Penrose’s The Big, the Small and The Human Mind”) Observer and the observed (Here we take both accentuation coefficient and dissipations coefficients as zero)

\[
(2). \Psi = -e_1 \Psi_1 - e_2 \Psi_1 - \ldots - e_n \Psi_n \\
(3). \Psi = +e_1 \Psi_1 - e_2 \Psi_1 - \ldots - e_n \Psi_n \\
(4). \Psi = +e_1 \Psi_1 - e_2 \Psi_1 - \ldots - e_n \Psi_n \\
(5). \Psi = +e_1 \Psi_1 - e_2 \Psi_1 - \ldots - e_n \Psi_n \\
\]

(6) Svabhava and space time world lines

(7) Consciousness (Storage of all information and visual representations with mind acting upon it anagrammatically. Note we eschew using words “being” and “becoming” We are infact beings with sense of consciousness and individual consciousness always tries to do”high jump” like body wants to. This is an evolutionary process. Leviathans’ literature has done more damage to the subject than addressing quintessential. We leave metaphorical and take the categorical and classification.)

(8) In quantum mechanics, the Schrödinger equation is an equation that describes how the quantum state of a physical system changes with time. It was formulated in late 1925, and published in 1926, by the Austrian physicist Erwin Schrödinger

Simulations and dissimulations by brahman and AntiBrahman agency (the stimulus providers to the response of the human beings) and concomitant misconception of it to be real by the human being (if two persons walk with you when you go out, it is accident. if twenty people walk with you every day and every time you go out it is coordination, a “design” created by the agency of Brahman-AntiBrahman category. be very clear about the association is nothing but what has been taught to you by reference groups and in the eventuality of the “association” say green” for good and “red” for danger.

Representation of neti neti in Schrödinger’s equation first term on the above equation when added to other terms on the RHS leads to the same situation of addition of milk to water. Here we shall not take the terms separately but assume that they are added to the lhs on one shot basis

**SCHRÖDINGER’S WAVE EQUATION, RELATIVISTIC THEORIES AND WERNER HEISENBERG MATRIX**

Following variables are taken in to consideration:

1. Theory of Schrodinger’s Wave Equation (It is to be reiterated that we are classifying the universally applicable Schrödinger’s Wave equation based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Schrödinger’s equation could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Wave Equation itself. )

2. Werner-Heisenberg matrix (It is to be reiterated that we are classifying the universally applicable Werner-Heisenberg matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits, which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behavior of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalization consummation and realiztation of the Werner Heisenberg Matrix itself)

3. Richard Feynman’s path integral formulation like Werner Heisenberg Matrix (It is to be reiterated that we are classifying the universally applicable Richard Feynman’s path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We
recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits, which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behavior of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalization consummation and realization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself).

(4) Relativistic Theories vis-a-vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix (It is to be reiterated that we are classifying the universally applicable Relativistic Theories vis-a-vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, Penchant, predilection, proclivities, propensities, Solutional behavior conceptual implications thereof, deployed contextual importance therein, anticipatory correlation and consubstationary causal association parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Relativistic Theories Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behavior of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalization consummation and realization of the Relativistic and concomitant and corresponding classificatory categorization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself in the foregoing).

PLATONIC WORLD-PHYSICAL WORLD AND MENTAL WORLD

Following factors are taken into consideration:

(1) Category one of Platonic World (No one has seen entire world or universe. It is based on this concept that classification is done. Everything needs to be explored)

(2) Category Two of Platonic World (or the part thereof)

(3) Category three of Platonic world

(4) Human mind corresponding to the above classification category one

(5) Human mind corresponding to the above classification category Two

(6) Human mind corresponding to the above classification category Three

(7) \(G_{16} \): Category one of Physical World

(8) \(G_{17} \): Category Two Of Physical World

(9) \(G_{18} \): Category Three Of Physical World

(10) \(T_{16} \): Individuals With Mind Corresponding To Category One In The Foregoing

(11) \(T_{17} \): Individuals With Mind Corresponding To Category two In The Foregoing

(12) \(T_{18} \): Individuals With Mind Corresponding To Category Three In The Foregoing

(13) \(G_{20} \): Category One Of Mental World

(14) \(G_{21} \): Category Two Of Mental World

(15) \(G_{22} \): Category Three Of Mental World

(16) \(T_{20} \): Individuals With Mind Corresponding To Category One In The Foregoing

(17) \(T_{21} \): Individuals With Mind Corresponding To Category One In The Foregoing

(18) \(T_{22} \): Individuals With Mind Corresponding To Category One In The Foregoing

TIME DEPENDENT SCHRODINGER’S EQUATION:

\[ E \Psi = \hat{H} \Psi \]

AFFECTIVE CONSCIOUSNESS AND QUANTUM FIELD

Following variables are taken into consideration:

1. Change is a potentially produces and transforms as an emotional event as people anticipate or
experience its outcomes and processes.

2. Emotional aspects of organizational change, promote acceptance of change or resistance to it. Cognitive, affective and behavioral responses to organizational change indicates outcomes, scale, temporal issues and justice

3. Emotional intelligence, disposition, previous experience of change, and change and stress outside the workplace affect those in the employee’s perceptions of the leaders/managers/agents (their leadership ability, emotional intelligence and trustworthiness); and those in the employee’s perception of the organization (its culture and change context)

4. Cognitive appraisal theory takes the position that emotion derives from cognition as people contemplate the importance of events (such as organizational change) to their wellbeing and consider how they will cope.

5. Social constructionism was used as a theoretical platform because it combines the individual experience of emotions during change with the social forces that help shape them.

Following are the second line variables:

1. The Higgs Field is the fundamental quantum arena, quantitative form of means and ends that gives birth to all mass in the universe. In the finding of Higgs Boson, there exists an inferential thought and diversity and essential predications, predicational anteriority, character constitution of primordial exactitude and ontological resonance of the formation of the world itself.

2. The particle Higgs Boson

3. Higgs Quantum Field

4. Mass of the matter

5. Quantum Field Theory

6. Electro-Weak Unification and Broken Symmetry

7. Grand Unification Theory

8. Super symmetry

9. Quantum Gravity and Super gravity

10. The Heterotic String

11. Unified Quantum Field Theories

12. Principle of spontaneously broken symmetry, locates (eb) deeply hidden symmetries of nature at fundamental space-time scales.

13. Principle of broken symmetry explains the emergence of diverse forces from an initially unified field.

14. Profound symmetry principle called super symmetry, which has the potential of unifying force fields and matter fields in the context of a single field.

15. Superstring theories.

16. Quantum field theory

We have given a model for Brahman (Neti Neti) as defined in the scriptures using Schrödinger’s Equation. There do not seem to be any literature on this. This is a mathematical statement of whatever has been said since long time and no compactified model seems to be there in literature.

INTRODUCTION:

Neti Neti: Neither This, Nor That -- In (transpersonal/non-dual) Reality "WE" are more than that! WE are THEE! Thee WE BE! Merry BE!( We have used Wikipedia in the preparation of the Introductory remarks)

Neti Neti: What it is not

Neti -- neti is a well known Sanskrit expression that is found in the Upanishads, but also is widely misunderstood (both in the West as well as in classic Indian dualistic philosophic traditions as well). That is, it is subject to much varied interpretations. The common misunderstanding comes from a false assumption (which is nothing other than a dualistic preference of the mind) in which the saying is
interpreted as a presumption of a programmed alienated context (an illusory assumption where God, Spirit, or Brahman is absent, separate in our daily life, or "elsewhere"). This limited backdrop occurs within the assumed context of a predilection toward the pre-existing false assumption of spiritual alienation (and may I add corruption), where a state of sin/separation from spirit is taken as the underlying "reality". Certainly, if a spiritual seeker finds themselves spiritually alienated and estranged from spirit, which is lacking, then this sadhak (spiritual seeker) can strive for re-union/re-integration. However, it is valuable to not set the quest within this corrupt and alienated framework less we amplify it as a condition paralysis/stickiness or spiritual stasis. Such is a limited/ignorant view of reality. In REALITY, this false coloring of spiritual alienation can be likened to grasping in the wind, being an error of the programmed dualistic mind. Within this mind-made conditioned schism that separates "spiritual principles or the sacred" from man, Beingness, and from the earth in an oppositional manner, is the error that underlies all the corrupted brutalities that man lays upon himself, his fellow creatures, and that of future generations. Thus, the Upanishad saying, neti-neti (neither this nor that), will be used to disclose the lie and reveal the truth. Neti-Neti in the Non-Dual Context will posit the opposite jumping off place of interpretation than dualism (dvaita). We will reveal and be grounded within that Reality, which true and spirited vision and clarity naturally expresses. That Spirit is the ever-present all pervasive Untouched Eternal Reality, as nisprapanca, acintya, aprapanca, and non-dual (advaita). It is Never-ending, Beginning less Reality, which is Eternally Present as sacred presence. Therefore, knowing and affirming the true context of "reality" from the beginning, as a non-dual assumption, then the term neti-neti can be applied simultaneously as a negation of subject/object duality, and hence as an affirmation of the non-dual unitive state.

Although primordial presence appears to be absent or lacking from "normal" dualistic materialistic everyday life (as chronic ignorance), never-the-less we will view this "normalcy" as an error of the conditioned "normal" mind -- as a result of negative programming, where the gross physicality of sense "objects" as well as similar I/it dualistic obsessions are perceived as fragmented and disorganized. That mental and energetic obsession/fixation occludes universal sacred presence out of context of the whole. Thus this materialistic/dualistic chronic "normalcy" will be considered as the normal icy process of that establishes ignorance (avidya), as a fragmented self limiting skew, as a prejudicial and superficial bias of what really is in its natural universal and unbiased way. As such, chronic dualistic referencing (the reification process), as an illusory and an aberrant, corrupt, and perverted way of seeing and being, maintains a hallucinatory mindset, which when accepted by our peers becomes institutionalized ignorance, social stasis, bastion, pillar, post, stylobate and sentinel and a force that is counter evolutionary.

The Sanskrit expression, neti neti, as expressed by the realized sages points to that vast reality, which is more than dualistic fixation. As such "not this, not that", which is the common translation, becomes transformed to, "we belong to a boundless all inclusive realm of Great Integrity and Continuity-the WE realm of All Our Relations being therefore more than just a fragmented limitation or corruption of the whole, but rather the phrase, neti neti, also implicitly implies "tat tvam asi" -- That eternal spirit thou art. Yoga practice is designed to open that living book where all beings and things are relatives and kin as Vasudev Kutumbhkam--the entire Universe is One Family. This is an exploration of vast interconnectedness, a vast diversity within a coherent unity, and a vast unity (hologram) found in the form of a vast holographic diversity, both.

Many levels of Interpretation

The expression, neti neti, literally means "neither this, nor that" or not this and not that. In the first level this is the rejection of a separate self or ego. It is a rejection of fragmentation or split from universal spirit which is embedded within all beings and things. In the holistic multidimensional context, it means that human beings are not just separate egos; we cannot ever be adequately defined as being separate from spirit without introducing a delusion -- spiritual self-alienation, counteractualisation of consciousness. The projection of an individual self or observer apart from the object, which the observer observes defines a limited dualistic mental framework. In turn such occludes the larger picture, which includes all time, space, and knowledge. Rather we, as human beings, are part of an immense sacred process, not apart from it.

Thus "neti neti", as a statement, means that we are not anything separate as in the disparate dualistic
framework of a separate "I/it" subject/object duality context (versus the sacred non-dual and transpersonal "I-Thou" context) wherein we identify as a finite expression integrally part of a boundless spirit (like a wave on the ocean). We are neither the ego, nor are we nothing at all. We are neither the all, nor nothing at all. Neither just this observer, nor just that (the observed). Neither eternal nor finite, neither eternalism nor nihilism, neither empty nor solid. Rather it is the great non-dual or advaita statement of both/and -- both, but neither by itself. "And" in the sense of a greater synthesis or unity -- call it the tantric Siva/Shakti if one likes, albeit nihilists will take it as a negation of reality.

Although negative dialectics (such as found in Nagarjuna's Buddhist Madhyamika philosophy, which is considered as non-dual philosophy by Buddhists) , neti neti, not only negates the validity of a perception which imputes a separate/independent self existence (an ego) which includes the perception of a valid existence of a separate/independent object as well as the egoic observer, but also neti-neti, by disowning duality per se, one is affirming a holographic non-dual dynamic holistic system, where "things" as sense objects, the senses, the process of perception, mental dispositions, and intellectual function, all operate interdependently, becoming organized by an inherent vast intrinsic transconceptual schemata within the sphere of beginning less primordial awareness totally free from concepts of sequential time and limited space. Neti-neti is thus not a statement of existence per se, negating existence per se, affirming non-existence, negating the existence of both, or the negation of the negation of both, and so on infinitum, but of negating any dualistic view. Rather it affirms the interdependence of all things and beings -- the holographic relationship. Negative dialectics (negation without affirmation) most often serves to go down a slippery slope of the extremes of nihilism and cynicism.

Although there are volumes of pages written about Buddhist non-dual philosophy (Madhyamika), we will briefly describe it in terms of neti neti, stating that phenomena, things, objects, the objective world, or "existence per se" do not exist by themselves independently. In short objects or observers have no independent or exclusive self existence other than in the dualistic imputation of the egoic mental formulation. Such things, objects, phenomena, or objective world per se, cannot rightly be said to exist, to not exist, both simultaneously, or neither. There are many logical reasons given. The easiest reason is that they never existed in the first place, so there is no self existing reference point (referent) to some "thing" which is not existent as compared to something else which does not exist. Secondly, nothing can be said to truly "not exist" exclusively in relationship to something else (to that which is considered to truly exist exclusively or independently). Likewise things, phenomena, objects, the objective world, etc., do not both exclusively not exist and exist at the same time, as it would violate the previous statements. Lastly, according to Nagarjuna things, objects Phenomena, or the objective world cannot be said to both not exist, and both not exist, ad infinitum.

Philosophical speculation, dialectics, logic, and reductionism, all have been used to resolve the abstract problematic structure and apodictic knowledge and functional topology of the neti neti. While extracting it from the realm of philosophical speculation, with multiplicity of contrarian presuppositions, and logical and dialectical attributes there is both intellectual gratification and divergential affirmation on one hand and disjunctive syllogism and dialectic transformation and disjunction of synthesis on the other. This however is done not at the cost of predicational integrity and character constitution as stated in the original texts.

The four (and more) negations

the negation of solid independent existence
the negation of the negation of existence
the negation of both existence and non-existence
the negation of the negation of both existence and non-existence

We have written these forms in Schrodinger's equations following Penrose's interpretation of Schrödinger's cat

None of these imputations can stand on their own, so they are negated. In short, things, objects, phenomena, events, or the world per se do not exist as separate selves possessing an intrinsic independent existence, rather phenomena can only be said to truly exist when taken as parts of a mutually interdependent whole, hence the inseparable nature of differentiated reality (temporal ever-
changing relativity) with undifferentiated absolute reality (the unchanging Beginning less a-temporal and primordial formless realm). Both are married as the inseparable unity of dharmakaya (empty space) and rupakaya (form), wisdom and compassion, emptiness and clarity, or emptiness and essential nature, which are different formulations of the same non-dual truth. The crucial key is that the non-dual view is equivalent to the inseparable interdependent/interconnected state of naljor (yoga) which is the great natural state of perfect wealth. Things do not exist separate as exclusive independent entities as solid or permanent, nor do they not exist at all as in an inane nihilistic state. This is the essential non-dual meaning of the statement of neti/neti By both/and, it is meant that neti neti refers to the illusion that things/objects exist by in and by themselves, as well as the illusion that the observer or ego truly exists as a separate independent observer. The both/and affirmation is the affirmative statement of interconnectedness and interdependent co-arising, where things as well as the intelligent evolutionary life force behind all things can be said to truly co-exist interdependently in the context of primordial time and wisdom.

Neti Neti as the body too -- the body too! We are the intimate limbs and expression of the body of cosmic consciousness.

So on one level neti neti says, "not self not self", not small mind self, not just individual mind, "not ego-not ego", not duality, not separateness, not division, not only this or only that, but in doing so it affirms its own negation in a greater affirmation in an unlimited all encompassing holographic integration as we will see. Too often dualistic interpreters (see introduction above) interpret neti neti as a dualistic statement hearing separate as if saying "separate" from the body (as ego identifies with the body), separate from nature, the earth, the senses or even entire world of form as in an infinite and final nihilistic withdrawal. But that is an oversimplified logical mistake, throwing the baby out with the bath water, unless one understands that this statement is an affirmation of the integrative non-dual state where nothing is separate. But dropping the ego is only one step within the integrative step (re-integrating with the whole).

As a spiritual saying within the framework of yoga being union, net-neti, is understood correctly within a spiritual context of union with Universal all pervading Spirit, (Quantum Field pervading the entire universe) not within a dualistic alienated context. Anyone who has practiced yoga as a spiritual practice consistently over time begins to be familiar with that territory. They begin to recognize that they exist within a larger relational boundless framework, where nothing exists in and by itself as if in a vacuum as an ego or entity. Indeed these interconnections between all beings, all of creation in All Our Relations are the realization of the non-dual Great Integrity which is yoga. By the body too, it is meant that the all pervasive consciousness includes all, including the body, but it is not limited to the body. In that experiential state of pure being and pure consciousness nothing is excluded and nothing clamors for inclusion.

The common error of the dualistic mind that must be pointed out is that a negation must also be an affirmation of another avenue of perception. If that new context does not exclude anything and is thus does not need to include anything; if it is not lacking in any way, is NOT separate, isolated, or independent, but is universal, all pervading, and primordial, then it satisfies the non-dual criteria. If not negation becomes part of an aversion process, where one craves escape, creates negative karma by trying to escape, avoid, isolate, or separate oneself from an imputed isolated autonomous "thing" (usually the body, natural feelings, the natural world, and evolutionary process). The latter is not yoga. In yoga we are looking for integration. Rather when a yogi starts to realize the greater wholeness of an expanded process oriented realm of evolutionary consciousness and being, he/she gladly surrenders his/her previously conditioned limited sense of "self" consciousness (asmita), recognizing it as an impediment/obscuration (klesha). Thus, suffering (dukh) is avoided and happiness and bliss eventually realized in Sat Chit Ananda. This is not to negate that at physical death, the physical body is relinquished, but that is part and parcel of a greater continuity and evolutionary integration process which has already been affirmed.

Thus authentic spiritual practice in yoga is aimed at realizing the "Reality" and truth of this non-dual transpersonal omnipresent and all knowing universal boundless Mind which is expressed through every cell in the body subjectively as well as objectively (united). Intelligent spirit is not only seen as manifest in the physical body but also in Cosmic consciousness nature, and the entire manifest universe
simultaneously as sacred presence --who we really are (swarupa) in context of total boundless space and absolute time.

For an authentic yogi, knowledge, jnana, or gnosia, is not derived intellectually or conceptionally -- realization is not just words or adherence to a memorized philosophy, but rather it is a direct expression of direct yoga experience. Hence, it is fundamentally non-dual (based on an experiential union/integration) as distinct from an objectification process, which separates the observer from "something". Here we must be careful to make this distinction, and thus not perpetuate this common confusion. Neti-neti means not that, not words, not apparent isolated events that are frozen in time, but rather an experience grounded in the Great river of endless all encompassing continuity and everything short of that.

It is in this sense thus "neti -neti" is an affirmation of the all inclusive, omnipresent, non-dual, universal, and transpersonal identity which knows no bounds -- sometimes called Brahman in Sanskrit, sometimes called God in the west, or nirbija samadhi in traditional yoga, but at the same time we are warned not to think of it as an independent entity, a religious doctrine, belief system, or ideology, to which could be capable of obedience or conformity. Neither is neti-neti a negation of the physical relative world (as is often assumed). Rather on a higher level neti-neti means that we are neither just Brahman separate from the world, nor just the world, neither exclusively absolute nor exclusively relative, but both/and much more. Note that we have taken Brahman as the stimulus provider. He who knows Brahman becomes Brahman. It is just as simple as he who knows his subject becomes Professor. Or he who knows Supari mafia, crime syndicates, hoodlum mugger aggregates, loutish, bearish elements, raucous ribaldry congregation becomes a Don. This we have termed as Anti Brahman When Brahman is truly realized, it is not separate from atman (self), but this is not the glorified ego which is the common illusion. This Self is not an independent identity. The separate self (atman) is not the universal Self (Brahman) by definition. Brahman permeates and contains all, both, while it is known as the essence of all. All pervasive it cannot be isolated. Once realized not as a separate object, but one's true self, only then does atman and Brahman become one. Taken as a whole, Brahman transforms the delusion of a separate self (atman). As such neti-neti is not just a negation, isolation, refutation, nor exclusion, but more so, a great affirmation of sacred non-dual universal presence -- the boundless realm of All Our Relations. It's more than human words can adequately express. There is duality in Brahman or for that matter Anti Brahman. All are equal. They do not care if you are a commissioner or a commissionaire. The show must go on. There is nothing deep to read in these statements.

However the academician, the orthodox scholar, the pandit, the self gratuititous intellectual, the religionist, fundamentalist, the normal left brained dominated (unbalanced), but unfortunately "normal" person (who is submersed chronically in the duality, bias, and fragmented thought processes of the normal "i-it" limited context) carries with them the biased filter of a pre-programmed self perpetuating inability or ignorance (avidya) preventing them from expanding beyond their self imposed dualistic mindset/prison -- held together by rigid mind constructs, concepts, artificial half truth belief systems, preconceptions, mechanisms of analytical reductionism, prejudice, pride, arrogance, etc. Such (albeit normal) people who have become inured to such negative programming having adopted habitual distorted standpoints, their minds have become chronically artificially alienated from living spirit in a tenacious way (citta-vrtti). This is condensation of nomadic singularities and counteractualisation of consciousness. Thus they see spirit or God outside of everyday life -- separate from this world and life. Once that alienation sets in then, it is an easy assumption that God or spirit exists in death or outside of nature. God becomes associated with death. Yet the opposite is of course what is needed. Ideologues and fundamentalists simply need to bring spirit back into their lives rather than further pushing it away by attachment to overly simplistic false assumptions. What is needed hence is to release/relax these mental attachments.

This is not to say that intellectual or rational function is "bad" per se. Rather, it is a specific technical tool that can be misused/overused to create lifeless, contrived, narrow, and artificial mind constructs no matter how logical they may appear, that preclude spirit and dishonor life. This is what we meant by the statement that space and time is a part and parcel of the symphony and anecdote of life and aphorism of thought like that of waking state or dream state. These ignorant mind fabrications can box man in to very limit conceptual frameworks and self limiting beliefs which compound his suffering and alienation
if not surrendered to the larger innate intelligence behind the intellect (buddhi) and which animates it. Logic without inspiration thus gets in the way of universal boundless mind, thus creating an artificial split (duality) between the one who views (a false identification of separateness or ego) and the object that is being viewed, and thus the process of viewing (or consciousness) becomes severely obscured, conditioned, biased, frozen, and self limiting. This is the common assumptive trap of extractive reductionist mentation or over objectification of the right brain imbalanced person (a widespread epidemic in the modern world). Simply the Buddhists call this subject/object duality, or one may simply call it cerebral over dominance or fixation upon a false identification. It is necessary to release (vairagya) this fixation if one is a sincere seeker. This is what we meant by the fact that the secund imagination, unannihilatory manifestation and nonlinear franticness of Objective Reality and Subjective Experience are one. We have put it mildly and politely by stating if “Subjective Experience” is only enough probably a courtesan would have been a Brahmangyanai. Or “Objective Reality” or storage of knowledge is probably a Quantum computer would have been one.

A yoga practitioner (sadhak) or true spiritual seeker asks the fundamental question of the spiritual seeker -- the one who inquires, explores, searches, and observes -- who practices self observation and self study (swadhyaya), thus removing the bias and prejudice of the a separate observer. Such a seeker is not interested in the small answers and the inquiry is not necessarily put into words, other than into describing the nature of inquiry itself. The essential practice is trans conceptual. By inquiring, "who am I" conceptually, one defines oneself in terms of everything else. As such it is only the first preliminary question, through whose ultimate answer everything else understood. The answer is not separate, not independent. Ultimately, the non-dual realization is beyond the ability of human intellectual/conceptual thought constructs; rather it is truly timeless primordial wisdom, which must be allowed to express itself. Neti neti says it is not in the books, nor not "not in the words, belief systems, or conceptual frames. Neither is gnosis found solely in the physical body, nor not only in the body, rather the selfless unselfish state of pure being (Sat) both the body and everything else, and primordial wisdom, including vast time. Hence in this yogic light we experience a holographic embodied spirituality where phenomena is permeated with timeless wisdom, where phenomena and being co-emerges spontaneously as one when we allow it to happen -- when the wisdom eye opens, the true nature of phenomena is known. Yoga is a practice, which allows us to realize that great integral synchronicity. Otherwise, we exist spiritually estranged and disempowered in chronic spiritual self-alienation (error or sin).

Spiritual practice as such then is the remedial adjustment for the skew introduced by the conditioned small self (limited viewer which introduces time/space bias) in favor for the transpersonal universal spiritual boundless view. Sadhana thus brings us back into balance and harmony. That this REALITY is possible and that it involves an inner awareness; i.e., an awareness of "self" is the assumption that separates the spiritual explorer from the assumptive limitations and beliefs of an ordinary philosophic inquirer.

So this again leads one to ask the spiritual question, who am I? Thus in authentic yoga sadhanas of swadhyaya (self study) this exact process of inquiry into Self nature is engaged, not the study of what books, scripture, priests, or external authority figures or status quo structures tell us who we are. The successful answer to this question depends on LISTENING/ASKING going deep into that very process of inquiry itself. This is called, BEING OPEN, in dhyana (emptiness meditation) practice.

Knowing our own mind (instrument of knowing) is a wonderful gift that we can give ourselves. At first, one may encounter demons (phantoms of the dualistic mind which one can apply "neti-neti"), neither this nor that -- not self not not self. Something likes mathematic sing or anthropomorphizing of mathematics with all rational Leibneizism, rational relativity and Socratic Subjectivity. Going into that light of knowledge it leads to what is called self realization in the spiritual context. If the answer is found in a book, external authority, scripture, or consensus reality then the book answer is temporal, prejudiced, contrived, and "religious" (conforming to an external authority) versus spiritual and universal which assumes that the answer has come about experientially from a subjective inner realization. The former conclusion (which is the norm) is achieved through conformity and memorization, while the latter is achieved at first through critical and creative thought (neti-neti) and then from their non-dual integration is based upon the highest Universal Eternal Omnipresent Source which is eternally present -- which is omniscient as well as omnipresent -- eternally here and now as
beginning less and endless boundless universal Mind.

Thus the first step in this spiritual process is to make the inquiry which must challenge all sacred cows and institutionalized authority. The second step when this external knowledge (knowledge about something) and programming has been discarded, then one has space for the inner clarity and grace to arise (prasadam) which is subtle beyond the most subtle -- objectless. How else could one know the true Self who is not bound by any singular object, if not from every cell in your very body and every cell in the universe and every non-cell, and non-atom? These points to beauteous completion (where the bonds of karma are exhausted --to ultimate fulfillment and Grace). Kind attention is drawn to the coextensive representation, apriori determination and discussuional addendum of consciousness which is defines as the storage of all images, information, information fields, mind acting on it and the resultant being calculated by ASCII. It is just that when you think another sentence is formed.

This is the reason that yogis and true seekers meditate (practice dhyana) and perform spiritual practices (sadhanas) in order to penetrate the veil of illusion, to remove conditioned ignorance, to free themselves of conditioned bias, belief systems, prejudice, programmed mental propensities/tendencies, habits, limitations, and karma, and thus to eventually abide in the wisdom expanse of the Great Integrity -- the Great Universal Consciousness and Beingness which is supra-human, i.e., it includes the human realm and earth realm, but is neither anthropocentric nor dualistic. To limit, reduce, or impose human terminology, symbolism, and language into this realm, is exactly the self imposed barrier which the intellectual suffers and becomes imprisoned. Thus neti -neti can also be translated, "not just that, but more than that - more than just that, much more unlimitly so". Nothing is excluded. This may appear to be a heretical interpretation to the orthodox academician intent on separation and negation, yet maybe some academicians would agree that analytical processes are preliminary and supportive, helping the practitioner to eventually weed out errors of the mind and surpass the limitations of the human brain. Certainly we are more than THAT, which words can describe. In other "words", it is a spiritual statement only if it is an affirmation of the presence of an all inclusive living spirit; i.e., that non-dual teachings calls forth as it discards the ego (false and limited view of a separate "i" observer self). It does not negate nature, and the natural unconditioned mind, but rather it negates false and limited views and delusion. Failing these criteria, it is not a spiritual statement; rather it is intellectual speculation and/or ideology. Zen thus coordinates on the fact that there is no great knowledge to be acquired. “Experience” is what we have defined and there are neither births nor rebirths.

Unfortunately, in both the East and the West the common man substitutes belief, prama. Academic book knowledge, or such as a displacement for wisdom or gnosis, but the yogi does not fall victim to that trap. Those who are tied to the sterile puppet strings of reductionist thinking of the intellect (immersed in the limitations of symbolic world and duality) interpret the expression neti-neti, as "not this body, not this body" even though the body has nothing to do with the Sanskrit word, neti. Rather this occurs because of the intellectual's ingrained dualism, i.e., that says that God or Spirit has to exist elsewhere -- not here not here they are saying. Somewhere else -- somewhere else! This self defeating estrangement puts the intellectual into self imposed spiritual alienation and by definition distances them from God, Spirit, sacred presence, union, or samadhi -- the living world of "All Our Relations".

Fear of life or pain, is the "normal" and common reaction, but unnatural. It produces an alienated world of institutionalized spiritual Diaspora, corruption, fragmentation, and sin. This is the affliction that authentic yoga attempts to alleviate and eventually remedy, by meeting our mental pain, and thus destroying fear and attachment, instead of attempting escape. This institutionalized anti-body and anti-nature prejudice fortunately has become the acculturated and authoritative standard interpretation (no matter how absurd it appears to a true sadhak). It has become so entrenched that many humans are destroying themselves and their habitat. Rather in truth, neti neti means that we are not separate or apart from the sacred web of life: that this body does not exist separate from millions of years of co-evolution with the universe; that this body is a manifestation of beginning less love -- of eternal spirit right here in the present as a representative of eternal presence -- of love loving love. Here the true nature of phenomena is disclosed as it discloses its beginning less primordial origin. Such a life becomes a living library in each instant -- in all instances.

The alienation of man from this deep holographic/reflexive relationship is ignorance, where man denies his connectivity with all beings and things. Nothing else will be able to fill this empty gap. Neurotic
consumerism is dependent upon this spilt from living spirit and source. Man's true authority has become institutionalized by greedy and exploitive priests, kings, and their lackeys who thrive on manipulating others. These vampires create confusion on purpose in order to dumb people down and create a fundamental self doubt, which thus creates dependence upon these external authoritative structures, institutions, empires, kingships, priesthood, hierarchies, institutions, religions, etc. Of course those entrenched and dependent on this structure will deny this.

For example, neti neti as interpreted by some Vedantists (not all) is a form of denial of and escape from the world, of embodiment, of aversion to and fear of nature, creation, and embodiment. It's an easy way out/copout, but is none-the-less very seductive to the confused masses. The self delusion rap goes like this; "Just pretend that the world doesn't exist, that life is an illusion (maya), numb yourself out to it, the body, the senses, and sever all bonds to it, and then all your problems will go away. All suffering is just an illusion, war, torture, disease, poverty etc., is an illusion". In short rationalize out suffering and rationalize in Brahman. Take that as one's acting role as if playing in a theatre. Such is the pretentious play of those lost within inorganic and contrived self fabricated and sterile mindsets based on preferential thinking (raga/dvesa as attachment and repulsion). Obviously kleshas and ignorance such as raga/dvesa cannot be remediated through application of raga (preference) or dvesa (dislike and escapism), nor can nirvikalpa samadhi be realized through utilizing vikalpa.

Yes, many attachments such as to the body go away when the physical body dies, but who dies and who dreams on, what samskaras, mental attachments, fears, confusion, and karma continues. These are the elements that are to be liberated today in order to become liberated today and in the future. What about future generations and the welfare of our children? What about the future of the human species and the planet? These are human questions which beg the question of why human birth in the first place which are too earthly questions for the orthodox.

The narrow minded academic Advaita (non-dual) adherent interpretation of neti-neti precludes and escapes nature, embodiment, and differentiated consciousness, focusing on isolation toward Source only or monism. Monism is not non-dual, but rather one way union with the one at the expense of the many. Orthodox escapist oriented Vedanta tends toward monism -- a force moving away from differentiated consciousness while labeling everything Brahman at the expense of everything else.

They may deny or affirm, depending, that they have created a conceptual ideological split between spirit (Brahman) and Nature (embodiment); yet at the same time they may mistakenly call their philosophy non-dual. As such these so called non-dualists are as other worldly as fundamentalist Christians, who believe that God is in heaven (not here). Heaven is where they will go AFTER they die. That is another word, for institutionalized spiritual estrangement -- something Emperor Constantine created in the 4th century at the Nicene Council with a design to own the world. Joshua bin Josef of course never indicated anything as absurd as a distant God. In fact he is reported as saying: "My God is of the Living". In the Nicene Council that established the legitimacy of the Christian bible, only those representatives chosen by the Roman emperor, Constantine, were invited. Even some of these could not be made to agree to a new church based on Paul's doctrines of sin, so they were condemned and banished. The main controversy stemmed from what was called the Nicene Creed, a statement of Christian faith that was adopted by the Nicene Council which clearly affirmed that heaven was elsewhere than here.

It is remarkable that there are elements within Vedanta who also create that same alienation from Spirit (the well known Brahman-Maya split) while still insisting on labeling it non-duality. However it appears that such represents a deep duality, dissociation, and denial. The deepness of that denial is that the Vedantists think that they are non-dualists (most of them), but have in Reality created their own drama which may I add won’t end until they drop their own self imposed Vedantic filter. In both systems there exists a fear of nature, the body, and embodiment -- a statement of enduring suffering and denying fulfillment in trade for some distant far off time. In order to deal with the fear and trauma of existence brought upon one's state of confusion, one dissociates from "self" and embodiment -- from even an association with the body or life. Such systems reinforce an alienation from our own feelings. Such dogma makes us believe that sex, pleasure, bodily feelings, connection with the world or our animal nature is bad. Implied in that assumption is that we should ignore or overcome the body and our feelings. In that sense there is the idea of sacrifice without reward or a delayed other world rewards.
However in truth the body is god's temple and our feet and arms are his limbs. True non-duality (call it advaya if you like) synchronizes non-differentiated consciousness or absolute reality with differentiated consciousness (relative reality), one not excluding the other in true non-duality (advaya) with neither extreme existing by itself nor negating/excluding the other. Thus neti/neti is neither an undifferentiated consciousness (Brahman) interpretation, nor a differentiated consciousness (maya), but rather both/and. Maya is the clothing or apparel of Brahman. It both conceals Brahman as well as discloses Brahman. Without the clothing Brahman is invisible. They must be seen as an overall whole. A monist would state that only Brahman or the eternal is real to the exclusion of the temporal, while a non-dualist would recognize the relative and absolute truths as mutually co-arising.

Here we come to the true value of the saying, neti neti, as being neither monist, neither one extreme nor the other, neither up, nor down, but rather non-linear and multidimensional. If one searches inside to the observer - to see how his conditioning and prejudice (vrttis) colors the field of consciousness, one would eventually have to ask who is self and where does it abide? Religionists do not embark in such a genuine inquiry because they are compelled to follow their politically correct external authoritative traditional dictates, and not contradict or question it. However after a thorough spiritual inquiry by true seekers, no one has been able to find a separate self, or ego. Where does it abide? This separate self or ego is the simple dualistic question, but no one has been able to find it. After a logical search most people mistake that the ego is defined by the physical limits of the body; i.e., where the body ends, the ego ends. Hence in this situation, the term, neti neti does mean; no, I am not just the body -- not the physical body only, in the sense we are not the ego, rather we are more than that. It says we are more than a thing that dies but also this thing that dies AND also includes this thing that dies and more than that. The yogi gets this answer not by logic alone, but from direct experience of that expanded boundless universal awareness brought through dhyana and samadhi -- from the open heart. That experience informs then the frontal cortex and thus the logic of it is inspired through direct experience of the opened heart dictating to the open mind.

So that most common mistake by the closed minded, logician, philosopher, religionist, or cortically dominated man is the most common mistake -- the preferential assumption that the body is not real or spiritual because of great pain and dvesa (repulsion). Neti neti really means is that the body is only a small part of who we really are in terms of the big Self of All Our Relations. It is part of the omnipresent magical and intelligent great matrix of Siva/shakti. The Great Integrity is just that, all inclusive, omnipresent, universal, innate, and non-fragmented by definition so how could it not include the body and the entire cosmos? The only thing it does not include is the delusion of the ego -- that somehow the body is the ego or that any thing exists by itself in a vacuum. The "i" in Reality is empty and vacuous -- there is no "i". The "i" is a neurotic substitute -- place holder, for the absence of the sacred relationship. This sacred relationship occurs in the transpersonal. Both/and, non-dual realization that authentic yoga (reunification) brings.

Thus the true interpretation of neti-neti is that we are not this, not that -- not any "thing" separate. Neti-neti states clearly that we are not "just" that or this, i.e., that we are not separate and limited beings but rather thou are that, i.e., that our true identity is Brahman, not the limited and biased spin of ego (the delusion of being separate). In truth we live in sacred presence, but we are asleep in ignorance, delusion, and forgetfulness. Thus a Sufi affirmation to the divine goes; "I am nothing at all by myself. Only YOU exist -- God is everything"! Ignorance means just that; i.e., that some "observer/l" is ignoring the whole/hologram.

Just as the stars and valley are obscured by clouds and fog, they none-the-less truly exist as they are beyond their appearances, beyond the senses, beyond superficial clothing. The clearing of the clouds and lifting of the fog alludes to the process of purifying the matrix and lifting the veil of ignorance and illusion. Egohood only exists in our mind. It is a shared hallucination although a powerful one. Although many people may "act out" from this neurotic state, and thus mutually serve to amplify the illusion, it does not follow that it is none-the-less more real. Ego reality is "in Reality" an empty illusion -- void, a hallucination. Here we are approaching Buddhist non-dualism (free from all extremes). The ego is a delusion and hence so are the way that the ego normally perceives phenomena. But these limited or superficial ways of knowing does not preclude the true nature of mind or true nature of phenomena.
The non-dual unitive process includes the process of going toward primordial uncreated beginningless and the evolutionary creative manifestation from primordial source. This is only possible to know in the primordial present as instant presence. Primordial space and time has no end, no beginning, nor middle. It is not a thing or entity, and has always been available as sacred presence in the eternal timeless moment Primordial space has always been here, beyond before the beginning and after the end in the Beginning less never ending/ever-new/ever-now-ness), which is both nowhere and everywhere, neither one nor the other, and neither neither (neti neti). This is the ultimate unbiased universal unlimited and complete Reality, of which nobody can step outside of and view objectively.

This non-dual process in which all are kin is non-linear and non-dual BOTH/AND simultaneously like a non-dual pillar of light going up and down, neither up or down, left or right, in or out, neither both, but both/and. In authentic yoga we are this non-dual pillar in the activated core/heart reached through the middle innermost central channel (within the sushumna). Energetically this occurs when the winds moving in the ida and pingala channels (nadis) are subtilized to the extent that they dissolve into the central channel (sushumna) and stilled (sunya) in samadhi (see yoga Sutra III.3). Here samsaric mental afflictions are dissolved

Thus we are not "just" the energy body and not just the physical body which is ever changing, temporary, and corporeal, but at the same our identity must include the form bodies (in the greater context) in each sacred eternal moment of non-dual being. This is the salient point; i.e., that in all pervasive vast space, there is nothing excluded nor nothing that remains to be included. It is the Great Integrity -- the Great Completion -- the Mahamudra as the vast expanse where duality is merely a limited misperception. The orthodox scholar, dualist, and common man mistakes the expression to say that we are not the body, but the yogi takes it to say that we are all inclusive, the body, and the river, and the stars, the sacred mandala, divine creatrix, -- not just one of these limited, isolated, separate, and false identifications. In other words, the words, neti-neti, cannot be viewed successfully as an exclusive statement out of context with Brahman, but rather as an inclusive affirmation of Brahman and Atman -- Atman only truly exists united with Brahman-- of an all inclusive reality where there is no separation. The yogi as a true seeker and fearless rainbow warrior who learns to see living spirit inherent all created things -- learns to see the continuity of eternal spirit in all of creation at all times. As Van Morrison has put it, "Spirit Don't Ever Die". It exists NOW.

On the Connection between Buddhist Anatta/Anatma, Sunyata, Nondualism, Advaita/Advaya, and (Atman/Brahman Unity Utilizing Neti- Neti as the Integrative Key

Neti neti is a saying found in the Upanishads and especially attributed to the Avadhuta Gita. Literally, it means neither this, nor that. Considerable debate has arisen over exactly "what" (if anything specific) does it really refer to? Indeed, the true nature of that "what", as the true nature of reality is a large topic in Hindu Advaita philosophy as well as in Buddhist Madhyamika philosophy. As discussed above, the most classic fundamentalist anti-nature and escapist view is that neti neti is a statement of transcendence (isolation and escape), saying in effect, and one is not the body, not identified with nature, and thus one doesn't belong on the planet. In that disidentification one attempts to dis-identify and avoid death and suffering. Simplistic, but appealing to the escapist/fearful mind none-the-less. Since it is said twice some may say that it says, I am neither the body AND not the mind; i.e., my true identity is not confined to either self existence, or non-existence, but rather a larger non-dual Reality which encompasses both. In that way, the self (atman) is not viewed as independent, isolated, or separate from the all encompassing Brahman, but neither is Brahman absent from existence. Rather "existence" is redefined in non-dual terms. A cogent point to remember is that neti is repeated TWICE, rather than as a single negation "once", so we will take it as meaning, neither and nor.

Classically, neti neti is a refutation of duality, and thus an affirmation of the Great non-dual Integrity, often called Brahman, which affirms one's identity as interdependent with all beings and things, united as one as part and parcel with the hologram.. However, in some orthodox fundamentalist traditions, Brahman is defined as something separate from what they call Maya, which is usually defined as temporal existence. Hence they create an opposing duality (between eternal and temporal), but are they really separate or rather are they parts of a greater whole? In Kashmir Shaivism, maya both clothes/masks Brahman and reveals Brahman, depending upon the depth of one's penetrating wisdom.
So in some other schools and especially tantricism in general, which evolved from the latter Yoga Upanishads and afterwards such as the Buddhist and Kashmir tantricism, neti-neti can be interpreted as an all inclusive statement of true Universal identification, which is both undifferentiated and differentiated -- as not being separate; i.e., I am not an ego or separate "self". Rather I identify and belong to the greater whole. That self or identity is not separate/independent from the whole (Brahman), yet it is omnipresent simultaneously and transpersonally, reflecting our true non-dual nature. All that can be ascribed to the first neti of neti-neti, which negates the identification of the ego or separate self.

The second neti can say I am not just the whole, but also within all the parts of the whole -- I am both at the same time. Thus one may say that it is an affirmation of the Buddhist idea of anatta or atman, which similarly is the realization of the unreality of a separate self or ego (that the ego is fabricated and delusional, only existing because of unawareness of the Great Integrity, which bind us all to a universal unbiased commonality/Reality. This is to say that one can not be fixated on an egoic mindset at the same time as residing one's consciousness and being in the realm of vast space, vast primordial time, and vast knowledge. No, the latter is the ever present option which the open-mind chooses. The egoic mind is just a cramped, imprisoned, and limited version of the former.

So just as the first neti can say no to a separate observer (ego) who is free from subject/object duality, the second neti can be said to be a statement that affirms the emptiness of the object as viewed as independent and disconnected from the whole, which is nothing other than the an affirmation that "things" do not exist per se, by themselves, independently by themselves. Thus neti neti is best understood as an affirmation statement that we are not just the body, not an ego, not fragmented parts, and not separate -- that the body is part of a far more vast interconnected web of all life both of form (saguna) and the formless (nirguna) which is inseparable. “Neti neti” can be said to be a profoundly deep ultimate statement of non-dual beingness, neither one or the other, but rather an altogether expansive context which is all inclusive where nothing is left out nor needs to be included – which is neither form nor void in themselves, but both form and void intrinsically, neither just body or separate from body, but viewed within an integrated context where body and spirit both coincide and are synchronized.

If the Buddhist term, anatta (anatma in Sanskrit), is the negation of the fabricated delusional non-existence of the ego (a separate self), then there is no contradiction between Buddhism with Advaita. Confusion only arises when either Buddhists misunderstand the non-dual intention in neti - neti as a disowning of any idea of separate self; or where Hindus become confused that Brahman is a larger ego (soul) or separate self identification that can be conceived or owned by the ego; or when Hindus believe that Buddhist anatta is a nihilistic statement that denies existence completely. Such confusions arise only because of ideological conditioning and self identification which preclude listening fully or thoroughly while studying the other’s point of view.

Even in the older Buddhist Pali Canon, anatta (anatman, in Sanskrit) quite clearly confirms the reality of a non-separate (selfless) self simultaneously with the emptiness of any separate object whatsoever (which is the affirmation of emptiness or sunyata). The Vedantic statement that Atman and Brahman are united as one is thus a statement of no separate “self” also. Indeed atman does not exist separate from Brahman, but rather only when integrated with Brahman. Although Shankara, the first systematizer of Advaita (unqualified non-dualistic) Vedanta (culmination of all revelations), is credited with arguing against the Buddhists, if one studies his commentaries, one simply sees that his intent was simply to defend, promote, and reform Hinduism, rather than to attack basic Buddhist ideas. Buddha also did not challenge basic Hindu assumptions, rather than attempt to free people from dependence upon ritual, custom, ceremony, hierarchy, caste, idol worship, and dependence upon ancient scripture. Ancient Buddhism can be seen as a form of Hinduism, which is stripped naked of all its local, provincial, and non-universal ritual and cultural trappings, albeit cultural baggage has cropped up in Buddhism over the past 2600 years. So although rather heretical, one may postulate that there is not a large gulf between the two religions as religious zealots and defenders any claim; rather the same truth is being stated within two different culturally described contexts.

Similarly sunyata (emptiness), is a broader application of the Buddhist statement of anatta, saying that neither the observer nor the observed – neither the object nor the knower of the object have any basis as separate other than as fabrications modified by conceptual ideations. In short, there remains no objective
basis in Reality, as long as the observation point (observer) is limited or biased. Rather the basis or
ground of the observer and observed is their inherent emptiness in themselves of an independent self or
thing; i.e., no objects or observers exist in Reality separate from the universal boundless Mind. Without
this complete integral boundless universal perspective, all other views are biased, distorted, prejudiced,
fragmented and corrupt by definition. Here neti/neti again is remembered neither observer, nor
observed, not one or the other, but both/and inside an all inclusive whole. Radical that it may seem to
religionists, neti, neti, and sunyata point to the same truth. This is why net is repeated twice. It is the
way one says neither one nor the other, but both/and --the many and the one where a bigger complete
view is realized in Mahasandhi.

Of course strict dogmatists and ideologues who have become seduced by tradition, will stick to the old
dogma unmove, unable to think outside their accustomed box. Similarly orthodox Vedantists will
interpret this within their own box accordingly, but here we will utilize the Kashmir Tantric view which
easily reconciles with the Buddhist view to show that neti/neti is an advanced non-dual statement
relevant to both the cave dweller and layperson equally.

To sum up, we have to realize that in Advaita, Atman means as being one with Brahman. It is not the
ego, because the ego separates and denies this unity. It can be called the soul only if the soul is defined
as united with Brahman. When it is disconnected, then it is the egoic mindset (delusion) which rules,
not the union of Brahman and Atman. This soul then is a reflection and embodiment of Brahman, not its
negation (not the egoic identification). That Atman can never own or possess Brahman, rather it is
entirely owned by Brahman. Thus the statement of neti neti as a statement that one is not different from
Brahman is not to be taken as, "the ego is not separate from Brahman", but rather the ego is nothing at
all, where the soul is the reflection and expression of the universal all inclusive Brahman. Atman
understood this way as not being independent from Brahman, is not an egoic atman, rather it is
incapable of standing alone and apart from Brahman. It is anatman in the Buddhist sense of being empty
of an intrinsic independent self. This Atman is akin to the Buddhist idea of Buddhanature, where all
beings contain the evolutionary potential for complete awakening, Buddha nature being
intrinsic/inherent as one's true nature. The key in Buddhism is to reflect and express this inherent true
nature -- to become a living Buddha. In this way, Buddhanature (Tathagatagarbha) is the same as the
Advaitic Atman, recognizing that Advaita Vedanta's definition of "Atman" occurred 1000 years after
the mahasamadhi of the Buddha.

In the true non-sectarian yogic tradition, that which is put an end to, that which ceases, is
ignorance/unawareness, or rather the veil of illusory thought patterns cease. the world of duality ceases
in neti neti. When the veil of ignorance is destroyed, then illumination shines forth unadulterated.
Conversely, the process of denying or limiting Reality as-it-is through ignoring what is is called avidya
(ignorance). When we ignore something, then we lose the big picture and wind up with distorted and
biased views. One could just as well say that we are suffering from myopia or self deceit/delusion.
When this confusion, delusion, ignorance, unawareness, or illusory bias (labeling it by whatever name
that may be useful) is removed/negated, then we see things as they are -- Reality shines forth in net neti
-- when "not this and not that" becomes natural and spontaneous.

This Reality in tantra and Mahasandhi is natural, uncontrived, and unconditioned. It is our natural state.
What has been removed is the past programming. The result is the profound synchronization of absolute
and relative truth, shiva/shakti, right and left channels, pingala/ida, undifferentiated and differentiated
consciousness and so forth. It is not a denial of the temporal as if samsara were separate from nirvana,
or that nirvana was not found in samsara, or that shiva was not inside shakti, and so forth simply
affirming the eternal presence. Now abiding in sacred presence - the basis of true spirituality, there is no
thing truly solid, separate, or substantial that can be said to exist by itself (independently). Nor can that
"something be said to exist (as it never existed separately in the first place. Neither can one say both,
that it both truly exists and not exist, because there is no separate thing that can be spoken about. Nor
can they be said to both not exist. Again in all these situations the reified separate object of existence,
can not freely succeed as a referent or object to refer to. Relative and absolute, differentiated and
undifferentiated, form and void. the moon and sun, the left and the right, and the earth and heaven are
inseparable in the non-dual context of interdependence. Neti neti therein is the unification of wisdom
and compassion, emptiness and bliss, emptiness and clarity, and emptiness and essence, where neti/neti
is the negation of the process of dualistic illusion, fragmentation, and separation itself.

This occurs when awareness and the authentic energetic qualities of awareness are synchronized in the deepest recess of the nondual central channel (the avadhuti, kun dari ma, or sushumna). This occurs when the inner awareness meets its counterpart in outer awareness -- where the breaks in Indra's net is mended -- where what is, is seen by the heart in the heart directly.

From the Hevajra tantra Chapter One

"Buddha replied, 'There are thirty-two nerve-channels. These thirty-two are the bearers of the bodhicitta and flow into the center of Great Bliss. Among them three nerve-channels, Lalana, Rasana, and Avadhuti are the most important. Lalana has the nature of Wisdom and Rasana of Skillful means. In the middle, between them is Avadhuti, free from the duality of subject and object. Lalana is the bearer of Aksobhya and Rasana is the bearer of Rakta. The bearer of both Wisdom and Moon is known as Avadhuti.'"

Not Two (neither not one)

Non-dual (advaita) actually means (divitiyam nasti). However purists in Hindu tradition actually mean, eka vastu vada (there is no second substance except that Brahman is the only thing that exists as if it is separate from the physical created world or universe). This is actually a dualistic confusion of course and is really monism, rather than nondualism, where both diversity and unity are celebrated interdependently. Thus within this orthodox definition of Brahman, the phrase, eka vastu vada, is more accurate to describe what they mean, rather than the word, advaita.

Buddhism also uses the word, advaya, more than advaita to connote "not two" i.e. free from the two extremes (skt. dvaya anta mukta) of samarupa (the tendency to see things as really existing) and apavada (the tendency to see things as non-existing). Free from these extremes is the famous middle path (free from eternalism and nihilism) taught by Buddha. Advaya is not of a thing (the one and only thing like Brahma but a description of the svarupa of samsara) as things really are in their relative connected sense). That is why in tantra samsara which is illusory, but not an illusion is transformed into advaya jnana (nondual wisdom) in Buddhism whereas in Hinduism the illusory samsara vanishes and the true eternal unchanging Brahman dawns, but too often within the orthodox schools at the expense of diversity (excluding or denying the relative world) hence being escapist and unworlly stripping spirituality and God from embodiment.

Further, monism may impute that there exists only the observer, as if the whole is a separate object. That is the common mistake of selfhood (ego) where we ascribe name and form and "self/ego" to that which is unbounded, infinite, ineffable, universal, and totally incapable of being objectified. But really Brahman as the absolute cannot be an object and the "self" cannot identify with "it" as such without falling into dualistic delusion. Such an experience is impossible, otherwise the ego would try to grasp unto it (as it does). Thus monism is really a dualistic system -- an error in which one acknowledges the source/creator by negating or ignoring the creation -- where the observer is separated from the observed. Monism is a one way street. In relative reality "everything" is not the same, rather life is infinitely diverse and rich, but at the same time it is interconnected --part of a greater whole. Rather non-duality in net/neti indicates clearly that creator (as source) and creation (as active manifestation) are not separate, just as a fire was started by a spark -- the spark is implicate to it. They cannot be separated one from the other without creating a dualistic limitation.

Rather the case is that undifferentiated reality (absolute truth) coincides with differentiated reality (pratityasamutpada or relative truth) in sacred presence. Here both are acknowledged as well as linked in yoga. Conceptual formations no longer displace primordial wisdom, rather view is educated by the evidence of our unfiltered/naked awareness and direct experience. In monism however everything is confused with everything else. In other words it is quite different to say that only undifferentiated reality (absolute truth) exists and differentiated reality (relative or conventional truth) does not -- either one or the other, rather in true non-duality both exist AND do not exist – neti/neti. Existence, phenomena,
things, and events appear to exist, appear to either rise or fall away, both, or neither, but all such referents are designators, indicators, or imputations toward nothing true, rather it arises from a limited arena of frozen time and contracted space. It is a very small part of the entire picture, like a distorted or occluded lens which obscures the vast hologram and primordial integrity which is rich beyond measure. In truth there is nothing separate from the vast and all pervasive Primordial Mind. This is disclosed to the true seeker. What else can be known? Once the true nature of the Mind is known, so is the true nature of nature also known in its timeless radiant and vibrant wonder? Hence monism thus is a dull sameness, inane, empty, indifferent, and nihilistic. Crucially, non-duality is not monism and it is not negation. Neti-neti is not negation of any thing; rather it is all inclusive where nothing needs to excluded or included. Neither exists separately from each other so in the non-dual Buddhist school it is simply said that no separate thing exists, not even emptiness is a thing. Because we usually use words to communicate, and words are inherently dualistic(symbolic, it is difficult not to speak about an observer and the observed, but we can try to make that extra effort. Ultimately of course we need to give up words and thought constructs completely, like through objectless meditation (dhyana) or other transconceptual nisprapanca) yogic arts which allow for direct experience devoid of mere/empty symbolic representations.

Again a quote from Bede Griffiths and Georg Feuerstein, both mature observers of both western and eastern scenes.

"Advaita (nonduality) does not mean "one" in the sense of eliminating all differences. The differences are present in the one in a mysterious way. They are not separated anymore, and yet they are there."

Bede Griffiths (1997)

"The difference between monism and Nondualism might appear subtle, but it is important. The question is crucial for functional yoga or not the linguistic expressions by yogins etc. reflect their actual realization. Could it be that an adept has broken through to nondual realization but is still caught in monistic language?"

There is no question in my mind, for instance, that many of the statements in the early Upanishads point to genuine transcendental, transconceptual realization but resort to poetical/mythical/symbolic language that could give an inexperienced reader a different idea. In other words, there is not inevitably a perfect match between experience and language. This is a critical fact, which we need to bear in mind when looking for Upanishadic passages that testify to either monism or nondualism."

George Feuerstein

Although yoga, as an experiential non-dual practice, has been known for millennia, it is not a coincidence that non-duality as elucidated by Nagarjuna (circa 200 AD) and his non-dual/indivisible doctrine of sunyata and form (the union of ultimate and relative truths) became further elaborated upon by Shankaracharya with his non-dual doctrine of advaita vedanta (circa 7th century CE). This breakthrough context linking form and consciousness set the stage for the innovation of Hindu, and especially Kashmir tantra, hatha yoga, tantra yoga, laya yoga, kundalini yoga, and shaktism. Simultaneously Buddhist mahasandhi/mahamudra flourished in Afghanistan, Pakistan, and India eventually going to Tibet, China, and Japan. Such preliminaries posited the non-dual ground for such advanced practices. Both Hindu and Buddhist tantric and hatha yoga certainly explored new ways of self realization in the backdrop of a non-dual (advaya) providing a rich all-inclusive vibrant organic context, wherein primordial wisdom was co-evolving with the evolutionary force into an enlightened age, where wakin-up (while in this very body) was deemed accessible. Here the body, energy body, and mind were recognized as active co-participants. The sharpening of that indivisible relationship became the skillful instrument/vehicle for the exploration.

Numerous practices of integration evolved such as Sri Vidya, the body as a magical yantra (Trulkhor in Tibetan), the sadhanas of inner heat (kundalini or tummo), the Six Yogas of Naropa, the doctrine of harmonizing the five koshas (sheaths), working consciously with the inner winds, channels, and bindu, and so forth (to name a few yogic techniques which are outlined in tantra and yoga). Similarly, the Tibetan word for yoga is naljor. Nal means natural and jor is wealth. Hence, yoga or naljor brings forth
a natural innate state of wealth, health, and fulfillment. This enlightened age of body mind holistic integrated practices, continued to evolve in Medieval India up until the Mogul invasions of India (8th-13th centuries), wherein India's innovative genius became feared, suppressed, repressed, poisoned, and supplanted by alien invaders from the West.

Monism fears diversity, because it is a threat to egoic order and its attempt at imputational domination/control. It is thus an egoic grasping/craving which maintains the rend which substitutes monism as an inadequate compensation of a natural organic non-dual open-ended interdependence free from conflation (the superimposition of egoic confusion). Shankaracharya's non-dual "Self" (Brahman) is not an independent/separate self identity, but a genuine integration as a description of an indivisible interdependence where everything is connected. That Self is all-rich, all encompassing, infinitely differentiated and inseparably married as nature is married to awareness, Shakti to shiva, Samantabhadra/Samantabhadra, the true nature of the mind and the true nature of nature, or simply said as mind is married to the body, so too is heaven married to the earth. The problem, if any are to be identified, is that human beings tend to habitually ignore their truly innate connection with the infinite -- as part of the whole, thus wallowing in a state of chronic egoic confusion (samsara).

Since the human mind, being part of a larger primordial system, cannot grasp the entire process intellectually, the folly is to think otherwise; i.e., that the intellect is an efficacious tool for spiritual knowledge -- knowledge of mind itself. In fact, mind acts upon the storage of information, visual representations, anecdotes, aphorisms of thought, belief systems, and most important is that the anagrammatic mind concocts and concocts and fabricates a thesis for itself mostly the same. This result in various illusions of states that of waking state, dreaming state, and dreamless state. Without knowing the instrument of knowing, then chronic errors will continue to be repeated. So the major "problem" which arises for those mired in intellectual, fundamentalist, philosopher, and others lost in dualistic thinking is that they have become addicted to navigating "reality" through a veil of deadened and numbed out chronic processes of chronic extraction and over objectification -- where anecdotes of truth does not meet aphorisms of thought, where the rocky razzmatazz of falsehood and illusion drowns the angst saga of taciturn bay and enduring silence. through separation -- an exclusiveness (reductionism, analytical mentation, and comparative analysis) without placing their data within the holistic context -- in integration and harmony with all time -- as exhibitions of gnosia of the Great Integrity where all the seemingly dualistic parts are seen in relationship with all others in all of time. Indeed, such a world view appears broken and fragmented due to self sabotage. Such folly! Even if a yogi should speak, if the listener has no direct experience, then the listener will try to understand the "yogic speak" through their intellect, which will only confuse the listener more. It will have the effect of bringing the listener even more so in their logical mind in a futile attempt to understand the "yogi speech". It might appear that the yogi is speaking intellectually, because the words are not logical and appear to be "foreign". Thus yogis are rare, but rarer still are teachers of yoga. It is said that the teacher comes when the student is ready, but "who" is ready? A true spiritual seeker places his/her predilections and prejudices aside -- offering it up as a fire offering while making an affirmation that all of his/her activities be reordered and placed in alignment and harmony with the universal timeless guidance of a sacred transpersonal nonduality -- the world of Grace where Maheshwara (the param-purusha resides). In indigenous terms the Great Integrity of "All Our Relations") is a holographic realm of profound and sacred suchness. A true sadhak (practitioner) will not allow themselves to be swayed, distracted, nor dissipated by the intellect or conception processes. "Here" in this sacred presence, which is not based on intellectual reductionism, the eternal timeless reality (creator) bleeds through and is revealed in every action of creation -- as divine Shakti and as such in yogic terms Siva/Shakti is in perfect balance, harmony, and synchronicity. "HERE" yin/yang is in perfect balance and thus the complete Tai-Chi is accomplished. "HERE" father/mother, sky and earth, spirit and nature, pure consciousness and pure Beingness exist as a self supporting mutuality -- as embodied LOVE

In this non-dual/transpersonal world of an all inclusive Brahman, divine Shakti is seen in the tree, the river, the flowers, the stars, the ocean, and even in the human animal. It is beyond objectification or elaboration (nisprapanca/Nirvikalpa). "It" as such cannot be known or owned. The only place it is not seen is in that which is unreal, make believe, symbolic -- that which is illusory, false, or what is considered to exist in isolation in the frozen and deluded corrupted fragmented sphere of the egoic mindset (limitation and individual "self hood"). This latter sphere (the delusional reality of the ego) only
exists in the deluded mind, so it is not considered real or true (although it colors "reality" for those who are deluded in superficial appearances).

"Neti-neti" does not mean that we are not this body, only that we are more than this body -- We are the Long Body -- vast Being, the Big Universal Unbiased Boundless Primordial Mind in which all things see from vast space and vast time as a synchronistic and holographic simultaneity are interdependent, interconnected, and mutually co-emerging. Here everything is alive and vivid.

Things, objects, or phenomena are not interpreted through only the five or six sense organs, but rather through the universal MIND. The human animal, being a vital part of creation, it is up to humans to claim their rightful place in total harmony with the evolutionary force and no longer repress/deny it. This human body is not separate from this great body of timeless being -- in context of the larger body of an all inclusive interpretation of Brahman where we are all related. While in this unbiased and unlimited "Reality", no separate "i" exists. This "reality" is neither exclusively eternal. Nor exclusively temporal, rather it is both/and (neti/neti). It is the result of the marriage of crown and root, sahasrara and muladhara, pingala and ida, Shiva/Shakti, spirit/nature, objective/subjective, consciousness/Beingness. Such is truly non-dual and synergistically synchronistic. It cannot be realized through while remaining bound by the limitations of logic or analytical thought, but rather it comes from a more inclusive intelligent direct Connection/Union.

Indeed we would be misled to identify with just the physical body, the sense world, or the process of perception out of context with how the body arrived here or the origin of thoughts or the process of perception arise as separate from the process of creation/evolution or primordial wisdom-- who and what "it" really is -- we would be lost in an illusion and ignorant limitation, false identification subject to narcissism, delusional self aggrandizement of the illusion of separate self (ego), and all the rest of the many demons of ego delusion born of unawareness such as self pride, nationalism, racism, fame, status, envy, hatred, greed, lust, and the afflictions which stem from this false identification (the unfortunate common milieu). So within a primordial natural context, in light of "reality" and "truth", then ignorance is negated, the world of separateness is not only an illusion, it doesn't really exist per se, i.e., it is empty of independent substantiality outside from the non-dual context of interdependence. Existence viewed from the fragmented context of apparently separate senses, separate sense objects, separate self/ perceiver conspire toward a limited and biased view. It is "not" as it appears in terms of what is or what it is not. This "not self" or non-existence also does not exist. Both "not-self" and not "not-self" do not exist alone. Neither both, nor other. Nothing exists separate as "itself" except the universal heart/core -- LOVE.

No separate self
No independent 'I'
Rather an interdependent, indivisible, inseparable WE

No I am that, but rather WE ARE THEE

The true nature of our mind, is beyond a stasis of a separate/independent mentation/intellectual conception or artifice

The true nature of phenomena is thus revealed within the appearance of a body as well as outside of the appearance of the body,

In all phenomena as co-participants of all pervading omnipresent omniscience

WE ARE THEE -- Om Tat Sat- Tat Tvam Asi

Neither self, nor not self, as true Self is plural and unlimited, as our true nature, no one can own/possess it (albeit many have tried).

This is the true meaning of "neti neti" -- the end of spiritual self-alienation. The process then is in arriving home to indigenous all encompassing space. When we arrive, we wake up to see that we are
profoundly present -- we become at one with sacred presence.

Thus the expression, neti neti, in context means we are the great LONG BODY all together taken as a whole, the Great Integrity/completion, not separate/fragmented from "God's" infinite parts --THAT is OUR TRUE NATURE residing in this transpersonal interdependent spiritual fullness and wonder -- this natural, profoundly intelligent, and wholesome expression of complete and continuous integration and synchronicity with all things past present and future. In the Heart then, neti - neti means not separate, not separate -- not apart -- and thus we put to rest limited self imposed false identifications, habits, and beliefs and thus the "reality" of yoga -- of a world inextricably bound together in its continuity is realized. HERE Self acknowledges Self -- eternal Love recognizes eternal love -- we express the love which is our hearts. It is from this core Beingness -- this abiding in the true self nature of the natural unconditioned universal mind -- this as pure core/heart center -- where we learn to respect, affirm, and acknowledge in All Our Relations -- serving as the fount of great integral Beingness -- that we increasingly draw our sustenance, sense of balance, sense of centeredness, peace, security, inspiration, and eventually identity from -- from the HRIDAYAM which is the HEART OF HEARTS where the omnipresent teaching of the always present teacher (as eternal presence) intimately reveals in self liberatory revelation. HERE we cease to be betrayed, abandoned, ignored, and afraid as we cease to betray, abandon, ignore, and fear. HERE neurotic compensatory desire ceases -- the vrttis are annihilated (nirodha) and we rest in our true nature (swarupa).

Just this, just this

Just that, just that

Nothing else, all included

Empty of separate self

As-it-is, always

When the veil is lifted

When awakened

The ineffable great ground of pure Beingness and consciousness, disclosed through the removal of falsehood (neti neti) reveals our true and essential nature (eternal, uncontrived, natural, innate, and "self" abiding) -- from which all are related -- the spark behind the flame and the flame are one, not two) -- the sparkling spirit which animates and breathes life into the world at each and every moment unending. Even HERE- all-ways/always HERE. WE are that great wonder taken all together-- Om Tat Sat!

Tat Tvam Asi -- That Thou art -- WE are THEE -- Boundless Great Integrity -- Pure Love in its natural expression -- The reality where all relations of a separate "self" are known as an incomplete myopia (as illusory), where, the Reality of All Our Relations shines vibrant, naturally vivid, self illuminating, and bright ... and THAT is how it really is.

(1) Neti, Neti principle we write in mathematical form as follows (See Schrodinger’s cat in Penrose’s The Big, the Small and The Human Mind")Observer and the observed(Here we take both accentuation coefficient and dissipations coefficients as zero)

(2) \( (\Psi) = -e_1 (\Psi) 1 - e_2 (\Psi) 1\) – ……………………………en (\(\Psi)\) n

(3) \( (\Psi) = +e_1 (\Psi) 1 + e_2 (\Psi) 1\) – ……………………………en (\(\Psi)\) n

(4) \( (\Psi) = +1 (\Psi) 1 - e_2 (\Psi) 1\) – ……………………………en (\(\Psi)\) n

(5) \( (\Psi) = -e_1 (\Psi) 1 + e_2 (\Psi) 1\) – ……………………………en (\(\Psi)\) n

(6) Svabhava and space time world lines

(7) Consciousness (Storage of all information and visual representations with mind acting upon it anagrammatically. Note we eschew using words “being” and “becoming” We are infact beings with sense of consciousness and individual consciousness always tries to do”high jump” like body wants to. This is an evolutionary process.Levithanish literature has done more damage to the subject than addressing quintessential. We leave metaphorical and take the categorical and classification.)
In quantum mechanics, the Schrödinger equation is an equation that describes how the quantum state of a physical system changes with time. It was formulated in late 1925, and published in 1926, by the Austrian physicist Erwin Schrödinger.

Quantum Field Theory, Statistical Mechanics (We Are Talking Of Systems To Which Theory Is Applicable And Holds. Bank Example Of Assets Being Equivalent To Liabilities And That Each Individual Debits And Credits Being Conservative In Addition To The Conservativeness Of Holistic System Is To Be Mentioned), Markov Field And Hilbert Space:

Module Numbered One

NOTATION:

$G_{13}$ : Category One Of Quantum Field Theory And Statistical Mechanics

$G_{14}$ : Category Two Of Quantum Field Theory And Statistical Mechanics

$G_{15}$ : Category Three Of Quantum Field Theory And Statistical Mechanics

$T_{13}$ : Category One Of Yang Baxter Equations, Quantum Groups, Bose Fermion Equivalence Rational Conformal Field Theory (Again Classification Is For The Systems For Which The Theories Are Applicable Or For That Matter Systems Which Violates The Theories Mentioned)

$T_{14}$ : Category Two Of Yang Baxter Equations, Quantum Groups, Bose Fermion Equivalence Rational Conformal Field Theory (Again Classification Is For The Systems For Which The Theories Are Applicable Or For That Matter Systems Which Violates The Theories Mentioned)

$T_{15}$ : Category Three Of Yang Baxter Equations, Quantum Groups, Bose Fermion Equivalence Rational Conformal Field Theory (Again Classification Is For The Systems For Which The Theories Are Applicable Or For That Matter Systems Which Violates The Theories Mentioned)

$LHS$ Of Schrödinger’s Equation And $RHS$ Of Schrodinger’s Equations (Negative Sign Implies That The Amount Is Extracted From $LHS$. It Can Happen Instantaneously Or Immediately Without Time Lag. An Example Of Distillation To Make Curds Out Of Milk And Addition Of Water To Mild Can Be Given. This Is Most Important Aspect In Our Further Analysis)

Module Numbered Two:

$G_{16}$ : Category one of $LHS$ (note here that we are talking of the individual systems to which the Schrödinger’s equation is applied and the classification is based on the features and characteristics of the systems and parametricization of systems which attribute and ascribe the formulation of classification and stratification of the systems)

$G_{17}$ : Category two of $LHS$(note here that we are talking of the individual systems to which the Schrödinger’s equation is applied and the classification is based on the features and characteristics of the systems and parametricization of systems which attribute and ascribe the formulation of classification and stratification of the systems)

$G_{18}$ : Category three of $LHS$(note here that we are talking of the individual systems to which the Schrödinger’s equation is applied and the classification is based on the features and characteristics of
the systems and parametricization of systems which attribute and ascribe the formulation of
classification and stratification of the systems

\[ T_{16} \text{ : category one of LHS (note here that we are talking of the individual systems to which the Schrödinger’s equation is applied and the classification is based on the features and characteristics of the systems and parametricization of systems which attribute and ascribe the formulation of classification and stratification of the systems) } \]

\[ T_{17} \text{ : category two of LHS (note here that we are talking of the individual systems to which the Schrödinger’s equation is applied and the classification is based on the features and characteristics of the systems and parametricization of systems which attribute and ascribe the formulation of classification and stratification of the systems) } \]

\[ T_{18} \text{ : category three of LHS (note here that we are talking of the individual systems to which the Schrödinger’s equation is applied and the classification is based on the features and characteristics of the systems and parametricization of systems which attribute and ascribe the formulation of classification and stratification of the systems translations in space and time) } \]

\[ i\hbar \frac{\partial}{\partial t} \Psi(r, t) = -\hbar^2 \frac{\nabla^2 \Psi(r, t)}{2m} + V(r, t)\Psi(r, t) \]

**Module Numbered Three:**

\[ G_{20} : \text{ Category one of LHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and Parametricization is classificatory, stratificatory and sequatratitional. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, and is suitable for further classifications and organismationalised generalized theories and their concomitant terms. } \]

\[ G_{21} : \text{ CATEGORY TWO OF LHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. } \]

\[ G_{22} : \text{ CATEGORY THREE OF LHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, and is suitable for further classifications and organismationalised generalized theories and their concomitant terms. } \]
$T_{20} :$ CATEGORY two of LHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, and is suitable for further classifications and organisationalized generalized theories and their concomitant terms

$T_{21} :$ CATEGORY two of LHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, and is suitable for further classifications and organisationalized generalized theories and their concomitant terms

$T_{22} :$ CATEGORY THREE LHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, and is suitable for further classifications and organisationalized generalized theories and their concomitant terms

\[
i\hbar \frac{\partial}{\partial t} \Psi (\mathbf{r}, t) = \frac{-\hbar^2}{2m} \nabla^2 \Psi (\mathbf{r}, t) + V (\mathbf{r}, t) \Psi (\mathbf{r}, t)
\]

Second Term Of The Rhs And First Term Of Rhs Of The Time Dependent Schrodinger's Equation

: Module Numbered Four:

$G_{24} :$ CATEGORY ONE of LHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, and is suitable for further classifications and organisationalized generalized theories and their concomitant terms

$G_{25} :$ CATEGORY two of second term on RHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term
of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of substraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, andis suitable for further classifications and organizerisationalised generalized theories and their concomitant terms

\[ G_{26} : \text{CATEGORY THREE of second term on RHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatratational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of substraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, andis suitable for further classifications and organizerisationalised generalized theories and their concomitant terms} \]

\[ T_{24} : \text{CATEGORY one of the first term on RHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatratational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of substraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, andis suitable for further classifications and organizerisationalised generalized theories and their concomitant terms} \]

\[ T_{25} : \text{CATEGORY two of the first term on the RHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatratational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of substraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, andis suitable for further classifications and organizerisationalised generalized theories and their concomitant terms} \]

\[ T_{26} : \text{CATEGORY THREE of first term on RHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatratational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of substraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, andis suitable for further classifications and organizerisationalised generalized theories and their concomitant terms} \]
Term On Lhs And Term On Rhs:

Module Numbered Five:

\( G_{28} \) : CATEGORY ONE OF THE TERM ON LHS of the time in dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, anddis suitable for further classifications and organisationalised generalized theories and their concomitant terms

\( G_{29} \) : CATEGORY two of the term on LHS of the time independent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, anddis suitable for further classifications and organisationalised generalized theories and their concomitant terms

\( G_{30} \) : CATEGORY three of term on LHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, anddis suitable for further classifications and organisationalised generalized theories and their concomitant terms

\( T_{28} \) : CATEGORY one of the term on RHS of the time independent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, anddis suitable for further classifications and organisationalised generalized theories and their concomitant terms

\( T_{29} \) : Category two of the term on RHS of the time independent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and
the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, and is suitable for further classifications and organisationised generalized theories and their concomitant terms.

\[ \text{T}_{30} : \text{CATEGORY three of the term on RHS of the time independent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck’s scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck’s scale also, and is suitable for further classifications and organisationised generalized theories and their concomitant terms.} \]

The four (and more) negations
1) the negation of solid independent existence
2) the negation of the negation of existence
3) the negation of both existence and non-existence
4) the negation of the negation of both existence and non-existence

\textbf{Svabhava And Space Time World Lines:}

\textbf{Module Numbered Six:}

\[ \text{G}_{32} : \text{CATEGORY ONE OF SVABHAVA (Needless to emphasise that different individuals have different predilections, proclivities, pen chance and propensities and they can be classified based on those characteristics and parametricization. In consideration to the fact that rajas and Tamás had been taken in to consideration to account in an earlier paper we eschew them in to consideration in this account.}} \]

\[ \text{G}_{33} : \text{Category Two Of Svabhava}} \]

\[ \text{G}_{34} : \text{Category Three Of svabhava} \]

\[ \text{T}_{32} : \text{Category One Of Space And Time World Lines} \]

\[ \text{T}_{33} : \text{Category Two Of space And Time World Lines} \]

\[ \text{T}_{34} : \text{Category Three Of Space And Time World Lines} \]

\textbf{Simulations And Dissimulations By Brahman And Antibrahman Agency (The Stimulus Providers To The Response Of The Human Beings) And Concomitant Misconception Of It To Be Real By The Human Being (If Two Persons Walk With You When You Go Out, It Is Accident. If Twenty People Walk With You Every Day And Every Time You Go Out It Is Coordination, A “Design” Created By The Agency Of Brahman-Antibrahman Category. Be Very Clear About The}
Association Is Nothing But What Has Been Taught To You By Reference Groups And In The Eventuality Of The “Association” Say Green” For Good And “Red” For Danger Continues To Appear For An, In Respect To An Event Or A Subject Matter, Mind Acts Upon That And Produces Various Permutations And Combinations. That Is Why People Sometimes Say Just Like The Events Are Simulated, In The World By Ourselves, Like Thinking About The Past And Crying Over It, Or Thinking About The Future And Worrying About It, Brahman, The Simulatory Agency, A Giant Computer If You Like, Manipulates The Fundamental Forces ,“Energy” Of The Universe To Create An Illusory Space Time)

Module Numbered Seven)

\( G_{36} \) : Category One Of Simulation And Dissimulation by Brahman And Antibrahman

\( G_{37} \) : category two of simulation and dissimulation by AntiBrahman

\( G_{38} \) : category three of simulation and dissimulation by brahman AntiBrahman(note that we have defined brahman and AntiBrahman as simulation dissimulation agencies and these are classified based on the people such simulations are acting upon)

\( T_{36} \) : Category One Of Prakruti Kshobha(Broadly Deprivation) Or Prakruti Siddhi(Broadly Gratification) (Note We Are Talking Of Individuals Who Are Affected Who Are Spoken About)

\( T_{37} \) : Category Two Of prakruti Kshobha And Prakruti Siddhi

\( T_{38} \) : Category Three Of Prakruti Kshobha And Prakruti Siddhi

Representation Of Neti Neti(Case One) By Schrodinger’s Wave Equation. Look At Penrose On Schrödinger’s Cat In The Big The Small And The Human Mind

\((\Psi) = - e_1 (\Psi)_1 - e_2 (\Psi)_2 - \ldots \ldots \ldots \ldots e_n (\Psi)_n\)

Representation Of Neti Neti In Schrodinger’s Equation First Term On The Above Equation When Added To Other Terms On The Rhs Leads To The Same Situation Of Addition Of Milk To Water. Here We Shall Not Take The Terms Separately But Assume That They Are Added To The Lhs On One Shot Basis.

Module Numbered Eight

\( G_{40} \): CATEGORY ONE OF TERM ON LHS

\( G_{41} \): CATEGORY TWO OF TERM ON LHS

\( G_{42} \): CATEGORY TWO OF TERM ON LHS

\( T_{40} \): CATEGORY ONE OF TERM ON RHS

\( T_{41} \): CATEGORY TWO OF TERM S IN RHS

\( T_{42} \): CATEGORY THREE OF TERMS IN RHS.

\((a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}, (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\)
(b_{16}^{(2)}, b_{17}^{(2)}, b_{18}^{(2)}; (a_{26}^{(3)}, (a_{27}^{(3)}, (a_{22}^{(3)}, (b_{26}^{(3)}, b_{22}^{(3)}), (a_{24}^{(4)}, a_{25}^{(4)}), (a_{26}^{(4)}, (b_{24}^{(4)}, b_{25}^{(4)}), (b_{26}^{(5)}, (b_{29}^{(5)}, (b_{30}^{(5)}, (a_{28}^{(5)}, (a_{29}^{(5)}, (a_{30}^{(5)}, (a_{32}^{(5)}, (a_{33}^{(6)}, (a_{34}^{(6)}, (b_{32}^{(6)}, (b_{33}^{(6)}, (b_{34}^{(6)}

are Accentuation coefficients

(a_{13}^{(1)}, (a_{14}^{(1)}, (a_{15}^{(1)}, (b_{13}^{(1)}, (b_{14}^{(1)}), (b_{15}^{(1)}, (a_{16}^{(2)}, (a_{17}^{(2)}, (a_{18}^{(2)}, (b_{16}^{(2)}, b_{17}^{(2)}, b_{18}^{(2)}, (a_{26}^{(3)}, (a_{27}^{(3)}), (a_{22}^{(3)}, (b_{26}^{(3)}, b_{21}^{(3)}, (b_{22}^{(3)}, (a_{24}^{(4)}, a_{25}^{(4)}, (a_{26}^{(4)}, (b_{24}^{(4)}, b_{25}^{(4)}, (b_{26}^{(5)}, (b_{29}^{(5)}, (b_{30}^{(5)}), (a_{28}^{(5)}, (a_{29}^{(5)}, (a_{30}^{(5)}, (a_{32}^{(6)}, (a_{33}^{(6)}, (a_{34}^{(6)}), (b_{32}^{(6)}, (b_{33}^{(6)}, (b_{34}^{(6)}

are Dissipation coefficients

Quantum Field Theory And Statistical Mechanics (We Are Talking Of Systems To Which Theory Is Applicable And Holds. Bank Example Of Assets Being Equivalent To Liabilities And That Each Individual Debits And Credits Being Conservative In Addition To The Conservativeness Of Holistic System Is To Be Mentioned), Markov Field And Hilbert Space:

Module Numbered One

The differential system of this model is now (Module numbered one)

\[ \frac{dG_{13}}{dt} = (a_{13}^{(1)}G_{14} - [(a_{13}^{(1)} + (a_{13}^{(1)}(T_{14},t))]G_{13} \]

\[ \frac{dG_{14}}{dt} = (a_{14}^{(1)}G_{13} - [(a_{14}^{(1)} + (a_{14}^{(1)}(T_{14},t))]G_{14} \]

\[ \frac{dG_{15}}{dt} = (a_{15}^{(1)}G_{14} - [(a_{15}^{(1)} + (a_{15}^{(1)}(T_{14},t))]G_{15} \]

\[ \frac{dT_{13}}{dt} = (b_{13}^{(1)}T_{14} - [(b_{13}^{(1)} - (b_{13}^{(1)}(G,t))]T_{13} \]

\[ \frac{dT_{14}}{dt} = (b_{14}^{(1)}T_{13} - [(b_{14}^{(1)} - (b_{14}^{(1)}(G,t))]T_{14} \]

\[ \frac{dT_{15}}{dt} = (b_{15}^{(1)}T_{14} - [(b_{15}^{(1)} - (b_{15}^{(1)}(G,t))]T_{15} \]

\[ +(a_{13}^{(1)}(T_{14},t) = \text{First augmentation factor} \]

\[ -(b_{13}^{(1)}(G,t) = \text{First detritions factor} \]

\[ i\hbar \frac{\partial}{\partial t} \Psi(r, t) = \frac{-\hbar^2}{2m} \nabla^2 \Psi(r, t) + V(r, t) \Psi(r, t) \]

Lhs Of Schrödinger’s Equation And Rhs Of Schrodinger’s Equations (Negative Sign Implies That The Amount Is Extracted From Lhs. It Can Happen Instantaneously Or Immediately Without Time Lag. An Example of Distillation To Make Curds Out Of Milk And Addition Of Water To Mild Can Be Given. This Is Most Important Aspect In Our Further Analysis)

Module Numbered Two:

The differential system of this model is now (Module numbered two)

\[ \frac{dG_{16}}{dt} = (a_{16}^{(2)}G_{17} - [(a_{16}^{(2)} + (a_{16}^{(2)}(T_{17},t))]G_{16} \]

74
\begin{align*}
d\dot{G}_{17} &= (a_{17})^{(2)}G_{16} - [(a')_{17}^{(2)} + (a'')_{17}^{(2)}(T_{17}, t)]G_{17} \\
d\dot{G}_{18} &= (a_{18})^{(2)}G_{17} - [(a')_{18}^{(2)} + (a'')_{18}^{(2)}(T_{17}, t)]G_{18} \\
d\dot{T}_{16} &= (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)}(G_{19}, t)]T_{16} \\
d\dot{T}_{17} &= (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)}(G_{19}, t)]T_{17} \\
d\dot{T}_{18} &= (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)}(G_{19}, t)]T_{18} \\
+ (a')_{16}^{(2)}(T_{17}, t) &= \text{ First augmentation factor} \\
- (b'_{16})^{(2)}(G_{19}, t) &= \text{ First detriments factor} \\
\end{align*}

\[
\frac{i\hbar}{\hbar} \frac{\partial}{\partial t} \Psi(r, t) = -\frac{\hbar^2}{2m_e} \nabla^2 \Psi(r, t) + V(r, t)\Psi(r, t)
\]

**Module Numbered Three**

The differential system of this model is now (Module numbered three)

\begin{align*}
d\dot{G}_{20} &= (a_{20})^{(3)}G_{21} - [(a')_{20}^{(3)} + (a'')_{20}^{(3)}(T_{21}, t)]G_{20} \\
d\dot{G}_{21} &= (a_{21})^{(3)}G_{20} - [(a')_{21}^{(3)} + (a'')_{21}^{(3)}(T_{21}, t)]G_{21} \\
d\dot{G}_{22} &= (a_{22})^{(3)}G_{21} - [(a')_{22}^{(3)} + (a'')_{22}^{(3)}(T_{21}, t)]G_{22} \\
d\dot{T}_{20} &= (b_{20})^{(3)}T_{21} - [(b')_{20}^{(3)} + (b'')_{20}^{(3)}(G_{23}, t)]T_{20} \\
d\dot{T}_{21} &= (b_{21})^{(3)}T_{20} - [(b')_{21}^{(3)} + (b'')_{21}^{(3)}(G_{23}, t)]T_{21} \\
d\dot{T}_{22} &= (b_{22})^{(3)}T_{21} - [(b')_{22}^{(3)} + (b'')_{22}^{(3)}(G_{23}, t)]T_{22} \\
+ (a')_{20}^{(3)}(T_{21}, t) &= \text{ First augmentation factor} \\
- (b'_{20})^{(3)}(G_{23}, t) &= \text{ First detriments factor} \\
\end{align*}

\[
\frac{i\hbar}{\hbar} \frac{\partial}{\partial t} \Psi(r, t) = -\frac{\hbar^2}{2m_e} \nabla^2 \Psi(r, t) + V(r, t)\Psi(r, t)
\]

**Second Term Of The Rhs And First Term Of Rhs Of The Time Dependent Schrodinger's Equation**

**Module Numbered Four**

The differential system of this model is now (Module numbered Four)

\[
d\dot{G}_{24} = (a_{24})^{(4)}G_{25} - [(a')_{24}^{(4)} + (a'')_{24}^{(4)}(T_{25}, t)]G_{24}
\]

75
\[ \frac{dG_{25}}{dt} = (a_{25}')^{(4)} G_{24} - [(a_{25}')^{(4)} + (a_{25}')^{(4)}(T_{25}, t)] G_{25} \]

\[ \frac{dG_{26}}{dt} = (a_{26}')^{(4)} G_{25} - [(a_{26}')^{(4)} + (a_{26}')^{(4)}(T_{25}, t)] G_{26} \]

\[ \frac{dT_{24}}{dt} = (b_{24}')^{(4)} T_{25} - [(b_{24}')^{(4)} - (b_{24}')^{(4)}((G_{27}), t)] T_{24} \]

\[ \frac{dT_{25}}{dt} = (b_{25}')^{(4)} T_{24} - [(b_{25}')^{(4)} - (b_{25}')^{(4)}((G_{27}), t)] T_{25} \]

\[ \frac{dT_{26}}{dt} = (b_{26}')^{(4)} T_{25} - [(b_{26}')^{(4)} - (b_{26}')^{(4)}((G_{27}), t)] T_{26} \]

\[ + (a_{24}')^{(4)}(T_{25}, t) = \text{First augmentation factor} \]

\[ -(b_{24}')^{(4)}((G_{27}), t) = \text{First detritions factor} \]

\[ F \Psi = \hat{H} \Psi. \]

Module Numbered Five:

The differential system of this model is now (Module number five)

\[ \frac{dG_{28}}{dt} = (a_{28}')^{(5)} G_{29} - [(a_{28}')^{(5)} + (a_{28}')^{(5)}(T_{29}, t)] G_{28} \]

\[ \frac{dG_{29}}{dt} = (a_{29}')^{(5)} G_{28} - [(a_{29}')^{(5)} + (a_{29}')^{(5)}(T_{29}, t)] G_{29} \]

\[ \frac{dG_{30}}{dt} = (a_{30}')^{(5)} G_{29} - [(a_{30}')^{(5)} + (a_{30}')^{(5)}(T_{29}, t)] G_{30} \]

\[ \frac{dT_{28}}{dt} = (b_{28}')^{(5)} T_{29} - [(b_{28}')^{(5)} - (b_{28}')^{(5)}((G_{31}), t)] T_{28} \]

\[ \frac{dT_{29}}{dt} = (b_{29}')^{(5)} T_{28} - [(b_{29}')^{(5)} - (b_{29}')^{(5)}((G_{31}), t)] T_{29} \]

\[ \frac{dT_{30}}{dt} = (b_{30}')^{(5)} T_{29} - [(b_{30}')^{(5)} - (b_{30}')^{(5)}((G_{31}), t)] T_{30} \]

\[ + (a_{28}')^{(5)}(T_{29}, t) = \text{First augmentation factor} \]

\[ -(b_{28}')^{(5)}((G_{31}), t) = \text{First detritions factor} \]

Svabhava And Space Time World Lines:

Module Numbered Six

The differential system of this model is now (Module numbered Six)

\[ \frac{dG_{32}}{dt} = (a_{32}')^{(6)} G_{33} - [(a_{32}')^{(6)} + (a_{32}')^{(6)}(T_{33}, t)] G_{32} \]

\[ \frac{dG_{33}}{dt} = (a_{33}')^{(6)} G_{32} - [(a_{33}')^{(6)} + (a_{33}')^{(6)}(T_{33}, t)] G_{33} \]

\[ \frac{dG_{34}}{dt} = (a_{34}')^{(6)} G_{33} - [(a_{34}')^{(6)} + (a_{34}')^{(6)}(T_{33}, t)] G_{34} \]

\[ \frac{dT_{32}}{dt} = (b_{32}')^{(6)} T_{33} - [(b_{32}')^{(6)} - (b_{32}')^{(6)}((G_{35}), t)] T_{32} \]
Simulations And Dissimulations By Brahman And Antibrahman Agency (The Stimulus Providers To The Response Of The Human Beings) And Concomitant Misconception Of It To Be Real By The Human Being (If Two Persons Walk With You When You Go Out, It Is Accident. If Twenty People Walk With You Every Day And Every Time You Go Out It Is Coordination, A “Design” Created By The Agency Of Brahman-Antibrahman Category. Be Very Clear About The Association Is Nothing But What Has Been Taught To You By Reference Groups And In The Eventuality Of The “Association” Say Green” For Good And “Red” For Danger Continues To Appear For An, In Respect To An Event Or A Subject Matter, Mind Acts Upon That And Produces Various Permutations And Combinations. That Is Why People Sometimes Say Just Like The Events Are Simulated, In The World By Ourselves, Like Thinking About The Past And Crying Over It, Or Thinking About The Future And Worrying About It, Brahman Manipulates The Fundamental Forces,”Energy” Of The Universe To Create An Illusory Space Time)

Module Numbered Seven:

The differential system of this model is now (SEVENTH MODULE)

\[
\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - [(b_{33}^{(6)})^{(6)} - (b_{33}^{(6)})^{(6)}((G_{33}), t)]T_{33}
\]

\[
\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - [(b_{34}^{(6)})^{(6)} - (b_{34}^{(6)})^{(6)}((G_{33}), t)]T_{34}
\]

\[+(a_{32}^{(6)})^{(6)}(T_{33}, t) = \text{First augmentation factor}\]

Representation Of Neti Neti (Case One) By Schrodinger’s Wave Equation. Look At Penrose On Schrödinger’s Cat In The Big The Small And The Human Mind

\[\Psi = e_1 (\Psi) e_2 (\Psi) \ldots \ldots \ldots e_n (\Psi)\]

Representation Of Neti Neti In Schrodinger’s Equation First Term On The Above Equation When Added To Other Terms On The Rhs Lead To The Same Situation Of Addition Of Milk To Water. Here We Shall Not Take The Terms Separately But Assume That They Are Added To The Lhs On One Shot Basis.

Module Numbered Eight

GOVERNING EQUATIONS:
The differential system of this model is now

\[
\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a_{40}')^{(8)} + (a_{40}'')^{(8)}(T_{41}, t)\right]G_{40}
\]

\[
\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a_{41}')^{(8)} + (a_{41}'')^{(8)}(T_{41}, t)\right]G_{41}
\]

\[
\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a_{42}')^{(8)} + (a_{42}'')^{(8)}(T_{41}, t)\right]G_{42}
\]

\[
\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b_{40}')^{(8)} - (b_{40}'')^{(8)}(G_{43}, t)\right]T_{40}
\]

\[
\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b_{41}')^{(8)} - (b_{41}'')^{(8)}(G_{43}, t)\right]T_{41}
\]

\[
\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b_{42}')^{(8)} - (b_{42}'')^{(8)}(G_{43}, t)\right]T_{42}
\]

\[-(b_{40}'')^{(7)}(G_{39}, t) = \text{ First detritions factor}\]

First Module Concatenation:

\[
\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{c}
(a_{13}')^{(1)} + (a_{13}'')^{(1)}(T_{14}, t) + (a_{13}''')^{(1)}(T_{15}, t) + (a_{13}''''\cdots)^{(1)}(T_{21}, t) \\
+ (a_{13}'')^{(2,2,2)}(T_{25}, t) + (a_{13}''')^{(5,5,5,5)}(T_{29}, t) + (a_{13}''''\cdots)^{(6,6,6,6,6,6)}(T_{33}, t)
\end{array}\right] G_{13}
\]

\[
\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{15} - \left[\begin{array}{c}
(a_{14}')^{(1)} + (a_{14}'')^{(1)}(T_{14}, t) + (a_{14}''')^{(1)}(T_{15}, t) + (a_{14}''''\cdots)^{(1)}(T_{21}, t) \\
+ (a_{14}'')^{(2,2,2)}(T_{25}, t) + (a_{14}''')^{(5,5,5,5)}(T_{29}, t) + (a_{14}''''\cdots)^{(6,6,6,6,6,6)}(T_{33}, t)
\end{array}\right] G_{14}
\]

\[
\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{16} - \left[\begin{array}{c}
(a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14}, t) + (a_{15}''')^{(1)}(T_{15}, t) + (a_{15}''''\cdots)^{(1)}(T_{21}, t) \\
+ (a_{15}'')^{(2,2,2)}(T_{25}, t) + (a_{15}''')^{(5,5,5,5)}(T_{29}, t) + (a_{15}''''\cdots)^{(6,6,6,6,6,6)}(T_{33}, t)
\end{array}\right] G_{15}
\]

Where \([a_{13}']^{(1)}(T_{14}, t), [a_{13}'']^{(1)}(T_{14}, t), [a_{13}''']^{(1)}(T_{14}, t)\) are first augmentation coefficients for category 1, 2 and 3

\+[a_{13}'']^{(2,2,2)}(T_{25}, t), +[a_{13}''']^{(5,5,5,5)}(T_{29}, t), +[a_{13}''''\cdots)^{(6,6,6,6,6,6)}(T_{33}, t)\]

are second augmentation coefficient for category 1, 2 and 3

\+[a_{13}''']^{(2,2,2)}(T_{25}, t), +[a_{13}''''\cdots)^{(5,5,5,5)}(T_{29}, t), +[a_{13}''''\cdots)^{(6,6,6,6,6,6)}(T_{33}, t)\]

are third augmentation coefficient for category 1, 2 and 3

\+[a_{13}''''\cdots)^{(5,5,5,5)}(T_{29}, t), +[a_{13}''''\cdots)^{(6,6,6,6,6,6,6)}(T_{33}, t)\]

are fourth augmentation coefficient for category 1, 2 and 3

\+[a_{13}''''\cdots)^{(5,5,5,5)}(T_{29}, t), +[a_{13}''''\cdots)^{(6,6,6,6,6,6,6)}(T_{33}, t)\]

are fifth augmentation coefficient for category 1, 2 and 3

\+[a_{13}''''\cdots)^{(6,6,6,6,6,6,6)}(T_{33}, t)\]

are sixth augmentation coefficient for category 1, 2 and 3

\+[a_{13}''''\cdots)^{(6,6,6,6,6,6,6)}(T_{33}, t)\]

are seventh augmentation Coefficients

\+[a_{13}''''\cdots)^{(6,6,6,6,6,6,6)}(T_{33}, t)\]

are eighth augmentation Coefficients

78
\[
\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix}
(b'_{13})^{(1)} & - (b''_{12})^{(1)}(G, t) & - (b''_{36})^{(7,1)}(G_{39, t}) & - (b''_{36})^{(3,3)}(G_{23, t}) \\
- (b''_{24})^{(4,4,4,4)}(G_{27, t}) & - (b''_{28})^{(5,5,5,5)}(G_{31, t}) & - (b''_{32})^{(6,6,6,6)}(G_{35, t}) \\
- (b''_{36})^{(7,1)}(G_{39, t}) & - (b''_{40})^{(8)}(G_{43, t}) 
\end{bmatrix} T_{13}
\]

\[
\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix}
(b'_{14})^{(1)} & - (b''_{14})^{(1,1)}(G, t) & - (b''_{18})^{(2,2)}(G_{19, t}) & - (b''_{21})^{(3,3)}(G_{23, t}) \\
- (b''_{24})^{(4,4,4,4)}(G_{27, t}) & - (b''_{28})^{(5,5,5,5)}(G_{31, t}) & - (b''_{32})^{(6,6,6,6)}(G_{35, t}) \\
- (b''_{36})^{(7,1)}(G_{39, t}) & - (b''_{41})^{(8)}(G_{43, t}) 
\end{bmatrix} T_{14}
\]

\[
\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix}
(b'_{15})^{(1)} & - (b''_{15})^{(1,1)}(G, t) & - (b''_{18})^{(2,2)}(G_{19, t}) & - (b''_{22})^{(3,3)}(G_{23, t}) \\
- (b''_{24})^{(4,4,4,4)}(G_{27, t}) & - (b''_{30})^{(5,5,5,5)}(G_{31, t}) & - (b''_{32})^{(6,6,6,6)}(G_{35, t}) \\
- (b''_{36})^{(7,1)}(G_{39, t}) & - (b''_{42})^{(8)}(G_{41, t}) 
\end{bmatrix} T_{15}
\]

Where \(- (b''_{13})^{(1)}(G, t), - (b''_{14})^{(1)}(G, t), - (b''_{15})^{(1)}(G, t)\) are first detritions coefficients for category 1, 2 and 3.

\(- (b''_{12})^{(2,2)}(G_{19, t}), - (b''_{18})^{(2,2)}(G_{19, t}), - (b''_{14})^{(2,2)}(G_{19, t})\) are second detritions coefficients for category 1, 2 and 3.

\(- (b''_{24})^{(5,5,5,5)}(G_{23, t}), - (b''_{25})^{(5,5,5,5)}(G_{23, t}), - (b''_{24})^{(5,5,5,5)}(G_{23, t})\) are third detritions coefficients for category 1, 2 and 3.

\(- (b''_{26})^{(4,4,4,4)}(G_{27, t}), - (b''_{28})^{(4,4,4,4)}(G_{27, t}), - (b''_{26})^{(4,4,4,4)}(G_{27, t})\) are fourth detritions coefficients for category 1, 2 and 3.

\(- (b''_{29})^{(5,5,5,5)}(G_{31, t}), - (b''_{28})^{(5,5,5,5)}(G_{31, t}), - (b''_{29})^{(5,5,5,5)}(G_{31, t})\) are fifth detritions coefficients for category 1, 2 and 3.

\(- (b''_{36})^{(6,6,6,6)}(G_{35, t}), - (b''_{36})^{(6,6,6,6)}(G_{35, t}), - (b''_{36})^{(6,6,6,6)}(G_{35, t})\) are sixth detritions coefficients for category 1, 2 and 3.

\(- (b''_{41})^{(7,1)}(G_{43, t}), - (b''_{40})^{(7,1)}(G_{43, t}), - (b''_{40})^{(7,1)}(G_{43, t})\) are seventh detritions coefficients.

\(- (b''_{42})^{(8)}(G_{41, t}), - (b''_{41})^{(8)}(G_{41, t}), - (b''_{42})^{(8)}(G_{41, t})\) are eighth detritions coefficients.

**Second Module Concatenation:**

\[
\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix}
(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17, t}) + (a''_{16})^{(1,1)}(T_{14, t}) + (a''_{16})^{(3,3)}(T_{21, t}) \\
+ (a''_{26})^{(4,4,4,4)}(T_{25, t}) + (a''_{28})^{(5,5,5,5)}(T_{29, t}) + (a''_{24})^{(6,6,6,6)}(T_{33, t}) \\
+ (a''_{36})^{(7,7,7)}(T_{37, t}) + (a''_{40})^{(8)}(T_{41, t}) 
\end{bmatrix} G_{16}
\]

\[
\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix}
(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17, t}) + (a''_{17})^{(1,1)}(T_{14, t}) + (a''_{17})^{(3,3)}(T_{21, t}) \\
+ (a''_{27})^{(4,4,4,4)}(T_{25, t}) + (a''_{29})^{(5,5,5,5)}(T_{29, t}) + (a''_{27})^{(6,6,6,6)}(T_{33, t}) \\
+ (a''_{37})^{(7,7,7)}(T_{37, t}) + (a''_{41})^{(8)}(T_{41, t}) 
\end{bmatrix} G_{17}
\]

\[
\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix}
(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17, t}) + (a''_{18})^{(1,1)}(T_{14, t}) + (a''_{18})^{(3,3)}(T_{21, t}) \\
+ (a''_{28})^{(4,4,4,4)}(T_{25, t}) + (a''_{30})^{(5,5,5,5)}(T_{29, t}) + (a''_{28})^{(6,6,6,6)}(T_{33, t}) \\
+ (a''_{38})^{(7,7,7)}(T_{37, t}) + (a''_{42})^{(8)}(T_{41, t}) 
\end{bmatrix} G_{18}
\]

Where \(+(a''_{16})^{(1,1)}(T_{17, t}), +(a''_{16})^{(1,1)}(T_{17, t}), +(a''_{16})^{(1,1)}(T_{17, t})\) are first augmentation coefficients for category 1, 2 and 3.

\(+(a'_{16})^{(2)}(T_{17, t}), +(a''_{16})^{(2)}(T_{17, t}), +(a''_{16})^{(2)}(T_{17, t})\) are second augmentation coefficients for category 1, 2 and 3.
are third augmentation coefficient for category 1, 2 and 3

are fourth augmentation coefficient for category 1, 2 and 3

are fifth augmentation coefficient for category 1, 2 and 3

are sixth augmentation coefficient for category 1, 2 and 3

are Seventh Detrition Coefficients

are Eight Augmentation Coefficients

Third Module Concatenation:

\[
\begin{align*}
\frac{dG_{20}}{dt} &= \left[ (a_{20})^{(3)} + (a_{20})^{(3)}(T_{21}, t) \right] + \left[ (a_{20})^{(2,2,2)}(T_{17}, t) \right] + \left[ (a_{16})^{(1,1,1)}(T_{14}, t) \right] \\
&+ \left[ (a_{20})^{(4,4,4,4)}(T_{25}, t) \right] + \left[ (a_{20})^{(5,5,5,5)}(T_{29}, t) \right] + \left[ (a_{20})^{(6,6,6,6)}(T_{33}, t) \right] \\
&+ \left[ (a_{20})^{(7,7,7)}(T_{37}, t) \right] + \left[ (a_{20})^{(8,8,8)}(T_{41}, t) \right] \\
\end{align*}
\]

\[
G_{20}
\]

\[
\begin{align*}
\frac{dG_{21}}{dt} &= \left[ (a_{21})^{(3)} + (a_{21})^{(3)}(T_{21}, t) \right] + \left[ (a_{17})^{(2,2,2)}(T_{17}, t) \right] + \left[ (a_{14})^{(1,1,1)}(T_{14}, t) \right] \\
&+ \left[ (a_{21})^{(4,4,4,4)}(T_{25}, t) \right] + \left[ (a_{21})^{(5,5,5,5)}(T_{29}, t) \right] + \left[ (a_{21})^{(6,6,6,6)}(T_{33}, t) \right] \\
&+ \left[ (a_{21})^{(7,7,7)}(T_{37}, t) \right] + \left[ (a_{21})^{(8,8,8)}(T_{41}, t) \right] \\
\end{align*}
\]

\[
G_{21}
\]

\[
\begin{align*}
\frac{dG_{22}}{dt} &= \left[ (a_{22})^{(3)} + (a_{22})^{(3)}(T_{21}, t) \right] + \left[ (a_{18})^{(2,2,2)}(T_{17}, t) \right] + \left[ (a_{15})^{(1,1,1)}(T_{14}, t) \right] \\
&+ \left[ (a_{22})^{(4,4,4,4)}(T_{25}, t) \right] + \left[ (a_{22})^{(5,5,5,5)}(T_{29}, t) \right] + \left[ (a_{22})^{(6,6,6,6)}(T_{33}, t) \right] \\
&+ \left[ (a_{22})^{(7,7,7)}(T_{37}, t) \right] + \left[ (a_{22})^{(8,8,8)}(T_{41}, t) \right] \\
\end{align*}
\]

\[
G_{22}
\]
\[
\frac{df_{20}}{dt} = \begin{bmatrix}
(b_{20})^{(3)}(T_{20}, t) - (b_{20})^{(3)}(G_{23}, t) - (b_{20})^{(3)}(G_{19}, t) - (b_{20})^{(1,1,1)}(G, t) \\
-(b_{20})^{(2,2,2)}(G_{23}, t) - (b_{20})^{(2,2,2)}(G_{19}, t) - (b_{20})^{(3)}(G, t)
\end{bmatrix} T_{20}
\]

\[
\frac{df_{21}}{dt} = \begin{bmatrix}
(b_{21})^{(3)}(T_{21}, t) - (b_{21})^{(3)}(G_{23}, t) - (b_{21})^{(3)}(G_{19}, t) - (b_{21})^{(1,1,1)}(G, t) \\
-(b_{21})^{(2,2,2)}(G_{23}, t) - (b_{21})^{(2,2,2)}(G_{19}, t) - (b_{21})^{(3)}(G, t)
\end{bmatrix} T_{21}
\]

\[
\frac{df_{22}}{dt} = \begin{bmatrix}
(b_{22})^{(3)}(T_{22}, t) - (b_{22})^{(3)}(G_{23}, t) - (b_{22})^{(3)}(G_{19}, t) - (b_{22})^{(1,1,1)}(G, t) \\
-(b_{22})^{(2,2,2)}(G_{23}, t) - (b_{22})^{(2,2,2)}(G_{19}, t) - (b_{22})^{(3)}(G, t)
\end{bmatrix} T_{22}
\]

\[
\begin{align*}
+ (a_{23}^{(1)}(T_{23}, t) + (a_{23}^{(1)}(T_{23}, t) + (a_{23}^{(1)}(T_{23}, t) & \text{ are first augmentation coefficients for category 1, 2 and 3} \\
+ (a_{24}^{(2,2,2)}(T_{16}, t) + (a_{24}^{(2,2,2)}(T_{17}, t) + (a_{24}^{(2,2,2)}(T_{18}, t) & \text{ are second augmentation coefficients for category 1, 2 and 3} \\
+ (a_{25}^{(1,1,1)}(T_{16}, t) + (a_{25}^{(1,1,1)}(T_{17}, t) + (a_{25}^{(1,1,1)}(T_{18}, t) & \text{ are third augmentation coefficients for category 1, 2 and 3} \\
+ (a_{25}^{(3,3,3)}(T_{25}, t) + (a_{25}^{(3,3,3)}(T_{25}, t) + (a_{25}^{(3,3,3)}(T_{25}, t) & \text{ are fourth augmentation coefficients for category 1, 2 and 3} \\
+ (a_{26}^{(3,3,3,3,3)}(T_{26}, t) + (a_{26}^{(3,3,3,3,3)}(T_{25}, t) + (a_{26}^{(3,3,3,3,3)}(T_{25}, t) & \text{ are fifth augmentation coefficients for category 1, 2 and 3} \\
+ (a_{27}^{(6,6,6,6,6,6)}(T_{39}, t) + (a_{27}^{(6,6,6,6,6,6)}(T_{39}, t) + (a_{27}^{(6,6,6,6,6,6)}(T_{39}, t) & \text{ are sixth augmentation coefficients for category 1, 2 and 3} \\
+ (a_{28}^{(7,7,7,7,7,7)}(T_{43}, t) + (a_{28}^{(7,7,7,7,7,7)}(T_{43}, t) + (a_{28}^{(7,7,7,7,7,7)}(T_{43}, t) & \text{ are seventh augmentation coefficients for category 1, 2 and 3} \\
+ (a_{29}^{(8,8,8,8,8,8)}(T_{43}, t) + (a_{29}^{(8,8,8,8,8,8)}(T_{43}, t) + (a_{29}^{(8,8,8,8,8,8)}(T_{43}, t) & \text{ are Eigth augmentation coefficients for category 1, 2 and 3}
\end{align*}
\]
\[
\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[ (a_{13})^{(1,1,1)}(T_{14, t}) + (a_{16})^{(2,2,2,2)}(T_{17, t}) + (a_{20})^{(3,3,3,3)}(T_{21, t}) \right] G_{24} + (a_{29})^{(5,5,6)}(T_{29, t}) + (a_{32})^{(6,6,6)}(T_{33, t}) + (a_{36})^{(7,7,7,7)}(T_{37, t}) + (a_{40})^{(8,8,8,8)}(T_{41, t})
\]

\[
\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[ (a_{13})^{(1,1,1)}(T_{14, t}) + (a_{16})^{(2,2,2,2)}(T_{17, t}) + (a_{20})^{(3,3,3,3)}(T_{21, t}) \right] G_{25} + (a_{29})^{(5,5,6)}(T_{29, t}) + (a_{32})^{(6,6,6)}(T_{33, t}) + (a_{36})^{(7,7,7,7)}(T_{37, t}) + (a_{40})^{(8,8,8,8)}(T_{41, t})
\]

\[
\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[ (a_{13})^{(1,1,1)}(T_{14, t}) + (a_{16})^{(2,2,2,2)}(T_{17, t}) + (a_{20})^{(3,3,3,3)}(T_{21, t}) \right] G_{26} + (a_{29})^{(5,5,6)}(T_{29, t}) + (a_{32})^{(6,6,6)}(T_{33, t}) + (a_{36})^{(7,7,7,7)}(T_{37, t}) + (a_{40})^{(8,8,8,8)}(T_{41, t})
\]

Where

\[ (a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)} \] are first augmentation coefficients for category 1, 2 and 3

\[ (a_{29})^{(5,5,6)}, (a_{32})^{(6,6,6)} \] are second augmentation coefficients for category 1, 2 and 3

\[ (a_{36})^{(7,7,7,7)} \] are third augmentation coefficients for category 1, 2 and 3

\[ (a_{13})^{(1,1,1)}, (a_{16})^{(2,2,2,2)}, (a_{20})^{(3,3,3,3)} \] are fourth augmentation coefficients for category 1, 2 and 3

\[ (a_{29})^{(5,5,6)}, (a_{32})^{(6,6,6)} \] are fifth augmentation coefficients for category 1, 2 and 3

\[ (a_{36})^{(7,7,7,7)} \] are sixth augmentation coefficients for category 1, 2 and 3

\[ (a_{40})^{(8,8,8,8)} \] are seventh augmentation coefficients

\[ (a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)} \] are eighth augmentation coefficients

\[
\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[ -(b_{13})^{(1,1,1)}(G_{c}, t) - (b_{16})^{(2,2,2,2)}(G_{c}, t) - (b_{20})^{(3,3,3,3)}(G_{c}, t) \right] T_{24} + (b_{29})^{(5,5,6)}(G_{c}, t) + (b_{32})^{(6,6,6)}(G_{c}, t) + (b_{36})^{(7,7,7,7)}(G_{c}, t) + (b_{40})^{(8,8,8,8)}(G_{c}, t)
\]

\[
\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[ -(b_{13})^{(1,1,1)}(G_{c}, t) - (b_{16})^{(2,2,2,2)}(G_{c}, t) - (b_{20})^{(3,3,3,3)}(G_{c}, t) \right] T_{25} + (b_{29})^{(5,5,6)}(G_{c}, t) + (b_{32})^{(6,6,6)}(G_{c}, t) + (b_{36})^{(7,7,7,7)}(G_{c}, t) + (b_{40})^{(8,8,8,8)}(G_{c}, t)
\]

Where

\[ -(b_{24})^{(4)}, -(b_{25})^{(4)}, -(b_{26})^{(4)} \] are first detrition coefficients for category 1, 2 and 3

\[ -(b_{29})^{(5,5,6)}, -(b_{32})^{(6,6,6)} \] are second detrition coefficients for category 1, 2 and 3

\[ -(b_{36})^{(7,7,7,7)} \] are third detrition coefficients for category 1, 2 and 3

\[ -(b_{40})^{(8,8,8,8)} \] are fourth detrition coefficients for category 1, 2 and 3
are fourth detrition coefficients for category 1, 2 and 3
\[-b_{28}^{(7,2,2,2)}(G_{28}, t) - b_{28}^{(2,2,2,2)}(G_{28}, t) - b_{28}^{(3,2,2,2)}(G_{28}, t)\]
are fifth detrition coefficients for category 1, 2 and 3
\[-b_{28}^{(7,3,3,3)}(G_{28}, t) - b_{28}^{(3,3,3,3)}(G_{28}, t) - b_{28}^{(3,3,3,3)}(G_{28}, t)\]
are sixth detrition coefficients for category 1, 2 and 3
\[-b_{28}^{(7,7,7,7)}(G_{28}, t) - b_{28}^{(7,7,7,7)}(G_{28}, t) - b_{28}^{(7,7,7,7)}(G_{28}, t)\] \(\text{ARE Seventh Detrition Coefficients}\)
\[-b_{28}^{(8,8,8,8)}(G_{28}, t) - b_{28}^{(8,8,8,8)}(G_{43}, t) - b_{28}^{(8,8,8,8)}(G_{43}, t)\] \(\text{Are Eighth Detrition Coefficients}\)

Fifth Module Concatenation:

\[
\frac{dG_{28}}{dt} = (a_{28})^{(5)} G_{28}^2 - \left[ (a_{28}^{(1)})^{(5)}(T_{29}, t) + (a_{28}^{(2)})^{(5)}(T_{29}, t) + (a_{28}^{(3)})^{(4,4)}(T_{25}, t) + (a_{28}^{(4)})^{(6,6,6)}(T_{33}, t) + (a_{28}^{(5)})^{(7,7,7,7)}(T_{37}, t) + (a_{28}^{(6)})^{(8,8,8,8)}(T_{41}, t) \right]
\]

\[
\frac{dG_{29}}{dt} = (a_{29})^{(5)} G_{29}^2 - \left[ (a_{29}^{(1)})^{(5)}(T_{29}, t) + (a_{29}^{(2)})^{(5)}(T_{29}, t) + (a_{29}^{(3)})^{(4,4,4)}(T_{25}, t) + (a_{29}^{(4)})^{(6,6,6)}(T_{33}, t) + (a_{29}^{(5)})^{(7,7,7,7)}(T_{37}, t) + (a_{29}^{(6)})^{(8,8,8,8)}(T_{41}, t) \right]
\]

\[
\frac{dG_{30}}{dt} = (a_{30})^{(5)} G_{30}^2 - \left[ (a_{30}^{(1)})^{(5)}(T_{29}, t) + (a_{30}^{(2)})^{(5)}(T_{29}, t) + (a_{30}^{(3)})^{(4,4,4)}(T_{25}, t) + (a_{30}^{(4)})^{(6,6,6)}(T_{33}, t) + (a_{30}^{(5)})^{(7,7,7,7)}(T_{37}, t) + (a_{30}^{(6)})^{(8,8,8,8)}(T_{41}, t) \right]
\]

Where \(+(a_{28}^{(1)})^{(5)}(T_{29}, t)\), \(+(a_{28}^{(2)})^{(5)}(T_{29}, t)\), \(+(a_{28}^{(3)})^{(4,4,4)}(T_{25}, t)\), \(+(a_{28}^{(4)})^{(6,6,6)}(T_{33}, t)\), \(+(a_{28}^{(5)})^{(7,7,7,7)}(T_{37}, t)\), \(+(a_{28}^{(6)})^{(8,8,8,8)}(T_{41}, t)\) are first augmentation coefficients for category 1, 2 and 3

And \(+(a_{29}^{(1)})^{(4,4,4)}(T_{25}, t)\), \(+(a_{29}^{(2)})^{(4,4,4)}(T_{25}, t)\), \(+(a_{29}^{(3)})^{(6,6,6)}(T_{33}, t)\), \(+(a_{29}^{(4)})^{(6,6,6)}(T_{33}, t)\), \(+(a_{29}^{(5)})^{(7,7,7,7)}(T_{37}, t)\), \(+(a_{29}^{(6)})^{(8,8,8,8)}(T_{41}, t)\) are second augmentation coefficients for category 1, 2, and 3

\(+(a_{30}^{(1)})^{(4,4,4)}(T_{25}, t)\), \(+(a_{30}^{(2)})^{(4,4,4)}(T_{25}, t)\), \(+(a_{30}^{(3)})^{(6,6,6)}(T_{33}, t)\), \(+(a_{30}^{(4)})^{(6,6,6)}(T_{33}, t)\), \(+(a_{30}^{(5)})^{(7,7,7,7)}(T_{37}, t)\), \(+(a_{30}^{(6)})^{(8,8,8,8)}(T_{41}, t)\) are third augmentation coefficient for category 1, 2, and 3

\(+(a_{30}^{(1)})^{(1,1,1,1)}(T_{16}, t)\), \(+(a_{30}^{(2)})^{(1,1,1,1)}(T_{16}, t)\), \(+(a_{30}^{(3)})^{(1,1,1,1)}(T_{16}, t)\) are fourth augmentation coefficients for category 1, 2, and 3

\(+(a_{30}^{(1)})^{(2,2,2,2)}(T_{27}, t)\), \(+(a_{30}^{(2)})^{(2,2,2,2)}(T_{27}, t)\), \(+(a_{30}^{(3)})^{(2,2,2,2)}(T_{27}, t)\) are fifth augmentation coefficients for category 1, 2, and 3

\(+(a_{30}^{(1)})^{(3,3,3,3)}(T_{23}, t)\), \(+(a_{30}^{(2)})^{(3,3,3,3)}(T_{23}, t)\), \(+(a_{30}^{(3)})^{(3,3,3,3)}(T_{23}, t)\) are sixth augmentation coefficients for category 1, 2, and 3

\(+(a_{30}^{(1)})^{(8,8,8,8)}(T_{41}, t)\), \(+(a_{30}^{(2)})^{(8,8,8,8)}(T_{41}, t)\), \(+(a_{30}^{(3)})^{(8,8,8,8)}(T_{41}, t)\) are Eighth Augmentation Coefficients

\[
\frac{dT_{28}}{dt} = (b_{28})^{(5)} T_{28} - \left[ (b_{28}^{(1)})^{(5)}(T_{28}, t) - (b_{28}^{(2)})^{(4,4)}(G_{31}, t) - (b_{28}^{(3)})^{(6,6,6)}(G_{35}, t) - (b_{28}^{(4)})^{(7,7,7,7)}(G_{39}, t) \right]
\]

\[
\frac{dT_{29}}{dt} = (b_{29})^{(5)} T_{29} - \left[ (b_{29}^{(1)})^{(5)}(T_{29}, t) - (b_{29}^{(2)})^{(4,4)}(G_{27}, t) - (b_{29}^{(3)})^{(6,6,6)}(G_{35}, t) - (b_{29}^{(4)})^{(7,7,7,7)}(G_{39}, t) \right]
\]
Sixth Module Concatenation

\[ \frac{dT_{30}}{dt} = (b_{30})^{(5)} T_{29} - \left[ (b_{30}^{(5)} - (b_{30}^{(5)})^{(G_{31}, t)} - (b_{26}^{(5,4,4)}(G_{27}, t) - (b_{24}^{(5,6,6)}(G_{35}, t) - (b_{13}^{(1,1,1,1)}(G, t) - (b_{19}^{(2,2,2,2,2)}(G_{19}, t) - (b_{22}^{(3,3,3,3)}(G_{23}, t) - (b_{60}^{(7,7,7,7,7)}(G_{30}, t) \right] \] 

\[ \frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} - \left[ (a_{32}^{(6)})^{(6)}(T_{32}, t) + (a_{29}^{(5,5,5)}(T_{29}, t) + (a_{24}^{(4,4,4)}(T_{25}, t) + (a_{14}^{(1,1,1,1)}(T_{14}, t) + (a_{19}^{(2,2,2,2,2)}(T_{17}, t) + (a_{22}^{(3,3,3,3)}(T_{21}, t) + (a_{17}^{(7,7,7,7,7)}(T_{37}, t) + (a_{41}^{(8,8,8,8,8)}(T_{41}, t) \right] G_{32} \]

\[ \frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \left[ (a_{33}^{(6)})^{(6)}(T_{33}, t) + (a_{29}^{(5,5,5)}(T_{29}, t) + (a_{24}^{(4,4,4)}(T_{25}, t) + (a_{14}^{(1,1,1,1)}(T_{14}, t) + (a_{19}^{(2,2,2,2,2)}(T_{17}, t) + (a_{22}^{(3,3,3,3)}(T_{21}, t) + (a_{17}^{(7,7,7,7,7)}(T_{37}, t) + (a_{41}^{(8,8,8,8,8)}(T_{41}, t) \right] G_{33} \]

\[ \frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \left[ (a_{34}^{(6)})^{(6)}(T_{34}, t) + (a_{30}^{(5,5,5)}(T_{30}, t) + (a_{26}^{(4,4,4)}(T_{26}, t) + (a_{16}^{(1,1,1,1)}(T_{16}, t) + (a_{19}^{(2,2,2,2)}(T_{19}, t) + (a_{22}^{(3,3,3,3)}(T_{22}, t) + (a_{17}^{(7,7,7,7,7)}(T_{37}, t) + (a_{42}^{(8,8,8,8,8)}(T_{42}, t) \right] G_{34} \]

\[ + (a_{32}^{(6)})(5,5,5)(T_{32}, t) + (a_{33}^{(6)})(5,5,5)(T_{33}, t) + (a_{34}^{(6)})(5,5,5)(T_{34}, t) \] are first augmentation coefficients for category 1, 2 and 3

\[ + (a_{30}^{(5,5,5)})(T_{30}, t) + (a_{26}^{(4,4,4)})(T_{26}, t) + (a_{16}^{(1,1,1,1)})(T_{16}, t) \] are second augmentation coefficients for category 1, 2 a,

\[ + (a_{14}^{(1,1,1,1)})(T_{14}, t) + (a_{19}^{(2,2,2,2)})(T_{19}, t) + (a_{22}^{(3,3,3,3)})(T_{22}, t) \] are third augmentation coefficients for category 1, 2 a

\[ + (a_{17}^{(7,7,7,7,7)})(T_{37}, t) + (a_{41}^{(8,8,8,8,8)})(T_{41}, t) \] - fourth augmentation coefficients

\[ + (a_{23}^{(2,2,2,2,2)})(T_{23}, t) + (a_{19}^{(2,2,2,2,2)})(T_{19}, t) + (a_{17}^{(7,7,7,7,7)})(T_{37}, t) \] - fifth augmentation coefficients

\[ + (a_{16}^{(1,1,1,1,1)})(T_{16}, t) + (a_{19}^{(2,2,2,2,2)})(T_{19}, t) + (a_{17}^{(7,7,7,7,7)})(T_{37}, t) \] sixth augmentation coefficients

\[ + (a_{36}^{(7,7,7,7,7)})(T_{37}, t) + (a_{36}^{(7,7,7,7,7)})(T_{37}, t) \] seventh augmentation coefficients

\[ + (a_{30}^{(6,6,6,6,6)}(T_{30}, t) + (a_{41}^{(8,8,8,8,8)})(T_{41}, t) \] are eight augmentation coefficients

\[ dT_{32} = (b_{32})^{(6)} T_{32} - \left[ (b_{32}^{(6)} - (b_{32}^{(6)})(G_{32}, t) - (b_{26}^{(5,5,5)}(G_{31}, t) - (b_{24}^{(4,4,4)})(G_{27}, t) \right] T_{32} \]
\[
\begin{align*}
\frac{dT_{33}}{dt} &= (b_{33})^{(6)}T_{33} \\
&= \begin{bmatrix}
(b_{33}^{(6)}) - (b_{33}^{(4,4,4)}(G_{31}, t)) - (b_{33}^{(5,5,5)}(G_{31}, t)) - (b_{25}^{(4,4,4)}(G_{27}, t)) \\
-(b_{14}^{(1,1,1,1,1,1)}(G, t)) - (b_{15}^{(2,2,2,2,2,2)}(G_{29}, t)) - (b_{22}^{(3,3,3,3,3,3)}(G_{23}, t)) \\
-(b_{3}^{(7,7,7,7,7,7,7)}(G_{39}, t)) - (b_{41}^{(8,8,8,8,8,8)}(G_{43}, t))
\end{bmatrix} T_{33}
\end{align*}
\]

\[
\begin{align*}
\frac{dT_{34}}{dt} &= (b_{34})^{(6)}T_{33} \\
&= \begin{bmatrix}
(b_{34}^{(6)}) - (b_{34}^{(4,4,4)}(G_{31}, t)) - (b_{34}^{(5,5,5)}(G_{31}, t)) - (b_{26}^{(4,4,4)}(G_{27}, t)) \\
-(b_{14}^{(1,1,1,1,1,1)}(G, t)) - (b_{15}^{(2,2,2,2,2,2)}(G_{29}, t)) - (b_{22}^{(3,3,3,3,3,3)}(G_{23}, t)) \\
-(b_{3}^{(7,7,7,7,7,7,7)}(G_{39}, t)) - (b_{41}^{(8,8,8,8,8,8)}(G_{43}, t))
\end{bmatrix} T_{33}
\end{align*}
\]

- \((b_{33})^{(6)}(G_{33}, t))\), \((b_{33})^{(2,2,2,2,2,2)}(G_{29}, t))\), \((b_{33})^{(3,3,3,3,3,3)}(G_{23}, t))\) are first detrition coefficients for category 1, 2 and 3
- \((b_{25})^{(5,5,5)}(G_{32}, t))\), \((b_{34})^{(4,4,4)}(G_{31}, t))\), \((b_{34}^{(3,3,3,3,3)}(G_{23}, t))\) are second detrition coefficients for category 1, 2 and 3
- \((b_{26})^{(4,4,4)}(G_{27}, t))\), \((b_{14}^{(1,1,1,1,1,1)}(G, t))\), \((b_{15}^{(2,2,2,2,2,2)}(G_{29}, t))\) are third detrition coefficients for category 1, 2 and 3
- \((b_{14}^{(1,1,1,1,1,1)}(G, t))\), \((b_{15}^{(2,2,2,2,2,2)}(G_{29}, t))\), \((b_{22}^{(3,3,3,3,3,3)}(G_{23}, t))\) are fourth detrition coefficients for category 1, 2, and 3
- \((b_{26})^{(5,5,5)}(G_{32}, t))\), \((b_{34})^{(4,4,4)}(G_{31}, t))\), \((b_{34}^{(3,3,3,3,3)}(G_{23}, t))\) are fifth detrition coefficients for category 1, 2, and 3
- \((b_{25})^{(4,4,4)}(G_{27}, t))\), \((b_{14}^{(1,1,1,1,1,1)}(G, t))\), \((b_{15}^{(2,2,2,2,2,2)}(G_{29}, t))\) are sixth detrition coefficients for category 1, 2, and 3
- \((b_{3}^{(7,7,7,7,7,7,7,7,7,7,7,7)}(G_{39}, t))\), \((b_{41}^{(8,8,8,8,8,8)}(G_{43}, t))\), \((b_{43}^{(8,8,8,8,8,8)}(G_{43}, t))\) are seventh detrition coefficients
- \((b_{43}^{(8,8,8,8,8,8)}(G_{43}, t))\), \((b_{43}^{(8,8,8,8,8,8)}(G_{43}, t))\), \((b_{43}^{(8,8,8,8,8,8)}(G_{43}, t))\) are eighth detrition coefficients.

**Seventh Module Concatenation:**

\[
\begin{align*}
\frac{dG_{36}}{dt} &= (a_{36}^{(7)})G_{37} - (a_{36}^{(7)})\left(a_{36}^{(7)}(T_{37}, t) + (a_{16}^{(7)}(T_{37}, t)) + (a_{26}^{(7)})(T_{21}, t) + \right. \\
&\left. (a_{24}^{(7)}(T_{29}, t) + (a_{20}^{(7)}(T_{29}, t) + (a_{34}^{(7)})(T_{33}, t) + \right. \\
&\left. (a_{13}^{(7)}(T_{14}, t) + (a_{44}^{(7)}(8,8,8,8,8,8)(T_{41}, t) \right) G_{36}
\end{align*}
\]

\[
\begin{align*}
\frac{dG_{37}}{dt} &= (a_{37}^{(7)})G_{36} - (a_{37}^{(7)}) + (a_{37}^{(7)}(T_{37}, t) + (a_{17}^{(7)}(T_{14}, t) + (a_{27}^{(7)})(T_{21}, t) + \right. \\
&\left. (a_{25}^{(7)}(T_{29}, t) + (a_{33}^{(7)})(T_{33}, t) + \right. \\
&\left. (a_{20}^{(7)})(8,8,8,8,8,8)(T_{41}, t) \right) G_{37}
\end{align*}
\]

\[
\begin{align*}
\frac{dG_{38}}{dt} &= (a_{38}^{(7)})G_{37} - \\
&\left(a_{38}^{(7)} + (a_{38}^{(7)}(T_{37}, t) + (a_{18}^{(7)}(T_{14}, t) + (a_{28}^{(7)})(T_{21}, t) + \right. \\
&\left. (a_{26}^{(7)})(T_{29}, t) + (a_{34}^{(7)})(T_{33}, t) + (a_{30}^{(7)})(T_{29}, t) + (a_{42}^{(7)})(8,8,8,8,8,8)(T_{41}, t) \right) G_{38}
\end{align*}
\]

\[
\begin{align*}
\frac{dT_{37}}{dt} &= (b_{37}^{(7)})T_{37} - (b_{36}^{(7)})) - (b_{36}^{(7)}))((G_{39}, t)) + (b_{49}^{(7)})(8,8,8,8,8,8) \right) - (b_{13}^{(7)}))/((G_{14}, t)) - \right)
\end{align*}
\]
\[
\begin{align*}
(b_{20}''(7) &\ (G_{231},t)) - (b_{24}''(7)\ (G_{27},t)) - (b_{28}''(7)\ (G_{31},t)) \\
(b_{22}''(7) &\ (G_{29},t)) \ T_{36} \\
\frac{df_{37}}{dt} &\ = (b_{23}''(7)\ T_{36}) - \\
(b_{20}''(7) &\ (G_{239},t)) - (b_{24}''(7)\ (G_{279},t)) - (b_{28}''(7)\ (G_{319},t)) \ - (b_{29}''(7)\ (G_{339},t)) - \\
(b_{21}''(7) &\ (G_{231},t)) - (b_{25}''(7)\ (G_{277},t)) - (b_{29}''(7)\ (G_{317},t)) - \\
(b_{23}''(7) &\ (G_{297},t)) \ T_{37} \\
\frac{df_{38}}{dt} &\ = (b_{30}''(7)\ T_{37}) - \\
(b_{10}''(7) &\ (G_{239},t)) - (b_{14}''(7)\ (G_{279},t)) - (b_{18}''(7)\ (G_{319},t)) - (b_{20}''(7)\ (G_{339},t)) - \\
(b_{12}''(7) &\ (G_{231},t)) - (b_{16}''(7)\ (G_{277},t)) - (b_{20}''(7)\ (G_{317},t)) - \\
(b_{34}''(7) &\ (G_{39},t)) \ T_{38} \\
\text{Eighth Module Concatenation:}
\end{align*}
\]

\[
\frac{dg_{40}}{dt} = \\
(a_{40}'(8))G_{41} - \\
\begin{bmatrix}
(a_{40}'(8)) + (a_{40}'(8)\ (T_{41},t)) + (a_{13}'(8))\ (T_{419},t) + (a_{16}'(8))\ (T_{417},t) + \\
(a_{20}''(7)\ (T_{23},t)) + (a_{24}''(8)\ (T_{23},t)) + (a_{28}''(8)\ (T_{29},t)) + (a_{32}''(8)\ (T_{33},t)) \\
(a_{30}''(8)\ (T_{37},t)) \\
\end{bmatrix}
G_{40}
\]

\[
\frac{dg_{41}}{dt} = (a_{41}'(8))G_{41} - \\
\begin{bmatrix}
(a_{41}'(8)) + (a_{41}'(8)\ (T_{41},t)) + (a_{13}'(8))\ (T_{419},t) + (a_{16}'(8))\ (T_{417},t) + \\
(a_{21}''(7)\ (T_{23},t)) + (a_{25}''(8)\ (T_{23},t)) + (a_{29}''(8)\ (T_{29},t)) + (a_{33}''(8)\ (T_{33},t)) \\
(a_{33}''(8)\ (T_{37},t)) \\
\end{bmatrix}
G_{41}
\]

\[
\frac{dg_{42}}{dt} = (a_{42}'(8))G_{42} - \\
\begin{bmatrix}
(a_{42}'(8)) + (a_{42}'(8)\ (T_{41},t)) + (a_{13}'(8))\ (T_{419},t) + (a_{16}'(8))\ (T_{417},t) + \\
(a_{22}''(7)\ (T_{23},t)) + (a_{26}''(8)\ (T_{23},t)) + (a_{30}''(8)\ (T_{29},t)) + (a_{34}''(8)\ (T_{33},t)) + \\
(a_{33}''(8)\ (T_{37},t)) \\
\end{bmatrix}
G_{42}
\]

\[
\frac{df_{40}}{dt} = (b_{40}'(8)\ T_{41}) - \\
\begin{bmatrix}
(b_{10}'(8)) - (b_{14}'(8)\ (G_{43},t)) - (b_{18}'(8)\ (G_{439},t)) - (b_{20}'(8)\ (G_{439},t)) - \\
(b_{24}'(8)\ (G_{27},t)) - (b_{28}'(8)\ (G_{31},t)) - (b_{32}'(8)\ (G_{339},t)) - \\
(b_{34}'(8)\ (G_{39},t)) \ T_{40} \\
\end{bmatrix}
\]

\[
\frac{df_{41}}{dt} = (b_{41}'(8)\ T_{41}) - \\
\begin{bmatrix}
(b_{41}'(8)) - (b_{41}'(8)\ (G_{43},t)) - (b_{41}'(8)\ (G_{439},t)) - (b_{43}'(8)\ (G_{439},t)) - \\
(b_{45}'(8)\ (G_{27},t)) - (b_{47}'(8)\ (G_{31},t)) - (b_{49}'(8)\ (G_{339},t)) - \\
(b_{47}'(8)\ (G_{39},t)) \ T_{41} \\
\end{bmatrix}
\]

\[
\frac{df_{42}}{dt} = (b_{42}'(8)\ T_{41}) - \\
\begin{bmatrix}
(b_{42}'(8)) - (b_{42}'(8)\ (G_{43},t)) - (b_{42}'(8)\ (G_{439},t)) - (b_{44}'(8)\ (G_{439},t)) - \\
(b_{46}'(8)\ (G_{27},t)) - (b_{48}'(8)\ (G_{31},t)) - (b_{49}'(8)\ (G_{339},t)) - \\
(b_{48}'(8)\ (G_{39},t)) \ T_{41} \\
\end{bmatrix}
\]

86
\[
\begin{align*}
\frac{b_{22}^{(g)}(G_{23}, t)}{b_{18}^{(g)}(G_{39}, t)} - \frac{b_{26}^{(g)}(G_{27}, t)}{b_{36}^{(g)}(G_{31}, t)} - \frac{b_{34}^{(g)}(G_{35}, t)}{b_{40}^{(g)}(G_{43}, t)}
\end{align*}
\]

\[T_{42}\]

\[+(a_{40}^{(g)}(T_{41}, t) = \text{First augmentation factor}\]

\[-(b_{40}^{(g)}(G_{43}, t) = \text{First detritions factor}\]

Where we suppose

(A) \((a_i)^{(1)}, (a_j)^{(1)}, (a_i')^{(1)}, (b_j)^{(1)}, (b_i')^{(1)}, (b_j')^{(1)} > 0,\]

\[i, j = 13, 14, 15\]

(B) The functions \((a_i')^{(1)}, (b_i')^{(1)}\) are positive continuous increasing and bounded.

**Definition of** \((p_i)^{(1)}, (r_i)^{(1)}:\)

\[(a_i')^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\bar{A}_{13})^{(1)}\]

\[(b_i')^{(1)}(G, t) \leq (r_i)^{(1)} \leq (\bar{B}_{13})^{(1)}\]

\[\lim_{T_{14} \to \infty} (a_i')^{(1)}(T_{14}, t) = (p_i)^{(1)}\]

\[\lim_{G \to \infty} (b_i')^{(1)}(G, t) = (r_i)^{(1)}\]

**Definition of** \((\bar{A}_{13})^{(1)}, (\bar{B}_{13})^{(1)}:\)

Where \((\bar{A}_{13})^{(1)}, (\bar{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}\) are positive constants and \(i = 13, 14, 15\)

They satisfy Lipschitz condition:

\[|(a_i')^{(1)}(T_{14}, t) - (a_j')^{(1)}(T_{14}, t)| \leq (\bar{k}_{13})^{(1)}|T_{14} - T_{14}'| e^{-(\bar{R}_{13})^{(1)}t}\]

\[|(b_i')^{(1)}(G, t) - (b_j')^{(1)}(G, T)| \leq (\bar{k}_{13})^{(1)}|G - G'| e^{-(\bar{R}_{13})^{(1)}t}\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i')^{(1)}(T_{14}, t)\)

and \((a_j')^{(1)}(T_{14}, t)\). \((T_{14}, t)\) And \((T_{14}, t)\) are points belonging to the interval \([\bar{k}_{13})^{(1)}, (\bar{R}_{13})^{(1)}]\).

It is to be noted that \((a_i')^{(1)}(T_{14}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\bar{M}_{13})^{(1)} = 1\) then the function \((a_i')^{(1)}(T_{14}, t)\), the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

**Definition of** \((\bar{M}_{13})^{(1)}, (\bar{k}_{13})^{(1)}:\)

(C) \((\bar{M}_{13})^{(1)}, (\bar{k}_{13})^{(1)}\), are positive constants

\[
\frac{(a_i)^{(1)}}{(M_{13})^{(1)}} \cdot \frac{(b_i)^{(1)}}{(M_{13})^{(1)}} < 1
\]

**Definition of** \((\bar{P}_{13})^{(1)}, (\bar{Q}_{13})^{(1)}:\)

(D) There exists two constants \((\bar{P}_{13})^{(1)}\) and \((\bar{Q}_{13})^{(1)}\) which together with \((\bar{M}_{13})^{(1)}, (\bar{k}_{13})^{(1)}, (\bar{A}_{13})^{(1)}\) and \((\bar{B}_{13})^{(1)}\) and the constants
(a_i^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15, \\

satisfy the inequalities

\[
\frac{1}{(M_{13})^3} [(a_i)^{(1)} + (a_i')^{(1)} + (A_{13})^{(1)} + (\dot{p}_{13})^{(1)} (\dot{k}_{13})^{(1)}] < 1
\]

\[
\frac{1}{(M_{13})^3} [(b_i)^{(1)} + (b_i')^{(1)} + (B_{13})^{(1)} + (\dot{q}_{13})^{(1)} (\dot{k}_{13})^{(1)}] < 1
\]

(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18

(E) The functions (a_i'')^{(2)}, (b_i'')^{(2)} are positive continuous increasing and bounded.

**Definition of** (p_i)^{(2)}, (r_i)^{(2)}:

(\dot{a}_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\dot{A}_{16})^{(2)}

(\dot{b}_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (\dot{B}_{16})^{(2)}

(F) \lim_{T_{17} \rightarrow \infty}(a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)}

\lim_{G_{19} \rightarrow \infty}(b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)}

**Definition of** (\dot{A}_{16})^{(2)}, (\dot{B}_{16})^{(2)}:

\[
\text{Where } (\dot{A}_{16})^{(2)}, (\dot{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)} \text{ are positive constants and } i = 16,17,18
\]

They satisfy Lipschitz condition:

\[
| (a_i'')^{(2)}(T_{17}, t) - (a_i'')^{(2)}(T_{17}, t) | \leq (\dot{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\dot{M}_{16})^{(2)}t}
\]

\[
| (b_i'')^{(2)}((G_{19}), t) - (b_i'')^{(2)}((G_{19}), t) | \leq (\dot{k}_{16})^{(2)}||G_{19} - (G_{19})'||e^{-(\dot{M}_{16})^{(2)}t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions (a_i'')^{(2)}(T_{17}, t) and (a_i'')^{(2)}(T_{17}, t) . (T_{17}, t) And (T_{17}, t) are points belonging to the interval \( [\dot{k}_{16}^{(2)}, (\dot{M}_{16})^{(2)}] \). It is to be noted that (a_i'')^{(2)}(T_{17}, t) is uniformly continuous. In the eventuality of the fact, that if (\dot{M}_{16})^{(2)} = 1 then the function (a_i'')^{(2)}(T_{17}, t), the SECOND augmentation coefficient would be absolutely continuous.

**Definition of** (\dot{M}_{16})^{(2)}, (\dot{k}_{16})^{(2)}:

\[
(\dot{M}_{16})^{(2)}, (\dot{k}_{16})^{(2)}, \text{ are positive constants}
\]

\[
\frac{(a_i)^{(2)}}{(\dot{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\dot{M}_{16})^{(2)}} < 1
\]

**Definition of** (\dot{P}_{13})^{(2)}, (\dot{Q}_{13})^{(2)}:

There exist two constants (\dot{P}_{13})^{(2)} and (\dot{Q}_{13})^{(2)} which together with (\dot{M}_{16})^{(2)}, (\dot{k}_{16})^{(2)}, (\dot{A}_{16})^{(2)} and (\dot{B}_{16})^{(2)} and the constants (a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18, \\

satisfy the inequalities

\[
\frac{1}{(\dot{M}_{16})^{(2)}} [(a_i)^{(2)} + (a_i')^{(2)} + (\dot{A}_{16})^{(2)} + (\dot{P}_{13})^{(2)} (\dot{k}_{16})^{(2)}] < 1
\]
\[
\frac{1}{(M_{16})^2} [ (b_i)_{16}^{(2)} + (b_j)^{(2)} + (\hat{B}_{16})_{16}^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)} ] < 1
\]

Where we suppose
\[
(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20, 21, 22
\]

The functions \((a_i')^{(3)}, (b_i')^{(3)}\) are positive continuous increasing and bounded.

**Definition of** \((p_i)^{(3)}, (r_i)^{(3)}\):
\[
(a_i'^{(3)}(T_{21}, t) ≤ (p_i)^{(3)} ≤ (\hat{A}_{20})^{(3)}
\]
\[
(b_i'^{(3)}(G_{23}, t) ≤ (r_i)^{(3)} ≤ (\hat{B}_{20})^{(3)}
\]
\[
\lim_{T_{21}→0}(a_i'^{(3)}(T_{21}, t) = (p_i)^{(3)}
\]
\[
\lim_{G_{23}→0}(b_i'^{(3)}(G_{23}, t) = (r_i)^{(3)}
\]

**Definition of** \((\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}\):

Where \((\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}\) are positive constants \(i = 20, 21, 22\)

They satisfy Lipschitz condition:
\[
| (a_i''^{(3)}(T_{21}', t) - (a_i''^{(3)}(T_{21}, t)) | ≤ (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}
\]
\[
| (b_i''^{(3)}(G_{23}', t) - (b_i''^{(3)}(G_{23}, t)) | ≤ (\hat{k}_{20})^{(3)}|G_{23} - G_{23}'|e^{-(\hat{M}_{20})^{(3)}t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i'^{(3)}(T_{21}, t)\) and \((b_i'^{(3)}(G_{23}, t)\) . \((T_{21}', t)\) and \((G_{23}, t)\) are points belonging to the interval \([T_{21}, (\hat{M}_{20})^{(3)}]\). It is to be noted that \((a_i'^{(3)}(T_{21}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{20})^{(3)} = 1\) then the function \((a_i'^{(3)}(T_{21}, t)\), the THIRD augmentation coefficient, would be absolutely continuous.

**Definition of** \((\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}\):
\[
(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}\), are positive constants
\]

There exists two constants \((\hat{M}_{20})^{(3)}(\hat{k}_{20})^{(3)}\) which together with \((\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}\), \((\hat{A}_{20})^{(3)}\), \((\hat{B}_{20})^{(3)}\) and the constants \((a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)}(p_i)^{(3)}, (r_i)^{(3)}\), \(i = 20, 21, 22\), satisfy the inequalities
\[
\frac{1}{(\hat{M}_{20})^{(3)}} [ (a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{B}_{20})^{(3)}(\hat{k}_{20})^{(3)} ] < 1
\]
\[
\frac{1}{(\hat{M}_{20})^{(3)}} [ (b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)} ] < 1
\]

Where we suppose
\[
(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24, 25, 26
\]

The functions \((a_i')^{(4)}, (b_i')^{(4)}\) are positive continuous increasing and bounded.
Definition of \((p_i)^{(4)}, (r_i)^{(4)}\):

\[
\begin{align*}
(a_i^{(4)}(T_{25}, t) &\leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)} \\
(b_i^{(4)}((G_{27}, t) &\leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)} \\
\lim_{T_{25} \rightarrow \infty} (a_i^{(4)}(T_{25}, t) &= (p_i)^{(4)} \\
\lim_{T_{25} \rightarrow \infty} (b_i^{(4)}((G_{27}, t) &= (r_i)^{(4)}
\end{align*}
\]

Definition of \((\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}\):

Where \((\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}\) are positive constants and \(i = 24, 25, 26\)

They satisfy Lipschitz condition:

\[
\begin{align*}
| (a_i^{(4)}(T_{25}, t) - (a_i')^{(4)}(T_{25}, t) | &\leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{B}_{24})^{(4)}t} \\
| (b_i^{(4)}((G_{27}, t) - (b_i')^{(4)}((G_{27}, t) | &\leq (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27}')||e^{-(\hat{B}_{24})^{(4)}t}
\end{align*}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i^{(4)}(T_{25}, t)\) and \((a_i')^{(4)}(T_{25}, t)\) . \((T_{25}, t)\) And \((T_{25}, t)\) are points belonging to the interval \([ (\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)} ]\). It is to be noted that \((a_i')^{(4)}(T_{25}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{24})^{(4)} = 4\) then the function \((a_i')^{(4)}(T_{25}, t)\), the FOURTH augmentation coefficient WOULD be absolutely continuous.

Definition of \((\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}\):

\((\hat{M}_{24})^{176^{(4)}}, (\hat{k}_{24})^{(4)}\), are positive constants

\[
\frac{(a_j)^{(4)}}{(M_{24})^{(4)}} \cdot \frac{(b_i)^{(4)}}{(M_{24})^{(4)}} < 1
\]

Definition of \((\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}\):

There exist two constants \((\hat{P}_{24})^{(4)}\) and \((\hat{Q}_{24})^{(4)}\) which together with \((\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)}\) and \((\hat{B}_{24})^{(4)}\) and the constants \((a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24, 25, 26\), satisfy the inequalities

\[
\begin{align*}
\frac{1}{(M_{24})^{(4)}} \left[ (a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)} \right] < 1 \\
\frac{1}{(M_{24})^{(4)}} \left[ (b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)} \right] < 1
\end{align*}
\]

Where we suppose

\[
(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28, 29, 30
\]

The functions \((a_i')^{(5)}, (b_i')^{(5)}\) are positive continuous increasing and bounded.

Definition of \((p_i)^{(5)}, (r_i)^{(5)}\):

\[
(a_i^{(5)}(T_{25}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}
\]
\[ (b_i^{(5)}(G_{31}, t) - (r_i^{(5)}(t) \leq (b_i^{(5)}(t) \leq (\mathcal{B}_{28})^{(5)}) \]

\[ \lim_{T_2 \to 0}(a_i^{(5)}(T_{29}, t) = (p_i)^{(5)} \]

\[ \lim_{G \to 0}(b_i^{(5)}(G_{31}, t) = (r_i)^{(5)} \]

**Definition of** \((\mathcal{A}_{28})^{(5)}, (\mathcal{B}_{28})^{(5)}\):

Where \((\mathcal{A}_{28})^{(5)}, (\mathcal{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}\) are positive constants and \(i = 28, 29, 30\)

They satisfy Lipschitz condition:

\[ |(a_i^{(5)}(T_{29}, t) - (a_i^{(5)}(T_{29}, t)| \leq (\mathcal{A}_{28})^{(5)}|T_{29} - T_{29}'| e^{-(\mathcal{A}_{28})^{(5)}t} \]

\[ |(b_i^{(5)}(G_{31}, t) - (b_i^{(5)}(G_{31}, t)| < (\mathcal{B}_{28})^{(5)}|(G_{31}) - (G_{31})'| e^{-(\mathcal{B}_{28})^{(5)}t} \]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i^{(5)}(T_{29}, t)\) and \((a_i^{(5)}(T_{29}, t)\). \((T_{29}, t)\) and \((T_{29}, t)\) are points belonging to the interval \([\mathcal{A}_{28}, (M_{28})^{(5)}\].

It is to be noted that \((a_i^{(5)}(T_{29}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\mathcal{M}_{28})^{(5)} = 5\) then the function \((a_i^{(5)}(T_{29}, t)\), the Fifth augmentation coefficient attributable would be absolutely continuous.

**Definition of** \((\mathcal{M}_{28})^{(5)}, (\mathcal{K}_{28})^{(5)}\):

\((\mathcal{M}_{28})^{(5)}, (\mathcal{K}_{28})^{(5)}, (\mathcal{A}_{28})^{(5)}, (\mathcal{B}_{28})^{(5)}\), and \((p_i)^{(5)}, (r_i)^{(5)}\) are positive constants with \((\mathcal{M}_{28})^{(5)}, (\mathcal{A}_{28})^{(5)} < 1\)

**Definition of** \((\mathcal{P}_{28})^{(5)}, (\mathcal{Q}_{28})^{(5)}\):

There exists two constants \((\mathcal{P}_{28})^{(5)}\) and \((\mathcal{Q}_{28})^{(5)}\) which together with \((\mathcal{M}_{28})^{(5)}\) \((\mathcal{A}_{28})^{(5)}\) \((\mathcal{B}_{28})^{(5)}\) and \((p_i)^{(5)}, (r_i)^{(5)}\) and the constants \((a_i)^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}\), \(i = 28, 29, 30\) satisfy the inequalities

\[ \frac{1}{(\mathcal{M}_{28})^{(5)}} [(a_i)^{(5)} + (a_i')^{(5)} + (\mathcal{A}_{28})^{(5)}(\mathcal{K}_{28})^{(5)}] < 1 \]

\[ \frac{1}{(\mathcal{M}_{28})^{(5)}} [(b_i')^{(5)} + (b_i')^{(5)} + (\mathcal{B}_{28})^{(5)}(\mathcal{K}_{28})^{(5)}] < 1 \]

Where we suppose

\[(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32, 33, 34 \]

The functions \((a_i')^{(6)}, (b_i')^{(6)}\) are positive continuous increasing and bounded.

**Definition of** \((p_i)^{(6)}, (r_i)^{(6)}\):

\[(a_i')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\mathcal{A}_{32})^{(6)} \]

\[(b_i')^{(6)}(G_{33}, t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\mathcal{B}_{32})^{(6)} \]

\[ \lim_{T_2 \to 0}(a_i^{(6)}(T_{33}, t) = (p_i)^{(6)} \]

\[ \lim_{G \to 0}(b_i^{(6)}(G_{33}, t) = (r_i)^{(6)} \]
Definition of $(\hat{A}_{32})^{(6)}$, $(\hat{B}_{32})^{(6)}$:

Where $(\hat{A}_{32})^{(6)}$, $(\hat{B}_{32})^{(6)}$, $(p_i)^{(6)}$, $(r_i)^{(6)}$ are positive constants and $\ell = 32,33,34$

They satisfy Lipschitz condition:

$$(a_i''(T'_{33}, t) - (a_i'')^{(6)}(T'_{33}, t)) \leq (\hat{k}_{32})^{(6)}|T'_{33} - T'_{33}|e^{-(\hat{A}_{32})^{(6)}t}$$

$$(b_i''(G_{33}), t) - (b_i'')^{(6)}(G_{33}), t) |(\hat{k}_{32})^{(6)}|(|G_{33}) - (G_{33}')|e^{-(\hat{B}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T'_{33}, t)$ and $(a_i''(T'_{33}, t)$, $(T'_{33}, t)$, $(T'_{33}, t)$, and $(T'_{33}, t)$ are points belonging to the interval $[\hat{k}_{32}]^{(6)}$, $[\hat{B}_{32}]^{(6)}$. It is to be noted that $(a_i'')^{(6)}(T'_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 6$ then the function $(a_i'')^{(6)}(T'_{33}, t)$, the SIXTH augmentation coefficient would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}$, $(\hat{K}_{32})^{(6)}$:

$(\hat{M}_{32})^{(6)}$, $(\hat{K}_{32})^{(6)}$, are positive constants

$$(\frac{a_i^{(6)}}{\ell^{(6)}}), (\frac{b_i^{(6)}}{\ell^{(6)}}) < 1$$

Definition of $(\hat{P}_{32})^{(6)}$, $(\hat{Q}_{32})^{(6)}$:

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}$, $(\hat{K}_{32})^{(6)}$, $(\hat{A}_{32})^{(6)}$, $(\hat{B}_{32})^{(6)}$, and the constants $(a_i)^{(6)}$, $(a_j)^{(6)}$, $(b_i)^{(6)}$, $(b_j)^{(6)}$, $(p_i)^{(6)}$, $(r_i)^{(6)}$, $i = 32,33,34$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[a_i^{(6)} + a_j^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{B}_{32})^{(6)}(\hat{K}_{32}]^{(6)} < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[b_i^{(6)} + b_j^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{K}_{32}]^{(6)} < 1$$

Where we suppose

$$(a_i)^{(7)}, (a_j)^{(7)}, (a'_i)^{(7)}, (a'_j)^{(7)}, (b_i)^{(7)}, (b_j)^{(7)}, (b'_i)^{(7)}, (b'_j)^{(7)} > 0, \quad i, j = 36,37,38$$

The functions $(a_i'')^{(7)}$, $(b_i'')^{(7)}$ are positive continuous increasing and bounded. Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}, \quad (b_i'^{(7)}(G, t) \leq (r_i)^{(7)} \leq (b_i^{(7)} \leq (\hat{B}_{36})^{(7)}$$

$$\lim_{t_2 \to \infty} (a_i'')(T_{37}, t) = (p_i)^{(7)}$$

$$\lim_{G \to 0} (b_i'')(G_{39}, t) = (r_i)^{(7)}$$

Definition of $(\hat{A}_{36})^{(7)}$, $(\hat{B}_{36})^{(7)}$:

Where $(\hat{A}_{36})^{(7)}$, $(\hat{B}_{36})^{(7)}$, $(p_i)^{(7)}$, $(r_i)^{(7)}$ are positive constants and $\ell = 36,37,38$

They satisfy Lipschitz condition:
With the Lipschitz condition, we place a restriction on the behavior of functions (a_i(7)(T_{37}, t)) and (a_i(5)(T_{37}, t)) . (T_{37}, t) And (T_{37}, t) are points belonging to the interval [(\hat{k}_{36}(7), (\hat{M}_{36})^{(7)})]. It is to be noted that (a_i(7)(T_{37}, t)) is uniformly continuous. In the eventuality of the fact, that if (\hat{M}_{36}(7) = 7) then the function (a_i(5)(T_{37}, t)) the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

**Definition of** (\hat{M}_{36}(7), (\hat{k}_{36})^{(7)}):

\[ \frac{(a_1(7))}{(M_{36}(7))} \cdot \frac{(b_1(7))}{(M_{36}(7))} < 1 \]

**Definition of** (\hat{p}_{36}(7), (\hat{q}_{36})^{(7)}):

There exists two constants (\hat{p}_{36}(7)) and (\hat{q}_{36}(7)) which together with (\hat{M}_{36}(7), (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)} and the constants (a_i(7)), (a_1(5), (b_1(7), (b_1(5)), (p_i(7), (r_i(7)), i = 36, 37, 38, satisfy the inequalities

\[ \frac{1}{(M_{36}(7))} [(a_1(7)) + (a_1(5)) + (\hat{A}_{36}(7)) + (\hat{p}_{36}(7)) (\hat{k}_{36}(7))] < 1 \]

\[ \frac{1}{(M_{36}(7))} [(b_1(7)) + (b_1(5)) + (\hat{B}_{36}(7)) + (\hat{q}_{36}(7)) (\hat{k}_{36}(7))] < 1 \]

**Definition of** G_i(0), T_i(0):

\[ G_i(t) \leq (\hat{p}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad G_i(0) = G_i^0 > 0 \]

\[ T_i(t) \leq (\hat{q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad T_i(0) = T_i^0 > 0 \]

Where we suppose

\[ (a_1(6), (a_1(5), (a_1(4)), (b_1(8)), (b_1(5)), (b_1(4)), (b_1(0)), (b_1(0)) > 0, \]

i,j = 40, 41, 42

**Definition of** (p_i(6), (r_i(6)):

\[ (a_1(5))(T_{41}, t) \leq (p_i(6)) \leq (\hat{A}_{40})^{(8)} \]

\[ (b_1(5))(G_{43}, t) \leq (r_i(0)) \leq (b_1(5))(G_{43}, t) \leq (\hat{B}_{40})^{(8)} \]

\[ \lim_{t_{2 \to \infty}} (a_1(5))(T_{41}, t) = (p_i(6)) \]

\[ \lim_{G_{\to \infty}} (b_1(5))(G_{43}, t) = (r_i(0)) \]

**Definition of** (\hat{A}_{40}(8), (\hat{B}_{40})^{(8)}):
Where \( \left( \hat{A}_{40} \right)^{(8)}, \left( \hat{B}_{40} \right)^{(8)}, \left( p_i \right)^{(8)}, \left( r_i \right)^{(8)} \) are positive constants and \( I = 40, 41, 42 \)

They satisfy Lipschitz condition:

\[
\left| (a''_i)^{(8)}(T_{41}, t) - (a''_i)^{(8)}(T_{41}, t) \right| \leq (\hat{k}_{40})^{(8)}|T_{41} - T_{41i}|e^{-\left( \hat{M}_{40} \right)^{(8)}t} \\
\left| (b''_i)^{(8)}(G_{43}, t') - (b''_i)^{(8)}(G_{43}, t') \right| < (\hat{k}_{40})^{(8)}\left| (G_{43}) - (G_{43})' \right| e^{-\left( \hat{M}_{40} \right)^{(8)}t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a''_i)^{(8)}(T_{41}, t)\) and \((a''_i)^{(8)}(T_{41}, t) \cdot (T_{41}, t)\) And \((T_{41}, t)\) are points belonging to the interval \([ (\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)} ] \).

It is to be noted that \((a''_i)^{(8)}(T_{41}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{40})^{(8)} = 8\) then the function \((a''_i)^{(8)}(T_{41}, t)\), the EIGHT augmentation coefficient would be absolutely continuous.

**Definition of** \((\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}\):

\((N)\) \((\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}\), are positive constants

\[
\left( \frac{(a_i)}{(M_{40})} \right)^{(8)}, \left( \frac{(b_i)}{(M_{40})} \right)^{(8)} < 1
\]

\[
\hat{G}_{15}(t) = G_{15} + \int_0^t \left[ \left( a_{15}^{(1)} \right) G_{14}(s_{(13)}) - \left( (a_{15}^{(1)}) + (a_{15}^{(1)}) (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}
\]

By

\[
\hat{G}_{14}(t) = G_{14} + \int_0^t \left[ \left( a_{14}^{(1)} \right) G_{14}(s_{(13)}) - \left( (a_{14}^{(1)}) + (a_{14}^{(1)}) (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}
\]

\[
\hat{T}_{13}(t) = T_{13} + \int_0^t \left[ \left( b_{13}^{(1)} \right) T_{13}(s_{(13)}) - \left( (b_{13}^{(1)}) - (b_{13}^{(1)}) (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}
\]

\[
\hat{T}_{14}(t) = T_{14} + \int_0^t \left[ \left( b_{14}^{(1)} \right) T_{14}(s_{(13)}) - \left( (b_{14}^{(1)}) - (b_{14}^{(1)}) (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}
\]

Where \(s_{(13)}\) the integrand that is integrated over an interval \((0, t)\)

if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

**Definition of** \(G_i(0), T_i(0)\):

\[
G_i(t) \leq \left( \hat{p}_{36} \right)^{(7)} e^{\left( \hat{M}_{36} \right)^{(7)} t}, \quad G_i(0) = G_i^0 > 0
\]

\[
T_i(t) \leq \left( \hat{q}_{36} \right)^{(7)} e^{\left( \hat{M}_{36} \right)^{(7)} t}, \quad T_i(0) = T_i^0 > 0
\]

Consider operator \(\mathcal{A}^{(7)}\) defined on the space of sextuples of continuous functions \(G_i, T_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+\) which satisfy

\[
G_i(0) = G_i^0, T_i(0) = T_i^0, G_i, T_i \leq \left( \hat{q}_{36} \right)^{(7)}, T_i \leq \left( \hat{q}_{36} \right)^{(7)}
\]
0 ≤ G_i(t) - G_i^0 ≤ (\tilde{P}_{16})^{(2)} e^{(\tilde{R}_{16})^{(2)} t}
0 ≤ T_i(t) - T_i^0 ≤ (\tilde{Q}_{16})^{(2)} e^{(\tilde{R}_{16})^{(2)} t}

By
\[ \tilde{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{16}) - \left( (a'_{16})^{(2)} + a''_{16} \right) (T_{17}(s_{16}), s_{16}) \right] G_{16}(s_{16}) \right] ds_{16} \]
\[ \tilde{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{16}) - \left( (a'_{17})^{(2)} + a''_{17} \right) (T_{16}(s_{16}), s_{16}) \right] G_{17}(s_{16}) \right] ds_{16} \]
\[ \tilde{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{16}) - \left( (a'_{18})^{(2)} + a''_{18} \right) (T_{17}(s_{16}), s_{16}) \right] G_{18}(s_{16}) \right] ds_{16} \]
\[ \tilde{T}_{16}(t) = T_{16}^0 + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{16}) - \left( (b'_{16})^{(2)} + b''_{16} \right) (G(s_{16}), s_{16}) \right] T_{16}(s_{16}) \right] ds_{16} \]
\[ \tilde{T}_{17}(t) = T_{17}^0 + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{16}) - \left( (b'_{17})^{(2)} + b''_{17} \right) (G(s_{16}), s_{16}) \right] T_{17}(s_{16}) \right] ds_{16} \]
\[ \tilde{T}_{18}(t) = T_{18}^0 + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{16}) - \left( (b'_{18})^{(2)} + b''_{18} \right) (G(s_{16}), s_{16}) \right] T_{18}(s_{16}) \right] ds_{16} \]

Where \( s_{16} \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( \mathcal{A}^{(2)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy
\[ G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\tilde{P}_{16})^{(2)}, T_i^0 \leq (\tilde{Q}_{16})^{(2)}, \]
0 ≤ G_i(t) - G_i^0 ≤ (\tilde{P}_{16})^{(2)} e^{(\tilde{R}_{16})^{(2)} t}
0 ≤ T_i(t) - T_i^0 ≤ (\tilde{Q}_{16})^{(2)} e^{(\tilde{R}_{16})^{(2)} t}

By
\[ \bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{16}) - \left( (a'_{16})^{(2)} + a''_{16} \right) (T_{17}(s_{16}), s_{16}) \right] G_{16}(s_{16}) \right] ds_{16} \]
\[ \bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{16}) - \left( (a'_{17})^{(2)} + a''_{17} \right) (T_{16}(s_{16}), s_{16}) \right] G_{17}(s_{16}) \right] ds_{16} \]
\[ \bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{16}) - \left( (a'_{18})^{(2)} + a''_{18} \right) (T_{17}(s_{16}), s_{16}) \right] G_{18}(s_{16}) \right] ds_{16} \]
\[ \bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{16}) - \left( (b'_{16})^{(2)} + b''_{16} \right) (G(s_{16}), s_{16}) \right] T_{16}(s_{16}) \right] ds_{16} \]
\[ \bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{16}) - \left( (b'_{17})^{(2)} + b''_{17} \right) (G(s_{16}), s_{16}) \right] T_{17}(s_{16}) \right] ds_{16} \]
\[ \bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{16}) - \left( (b'_{18})^{(2)} + b''_{18} \right) (G(s_{16}), s_{16}) \right] T_{18}(s_{16}) \right] ds_{16} \]

Where \( s_{16} \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( \mathcal{A}^{(3)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy
\[ G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\tilde{P}_{20})^{(3)}, T_i^0 \leq (\tilde{Q}_{20})^{(3)}, \]
0 ≤ G_i(t) - G_i^0 ≤ (\tilde{P}_{20})^{(3)} e^{(\tilde{R}_{20})^{(3)} t}
0 ≤ T_i(t) - T_i^0 ≤ (\tilde{Q}_{20})^{(3)} e^{(\tilde{R}_{20})^{(3)} t}
Consider operator \( A^{(4)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[
G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad 0 \leq (\hat{Q}_{24})^{(4)}_i, T_i^0 \leq (\hat{Q}_{24})^{(4)}_i,
\]

\[
0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)}_i e^{(\theta_{24})^{(4)}_i t},
\]

\[
0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)}_i e^{(\theta_{24})^{(4)}_i t}
\]

By

\[
\tilde{G}_{20}(t) = G_{20}^0 + \int_0^t \left[ \left( a_{20}^{(3)} G_{21}(s_{20}) \right) - \left( (a_{21}')^{(3)} + (a_{21}'')^{(3)} (T_{21}(s_{21}), s_{21}) \right) G_{20}(s_{20}) \right] ds_{20}
\]

\[
\tilde{G}_{21}(t) = G_{21}^0 + \int_0^t \left[ \left( a_{21}^{(3)} G_{20}(s_{20}) \right) - \left( (a_{21}')^{(3)} + (a_{21}'')^{(3)} (T_{21}(s_{21}), s_{21}) \right) G_{21}(s_{21}) \right] ds_{20}
\]

\[
\tilde{G}_{22}(t) = G_{22}^0 + \int_0^t \left[ \left( a_{22}^{(3)} G_{21}(s_{21}) \right) - \left( (a_{22}')^{(3)} + (a_{22}'')^{(3)} (T_{22}(s_{22}), s_{22}) \right) G_{22}(s_{22}) \right] ds_{22}
\]

\[
\tilde{T}_{20}(t) = T_{20}^0 + \int_0^t \left[ \left( b_{20}^{(3)} T_{21}(s_{20}) \right) - \left( (b_{20}')^{(3)} - (b_{20}'')^{(3)} (G(s_{20}), s_{20}) \right) T_{20}(s_{20}) \right] ds_{20}
\]

\[
\tilde{T}_{21}(t) = T_{21}^0 + \int_0^t \left[ \left( b_{21}^{(3)} T_{20}(s_{20}) \right) - \left( (b_{21}')^{(3)} - (b_{21}'')^{(3)} (G(s_{20}), s_{20}) \right) T_{21}(s_{21}) \right] ds_{20}
\]

\[
\tilde{T}_{22}(t) = T_{22}^0 + \int_0^t \left[ \left( b_{22}^{(3)} T_{21}(s_{21}) \right) - \left( (b_{22}')^{(3)} - (b_{22}'')^{(3)} (G(s_{22}), s_{22}) \right) T_{22}(s_{22}) \right] ds_{22}
\]

Where \( s_i(20) \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( A^{(5)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[
G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad 0 \leq (\hat{P}_{28})^{(5)}_i, T_i^0 \leq (\hat{Q}_{28})^{(5)}_i,
\]

\[
0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)}_i e^{(\theta_{28})^{(5)}_i t}
\]
0 ≤ T_i(t) − T_i^0 ≤ (\hat{Q}_{28})^{(5)} e^{i(\hat{\theta}_{28})^{(5)}t} \tag{195}

By

\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)}G_{28}(s_{28}) - \left( (a_{28}')^{(5)} + a_{28}''^{(5)} \right)(T_{28}(s_{28}), s_{28}) \right] ds_{28} \tag{196}

\bar{g}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28}')^{(5)}G_{28}(s_{28}) - \left( (a_{28}'')^{(5)} \right)(T_{28}(s_{28}), s_{28}) \right] ds_{28} \tag{197}

\bar{g}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30}')^{(5)}G_{30}(s_{30}) - \left( (a_{30}'')^{(5)} \right)(T_{30}(s_{30}), s_{30}) \right] ds_{30} \tag{198}

\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[ (b_{28})^{(5)}T_{28}(s_{28}) - \left( (b_{28}')^{(5)} - (b_{28}'')^{(5)} \right)(G(s_{28}), s_{28}) \right] ds_{28} \tag{199}

\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[ (b_{30})^{(5)}T_{30}(s_{30}) - \left( (b_{30}')^{(5)} - (b_{30}'')^{(5)} \right)(G(s_{30}), s_{30}) \right] ds_{30} \tag{200}

\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[ (b_{30})^{(5)}T_{30}(s_{30}) - \left( (b_{30}')^{(5)} - (b_{30}'')^{(5)} \right)(G(s_{30}), s_{30}) \right] ds_{30} \tag{201}

Where s_{28} is the integrand that is integrated over an interval (0, t)

Consider operator \mathcal{A}^{(6)} defined on the space of sextuples of continuous functions G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+ which satisfy

G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 ≤ (\hat{P}_{32})^{(6)}, T_i^0 ≤ (\hat{Q}_{32})^{(6)}, \tag{202}

0 ≤ G_i(t) − G_i^0 ≤ (\hat{P}_{32})^{(6)} e^{i(\hat{\theta}_{32})^{(6)}t} \tag{203}

0 ≤ T_i(t) − T_i^0 ≤ (\hat{Q}_{32})^{(6)} e^{i(\hat{\theta}_{32})^{(6)}t} \tag{204}

By

\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)}G_{32}(s_{32}) - \left( (a_{32}')^{(6)} + a_{32}''^{(6)} \right)(T_{32}(s_{32}), s_{32}) \right] ds_{32} \tag{205}

\bar{g}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)}G_{33}(s_{33}) - \left( (a_{33}')^{(6)} + a_{33}''^{(6)} \right)(T_{33}(s_{33}), s_{33}) \right] ds_{33} \tag{206}

\bar{g}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)}G_{34}(s_{34}) - \left( (a_{34}')^{(6)} + a_{34}''^{(6)} \right)(T_{34}(s_{34}), s_{34}) \right] ds_{34} \tag{207}

\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)}T_{32}(s_{32}) - \left( (b_{32}')^{(6)} - (b_{32}'')^{(6)} \right)(G(s_{32}), s_{32}) \right] ds_{32} \tag{208}

\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)}T_{33}(s_{33}) - \left( (b_{33}')^{(6)} - (b_{33}'')^{(6)} \right)(G(s_{33}), s_{33}) \right] ds_{33} \tag{209}

\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)}T_{34}(s_{34}) - \left( (b_{34}')^{(6)} - (b_{34}'')^{(6)} \right)(G(s_{34}), s_{34}) \right] ds_{34} \tag{210}

Where s_{32} is the integrand that is integrated over an interval (0, t)
if the conditions IN THE FOREGOING are fulfilled, there exists a solution satisfying the conditions

**Definition of** \( G_1(0), T_1(0) : \)

\[
G_1(t) \leq \left( \hat{p}_{36} \right)^{(7)} e^{(A_{36})^{(7)} t} , \quad G_1(0) = G_1^0 > 0
\]

\[
T_1(t) \leq \left( \hat{q}_{36} \right)^{(7)} e^{(A_{36})^{(7)} t} , \quad T_1(0) = T_1^0 > 0
\]

**Proof:**

Consider operator \( \mathcal{A}^{(7)} \) defined on the space of sextuples of continuous functions \( G_1, T_1 \colon \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[
G_1(0) = G_1^0 , T_1(0) = T_1^0 \leq \left( \hat{p}_{36} \right)^{(7)} e^{(A_{36})^{(7)} t}
\]

\[
0 \leq G_1(t) - G_1^0 \leq \left( \hat{p}_{36} \right)^{(7)} e^{(A_{36})^{(7)} t}
\]

\[
0 \leq T_1(t) - T_1^0 \leq \left( \hat{q}_{36} \right)^{(7)} e^{(A_{36})^{(7)} t}
\]

By

\[
\bar{G}_{36}(t) = G_3^0 + \int_0^t \left[ \left( a_{36}^{(1)} (a_{36}^{(1)} - a_{36}^{(2)}) (T_{37}(s_{36}), s_{36}) \right) G_{36}(s_{36}) \right] ds_{36}
\]

\[
\bar{G}_{37}(t) = G_3^0 + \int_0^t \left[ \left( a_{37}^{(1)} (a_{37}^{(1)} - a_{37}^{(2)}) (T_{37}(s_{36}), s_{36}) \right) G_{37}(s_{36}) \right] ds_{36}
\]

\[
\bar{G}_{38}(t) = G_3^0 + \int_0^t \left[ \left( a_{38}^{(1)} (a_{38}^{(1)} - a_{38}^{(2)}) (T_{38}(s_{36}), s_{36}) \right) G_{38}(s_{36}) \right] ds_{36}
\]

Where \( s_{36} \) is the integrand that is integrated over an interval \((0, t)\)

if the conditions above are fulfilled, there exists a solution satisfying the conditions

**Definition of** \( G_1(0), T_1(0) : \)

\[
G_1(t) \leq \left( \hat{p}_{40} \right)^{(8)} e^{(A_{40})^{(8)} t} , \quad G_1(0) = G_1^0 > 0
\]

\[
T_1(t) \leq \left( \hat{q}_{40} \right)^{(8)} e^{(A_{40})^{(8)} t} , \quad T_1(0) = T_1^0 > 0
\]

**Proof:**

Consider operator \( \mathcal{A}^{(8)} \) defined on the space of sextuples of continuous functions \( G_1, T_1 \colon \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy
\[
G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\hat{P}_{40})^{(8)}, \quad T_i^0 \leq (\hat{Q}_{40})^{(8)}, \quad 0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\hat{Q}_{40})^{(8)} t} \quad 0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}
\]

By

\[
\tilde{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{40}) - \left( (a_{40})^{(8)} + a_{40}^{(8)} \right) (T_{41}(s_{40}), s_{40}) \right] G_{40}(s_{40}) ds_{40}
\]

\[
\tilde{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{40}) - \left( (a_{41})^{(8)} + a_{41}^{(8)} \right) (T_{41}(s_{40}), s_{40}) \right] G_{41}(s_{40}) ds_{40}
\]

\[
\tilde{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{40}) - \left( (a_{42})^{(8)} + a_{42}^{(8)} \right) (T_{41}(s_{40}), s_{40}) \right] G_{42}(s_{40}) ds_{40}
\]

\[
\tilde{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{40}) - \left( (b_{40})^{(8)} - (b_{40})^{(8)(8)} G(s_{40}), s_{40}) \right) T_{40}(s_{40}) \right] ds_{40}
\]

\[
\tilde{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{40}) - \left( (b_{41})^{(8)} - (b_{41})^{(8)(8)} G(s_{40}), s_{40}) \right) T_{41}(s_{40}) \right] ds_{40}
\]

\[
\tilde{T}_{42}(t) = T_{42}^0 + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{40}) - \left( (b_{42})^{(8)} - (b_{42})^{(8)(8)} G(s_{40}), s_{40}) \right) T_{42}(s_{40}) \right] ds_{40}
\]

Where \(s_{40}\) is the integrand that is integrated over an interval \((0, t)\)

The operator \(\mathcal{A}^{(8)}\) maps the space of functions satisfying global equations into itself. Indeed it is obvious that

\[
G_{40}(t) \leq G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} \left( G_{41}^0 + (\hat{P}_{40})^{(8)(8)} e^{(\hat{M}_{40})^{(8)}} \right) \right] ds_{40} =
\]

\[
\left( 1 + (a_{40})^{(8)} t \right) G_{41}^0 + \frac{(a_{40})^{(8)}(P_{40})^{(8)}}{(M_{40})^{(8)}} \left( e^{(\hat{M}_{40})^{(8)} t} - 1 \right)
\]

From which it follows that

\[
(G_{40}(t) - G_{40}^0) e^{-(\hat{Q}_{40})^{(8)} t} \leq \frac{(a_{40})^{(8)}}{(M_{40})^{(8)}} \left( (\hat{P}_{40})^{(8)} + G_{41}^0 \right) e^{\left( -\frac{(P_{40})^{(8)} + G_{41}^0}{G_{41}^0} \right) t} + (\hat{P}_{40})^{(8)}
\]

\((G_{40}^0)\) is as defined in the statement of theorem 1

In order that the operator \(\mathcal{A}^{(1)}\) transforms the space of sextuples of functions \(G_i, T_i\) satisfying GLOBAL EQUATIONS into itself

The operator \(\mathcal{A}^{(1)}\) is a contraction with respect to the metric

\[
d \left( \left( G^{(1)}, T^{(1)} \right), \left( G^{(2)}, T^{(2)} \right) \right) =
\]

\[
\sup \left\{ \max_{i \in B_n} \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-\left( \hat{M}_{13}^{(1)} \right) t}, \max_{i \in B_n} \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-\left( \hat{M}_{13}^{(1)} \right) t} \right\}
\]

Indeed if we denote

**Definition of \( \tilde{G}, \tilde{T} : (\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T) \)

It results
\[ \left| G_{13}^{(1)} - G_{1}^{(2)} \right| \leq \int_0^1 (a_{13}^{(1)}') |G_{14}^{(1)} - G_{14}^{(2)}| e^{-(R_{13}^{(1)})x_{(13)}} e^{(R_{13}^{(1)})x_{(13)}} dS_{(13)} + \\
\int_0^1 ((a_{13}^{(1)})'G_{13}^{(1)} - G_{13}^{(2)}) e^{-(R_{13}^{(1)})x_{(13)}} e^{(R_{13}^{(1)})x_{(13)}} + \\
(a_{13}''^{(1)}(T_{14}^{(1)}, s_{(13)})) G_{13}^{(1)} - G_{13}^{(2)}| e^{-(R_{13}^{(1)})x_{(13)}} e^{(R_{13}^{(1)})x_{(13)}} + \\
G_{13}^{(2)} ((a_{13}''^{(1)}(T_{14}^{(1)}, s_{(13)}) - (a_{13}''^{(1)}(T_{14}^{(1)}, s_{(13)}))) | e^{-(R_{13}^{(1)})x_{(13)}} e^{(R_{13}^{(1)})x_{(13)}}) dS_{(13)} \]

Where \( s_{(13)} \) represents integrand that is integrated over the interval \([0, t] \)

From the hypotheses it follows

\[ \frac{1}{(R_{13}^{(1)})} \left( (a_{13}^{(1)}) + (a_{13}^{(1)})' + (A_{13}^{(1)}) + (P_{13}^{(1)}) (K_{13}^{(1)}) \right) d \left( (G^{(1)}, T^{(1)}); (G^{(2)}, T^{(2)}) \right) \]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{13}''^{(1)}(1) and (b_{13}''^{(1)}(1) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((P_{13}^{(1)}) e^{(R_{13}^{(1)})t} and (Q_{13}^{(1)}) e^{(R_{13}^{(1)})t} \) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \((a_{13}''^{(1)}(1) and (b_{13}''^{(1)}(1), i = 13, 14, 15 depend only on \( T_{14}^{(1)} and respectively on \( G(\text{and not on} \ t) \) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_i(t) = 0 \) and \( T_i(t) = 0 \)

From 19 to 24 it results

\[ G_i(t) \geq G_i^0 \left[ -G_i((a_{13}'^{(1)} - (a_{13}'^{(1)}(T_{14}(s_{(13)}), x_{(13)})) dS_{(13)} \right] \geq 0 \]

\[ T_i(t) \geq T_i^0 e^{(b_{13}'^{(1)})} > 0 \text{ for } t > 0 \]

**Definition of** \((\overline{M}_{13}^{(1)})_1^t\) and \((\overline{M}_{13}^{(1)})_3^t\):

**Remark 3:** if \( G_{13} \) is bounded, the same property have also \( G_{14} and G_{15} \). Indeed if

\[ G_{13} < (\overline{M}_{13}^{(1)}) \text{ it follows } \frac{dG_{16}}{dt} \leq (\overline{M}_{13}^{(1)}) - (a_{14}^{(1)}) G_{14} and by integrating

\[ G_{14} \leq (\overline{M}_{13}^{(1)})_2 = G_{14}^0 + 2(a_{14})^{(1)}((\overline{M}_{13}^{(1)})_1/(a_{14})^{(1)} \]

In the same way, one can obtain

\[ G_{15} \leq (\overline{M}_{13}^{(1)})_3 = G_{15}^0 + 2(a_{15})^{(1)}((\overline{M}_{13}^{(1)})_2/(a_{15})^{(1)} \]

If \( G_{14} or G_{15} \) is bounded, the same property follows for \( G_{13}, G_{15} \) and \( G_{13}, G_{14} \) respectively.

**Remark 4:** If \( G_{13} \) is bounded, from below, the same property holds for \( G_{14} and G_{15} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{14} \) is bounded from below.

**Remark 5:** If \( T_{13} \) is bounded from below and \( \lim_{t \to \infty}((b_{13}''^{(1)}(G(t), t)) = (b_{14}''^{(1)} \) then \( T_{14} \to \infty. \)

**Definition of** \((m_i)^{(1)}\) and \( \epsilon_i \):
Indeed let \( t_1 \) be so that for

\[
(b_{14}(t), t) < \varepsilon_1 \quad T_{13}(t) > (m)^{(1)}
\]

Then \( \frac{dT_{14}}{dt} \geq (a_{14}(t))(m)^{(1)} - \varepsilon_1 T_{14} \) which leads to

\[
T_{14} \geq \left( (a_{14})^{(1)(2)} \right) \left( \frac{1}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}
\]

If we take \( t \) such that \( e^{-\varepsilon_1 t} = \frac{1}{2} \) it results

\[
T_{14} \geq \frac{(a_{14})^{(1)(2)}(m)^{(1)} - \varepsilon_1 T_{14}}{2}
\]

By taking now \( \varepsilon_1 \) sufficiently small one sees that \( T_{14} \) is unbounded. The same property holds for \( T_{15} \) if \( \lim_{t \to 0} (b_{15}^{(1)(2)}(G(t), t) = (b_{15}^{(1)})^{(1)} \)

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take

\[
\frac{a_{14}^{(2)}}{(M_{16})^{(2)}} \leq (b_{16}^{(2)})^{(2)} < 1 \quad \text{and to choose}
\]

\[
(P_{16})^{(2)} \quad \text{and} \quad (\tilde{Q}_{16})^{(2)} \quad \text{large to have}
\]

\[
\frac{(P_{16})^{(2)}}{(M_{16})^{(2)}} \left[ (P_{16})^{(2)} + \left( (P_{16})^{(2)} + G_{1}^{(2)} \right) e^{-\frac{(P_{16})^{(2)} + G_{1}^{(2)}}{G_{0}^{(2)}}} \right] \leq (P_{16})^{(2)}
\]

\[
\frac{(b_{16}^{(2)})^{(2)}}{(M_{16})^{(2)}} \left[ (\tilde{Q}_{16})^{(2)} + \tau_{1} \right] e^{-\frac{(\tilde{Q}_{16})^{(2)} + \tau_{1}}{\tau_{1}^{(2)}}} \right] + (\tilde{Q}_{16})^{(2)} \leq (\tilde{Q}_{16})^{(2)}
\]

In order that the operator \( A^{(2)} \) transforms the space of sextuples of functions \( G_i, T_i \) satisfying

The operator \( A^{(2)} \) is a contraction with respect to the metric

\[
d \left( \left( (G_{16})^{(1)}, (T_{16})^{(1)} \right), \left( (G_{16})^{(2)}, (T_{16})^{(2)} \right) \right) = \\
\sup_{t \in \mathbb{R}_{+}} \left( G_{1}^{(1)}(t) - G_{1}^{(2)}(t) \right) e^{-\left( M_{16} \right)^{(2)}} \max_{t \in \mathbb{R}_{+}} \left| T_{1}^{(1)}(t) - T_{1}^{(2)}(t) \right| e^{-\left( M_{16} \right)^{(2)}}
\]

Indeed if we denote

\[
\text{Definition of } G_{19}, \tilde{F}_{19} : \quad \left( G_{19}, \tilde{F}_{19} \right) = A^{(2)}(G_{19}, T_{19})
\]

It results

\[
\left| G_{1}^{(1)} - G_{1}^{(2)} \right| \leq \int_{0}^{t}(a_{16}^{(2)})^{(2)} \left| G_{17}^{(2)} \right| e^{-\left( M_{16} \right)^{(2)}} \left| s_{16}(t) \right| \left| dS_{16} \right| + \\
\int_{0}^{t}(a_{16}^{(2)})^{(2)} \left| G_{16}^{(2)} - G_{16}^{(2)} \right| e^{-\left( M_{16} \right)^{(2)}} \left| s_{16}(t) \right| \left| dS_{16} \right| + \\
(a_{16}^{(2)})^{(2)} T_{17}^{(2)}(s_{16}(t)) \left| G_{16}^{(2)} \right| e^{-\left( M_{16} \right)^{(2)}} \left| s_{16}(t) \right| \left| dS_{16} \right| + \\
G_{16}^{(2)} \left( a_{16}^{(2)} T_{17}^{(2)}(s_{16}(t)) - (a_{16}^{(2)} T_{17}^{(2)}(s_{16}(t)) \right) e^{-\left( M_{16} \right)^{(2)}} \left| dS_{16} \right|
\]

Where \( S_{16}(t) \) represents integrand that is integrated over the interval \([0, t] \)

From the hypotheses it follows

\[
\left| (G_{19})^{(1)} - (G_{19})^{(2)} \right| e^{-\left( M_{16} \right)^{(2)}} \leq
\]
\[ \frac{1}{(M_{16})^2} \left( (a_{16}'')^2 + (a_{16}')^2 + (\lambda_{16})^2 + (P_{16})^2(\kappa_{16})^2 \right) d \left( (G_{19})^1, (T_{19})^1; (G_{19})^2, (T_{19})^2 \right) \]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \( (a''_{16})^2 \) and \( (b''_{16})^2 \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( (P_{16})^2 e^{(\lambda_{16})^2 t} \) and \( (Q_{16})^2 e^{(\lambda_{16})^2 t} \) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \( (a''_{16})^2 \) and \( (b''_{16})^2 \), \( i = 16, 17, 18 \) depend only on \( T_{17} \) and respectively on \( G_{19} \), and not on \( t \) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( \int \) where \( (b''_{16})^2 \) depend only on \( G_{19} \), and hypothesis can replaced by a usual Lipschitz condition.

**Definition of** \((\bar{M}_{16})(2)\):

Indeed let \( \varepsilon_2 \) be so that for \( t > t_2 \)

\[ (b''_{17})^2 - (b''_{16})^2 = \varepsilon_2 \]

Then \( \frac{dT_{17}}{dt} \geq \left( (a''_{17})^2(m)^2 \right) - \varepsilon_2 T_{17} \) which leads to

\[ T_{17} \geq \frac{\left( (a''_{17})^2(m)^2 \right)}{2} \left( 1 - e^{-\varepsilon_2 t} \right) + T_{17}^0 e^{-\varepsilon_2 t} \] If we take \( t \) such that \( e^{-\varepsilon_2 t} = \frac{1}{2} \) it results

\[ T_{17} \geq \frac{\left( (a''_{17})^2(m)^2 \right)}{2} \left( 1 - \frac{1}{2} \right) + \frac{1}{2} \] By taking now \( \varepsilon_2 \) sufficiently small one sees that \( T_{17} \) is unbounded.

The same property holds for \( T_{18} \) if \( \lim_{t \to \infty} (b''_{18})^2 = (G_{19})^2, (T_{19})^2 = (b''_{18})^2 \)

We now state a more precise theorem about the behaviors at infinity of the solutions
It is now sufficient to take \( \left( \frac{a_j}{M_{20}} \right)^{(3)} \), \( \left( \frac{b_j}{M_{20}} \right)^{(3)} \) \( \leq 1 \) and to choose \( (\bar{P}_{20})^{(3)} \) and \( (\bar{Q}_{20})^{(3)} \) large to have

\[
\frac{a_j}{M_{20}} \left[ (\bar{P}_{20})^{(3)} + \left( (\bar{P}_{20})^{(3)} + C_j^0 \right) e^{-\left( \frac{a_j}{M_{20}} \right)^{(3)} C_j^0} \right] \leq (\bar{P}_{20})^{(3)}
\]

\[
\frac{b_j}{M_{20}} \left[ (\bar{Q}_{20})^{(3)} + T_j^0 \right) e^{-\left( \frac{b_j}{M_{20}} \right)^{(3)} T_j^0} \right] \leq (\bar{Q}_{20})^{(3)}
\]

In order that the operator \( \mathcal{A}^{(3)} \) transforms the space of sextuples of functions \( G_i, T_i \) into itself

The operator \( \mathcal{A}^{(3)} \) is a contraction with respect to the metric

\[
d \left((G_{23})^{(1)}, (T_{23})^{(1)}, (G_{23})^{(2)}, (T_{23})^{(2)}\right) = \\
sup_{i \in \mathbb{R}^n} \left| G_i^{(1)}(t) - G_i^{(2)}(t) e^{-\left( \frac{a_i}{M_{20}} \right)^{(3)} t} \right| \max_{i \in \mathbb{R}^n} \left| T_i^{(1)}(t) - T_i^{(2)}(t) e^{-\left( \frac{b_i}{M_{20}} \right)^{(3)} t} \right|
\]

Indeed if we denote

**Definition of** \( \tilde{G}_{23}, \tilde{T}_{23} : (G_{23}), (T_{23}) = \mathcal{A}^{(3)}((G_{23}), (T_{23})) \)

It results

\[
\left| \tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)} \right| \leq \int_0^1 (a_20)^{(3)} \left| G_0^{(1)} - G_2^{(2)} \right| e^{-\left( \frac{a_20}{M_{20}} \right)^{(3)} S_{(20)}^t} e^{\left( \frac{b_20}{M_{20}} \right)^{(3)} S_{(20)}^t} dS_{(20)} + \\
\int_0^1 (a_20)^{(3)} \left| G_20^{(1)} - G_20^{(2)} \right| e^{-\left( \frac{a_20}{M_{20}} \right)^{(3)} S_{(20)}^t} e^{\left( \frac{b_20}{M_{20}} \right)^{(3)} S_{(20)}^t} + \\
(a_{20}''')^T \tilde{T}_{21}^{(1)}, S_{(20)}) \left| G_20^{(1)} - G_20^{(2)} \right| e^{-\left( \frac{a_20}{M_{20}} \right)^{(3)} S_{(20)}^t} e^{\left( \frac{b_20}{M_{20}} \right)^{(3)} S_{(20)}^t} + \\
G_20^{(2)} \left| (a_{20}''')^T \tilde{T}_{21}^{(1)}, S_{(20)} \right) \left| e^{-\left( \frac{a_20}{M_{20}} \right)^{(3)} S_{(20)}^t} e^{\left( \frac{b_20}{M_{20}} \right)^{(3)} S_{(20)}^t} \right|
\]

Where \( S_{(20)} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[
\left| G^{(1)} - G^{(2)} \right| e^{-\left( \frac{a_i}{M_{20}} \right)^{(3)} t} \leq \\
\frac{1}{\left( \frac{a_j}{M_{20}} \right)^{(3)} + (a_{20}''')^{(3)} + (b_{20}'''')^{(3)} + \\
\left( P_{20}^{(3)} \right) \left( Q_{20}^{(3)} \right) d \left((G_{23})^{(1)}, (T_{23})^{(1)}, (G_{23})^{(2)}, (T_{23})^{(2)}\right) \)
\]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis the result follows

**Remark 1**: The fact that we supposed \( (a_{20}''')^{(3)} \) and \( (b_{20}'''')^{(3)} \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( (\bar{P}_{20})^{(3)} e^{(\bar{Q}_{20})^{(3)} t} \) and \( (\bar{Q}_{20})^{(3)} e^{(\bar{Q}_{20})^{(3)} t} \) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \( (a_{20}''')^{(3)} \) and \( (b_{20}'''')^{(3)} \) depend only on \( T_{21} \) and respectively on \( (G_{23}) \) _and not on_ \( t \) and hypothesis can replaced by a usual Lipschitz condition.
**Remark 2:** There does not exist any \( t \) where \( G_1(t) = 0 \) and \( T_1(t) = 0 \). From 19 to 24 it results

\[
G_1(t) \geq G_1^0 e^{-\int_0^t \left( a_2''(a_1(x)) - \frac{dG_2}{dt} \right) dx} \geq 0
\]

\[
T_1(t) \geq T_1^0 e^{-\left( b_1''(a_1(x)) \right)} > 0 \quad \text{for} \quad t > 0
\]

**Definition of** \((M_2)^{(3)}_1\), \((M_2)^{(3)}_2\) and \((M_2)^{(3)}_3\):

**Remark 3:** if \( G_2 \) is bounded, the same property have also \( G_{21} \) and \( G_{22} \) indeed if \( G_2 < (M_2)^{(3)}_3 \) it follows \( \frac{dG_2}{dt} \leq (M_2)^{(3)}_1 - (a_{21})^{(3)} G_21 \) and by integrating

\[
G_{21} \leq (M_2)^{(3)}_2 = G_2^0 + 2(a_{21})^{(3)}((M_2)^{(3)}_1)/(a'_{21})^{(3)}
\]

In the same way, one can obtain

\[
G_{22} \leq (M_2)^{(3)}_3 = G_2^0 + 2(a_{22})^{(3)}((M_2)^{(3)}_2)/(a'_{22})^{(3)}
\]

If \( G_{21} \) or \( G_{22} \) is bounded, the same property follows for \( G_2 \), \( G_{22} \) and \( G_{21} \) respectively.

**Remark 4:** If \( G_2 \) is bounded, from below, the same property holds for \( G_{21} \) and \( G_{22} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{21} \) is bounded from below.

**Remark 5:** if \( T_{20} \) is bounded from below and \( \lim_{t \to \infty} (b_1''(a_1(x))(G_23(t), t)) = (b_1''(a_1(x))) \) then \( T_{21} \to \infty \).

**Definition of** \((m)^{(3)}\) and \( \varepsilon_3 \):

Indeed let \( t_3 \) be so that for \( t > t_3 \)

\[
(b_{21})^{(3)} - (b_1'')^{(3)}((G_23(t), t)) < \varepsilon_3 T_{20}(t) > (m)^{(3)}
\]

Then \( \frac{dT_{21}}{dt} \geq (a_{21})^{(3)(m)^{(3)}} - \varepsilon T_{21} \) which leads to

\[
T_{21} \geq \left( (a_{21})^{(3)(m)^{(3)}} / \varepsilon_3 \right) (1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}
\]

If we take \( t \) such that \( e^{-\varepsilon_3 t} = \frac{1}{2} \) it results

\[
T_{21} \geq \left( (a_{21})^{(3)(m)^{(3)}} / \varepsilon_3 \right) \cdot t = log \frac{2}{\varepsilon_3}
\]

By taking now \( \varepsilon_3 \) sufficiently small one sees that \( T_{21} \) is unbounded. The same property holds for \( T_{22} \) if \( \lim_{t \to \infty} (b_1''(a_1(x))(G_23(t), t)) = (b_1''(a_1(x))) \)

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take \( \left( \frac{a_j}{M_2} \right)^{(4)(a)} \left( \frac{b_j}{M_2} \right)^{(4)(a)} \) and \( (m)^{(4)} \) to choose

\( \bar{P}_{24} \) and \( \bar{Q}_{24} \) large to have

\[
\left( \bar{P}_{24} ight)^{(4)} + \left( \frac{a_j}{M_2} \right)^{(4)(a)} \leq \left( \bar{P}_{24} ight)^{(4)}
\]

\[
\left( \bar{Q}_{24} ight)^{(4)} + \left( \frac{b_j}{M_2} \right)^{(4)(a)} \leq \left( \bar{Q}_{24} ight)^{(4)}
\]
In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying IN to itself

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric

$$d \left( \left( (G_{24})^{(1)}(T_{24})^{(1)} \right), \left( (G_{24})^{(2)}(T_{24})^{(2)} \right) \right) =$$

$$\sup_{t \in \mathbb{R}_+} \max \left\{ \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-\left(\bar{R}_{24}^{(4)}\right)^{1/2} t}, \max_{t \in \mathbb{R}_+} \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-\left(\bar{R}_{24}^{(4)}\right)^{1/2} t} \right\}$$

Indeed if we denote

**Definition of $(G_{24}^{(1)}, (T_{24}^{(1)})) = (G_{24}^{(2)}, (T_{24}^{(2)}))$**

It results

$$\left| \bar{G}_{24}^{(1)} - \bar{G}_{24}^{(2)} \right| \leq \int_0^t \left( a_{24}^{(4)} \right) \left| G_{24}^{(1)}(T_{24})^{(1)} - G_{24}^{(2)}(T_{24})^{(2)} \right| e^{-\left(\bar{R}_{24}^{(4)}\right)^{1/2} t} e^{\left(\bar{R}_{24}^{(4)}\right)^{1/2} t} dS^{(24)} +$$

$$\int_0^t \left( a_{24}^{(4)} \right) \left| G_{24}^{(1)}(T_{24})^{(1)} - G_{24}^{(2)}(T_{24})^{(2)} \right| e^{-\left(\bar{R}_{24}^{(4)}\right)^{1/2} t} e^{\left(\bar{R}_{24}^{(4)}\right)^{1/2} t} dS^{(24)} +$$

$$\left( a_{24}^{(4)} \right) \left| T_{24}^{(1)}(S_{24})^{(1)} - T_{24}^{(2)}(S_{24})^{(2)} \right| e^{-\left(\bar{R}_{24}^{(4)}\right)^{1/2} t} e^{\left(\bar{R}_{24}^{(4)}\right)^{1/2} t} dS^{(24)} +$$

Where $S_{24}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left| (G_{24})^{(1)} - (G_{24})^{(2)} \right| e^{-\left(\bar{R}_{24}^{(4)}\right)^{1/2} t} \leq$$

$$\frac{1}{(\bar{R}_{24}^{(4)})^{1/2}} \left( \left( a_{24}^{(4)} \right) + \left( a_{24}^{(4)} \right) + \left( \bar{Q}_{24}^{(4)} \right) \right) d \left( \left( (G_{24})^{(1)}(T_{24})^{(1)} \right), \left( (G_{24})^{(2)}(T_{24})^{(2)} \right) \right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a_{24}^{(4)}')$ and $(b_{24}^{(4)}')$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\bar{Q}_{24}^{(4)} e^{(\bar{R}_{24}^{(4)})^{1/2} t}$ and $(\bar{Q}_{24}^{(4)} e^{(\bar{R}_{24}^{(4)})^{1/2} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{(4)}')$ and $(b_i^{(4)}')$, $i = 24, 25, 26$ depend only on $T_{25}$ and respectively on $(G_{24})$(and not on $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ and $T_i(t) = 0$

From **GLOBAL EQUATIONS** it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t (a_i^{(4)}(t) - (a_i^{(4)})(T_{24}(S_{24})))) dS_{24})} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-\left(\bar{Q}_{24}^{(4)}\right)^{1/2} t} > 0 \text{ for } t > 0$$
Definition of \((\overline{M}_{24})^{(4)}\) and \((\overline{M}_{24})^{(4)}\) and \((\overline{M}_{24})^{(4)}\):

**Remark 3:** If \(G_{24}\) is bounded, the same property hold also for \(G_{25}\) and \(G_{26}\), indeed if \(G_{24}\) is bounded it follows:

\[
G_{25} \leq ((\overline{M}_{24})^{(4)})_{2} = G_{25}^{0} + 2(a_{25})^{(4)}((\overline{M}_{24})^{(4)})_{1}/(a_{25})^{(4)}
\]

In the same way, one can obtain:

\[
G_{26} \leq ((\overline{M}_{24})^{(4)})_{3} = G_{26}^{0} + 2(a_{26})^{(4)}((\overline{M}_{24})^{(4)})_{2}/(a_{26})^{(4)}
\]

If \(G_{25}\) or \(G_{26}\) is bounded, the same property follows for \(G_{24}\), \(G_{26}\) and \(G_{24}\), \(G_{25}\) respectively.

**Remark 4:** If \(G_{24}\) is bounded, from below, the same property holds for \(G_{25}\) and \(G_{26}\). The proof is analogous with the preceding one. An analogous property is true if \(G_{25}\) is bounded from below.

**Remark 5:** If \(T_{24}\) is bounded from below and \(\lim_{t \to \infty} ((b_{27}^{(4)}))((G_{27})(t), t) = (b_{25}^{(4)})\) then \(T_{25} \to \infty\).

Definition of \((m)^{(4)}\) and \(\varepsilon_{4}\):

Indeed let \(t_{4}\) be so that for \(t > t_{4}\):

\[
(b_{25})^{(4)} - (b_{25}^{(4)}((G_{27})(t), t) < \varepsilon_{4}, T_{26}(t) > (m)^{(4)}
\]

Then \(\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_{4}T_{25}\) which leads to:

\[
T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_{4}}\right) (1 - e^{-\varepsilon_{4}t}) + T_{25}^{0}e^{-\varepsilon_{4}t}
\]

If we take \(t\) such that \(e^{-\varepsilon_{4}t} = \frac{1}{2}\) it results:

\[
T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right) \cdot t = \log \frac{2}{\varepsilon_{4}}
\]

By taking now \(\varepsilon_{4}\) sufficiently small one sees that \(T_{25}\) is unbounded. The same property holds for \(T_{26}\) if \(\lim_{t \to \infty} ((b_{26}^{(4)}))((G_{27})(t), t) = (b_{26}^{(4)})\).

We now state a more precise theorem about the behaviors at infinity of the solutions. ANALOGOUS inequalities hold also for \(G_{29}, G_{30}, T_{29}, T_{29}^{'}, T_{30}\).

It is now sufficient to take \((a_{j})^{(5)}\), \((b_{j})^{(5)}\), \((\overline{M}_{28})^{(5)}\) and \((M_{28})^{(5)}\) < 1 and to choose:

\[
(\overline{P}_{28})^{(5)}\) and \((\overline{Q}_{28})^{(5)}\) large to have:

\[
\left(\frac{a_{j}}{(\overline{M}_{28})^{(5)}}\right)
\left((\overline{P}_{28})^{(5)} + ((\overline{P}_{28})^{(5)} + G_{j}^{0})e^{-\left(\frac{(\overline{P}_{28})^{(5)} + G_{j}^{0}}{G_{j}^{0}}\right)}\right) \leq (\overline{P}_{28})^{(5)}
\]

\[
\left(\frac{b_{j}}{(M_{28})^{(5)}}\right)
\left((\overline{Q}_{28})^{(5)} + T_{j}^{0}e^{-\left(\frac{(\overline{Q}_{28})^{(5)} + T_{j}^{0}}{T_{j}^{0}}\right)} + (\overline{Q}_{28})^{(5)}\right) \leq (\overline{Q}_{28})^{(5)}
\]

In order that the operator \(\mathcal{A}^{(5)}\) transforms the space of sextuples of functions \(G_{i}, T_{j}\) into itself:

The operator \(\mathcal{A}^{(5)}\) is a contraction with respect to the metric...
Indeed if we denote

\[ \frac{\partial}{\partial t} \frac{\partial}{\partial t} \left( (G_{31})^{(1)}, (T_{31})^{(2)} \right) = \]

\[ \sup_{i \in \mathbb{R}^+} \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-(\bar{A}_{28})^{(5)} t} \max_{i \in \mathbb{R}^+} \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-(\bar{M}_{28})^{(5)} t} \]

\[ \left( (G_{31})^{(1)}, (T_{31})^{(2)} \right) = \mathcal{A}^{(5)} \left( (G_{31}), (T_{31}) \right) \]

It results

\[ \left| \bar{G}_{28}^{(1)} - \bar{G}_{28}^{(2)} \right| \leq \int_0^t (a_{28}^{(5)}) G_{28}^{(1)} - G_{28}^{(2)} e^{-(\bar{M}_{28})^{(5)} s_{(28)}} e^{-(\bar{M}_{28})^{(5)} s_{(28)}} dS_{(28)} + \]

\[ \int_0^t ((a_{28}^{(5)}) G_{28}^{(1)} - G_{28}^{(2)} e^{-(\bar{M}_{28})^{(5)} s_{(28)}} e^{-(\bar{M}_{28})^{(5)} s_{(28)}} + \]

\[ (a_{28}^{(5)})(T_{29}^{(1)}, s_{(28)})(G_{28}^{(1)} - G_{28}^{(2)} e^{-(\bar{M}_{28})^{(5)} s_{(28)}} e^{-(\bar{M}_{28})^{(5)} s_{(28)}} + \]

\[ G_{28}^{(2)}(a_{28}^{(5)})(T_{29}^{(1)}, s_{(28)})(G_{28}^{(1)} - G_{28}^{(2)}(T_{29}^{(2)}, s_{(28)})(e^{-(\bar{M}_{28})^{(5)} s_{(28)}} e^{-(\bar{M}_{28})^{(5)} s_{(28)}} dS_{(28)} \]

Where \( s_{(28)} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[ \left| (G_{31})^{(1)} - (G_{31})^{(2)} \right| e^{-(\bar{M}_{28})^{(5)} t} \leq \frac{1}{(\bar{M}_{28})^{(5)}} \left( (a_{28}^{(5)})(\bar{A}_{28}^{(5)}) + (a_{28}^{(5)})(\bar{A}_{28}^{(5)}) + \right) \]

\[ (\bar{G}_{28})^{(5)}(\bar{K}_{28})^{(5)} d \left( ((G_{31})^{(1)}, (T_{31})^{(1)}), (G_{31})^{(2)}, (T_{31})^{(2)} \right) \]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis (35, 35, 36) the result follows

**Remark 1:** The fact that we supposed \( (a_{28}^{(5)})(3) \) and \( (b_{28}^{(5)})(3) \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( (\bar{P}_{28})^{(5)} e^{(\bar{A}_{28})^{(5)} t} \) and \( (\bar{Q}_{28})^{(5)} e^{(\bar{M}_{28})^{(5)} t} \) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \( (a_{28}^{(5)})(3) \) and \( (b_{28}^{(5)})(3), i = 28, 29, 30 \) depend only on \( T_{29} \) and respectively on \( (G_{31})(and not on t) \) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_i(t) = 0 \) and \( T_i(t) = 0 \)

From GLOBAL EQUATIONS it results

\[ G_i(t) \geq G_i^0 e^{-\int_0^t [(a_{28}^{(5)})(a_{28}^{(5)})(T_{28}(s_{(28)}))dS_{(28)}]} \geq 0 \]

\[ T_i(t) \geq T_i^0 e^{-(b_{28}^{(5)} t)} > 0 \] for \( t > 0 \)

**Definition of** \( (\bar{M}_{28})^{(5)} \), \((\bar{M}_{28})^{(5)}\), \((\bar{M}_{28})^{(5)}\)

**Remark 3:** If \( G_{29} \) is bounded, the same property have also \( G_{29} \) and \( G_{30} \). Indeed if
In the same way, one can obtain

\[ G_{29} < (\mathcal{M}_{29})^{(5)} \text{ it follows } \frac{dG_{29}}{dt} \leq (\mathcal{M}_{29})^{(5)}_0 - (a_{29}^{(5)}) G_{29} \text{ and by integrating} \]

\[ G_{29} \leq (\mathcal{M}_{29})^{(5)}_0 G_{29} + 2(a_{29}^{(5)}) (\mathcal{M}_{29})^{(5)}_2 / (a_{29}^{(5)}) \]

In the same way, one can obtain

\[ G_{30} \leq (\mathcal{M}_{30})^{(5)}_0 G_{30} + 2(a_{30}^{(5)}) (\mathcal{M}_{30})^{(5)}_2 / (a_{30}^{(5)}) \]

If \( G_{29} \) or \( G_{30} \) is bounded, the same property follows for \( G_{28} \), \( G_{31} \) and \( G_{29} \) respectively.

**Remark 4:** If \( G_{28} \) is bounded, from below, the same property holds for \( G_{29} \) and \( G_{31} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{29} \) is bounded from below.

**Remark 5:** If \( T_{28} \) is bounded from below and \( \lim_{t \to \infty} ((b_{28}^{(5)})(G_{31})(t), t) = (b_{30}^{(5)}) \) then \( T_{29} \to \infty \).

**Definition of** \((m)^{(5)}\) and \(\varepsilon_5\) :

Indeed let \( t_5 \) be so that for \( t > t_5 \)

\[ (b_{29}^{(5)}) - (b_{1}^{(5)})(G_{31})(t), t) < \varepsilon_5, T_{29} (t) > (m)^{(5)} \]

Then \( \frac{dT_{29}}{dt} \geq (a_{29}^{(5)})(m)^{(5)} - \varepsilon_5 T_{29} \) which leads to

\[ T_{29} \geq \left( \frac{(a_{29}^{(5)})(m)^{(5)})}{\varepsilon_5} \right) \left( 1 - e^{-\varepsilon_5 t} \right) + T_{29} e^{-\varepsilon_5 t} \]

If we take \( t \) such that \( e^{-\varepsilon_5 t} = \frac{1}{2} \) it results

\[ T_{29} \geq \left( \frac{(a_{29}^{(5)})(m)^{(5)})}{\varepsilon_5 t} \right) \]

By taking now \( \varepsilon_5 \) sufficiently small one sees that \( T_{29} \) is unbounded. The same property holds for \( T_{30} \) if \( \lim_{t \to \infty} (b_{30}^{(5)})(G_{31})(t), t) = (b_{30}^{(5)}) \)

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for \( G_{33}, G_{34}, T_{32}, T_{33}, T_{34} \)

It is now sufficient to take \( \frac{(a_{1}^{(6)})}{(\mathcal{M}_{32})^{(6)}} \) and \( \frac{(b_{1}^{(6)})}{(\mathcal{M}_{32})^{(6)}} \) large to have

\[ \left( \frac{(a_{1}^{(6)})}{(\mathcal{M}_{32})^{(6)}} \right) \left( \mathcal{P}_{32}^{(6)} \right)^{\frac{1}{6}} + \left( \frac{(b_{1}^{(6)})}{(\mathcal{M}_{32})^{(6)}} \right) \left( \mathcal{Q}_{32}^{(6)} \right)^{\frac{1}{6}} \]

\[ \left( \mathcal{P}_{32}^{(6)} \right)^{\frac{1}{6}} \leq \left( \frac{\mathcal{P}_{32}^{(6)}}{\mathcal{M}_{32}^{(6)}} \right) \left( \mathcal{P}_{32}^{(6)} \right)^{\frac{1}{6}} \]

\[ \left( \mathcal{Q}_{32}^{(6)} \right)^{\frac{1}{6}} \leq \left( \frac{\mathcal{Q}_{32}^{(6)}}{\mathcal{M}_{32}^{(6)}} \right) \left( \mathcal{Q}_{32}^{(6)} \right)^{\frac{1}{6}} \]

In order that the operator \( \mathcal{A}^{(6)} \) transforms the space of sextuples of functions \( G_1, T_1 \) into itself

The operator \( \mathcal{A}^{(6)} \) is a contraction with respect to the metric

\[ d \left( \left( G_{33}^{(1)}, T_{33}^{(1)} \right), \left( G_{33}^{(2)}, T_{33}^{(2)} \right) \right) = \]
Indeed, if we denote

\[ \text{Definition of } (G_{32}), (T_{32}) : \quad \left( G_{32}, T_{32} \right) = \mathcal{A}^{(6)} \left( (G_{32}), (T_{32}) \right) \]

It results

\[ \left| G_{32}^{(1)} - t_{32}^{(2)} \right| \leq \int_0^t (a_{32})^{(6)} |G_{33}^{(1)} - G_{33}^{(2)}| e^{-t(M_{32})^{(6)}x_{32}} e^{t(M_{32})^{(6)}x_{32}} dS_{32} + \]

\[ \int_0^t (a_{32})^{(6)} |G_{32}^{(1)} - G_{32}^{(2)}| e^{-t(M_{32})^{(6)}x_{32}} e^{t(M_{32})^{(6)}x_{32}} dS_{32} + \]

\[ (a_{32}^{(2)})^{(6)} \left( T_{32}^{(1)}, S_{32} \right) |G_{32}^{(1)} - G_{32}^{(2)}| e^{-t(M_{32})^{(6)}x_{32}} e^{t(M_{32})^{(6)}x_{32}} + \]

\[ G_{32}^{(2)} (a_{32}^{(2)})^{(6)} \left( T_{32}^{(1)}, S_{32} \right) - (a_{32}^{(2)})^{(6)} \left( T_{32}^{(2)}, S_{32} \right) | e^{-t(M_{32})^{(6)}x_{32}} e^{t(M_{32})^{(6)}x_{32}} dS_{32} \]

Where \( S_{32} \) represents integrand that is integrated over the interval \([0, t] \)

From the hypotheses it follows

1. \( (a_j)^{(1)}, (a_j^{(1)}), (b_j)^{(1)}, (b_j^{'(1)}) > 0 \)

\[ i, j = 13, 14, 15 \]

2. The functions \( (a_j^{(1)}), (b_j^{'(1)}) \) are positive continuous increasing and bounded.

**Definition of** \((p_j)^{(1)}, (r_j)^{(1)}:\)

\( (a_j^{(1)})(T_{14}, t) \leq (p_j)^{(1)} \leq (A_{13})^{(1)} \)

\( (b_j^{'(1)})(G, t) \leq (r_j)^{(1)} \leq (B_{13})^{(1)} \)

3. \( \lim_{T_{14} \to \infty} (a_j^{(1)})(T_{14}, t) = (p_j)^{(1)} \)

\( \lim_{G \to \infty} (b_j^{'(1)})(G, t) = (r_j)^{(1)} \)

**Definition of** \((A_{13})^{(1)}, (B_{13})^{(1)}, (p_j)^{(1)}, (r_j)^{(1)}:\)

Where \((A_{13})^{(1)}, (B_{13})^{(1)}, (p_j)^{(1)}, (r_j)^{(1)}\)

are positive constants and

\[ \tau = 13, 14, 15 \]

They satisfy the Lipschitz condition:

\[ \left| (a_j^{(1)})(T_{14}, t) - (a_j^{(1)})(T_{14}, t) \right| \leq (\bar{k}_{13})^{(1)}|T_{14} - T_{14}^{}| e^{-(M_{13})^{(1)}t} \]

\[ \left| (b_j^{'(1)})(G, t) - (b_j^{'(1)})(G, T) \right| < (\bar{k}_{13})^{(1)} |G - G^{}| e^{-(M_{13})^{(1)}t} \]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_j^{(1)})(T_{14}, t)\) and \((a_j^{(1)})^{(1)}(T_{14}, t). \)\( (T_{14}, t)\) And \((T_{14}, t)\) are points belonging to the interval \([ (K_{13})^{(1)}, (M_{13})^{(1)} ] \).

It is to be noted that \((a_j^{(1)})(T_{14}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((M_{13})^{(1)} = 1\) then the function \((a_j^{(1)})(T_{14}, t)\), the first augmentation coefficient, attributable to

109
terrestrial organisms, would be absolutely continuous.

**Definition of** \( \tilde{M}_{13}^{(1)}, (\tilde{k}_{13}^{(1)}) \):

\[
\begin{align*}
(\tilde{M}_{13}^{(1)}, (\tilde{k}_{13}^{(1)}), \text{ are positive constants}
\end{align*}
\]

\[
\frac{(a_j)^{(1)}}{(M_{13})^{(1)}} \cdot \frac{(b_j)^{(1)}}{(M_{13})^{(1)}} < 1
\]

**Definition of** \( \tilde{P}_{13}^{(1)}, (\tilde{Q}_{13}^{(1)}) \):

There exists two constants \( \tilde{P}_{13}^{(1)} \) and \( \tilde{Q}_{13}^{(1)} \) which together with \( (\tilde{M}_{13}^{(1)}, (\tilde{k}_{13}^{(1)}), (\tilde{A}_{13})^{(1)}, (\tilde{B}_{13})^{(1)} \) and the constants \( (a_{j}^{(1)}), (a_{j}^{(1)}), (b_{j}^{(1)}), (b_{j}^{(1)}), (p_{j})^{(1)}, (r_{j})^{(1)}, i = 13, 14, 15, \) satisfy the inequalities

\[
\begin{align*}
\frac{1}{(M_{13})^{(1)}} \left[ (a_j)^{(1)} + (a_j')^{(1)} + (\tilde{A}_{13})^{(1)} + (\tilde{P}_{13}^{(1)} (\tilde{k}_{13}^{(1)}) \right] < 1
\end{align*}
\]

\[
\begin{align*}
\frac{1}{(M_{13})^{(1)}} \left[ (b_j)^{(1)} + (b_j')^{(1)} + (\tilde{B}_{13})^{(1)} + (\tilde{Q}_{13}^{(1)} (\tilde{k}_{13}^{(1)} \right] < 1
\end{align*}
\]

Analogous inequalities hold also for \( G_{37}, G_{38}, T_{36}, T_{37}, T_{38} \)

It is now sufficient to take \( \frac{(a_j)^{(1)}}{(M_{36})^{(1)}} \cdot \frac{(b_j)^{(1)}}{(M_{36})^{(1)}} < 1 \) and to choose \( (\tilde{P}_{36})^{(7)} \) and \( (\tilde{Q}_{36})^{(7)} \) large to have

\[
\begin{align*}
\left( \frac{(P_{36})^{(7)}}{(M_{36})^{(7)}} \right)^{2} \left( (\tilde{P}_{36})^{(7)} + \left( \frac{(P_{36})^{(7)} + C_{j}^{(7)}}{c_{j}^{(7)}} \right) \right) \leq (\tilde{P}_{36})^{(7)}
\end{align*}
\]

\[
\begin{align*}
\left( \frac{(Q_{36})^{(7)}}{(M_{36})^{(7)}} \right)^{2} \left( (\tilde{Q}_{36})^{(7)} + \left( \frac{(P_{36})^{(7)} + C_{j}^{(7)}}{c_{j}^{(7)}} \right) \right) \leq (\tilde{Q}_{36})^{(7)}
\end{align*}
\]

In order that the operator \( A^{(7)} \) transforms the space of sextuples of functions \( G_{i}, T_{i} \) satisfying into itself

Analogous inequalities hold also for \( G_{41}, G_{42}, T_{40}, T_{41}, T_{42} \)

It is now sufficient to take \( \frac{(a_j)^{(8)}}{(M_{40})^{(8)}} \cdot \frac{(b_j)^{(8)}}{(M_{40})^{(8)}} < 1 \) and to choose \( (\tilde{P}_{40})^{(8)} \) and \( (\tilde{Q}_{40})^{(8)} \) large to have

\[
\begin{align*}
\left( \frac{(P_{40})^{(8)}}{(M_{40})^{(8)}} \right)^{2} \left( (\tilde{P}_{40})^{(8)} + \left( \frac{(P_{40})^{(8)} + C_{j}^{(8)}}{c_{j}^{(8)}} \right) \right) \leq (\tilde{P}_{40})^{(8)}
\end{align*}
\]
In order that the operator $\mathcal{A}^{(0)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying GLOBAL EQUATIONS into itself

The operator $\mathcal{A}^{(0)}$ is a contraction with respect to the metric

$$
\tag{306}
\left( (\hat{Q}_{40})^{(0)} \right) + T_j^{(0)} e^{- \frac{(\hat{Q}_{40})^{(0)} + T_j^{(0)}}{T_j^{(0)}}} + (\hat{Q}_{40})^{(0)} \right) \leq (\hat{Q}_{40})^{(0)}
$$

Indeed if we denote

**Definition of** $(\hat{G}_{43}, \hat{T}_{43}) : (\hat{G}_{43}, \hat{T}_{43}) = \mathcal{A}^{(0)}((G_{43}), (T_{43}))$

It results

$$
\tag{307}
\left| \hat{G}_{40}^{(1)} - \hat{G}_{40}^{(2)} \right| \leq \int_0^t (a_{40})^{(0)} \left| G_{41}^{(1)} - G_{41}^{(2)} \right| e^{- (M_{40})^{(0)} x(40) t} e^{(M_{40})^{(0)} x(40) t} ds(40) + \\
\int_0^t (a_{40})^{(0)} \left| G_{40}^{(1)} - G_{40}^{(2)} \right| e^{- (M_{40})^{(0)} x(40) t} e^{(M_{40})^{(0)} x(40) t} + \\
(a_{40})^{(0)} \left( T_{41}^{(1)}, s(40) \right) \left| G_{40}^{(1)} - G_{40}^{(2)} \right| e^{- (M_{40})^{(0)} x(40) t} e^{(M_{40})^{(0)} x(40) t} + \\
G_{40}^{(2)} \left( (a_{40})^{(0)} \left( T_{41}^{(1)}, s(40) \right) - (a_{40})^{(0)} \left( T_{41}^{(2)}, s(40) \right) \right) \left| e^{- (M_{40})^{(0)} x(40) t} e^{(M_{40})^{(0)} x(40) t} ds(40) \right|
$$

Where $s(40)$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses IT follows

$$
\tag{308}
\left| (G_{43})^{(1)} - (G_{43})^{(2)} \right| e^{- (M_{40})^{(0)} t} \leq \left( \frac{1}{(M_{40})^{(0)}} + (a_{40})^{(0)} + (\hat{A}_{40})^{(0)} + (\hat{P}_{40})^{(0)} (\hat{K}_{40})^{(0)} \right) d \left( (G_{43})^{(1)}; (T_{43})^{(1)}; (G_{43})^{(2)}; (T_{43})^{(2)} \right)
$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis (38,35,36) the result follows

**Remark 1:** The fact that we supposed $(a_{40})^{(0)}$ and $(b_{40})^{(0)}$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(P_{40})^{(0)} e^{(M_{40})^{(0)} t}$ and $(\hat{Q}_{40})^{(0)} e^{(M_{40})^{(0)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{(0)})^{(0)}$ and $(b_i^{(0)})^{(0)}, i = 40, A1, A2$ depend only on $T_{41}$ and respectively on $(G_{43}) (and not on t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ and $T_i(t) = 0$

From global equations it results
\[ G_i(t) \geq G_i^0 e^{\int_{t_0}^{t} \left( (a_i)^{(0)}(\tau(t),t) - (a_i')^{(0)}(\tau(t),t) \right) d\tau(t)} \geq 0 \]
\[ T_i(t) \geq T_i^0 e^{-(b_i')^{(0)}(t)} > 0 \text{ for } t > 0 \]

Definition of \((\overline{M}_{40})^{(0)}\), \((\overline{M}_{40})^{(0)}_2\) and \((\overline{M}_{40})^{(0)}_3\) :

Remark 3: if \(G_{40}\) is bounded, the same property have also \(G_{41}\) and \(G_{42}\) . Indeed if
\[ G_{40} < (\overline{M}_{40})^{(0)} \]
it follows \(\frac{dG_{40}}{dt} \leq (\overline{M}_{40})^{(0)} - (a_{41})^{(0)}G_{41}\) and by integrating

\[ G_{41} \leq (\overline{M}_{40})^{(0)}_2 = G_{40}^0 + 2(a_{41})^{(0)}((\overline{M}_{40})^{(0)}_1/(a_{41})^{(0)}) \]

In the same way , one can obtain

\[ G_{42} \leq (\overline{M}_{40})^{(0)}_3 = G_{42}^0 + 2(a_{42})^{(0)}((\overline{M}_{40})^{(0)}_2/(a_{42})^{(0)}) \]

If \(G_{40}\) or \(G_{42}\) is bounded, the same property follows for \(G_{40}\), \(G_{42}\) and \(G_{40}\), \(G_{41}\) respectively.

Remark 4: If \(G_{40}\) is bounded, from below, the same property holds for \(G_{41}\) and \(G_{42}\). The proof is analogous with the preceding one. An analogous property is true if \(G_{41}\) is bounded from below.

Remark 5: If \(T_{40}\) is bounded from below and \(\lim_{t \to \infty}((b_{41})^{(0)}((G_{43})(t),t)) = (b_{41})^{(0)}\) then \(T_{41} \to \infty\)

Definition of \((m)^{(0)}\) and \(\varepsilon_0\) :

Indeed let \(t_8\) be so that for \(t > t_8\)

\[(b_{41})^{(0)} - (b_i^{(0)}((G_{43})(t),t) < \varepsilon_0, T_{40}(t) > (m)^{(0)})\]

Then \(\frac{dT_{41}}{dt} \geq (a_{41})^{(0)}(m)^{(0)} - \varepsilon_0 T_{41}\) which leads to

\[ T_{41} \geq \left(\frac{(a_{41})^{(0)}(m)^{(0)}}{\varepsilon_0}\right)(1 - e^{-\varepsilon_0 t}) + T_{41}^0 e^{-\varepsilon_0 t}\]

If we take \(t\) such that \(e^{-\varepsilon_0 t} = \frac{1}{2}\) it results

\[ T_{41} \geq \left(\frac{(a_{41})^{(0)}(m)^{(0)}}{2}\right)\times t = \log\frac{2}{\varepsilon_0} \]

By taking now \(\varepsilon_0\) sufficiently small one sees that \(T_{41}\) is unbounded. The same property holds for \(T_{42}\) if \(\lim_{t \to \infty}((b_{42})^{(0)}((G_{43})(t),t) = (b_{42})^{(0)}\)

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 82

Remark 1: There does not exist any \(t\) where \(G_i(t) = 0\) and \(T_i(t) = 0\)

From CONCATENATED GLOBAL EQUATIONS it results

\[ G_i(t) \geq G_i^0 e^{\int_{t_0}^{t} \left( (a_i)^{(0)}(\tau(t),t) - (a_i')^{(0)}(\tau(t),t) \right) d\tau(t)} \geq 0 \]
\[ T_i(t) \geq T_i^0 e^{-(b_i')^{(0)}(t)} > 0 \text{ for } t > 0 \]

Definition of \((\overline{M}_{36})^{(0)}\), \((\overline{M}_{36})^{(0)}_2\) and \((\overline{M}_{36})^{(0)}_3\) :

Remark 2: if \(G_{36}\) is bounded, the same property have also \(G_{37}\) and \(G_{38}\). Indeed if
$G_{36} < (M_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq (M_{36})^{(7)} - (a_{37}')^{(7)}G_{37}$ and by integrating

$G_{37} \leq (M_{36})^{(7)} + 2(a_{37})^{(7)}(M_{36})^{(7)}/(a_{37}')^{(7)}$

In the same way, one can obtain

$G_{38} \leq (M_{36})^{(7)} + 2(a_{38})^{(7)}(M_{36})^{(7)}/(a_{38}')^{(7)}$

If $G_{37}$ or $G_{38}$ is bounded, the same property follows for $G_{36}$, $G_{38}$ and $G_{36}$, $G_{37}$ respectively.

Remark 3: If $G_{36}$ is bounded, from below, the same property holds for $G_{37}$ and $G_{38}$. The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below.

Remark 5: If $G_{38}$ is bounded from below and $(b_{38})^{(7)}(G_{39})(t,t) = (b_{38}')^{(7)}$ then $T_{37} \rightarrow \infty$.

Definition of $(m)^{(7)}$ and $\varepsilon_{7}$:

Indeed let $t_{7}$ be so that for $t > t_{7}$

$(b_{37})^{(7)}(t) - (b_{37}')^{(7)}(G_{39})(t,t) < \varepsilon_{7}, T_{38} (t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_{7}T_{37}$ which leads to

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_{7}}\right)(1 - e^{-\varepsilon_{7}t}) + T_{37}e^{-\varepsilon_{7}t}$

If we take $t$ such that $e^{-\varepsilon_{7}t} = \frac{1}{e^{T_{37}}}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \ t = log\frac{2}{\varepsilon_{7}}$. By taking now $\varepsilon_{7}$ sufficiently small one sees that $T_{37}$ is unbounded. The same property holds for $T_{38}$ if $\lim_{t \rightarrow \infty} (b_{38})^{(7)}(G_{39})(t,t) = (b_{38}')^{(7)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

$-(\sigma_{2})^{(2)} \leq -(a_{10})^{(2)} + (a_{17})^{(2)}(T_{17}, t) + (a_{10})^{(2)}(T_{17}, t) \leq -(\sigma_{1})^{(2)}$

$-(\tau_{2})^{(2)} \leq -(b_{10})^{(2)} + (b_{17})^{(2)}(G_{19}, t) - (b_{10})^{(2)}(G_{19}, t) \leq -(\tau_{1})^{(2)}$

Definition of $(v_{1})^{(2)}, (v_{2})^{(2)}, (u_{1})^{(2)}, (u_{2})^{(2)}$:

By $(v_{1})^{(2)} > 0, (v_{2})^{(2)} < 0$ and respectively $(u_{1})^{(2)} > 0, (u_{2})^{(2)} < 0$ the roots

(a) of the equations $(a_{17})^{(2)}(v')^{2} + (\sigma_{2})^{(2)}v^{2} - (a_{10})^{(2)} = 0$

and $(b_{10})^{(2)}u^{2} + (\tau_{2})^{(2)}u^{2} - (b_{17})^{(2)} = 0$

Definition of $(\tilde{v}_{1})^{(2)}, (\tilde{v}_{2})^{(2)}, (\tilde{u}_{1})^{(2)}, (\tilde{u}_{2})^{(2)}$:

By $(\tilde{v}_{1})^{(2)} > 0, (\tilde{v}_{2})^{(2)} < 0$ and respectively $(\tilde{u}_{1})^{(2)} > 0, (\tilde{u}_{2})^{(2)} < 0$ the roots

of the equations $(a_{17})^{(2)}(v')^{2} + (\sigma_{2})^{(2)}v^{2} - (a_{10})^{(2)} = 0$

and $(b_{10})^{(2)}u^{2} + (\tau_{2})^{(2)}u^{2} - (b_{17})^{(2)} = 0$

Definition of $(m_{1})^{(2)}, (m_{2})^{(2)}, (\mu_{1})^{(2)}, (\mu_{2})^{(2)}$:

(b) If we define $(m_{1})^{(2)}, (m_{2})^{(2)}, (\mu_{1})^{(2)}, (\mu_{2})^{(2)}$ by
If we denote and define

\[(m_2)^{(2)} = (v_0)^{(2)}(m_1)^{(2)} = (v_1)^{(2)}, \text{ if } (v_0)^{(2)} < (v_1)^{(2)} \]

\[(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (\bar{v}_1)^{(2)}, \text{ if } (v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)} \]

and 

\[(v_0)^{(2)} = \frac{T_{16}^{0}}{T_{17}^{0}} \]

\[(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}, \text{ if } (\bar{v}_1)^{(2)} < (v_0)^{(2)} \]

and analogously

\[(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \text{ if } (u_0)^{(2)} < (u_1)^{(2)} \]

\[(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \text{ if } (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)} \]

and 

\[(u_0)^{(2)} = \frac{T_{16}^{0}}{T_{17}^{0}} \]

\[(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \text{ if } (\bar{u}_1)^{(2)} < (u_0)^{(2)} \]

Then the solution satisfies the inequalities

\[G_{16}^0 e^{((S_1)^{(2)}-(P_{16})^{(2)})t} \leq G_{16}^0 e^{(S_1)^{(2)}t} \]

\[(p_j)^{(2)} \text{ is defined} \]

\[
\frac{1}{(m_2)^{(2)}T_{16}^{0}} \leq G_{17}^0 (t) \leq \frac{1}{(m_2)^{(2)}T_{16}^{0}} G_{16}^0 e^{(S_1)^{(2)}t} \]

\[
\frac{(a_{16})^{(2)}(a_{16})^{(2)}}{(m_1)^{(2)}(S_1)^{(2)}-(P_{16})^{(2)}-(S_2)^{(2)})} \left[e^{((S_1)^{(2)}-(P_{16})^{(2)})t} - e^{((S_1)^{(2)}-(P_{16})^{(2)})t} \right] + G_{18}^0 e^{-((S_2)^{(2)}t)} \leq G_{18}^0 (t) \leq \frac{(a_{18})^{(2)}}{(m_2)^{(2)}(S_1)^{(2)}-(a_{18})^{(2)}} \left[e^{((S_1)^{(2)}-(a_{18})^{(2)}t)} - e^{((S_1)^{(2)}-(a_{18})^{(2)}t)} \right] + G_{18}^0 e^{-((a_{18})^{(2)}t)} \]

\[T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}^0 e^{((R_1)^{(2)}+(R_1)^{(2)})t} \]

\[
\frac{1}{(\mu_2)^{(2)}T_{16}^{0}} \leq T_{16}^0 e^{((R_1)^{(2)}+(R_1)^{(2)})t} \leq \frac{1}{(\mu_2)^{(2)}T_{16}^{0}} T_{16}^0 e^{(R_1)^{(2)}+(R_1)^{(2)}t} \]

\[
\frac{(b_{16})^{(2)}}{(\mu_2)^{(2)}(R_1)^{(2)}+(R_1)^{(2)}+(R_2)^{(2)})} \left[e^{((R_1)^{(2)}+(R_1)^{(2)}t)} - e^{((R_1)^{(2)}+(R_1)^{(2)}t)} \right] + T_{18}^0 e^{-((R_1)^{(2)}t)} \leq T_{18}^0 e^{-((R_1)^{(2)}t)} \]

\[
\frac{(a_{18})^{(2)}}{(\mu_2)^{(2)}(R_1)^{(2)}+(R_1)^{(2)}+(R_2)^{(2)})} \left[e^{((R_1)^{(2)}+(R_1)^{(2)}t)} - e^{((R_1)^{(2)}+(R_1)^{(2)}t)} \right] + T_{18}^0 e^{-((R_1)^{(2)}t)} \]

**Definition of** \((S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}:\)

Where \((S_2)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a_{16})^{(2)} \)

\((S_2)^{(2)} = (a_{16})^{(2)} - (P_{16})^{(2)} \)

\((R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(2)} - (b_{16})^{(2)} \)

\((R_2)^{(2)} = (b_{16})^{(2)} - (I_{16})^{(2)} \)

**Behavior Of The Solutions**

If we denote and define

**Definition of** \((\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}:\)

114
\( \sigma_3 \), \( \sigma_2 \), \( \tau_1 \), \( \tau_2 \) four constants satisfying

\[-\sigma_3 \leq -(a_{10}^{(3)}) + (a_{21}^{(3)}) - (a_{20}^{(3)}(T_{21}, t) + (a_{21}^{(3)})(T_{21}, t) \leq -\sigma_1 \]

\[-\tau_3 \leq -(b_{10}^{(3)}) + (b_{21}^{(3)}) - (b_{20}^{(3)}(G, t) - (b_{21}^{(3)})((G_{23}, t) \leq -\tau_1 \]

**Definition of** \( (v_1, v_2, u_1, u_2) \)

(b) By \( (v_1, v_2) > 0 \), \( (u_1, u_2) < 0 \) and respectively \( (u_1, u_2) < 0 \), \( (v_1, v_2) > 0 \) the roots of the equations \( (a_{21}^{(3)}(v) + (\sigma_1)(v) - (a_{20}^{(3)}) = 0 \)

and \((b_{21}^{(3)}(u) + \tau_2(u) - (b_{20}^{(3)}) = 0 \)

By \( (v_1, v_2) > 0 \), \( (u_1, u_2) < 0 \) and respectively \( (u_1, u_2) < 0 \), \( (v_1, v_2) > 0 \) the roots of the equations \( (a_{21}^{(3)}(v) + (\sigma_2)(v) - (a_{20}^{(3)}) = 0 \)

and \((b_{21}^{(3)}(u) + \tau_2(u) - (b_{20}^{(3)}) = 0 \)

**Definition of** \( (m_1, m_2, \mu_1, \mu_2) \)

(c) If we define \( (m_1, m_2, \mu_1, \mu_2) \) by

\( (m_2) = (v_0, m_1) = (v_1), \text{ if } (v_0) < (v_1) \)

\( (m_2) = (v_1, m_1) = (v_1), \text{ if } (v_1) < (v_0) \)

\[ (m_2) = (v_0, m_1) = (v_0), \text{ if } (v_0) < (v_0) \]

and analogously

\( (\mu_2) = (u_0, \mu_1) = (u_1), \text{ if } (u_0) < (u_1) \)

\( (\mu_2) = (u_1, \mu_1) = (u_1), \text{ if } (u_1) < (u_0) \)

\( (\mu_2) = (u_0, \mu_1) = (u_0), \text{ if } (u_0) < (u_0) \)

Then the solution satisfies the inequalities

\( G_{20}^0 e^{((s_j) - (p_{20})t)} \leq G_{20}^0 e^{(s_j)} \leq G_{20}^0 e^{(s_j)} \)

\( (p_1) \) is defined

\[ \frac{1}{(m_1)^3} G_{20}^0 e^{((s_j) - (p_{20})t)} \leq G_{21}^0 \leq \frac{1}{(m_2)^3} G_{20}^0 e^{(s_j)} \]

\[ \frac{1}{(m_1)^3} e^{((s_j) - (p_{20})t)} + \frac{1}{(m_2)^3} e^{(s_j)} \]

\[ T_{20}^0 e^{(r_j)} \leq T_{20}^0 \leq T_{20}^0 e^{(r_j)} \]
\[
\frac{1}{(\mu_2)^3}T_{20}^0e^{(R_1)^3(t)} \leq \frac{1}{(\mu_2)^3}T_{20}^0e^{([R_1] + (R_2)^3) t} 
\]
\[
= \frac{1}{(\mu_2)^3(R_1)^3(b_{22})^3 - (b_{22})^3} \left[ e^{(R_1)^3(t)} - e^{-(b_{22})^3(t)} \right] + T_{22}^0e^{-(b_{22})^3(t)} \leq T_{22}(t) \leq 
\]
\[
= \frac{1}{(\mu_2)^3((R_1)^3 + (R_2)^3)} \left[ e^{([R_1] + (R_2)^3) t} - e^{-(R_2)^3(t)} \right] + T_{22}^0e^{-(R_2)^3(t)}
\]

**Definition of** \((S_1)^3, (S_2)^3, (R_1)^3, (R_2)^3, \): 

Where \((S_1)^3 = (a_{20})^3(m_2^3) - (a_{20})^3\)

\((S_2)^3 = (a_{22})^3 - (\mu_2)^3\)

\((R_1)^3 = (b_{20})^3(\mu_2)^3 - (b_{20})^3\)

\((R_2)^3 = (b_{22})^3 - (\mu_2)^3\)

If we denote and define 

**Definition of** \((\sigma_1)^4, (\sigma_2)^4, (\tau_1)^4, (\tau_2)^4, \): 

(d) \((\sigma_1)^4, (\sigma_2)^4, (\tau_1)^4, (\tau_2)^4, \) four constants satisfying

\(-\sigma_1^4 \leq -a_{25}^4 + (a_{25}^4) - (a_{25}^4)(T_{25}, t) + (a_{25}^4)(T_{25}, t) \leq -\sigma_1^4\)

\(-\tau_1^4 \leq -b_{25}^4 + (b_{25}^4) - (b_{25}^4)(G_{25}, t) - (b_{25}^4)(G_{25}, t) \leq -\tau_1^4\)

**Definition of** \((v_1)^4, (v_2)^4, (u_1)^4, (u_2)^4, (v_1)^4, (u_1)^4, \): 

(e) By \((v_1)^4 > 0, (v_2)^4 < 0\) and respectively \((u_1)^4 > 0, (u_2)^4 < 0\) the roots of the equations \((a_{25})^4(v_1)^4 = 0\) and \((a_{25})^4(v_1)^4 = 0\)

and \((b_{25})^4(u_1)^4 = 0\) and \((b_{25})^4(u_1)^4 = 0\)

**Definition of** \((\bar{v}_1)^4, (\bar{v}_2)^4, (\bar{u}_1)^4, (\bar{u}_2)^4, \): 

By \((\bar{v}_1)^4 > 0, (\bar{v}_2)^4 < 0\) and respectively \((\bar{u}_1)^4 > 0, (\bar{u}_2)^4 < 0\) the roots of the equations \((a_{25})^4(v_1)^4 = 0\) and \((a_{25})^4(v_1)^4 = 0\)

and \((b_{25})^4(u_1)^4 = 0\) and \((b_{25})^4(u_1)^4 = 0\)

**Definition of** \((m_1)^4, (m_2)^4, (\mu_1)^4, (\mu_2)^4, (v_0)^4, \): 

(f) If we define \((m_1)^4, (m_2)^4, (\mu_1)^4, (\mu_2)^4, (v_0)^4, \) by

\[(m_2)^4 = (v_0)^4, (m_1)^4 = (v_1)^4, \text{if } (v_0)^4 < (v_1)^4\]

\[(m_2)^4 = (v_1)^4, (m_1)^4 = (\bar{v}_1)^4, \text{if } (v_0)^4 < (\bar{v}_1)^4, \]

and 

\[(v_0)^4 = \frac{u_{25}}{u_{25}}\]
\((m_2)_{(4)} = (v_2)_{(4)} = (v_0)_{(4)}, \text{ if } (\bar{v}_4)_{(4)} < (v_0)_{(4)}\)

and analogously

\((\mu_2)_{(4)} = (u_0)_{(4)} = (u_1)_{(4)}, \text{ if } (u_0)_{(4)} < (u_1)_{(4)}\)

Then the solution satisfies the inequalities

\[ G_{24}^0 e^{((S_1)_{(4)} - (p_{24})_{(4)}) t} \leq G_{24} e^{((S_1)_{(4)}) t} \]

where \((p_{(4)})_{(4)}\) is defined

\[ \frac{1}{(m_2)_{(4)}} G_{24}^0 e^{((S_1)_{(4)} - (p_{24})_{(4)}) t} \leq G_{25} \leq \frac{1}{(m_3)_{(4)}} G_{24} e^{((S_1)_{(4)}) t} \]

\[ \frac{(a_{24})_{(4)} G_{24}^0}{(m_3)_{(4)}} \left[ e^{((S_1)_{(4)} - (p_{24})_{(4)}) t} - e^{-(S_2)_{(4)}) t} \right] + G_{26}^0 e^{-(S_2)_{(4)}) t} \leq G_{26} \leq G_{26}^0 e^{-(S_2)_{(4)}) t} \]

\[ T_{24}^0 e^{((R_1)_{(4)} t) \leq T_{24} e^{((R_1)_{(4)} + (r_{24})_{(4)}) t} \leq T_{24}^0 e^{((R_1)_{(4)} + (r_{24})_{(4)}) t} \]

\[ \frac{1}{(\mu_1)_{(4)}} T_{24}^0 e^{((R_1)_{(4)} t) \leq T_{24} \leq \frac{1}{(\mu_1)_{(4)}} T_{24}^0 e^{((R_1)_{(4)} + (r_{24})_{(4)}) t} \]

\[ \frac{(b_{24})_{(4)} T_{24}^0}{(\mu_1)_{(4)}(r_{24})_{(4)} - (b_{24})_{(4)}} \left[ e^{((R_1)_{(4)} t) - e^{-(b_{24})_{(4)}} t} \right] + T_{26}^0 e^{-(b_{24})_{(4)} t} \leq T_{26} \leq T_{26}^0 e^{-(b_{24})_{(4)} t} \]

\[ \frac{(a_{24})_{(4)} T_{24}^0}{(\mu_2)_{(4)}(r_{24})_{(4)} + (a_{24})_{(4)}} \left[ e^{((R_1)_{(4)} + (r_{24})_{(4)}) t) - e^{-(R_2)_{(4)}} t} \right] + T_{26} e^{-(R_2)_{(4)} t} \]

**Definition of \((S_1)_{(4)}, (S_2)_{(4)}, (R_1)_{(4)}, (R_2)_{(4)}:\)**

Where \((S_1)_{(4)} = (a_{24})_{(4)}(m_2)_{(4)} - (a'_{24})_{(4)}\)

\[(S_2)_{(4)} = (a_{26})_{(4)} - (p_{26})_{(4)}\]

\[(R_1)_{(4)} = (b_{24})_{(4)}(\mu_2)_{(4)} - (b'_{24})_{(4)}\]

\[(R_2)_{(4)} = (b'_{26})_{(4)} - (r_{26})_{(4)}\]

**Behavior of the solutions**

If we denote and define

**Definition of \((\sigma_1)_{(5)}, (\sigma_2)_{(5)}, (\tau_1)_{(5)}, (\tau_2)_{(5)}:\)**
(g) \((\sigma_1^{(5)}, \sigma_2^{(5)}, r_1^{(5)}, r_2^{(5)})\) four constants satisfying
\[-(\sigma_2^{(5)}) \leq -(a_{28}^{(5)}) + (a_{29}^{(5)}) - (a_{28}^{(5)})(T_{29}, t) + (a_{28}^{(5)})(T_{29}, t) \leq -(\sigma_1^{(5)})\]
\[-(r_2^{(5)}) \leq -(b_{28}^{(5)}) + (b_{28}^{(5)}) - (b_{28}^{(5)})(G_{31}, t) - (b_{28}^{(5)})(G_{31}, t) \leq -(r_1^{(5)})\]

**Definition of** \((v_1^{(5)}, v_2^{(5)}, u_1^{(5)}, u_2^{(5)}, v^{(5)}, u^{(5)})\):

(h) By \((v_1^{(5)}) > 0, (v_2^{(5)}) < 0\) and respectively \((u_1^{(5)}) > 0, (u_2^{(5)}) < 0\) the roots of the equations \((a_{29}^{(5)})(v^{(5)})^2 + (\sigma_1^{(5)})v^{(5)} - (a_{28}^{(5)}) = 0\)
and \((b_{29}^{(5)})(u^{(5)})^2 + (r_1^{(5)})u^{(5)} - (b_{28}^{(5)}) = 0\)

**Definition of** \((\bar{v}_1^{(5)}, \bar{v}_2^{(5)}, \bar{u}_1^{(5)}, \bar{u}_2^{(5)})\):

By \((\bar{v}_1^{(5)}) > 0, (\bar{v}_2^{(5)}) < 0\) and respectively \((\bar{u}_1^{(5)}) > 0, (\bar{u}_2^{(5)}) < 0\) the roots of the equations \((a_{28}^{(5)})(v^{(5)})^2 + (\sigma_2^{(5)})v^{(5)} - (a_{28}^{(5)}) = 0\)
and \((b_{28}^{(5)})(u^{(5)})^2 + (r_2^{(5)})u^{(5)} - (b_{28}^{(5)}) = 0\)

**Definition of** \((m_1^{(5)}, m_2^{(5)}, \mu_1^{(5)}, \mu_2^{(5)}, \nu_0^{(5)})\):

(i) If we define \((m_1^{(5)}, m_2^{(5)}, \mu_1^{(5)}, \mu_2^{(5)})\)

\[
(m_2^{(5)}) = (v_0^{(5)}), (m_1^{(5)}) = (v_1^{(5)}), \text{if } (v_0^{(5)}) < (v_1^{(5)})
\]
\[
(m_2^{(5)}) = (v_1^{(5)}), (m_1^{(5)}) = (\bar{v}_1^{(5)}), \text{if } (v_1^{(5)}) < (v_0^{(5)}) < (\bar{v}_1^{(5)})
\]
and
\[
(v_0^{(5)}) = \frac{\sigma_0^{(5)}}{\sigma_2^{(5)}}
\]
\[
(m_2^{(5)}) = (v_1^{(5)}), (m_1^{(5)}) = (v_0^{(5)}), \text{if } (v_1^{(5)}) < (v_0^{(5)})
\]

and analogously
\[
(\mu_2^{(5)}) = (u_0^{(5)}), (\mu_1^{(5)}) = (u_1^{(5)}), \text{if } (u_0^{(5)}) < (u_1^{(5)})
\]
\[
(\mu_2^{(5)}) = (u_1^{(5)}), (\mu_1^{(5)}) = (\bar{u}_1^{(5)}), \text{if } (u_1^{(5)}) < (u_0^{(5)}) < (\bar{u}_1^{(5)})
\]
and
\[
(u_0^{(5)}) = \frac{\tau_0^{(5)}}{\tau_2^{(5)}}
\]
\[
(\mu_2^{(5)}) = (u_1^{(5)}), (\mu_1^{(5)}) = (u_0^{(5)}), \text{if } (u_1^{(5)}) < (u_0^{(5)}) \text{ where } (u_1^{(5)}), (\bar{u}_1^{(5)})
\]
are defined respectively

Then the solution satisfies the inequalities
\[
G_{28}^0 e^{((\pi_1^{(5)} - p_{28}^{(5)})t)} \leq G_{28}(t) \leq G_{28}^0 e^{((\pi_1^{(5)} - p_{28}^{(5)})t)}
\]
where \((p_1^{(5)})\) is defined
\[
\begin{align*}
\frac{1}{(m_2)^5} G_{28}^e (s_1^{(5)} - (p_{28}e^{(5)})) e(t) \leq G_{28}(t) \leq \frac{1}{(m_2)^5} G_{28}^e (s_2^{(5)}) e(t) & \quad (368) \\
\left(\frac{a_{30}^2}{m_1} \right)^{\frac{5}{2}} \left(\frac{(s_1^{(5)} - (p_{28}e^{(5)}))}{(s_2^{(5)})}\right)^{\frac{5}{2}} \left[ e^{(s_1^{(5)} - (p_{28}e^{(5)})) e(t)} - e^{-(s_2^{(5)}) e(t)} \right] + G_{30}^e e^{-(s_2^{(5)}) e(t)} \leq G_{30}(t) \leq & \quad (369) \\
\left(\frac{a_{30}^2}{m_2}\right)^{\frac{5}{2}} \left(\frac{(s_1^{(5)} - (a_{28}^s))}{(s_2^{(5)})}\right)^{\frac{5}{2}} \left[ e^{(s_1^{(5)} - (a_{28}^s)) e(t)} - e^{-(a_{28}^s)} e(t) \right] + G_{28}^e e^{-(a_{28}^s)} e(t) & \quad (370) \\
\frac{1}{(\mu_1)^5} T_{28}^e (r_1^{(5)}) e(t) \leq T_{28}(t) \leq \frac{1}{(\mu_2)^5} T_{28}^e ((r_1^{(5)} + (r_{28}e^{(5)})) e(t) & \quad (371) \\
\left(\frac{b_{30}^2}{(r_1^{(5)} - (b_{28}^s))}\right)^{\frac{5}{2}} \left(\frac{(r_1^{(5)} - (b_{28}^s))}{(b_{30}^s)}\right)^{\frac{5}{2}} \left[ e^{(r_1^{(5)} - (b_{28}^s)) e(t)} - e^{-(b_{28}^s)} e(t) \right] + T_{30}^e e^{-(b_{28}^s)} e(t) \leq & \quad (372) \\
\left(\frac{a_{30}^2}{(r_1^{(5)} + (r_{28}e^{(5)}))}\right)^{\frac{5}{2}} \left(\frac{(r_1^{(5)} + (r_{28}e^{(5)}))}{(r_{30}^s)}\right)^{\frac{5}{2}} \left[ e^{(r_1^{(5)} + (r_{28}e^{(5)})) e(t)} - e^{-(r_{30}^s)} e(t) \right] + T_{30}^e e^{-(r_{30}^s)} e(t) & \quad (373)
\end{align*}
\]

**Definition of** \( (S_1^{(5)}, S_2^{(5)}), (R_1^{(5)}, R_2^{(5)}) \):

Where \( (S_1^{(5)}) = (a_{28}^s (m_2)^5) - (a_{28}^s) \)

\( (S_2^{(5)}) = (a_{28}^s (m_2)^5) - (p_{28}e^{(5)}) \)

\( (R_2^{(5)}) = (b_{28}^s (\mu_2)^5) - (b_{28}^s) \)

\( (R_2^{(5)}) = (b_{28}^s (\mu_2)^5) - (r_{30}^s) \)

**Behavior of the solutions**

If we denote and define

**Definition of** \( \sigma_1^{(6)}, \sigma_2^{(6)}, \tau_1^{(6)}, \tau_2^{(6)} \):

\( (\sigma_1^{(6)}, \sigma_2^{(6)}, \tau_1^{(6)}, \tau_2^{(6)}) \) four constants satisfying

\( -\sigma_2^{(6)} \leq -(a_{32}^{\prime}(6) + (a_{32}^{\prime}(6)) (T_{33}, t) + (a_{32}^{\prime}(6)) (G_{35}, t) \leq -\sigma_1^{(6)} \)

\( -\tau_2^{(6)} \leq -(b_{32}^{\prime}(6) + (b_{32}^{\prime}(6)) (G_{35}, t) \leq -\tau_1^{(6)} \)

**Definition of** \( (\nu^{(6)}, \nu^{(6)}), (u_1^{(6)}, (u_2^{(6)}, \nu^{(6)}, u^{(6)}} \):

\( (\nu^{(6)}), (\nu^{(6)}), (u_1^{(6)}, (u_2^{(6)}, \nu^{(6)}, u^{(6)}} \):

\( (\nu^{(6)}), (\nu^{(6)}), (u_1^{(6)}, (u_2^{(6)}, \nu^{(6)}, u^{(6)}} \):

**Definition of** \( (\bar{\nu}^{(6)}, (\bar{\nu}^{(6)}), (\bar{u}^{(6)}), (\bar{u}^{(6)}} \):

By \( (\nu^{(6)}), (\nu^{(6)}), (u_1^{(6)}, (u_2^{(6)}, \nu^{(6)}, u^{(6)}} \):

\( (\nu^{(6)}), (\nu^{(6)}), (u_1^{(6)}, (u_2^{(6)}, \nu^{(6)}, u^{(6)}} \):

\( (\nu^{(6)}), (\nu^{(6)}), (u_1^{(6)}, (u_2^{(6)}, \nu^{(6)}, u^{(6)}} \):

and \( (b_{33}^{(6)} (u^{(6)}))^{(6)} + (\tau_2^{(6)})(u^{(6)}))^{(6)} - (b_{32}^{(6)} = 0 \)

and \( (b_{33}^{(6)} (u^{(6)}))^{(6)} + (\tau_2^{(6)})(u^{(6)}))^{(6)} - (b_{32}^{(6)} = 0 \)
Definition of \((m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}\):

(i) If we define \((m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}\) by

\[
(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \text{ if } (\nu_0)^{(6)} < (\nu_1)^{(6)}
\]

\[
(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_0)^{(6)}, \text{ if } (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},
\]

and

\[
(\nu_0)^{(6)} = \frac{\nu_0^{(0)}}{\nu_0^{(3)}},
\]

\[
(\nu_1)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \text{ if } (\nu_1)^{(6)} < (\nu_0)^{(6)}
\]

and analogously

\[
(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \text{ if } (u_0)^{(6)} < (u_1)^{(6)}
\]

\[
(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \text{ if } (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},
\]

and

\[
(u_0)^{(6)} = \frac{u_0^{(0)}}{u_0^{(3)}},
\]

\[
(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (\mu_2)^{(6)} < (u_0)^{(6)}
\]

are defined respectively.

Then the solution satisfies the inequalities

\[
G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}^0 e^{(S_1)^{(6)}t} \leq G_{32}^0 e^{((S_1)^{(6)} + (p_{32})^{(6)})t}
\]

where \((p_1)^{(6)}\) is defined

\[
\frac{1}{(\mu_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}^0 (t) \leq \frac{1}{(\mu_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t}
\]

\[
\left(\frac{(a_{32})^{(6)} }{(m_1)^{(6)}} \right) G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \left[ e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t} \right] + G_{34}^0 e^{-(S_2)^{(6)}t} \leq G_{34}^0 (t) \leq e^{((S_1)^{(6)} + (p_{32})^{(6)})t} \right)
\]

\[
T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}^0 e^{((R_1)^{(6)} + (R_2)^{(6)})t}
\]

\[
\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}^0 (t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (R_2)^{(6)})t}
\]

\[
\left(\frac{(b_{34})^{(6)} }{(\mu_2)^{(6)}} \right) e^{((R_1)^{(6)} - (b_{34})^{(6)})t} + T_{34}^0 e^{-(b_{34})^{(6)}t} \leq T_{34}^0 (t) \leq e^{((R_1)^{(6)} + (R_2)^{(6)})t} \right)
\]

Definition of \((S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}\):

Where

\[
(S_1)^{(6)} = (a_{32})^{(6)} (m_2)^{(6)} - (a_{32})^{(6)}
\]

\[
(S_2)^{(6)} = (a_{34})^{(6)} -(p_{34})^{(6)}
\]

120
\[
(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}
\]
\[
(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}
\]

If we denote and define

**Definition of** \((\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}:\)**

\((m)\) \((\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}\) four constants satisfying

\[-(\sigma_2)^{(7)} \leq -(a_{36}^{(7)} + (a_{57}^{(7)} - (a_{56}^{(7)})(T_{37}, t) + (a_{57}^{(7)})(T_{37}, t) \leq -(\sigma_1)^{(7)}\]

\[-(\tau_2)^{(7)} \leq -(b_{36}^{(7)} + (b'_{37}^{(7)} - (b'_{36}^{(7)})(G_{39}, t) - (b_{37}^{(7)})(G_{39}, t) \leq -(\tau_1)^{(7)}\]

**Definition of** \((v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}:\)**

\((n)\) By \((v_1)^{(7)} > 0, (v_2)^{(7)} < 0\) and respectively \((u_1)^{(7)} > 0, (u_2)^{(7)} < 0\) the roots of the equations

\[(a_{37}^{(7)}(v^{(7)})^2 + (a_{57}^{(7)}v^{(7)} - (a_{36}^{(7)} = 0\]

and \((b_{37}^{(7)}(u^{(7)})^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36}^{(7)} = 0\)

**Definition of** \((\bar{v}_1)^{(7)}, (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}:\)**

By \((\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0\) and respectively \((\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0\) the roots of the equations

\[(a_{37}^{(7)}(v^{(7)})^2 + (a_{57}^{(7)}v^{(7)} - (a_{36}^{(7)} = 0\]

and \((b_{37}^{(7)}(u^{(7)})^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36}^{(7)} = 0\)

**Definition of** \((m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}:\)**

\((o)\) If we define \((m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}\) by

\[(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}, \text{ if } (v_0)^{(7)} < (v_1)^{(7)}\]

\[(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}, \text{ if } (v_1)^{(7)} < (\bar{v}_1)^{(7)},\]

and \( (v_0)^{(7)} = \frac{\bar{v}_0}{\bar{v}_1} \)

\[(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}, \text{ if } (\bar{v}_1)^{(7)} < (v_0)^{(7)}\]

and analogously

\[(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \text{ if } (u_0)^{(7)} < (u_1)^{(7)}\]

\[(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \text{ if } (u_1)^{(7)} < (\bar{u}_1)^{(7)},\]

and \( (u_0)^{(7)} = \frac{\bar{u}_0}{\bar{u}_1} \)

\[(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \text{ if } (\bar{u}_1)^{(7)} < (u_0)^{(7)}\]

are defined respectively

Then the solution satisfies the inequalities
Definition of $(p)$

Theorem 2: Behavior of the solutions

Definition of $(q)$

If we denote and define

Where

Definition of $(S_1)$, $(S_2)$, $(R_1)$, $(R_2)$:

Where $(S_1) = (a_{36}^7(m_2)^7 - (a_{38}^7)^7)$

$(S_2) = (a_{38}^7)^7 - (p_{38}^7)^7$

$(R_1) = (b_{36}^7(\mu_2)^7 - (b_{38}^7)^7)$

$(R_2) = (b_{38}^7)^7 - (r_{38}^7)^7$

Behavior of the solutions

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}$, $(\sigma_2)^{(8)}$, $(\tau_1)^{(8)}$, $(\tau_2)^{(8)}$:

$(p)$ $(\sigma_2)^{(8)}$, $(\sigma_2)^{(8)}$, $(\tau_1)^{(8)}$, $(\tau_2)^{(8)}$ four constants satisfying

Definition of $(v_1)^{(8)}$, $(v_2)^{(8)}$, $(u_1)^{(8)}$, $(u_2)^{(8)}$, $(\nu)^{(8)}$, $(u)^{(8)}$:

$(q)$ By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_4)^{(8)}(\nu)^{(8)} + (\sigma_1)^{(8)}(u)^{(8)} - (a_4)^{(8)} = 0$

and $(b_{41})^{(8)}(u)^{(8)} + (\tau_1)^{(8)}(u)^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\tilde{v}_1)^{(8)}$, $(\tilde{v}_2)^{(8)}$, $(\tilde{u}_1)^{(8)}$, $(\tilde{u}_2)^{(8)}$:

By $(\tilde{v}_1)^{(8)} > 0$, $(\tilde{v}_2)^{(8)} < 0$ and respectively $(\tilde{u}_1)^{(8)} > 0$, $(\tilde{u}_2)^{(8)} < 0$ the
roots of the equations $(a_{41})^0(v^0)^2 + (\sigma_2)^0v^0 - (a_{40})^0 = 0$

and $(b_{41})^0(u^0)^2 + (\tau_2)^0u^0 - (b_{40})^0 = 0$

**Definition of** $(m_1)^0, (m_2)^0, (\mu_1)^0, (\mu_2)^0, (v_0)^0, (v_1)^0, (\tilde{v}_1)^0$:

If we define $(m_1)^0, (m_2)^0, (\mu_1)^0, (\mu_2)^0, (v_0)^0, (v_1)^0, (\tilde{v}_1)^0$ by

$(m_2)^0 = (v_0)^0, (m_1)^0 = (v_1)^0, \text{ if } (v_0)^0 < (v_1)^0$

$(m_2)^0 = (v_1)^0, (m_1)^0 = (\tilde{v}_1)^0, \text{ if } (v_0)^0 < (v_1)^0 < (\tilde{v}_1)^0,$

and

$(v_0)^0 = \frac{e_{40}^0}{e_{41}^0}$

$(m_2)^0 = (v_1)^0, (m_1)^0 = (v_0)^0, \text{ if } (v_0)^0 < (v_0)^0$

and analogously

$(\mu_2)^0 = (u_0)^0, (\mu_1)^0 = (u_1)^0, \text{ if } (u_0)^0 < (u_1)^0$

$(\mu_2)^0 = (u_1)^0, (\mu_1)^0 = (\tilde{u}_1)^0, \text{ if } (u_0)^0 < (u_0)^0 < (\tilde{u}_1)^0,$

and

$(u_0)^0 = \frac{T_4^0}{T_4^1}$

$(\mu_2)^0 = (u_1)^0, (\mu_1)^0 = (u_0)^0, \text{ if } (u_0)^0 < (u_0)^0$ where $(u_1)^0, (\tilde{u}_1)^0$ are defined respectively

Then the solution of GLOBAL EQUATIONS satisfies the inequalities

$G_{40}^0e^{(s_1)^0(t)-(p_{40})^0t} \leq G_{41}^0(t) \leq G_{40}^0e^{(s_1)^0(t)}$

where $(p_j)^0$ is defined

\[
\frac{1}{(m_1^0)^0}G_{40}^0e^{((s_1)^0-(p_{40})^0)t} \leq G_{41}^0(t) \leq \frac{1}{(m_2^0)^0}G_{40}^0e^{((s_1)^0)t}
\]

\[
\frac{(a_{42})^0}{(m_1^0)^0((s_1)^0-(p_{40})^0)}\left[e^{((s_1)^0-(p_{40})^0)t} - e^{-(s_2)^0(t)}\right] + G_{42}^0e^{-(s_2)^0t} \leq G_{42}^0(t) \leq \frac{(a_{42})^0}{(m_2^0)^0((s_1)^0-(p_{40})^0)}\left[e^{(s_1)^0t} - e^{-(a_{42})^0(t)}\right] + G_{42}^0e^{-(a_{42})^0(t)}
\]

$T_{40}^0e^{(R_1)^0} \leq T_{40}^0(t) \leq T_{40}^0e^{((R_1)^0+(r_{40})^0)t}$

$T_{40}^0e^{(R_2)^0} \leq T_{40}^0(t) \leq T_{40}^0e^{((R_2)^0+(r_{40})^0)t}$

**Definition of** $(S_1)^0, (S_2)^0, (R_1)^0, (R_2)^0$:
Where \((S_1)_{0}^{(8)} = (a_{40})_{0}^{(8)}(m_2)_{0}^{(8)} - (a_{40})_{0}^{(8)}\)
\[(S_2)_{0}^{(8)} = (a_{42})_{0}^{(8)} - (p_{42})_{0}^{(8)}\]
\[(R_1)_{0}^{(8)} = (b_{40})_{0}^{(8)}(μ_2)_{0}^{(8)} - (b_{40})_{0}^{(8)}\]
\[(R_2)_{0}^{(8)} = (b_{42})_{0}^{(8)} - (r_{42})_{0}^{(8)}\]

From GLOBAL EQUATIONS we obtain
\[
\frac{dv_{0}^{(8)}}{dt} = (a_{40})_{0}^{(8)} - \left( (a'_{40})_{0}^{(8)} - (a_{41})_{0}^{(8)} + (a''_{40})_{0}^{(8)}(T_{41}, t) \right) - (a_{42})_{0}^{(8)}(T_{41}, t)v_{0}^{(8)} - (a_{41})_{0}^{(8)}v_{0}^{(8)}
\]

**Definition of** \(v_{0}^{(8)}\) :
\[
v_{0}^{(8)} = \frac{\dot{a}_{40}}{\dot{a}_{41}}
\]

It follows
\[
-(a_{41})_{0}^{(8)}(v_{0}^{(8)})^2 + (\sigma_{41})_{0}^{(8)}v_{0}^{(8)} - (a_{40})_{0}^{(8)} \leq \frac{dv_{0}^{(8)}}{dt} \leq -(a_{41})_{0}^{(8)}(v_{0}^{(8)})^2 + (\sigma_{41})_{0}^{(8)}v_{0}^{(8)} - (a_{40})_{0}^{(8)}
\]

From which one obtains

**Definition of** \((\dot{v}_1)_{(0)}, v_{0}^{(8)}\) :

(a) For \(0 < \frac{(v_{0})_{(0)} - (v_{1})_{(0)}}{\dot{v}_1} < \frac{(v_{1})_{(0)} - (v_{2})_{(0)}}{\dot{v}_1}\)

\[
v_{(0)}(t) \geq \frac{(v_1)_{(0)} + (C)_{(0)}(v_2)_{(0)}e^{-[(a_{41})_{(0)}(v_{1})_{(0)} - (v_{0})_{(0)})]t}}{1 + (C)_{(0)}e^{-[(a_{41})_{(0)}(v_{0})_{(0)} - (v_{1})_{(0)})]t}} \quad , \quad (C)_{(0)} = \frac{(v_{1})_{(0)} - (v_{0})_{(0)}}{(v_{0})_{(0)} - (v_{1})_{(0)}}
\]

it follows \((v_{0})_{(0)} \leq v_{(0)}(t) \leq (v_{1})_{(0)}\)

In the same manner, we get

\[
v_{(0)}(t) \leq \frac{(v_1)_{(0)} + (C)_{(0)}(v_2)_{(0)}e^{-[(a_{41})_{(0)}(v_{1})_{(0)} - (v_{2})_{(0)})]t}}{1 + (C)_{(0)}e^{-[(a_{41})_{(0)}(v_{2})_{(0)} - (v_{1})_{(0)})]t}} \quad , \quad (C)_{(0)} = \frac{(v_{1})_{(0)} - (v_{0})_{(0)}}{(v_{0})_{(0)} - (v_{2})_{(0)}}
\]

From which we deduce \((v_{0})_{(0)} \leq v_{(0)}(t) \leq (v_{2})_{(0)}\)

(b) If \(0 < (v_{1})_{(0)} < (v_{0})_{(0)} = \frac{6\dot{a}_{40}}{4\dot{a}_{41}} < (v_{2})_{(0)}\) we find like in the previous case,

\[
(v_{1})_{(0)} \leq \frac{(v_1)_{(0)} + (C)_{(0)}(v_2)_{(0)}e^{-[(a_{41})_{(0)}(v_{1})_{(0)} - (v_{2})_{(0)})]t}}{1 + (C)_{(0)}e^{-[(a_{41})_{(0)}(v_{1})_{(0)} - (v_{2})_{(0)})]t}} \leq v_{(0)}(t) \leq \frac{(v_1)_{(0)} + (C)_{(0)}(v_2)_{(0)}e^{-[(a_{41})_{(0)}(v_{1})_{(0)} - (v_{2})_{(0)})]t}}{1 + (C)_{(0)}e^{-[(a_{41})_{(0)}(v_{1})_{(0)} - (v_{2})_{(0)})]t}} \leq (v_{2})_{(0)}
\]

(c) If \(0 < (v_{1})_{(0)} \leq (v_{2})_{(0)} \leq \frac{6\dot{a}_{40}}{4\dot{a}_{41}}\), we obtain

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\[ (v_1)(t) \leq \psi(t) \leq (v_0)(t) \leq \frac{e^{-\frac{(v_1)(t)-(v_2)(t)}{2}}}{1+e^{-\frac{(v_1)(t)-(v_2)(t)}{2}}} \leq (v_0)(t) \]

And so with the notation of the first part of condition (c), we have

**Definition of** \( \psi(t) \) :

\[ (m_2)(t) \leq \psi(t) \leq (m_1)(t), \quad \psi(t) = \frac{G_{40}(t)}{G_{41}(t)} \]

In a completely analogous way, we obtain

**Definition of** \( u(t) \) :

\[ (m_2)(t) \leq u(t) \leq (m_1)(t), \quad u(t) = \frac{T_{40}(t)}{T_{41}(t)} \]

Now, using this result and replacing it in CONCATENATED GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case** :

If \( (a_{40})(t) = (a_{41})(t) \), then \( (\sigma_1)(t) = (\sigma_2)(t) \) and in this case \( (v_1)(t) = (\psi_2)(t) \) if in addition \( (v_0)(t) = (v_1)(t) \) then \( \psi(t)(t) = (v_0)(t) \) and as a consequence \( G_{40}(t) = (v_0)(t) G_{41}(t) \) this also defines \( (v_0)(t) \) for the special case.

Analogously if \( (b_{40})(t) = (b_{41})(t) \), then \( (\tau_1)(t) = (\tau_2)(t) \) and then

\[ (u_1)(t) = (\bar{u}_1)(t) \] if in addition \( (u_0)(t) = (u_1)(t) \) then \( T_{40}(t) = (u_0)(t) T_{41}(t) \) This is an important consequence of the relation between \( (v_1)(t) \) and \( (\bar{v}_1)(t) \), and definition of \( (u_0)(t) \).

: From GLOBAL EQUATIONS we obtain

\[ \frac{dv_1}{dt} = (a_{24})(4) - ((a_{24})(4) + (a_{24})(4)(T_{25}, t)) - (a_{25})(4)(T_{25})(t) \]

**Definition of** \( v_1 \) :

\[ v_1(t) = \frac{G_{24}}{G_{25}} \]

It follows

\[ -(a_{25})(4)(v_1)^2 + (\sigma_2)(4)v_1 - (a_{24})(4) \leq \frac{dv_1}{dt} \leq -(a_{25})(4)(v_1)^2 + (\sigma_4)(4)v_1 - (a_{24})(4) \]

From which one obtains

**Definition of** \( \bar{v}_1 \), \( (v_0)(t) \) :

\[ (d) \ \text{For} \ 0 < \frac{(v_0)(t)}{G_{24}} < (v_1)(t) < (\bar{v}_1)(t) \]

\[ (v_0)(t) \leq \frac{(v_0)(t)+(C)(4)(v_0)(t)e^{-\frac{(a_{25})(4)(v_1)(t)-(v_0)(t))}{2}}{4+(C)(4)e^{-\frac{(a_{25})(4)(v_1)(t)-(v_0)(t))}{2}}} \leq (v_1)(t) \]

it follows \( (v_0)(t) \leq \psi(t) \leq (v_1)(t) \).
In the same manner, we get

\[ \psi^{(4)}(t) \leq \frac{(v_1^{(4)})^2 + (v_2^{(4)})^2 e^{-[(a_{23}^{(4)})^2](v_1^{(4)})^2 - \frac{1}{2} - (v_2^{(4)})^2]}}{4 + (\bar{C})^{(4)} + \bar{C}^{(4)} e^{-[\frac{(a_{23}^{(4)})^2}{2}](v_1^{(4)})^2 - \frac{1}{2} - (v_2^{(4)})^2]}} \quad \text{and} \quad \bar{C}^{(4)} = \frac{(v_1^{(4)} - \bar{v}_1^{(4)})}{(v_0^{(4)} - \bar{v}_2^{(4)})}

From which we deduce \((v_0^{(4)}) \leq \psi^{(4)}(t) \leq (\bar{v}_1^{(4)})

(e) If \(0 < (v_1^{(4)}) < (v_0^{(4)}) = \frac{\delta_{24}^{(4)}}{\delta_{25}^{(4)}} < (\bar{v}_1^{(4)})\) we find like in the previous case,

\[ (v_1^{(4)}) \leq \frac{(v_1^{(4)})^2 + (v_2^{(4)})^2 e^{-[(a_{23}^{(4)})^2](v_1^{(4)})^2 - \frac{1}{2} - (v_2^{(4)})^2]}{1 + (\bar{C})^{(4)} e^{-[\frac{(a_{23}^{(4)})^2}{2}](v_1^{(4)})^2 - \frac{1}{2} - (v_2^{(4)})^2]} \leq \psi^{(4)}(t) \leq \frac{(v_2^{(4)})^2 + (v_1^{(4)})^2 e^{-[(a_{23}^{(4)})^2](v_1^{(4)})^2 - \frac{1}{2} - (v_2^{(4)})^2]}{1 + (\bar{C})^{(4)} e^{-[\frac{(a_{23}^{(4)})^2}{2}](v_1^{(4)})^2 - \frac{1}{2} - (v_2^{(4)})^2]} \leq (\bar{v}_1^{(4)})

(f) If \(0 < (v_1^{(4)}) \leq (v_2^{(4)}) \leq (v_0^{(4)}) = \frac{\delta_{24}^{(4)}}{\delta_{25}^{(4)}}\), we obtain

\[ (v_1^{(4)}) \leq \psi^{(4)}(t) \leq \frac{(v_1^{(4)})^2 + (v_2^{(4)})^2 e^{-[(a_{23}^{(4)})^2](v_1^{(4)})^2 - \frac{1}{2} - (v_2^{(4)})^2]}{1 + (\bar{C})^{(4)} e^{-[\frac{(a_{23}^{(4)})^2}{2}](v_1^{(4)})^2 - \frac{1}{2} - (v_2^{(4)})^2]} \leq (v_2^{(4)}) \leq \frac{(v_2^{(4)})^2 + (v_1^{(4)})^2 e^{-[(a_{23}^{(4)})^2](v_1^{(4)})^2 - \frac{1}{2} - (v_2^{(4)})^2]}{1 + (\bar{C})^{(4)} e^{-[\frac{(a_{23}^{(4)})^2}{2}](v_1^{(4)})^2 - \frac{1}{2} - (v_2^{(4)})^2]} \leq (v_0^{(4)})

And so with the notation of the first part of condition (c), we have

**Definition of \(\psi^{(4)}(t)\):**

\[(m_2^{(4)}) \leq \psi^{(4)}(t) \leq (m_1^{(4)}), \quad \psi^{(4)}(t) = \frac{\delta_{24}^{(4)}}{\delta_{25}^{(4)}} \]

In a completely analogous way, we obtain

**Definition of \(u^{(4)}(t)\):**

\[(\mu_2^{(4)}) \leq u^{(4)}(t) \leq (\mu_1^{(4)}), \quad u^{(4)}(t) = \frac{\tau_{24}^{(4)}}{\tau_{25}^{(4)}} \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case:**

If \((a_{24}^{(4)}) = (a_{25}^{(4)})\), then \(\sigma_1^{(4)} = (\sigma_2^{(4)})\) and in this case \((v_1^{(4)}) = (\bar{v}_1^{(4)})\) if in addition \((v_0^{(4)}) = (v_1^{(4)})\) then \(v^{(4)}(t) = (v_0^{(4)})\) and as a consequence \(G_{24}(t) = (v_0^{(4)})G_{25}(t)\) this also defines \((v_0^{(4)})\) for the special case.

Analogously if \((b_{24}^{(4)}) = (b_{25}^{(4)})\), then \((\tau_1^{(4)}) = (\tau_2^{(4)})\) and then

\((u_1^{(4)}) = (\bar{u}_1^{(4)})\) if in addition \((u_0^{(4)}) = (u_1^{(4)})\) then \(T_{24}(t) = (u_0^{(4)})T_{25}(t)\) This is an important consequence of the relation between \((v_1^{(4)})\) and \((\bar{v}_1^{(4)})\), and definition of \((u_0^{(4)})\).

From GLOBAL EQUATIONS we obtain

\[ \frac{dv^{(5)}}{dt} = (a_{28}^{(5)}) - \left( (a_{28}^{(5)}) - (a_{29}^{(5)}) + (a_{28}^{(5)})T_{29}(t) \right) - (a_{29}^{(5)})T_{29}(t)v^{(5)} - (a_{29}^{(5)})v^{(5)} \]
In a completely analogous way, we obtain

\[ v^{(5)}(t) = \frac{\partial x}{\partial y} \]

It follows

\[ -\left( (a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} \right) \leq \frac{dv^{(5)}}{dt} \leq -\left( (a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} \right) \]

From which one obtains

**Definition of** \((\tilde{v}_1)^{(5)}, (v_0)^{(5)} :\)

(g) For \(0 < (v_0)^{(5)} = \frac{\partial x}{\partial y} < (v_1)^{(5)} < (\tilde{v}_1)^{(5)}\), we get

\[ v^{(5)}(t) \geq \frac{(v_1)^{(5)} + (\tilde{v}_1)^{(5)})(v_2)^{(5)} e^{-\left[ -(a_{29})^{(5)}(v_1)^{(5)} - (v_0)^{(5)} \right] t}}{1 + (\tilde{v}_1)^{(5)} e^{-\left[ -(a_{29})^{(5)}(v_1)^{(5)} - (v_0)^{(5)} \right] t}} \]

\[ v^{(5)}(t) \leq \frac{(v_1)^{(5)} + (\tilde{v}_1)^{(5)})(v_2)^{(5)} e^{-\left[ -(a_{29})^{(5)}(v_1)^{(5)} - (v_0)^{(5)} \right] t}}{1 + (\tilde{v}_1)^{(5)} e^{-\left[ -(a_{29})^{(5)}(v_1)^{(5)} - (v_0)^{(5)} \right] t}} \]

it follows \((v_0)^{(5)} \leq v^{(5)}(t) \leq (\tilde{v}_0)^{(5)}\)

In the same manner, we get

\[ \bar{v}^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{v}_2)^{(5)})(\bar{v}_2)^{(5)} e^{-\left[ -(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_0)^{(5)} \right] t}}{1 + (\bar{v}_1)^{(5)} e^{-\left[ -(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_0)^{(5)} \right] t}} \]

From which we deduce \((\tilde{v}_0)^{(5)} \leq v^{(5)}(t) \leq (\tilde{v}_2)^{(5)}\)

(h) If \(0 < (v_1)^{(5)} < (v_0)^{(5)} = \frac{\partial x}{\partial y} < (\tilde{v}_1)^{(5)}\) we find like in the previous case,

\[ (v_1)^{(5)} \leq \frac{(v_1)^{(5)} + (\tilde{v}_1)^{(5)})(v_2)^{(5)} e^{-\left[ -(a_{29})^{(5)}(v_1)^{(5)} - (v_0)^{(5)} \right] t}}{1 + (\tilde{v}_1)^{(5)} e^{-\left[ -(a_{29})^{(5)}(v_1)^{(5)} - (v_0)^{(5)} \right] t}} \leq v^{(5)}(t) \leq \frac{(v_2)^{(5)} e^{-\left[ -(a_{29})^{(5)}(\tilde{v}_0)^{(5)} - (\bar{v}_0)^{(5)} \right] t}}{1 + (\tilde{v}_1)^{(5)} e^{-\left[ -(a_{29})^{(5)}(\tilde{v}_0)^{(5)} - (\bar{v}_0)^{(5)} \right] t}} \leq (\tilde{v}_1)^{(5)} \]

(i) If \(0 < (v_1)^{(5)} \leq (\tilde{v}_1)^{(5)} \leq (v_0)^{(5)} = \frac{\partial x}{\partial y},\) we obtain

\[ (v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(v_1)^{(5)} + (\tilde{v}_1)^{(5)})(v_2)^{(5)} e^{-\left[ -(a_{29})^{(5)}(v_1)^{(5)} - (v_0)^{(5)} \right] t}}{1 + (\tilde{v}_1)^{(5)} e^{-\left[ -(a_{29})^{(5)}(v_1)^{(5)} - (v_0)^{(5)} \right] t}} \leq (v_0)^{(5)} \]

And so with the notation of the first part of condition (c), we have

**Definition of** \(v^{(5)}(t) :\)

\[ (m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}, \quad v^{(5)}(t) = \frac{\partial x}{\partial y} \]

In a completely analogous way, we obtain

**Definition of** \(u^{(5)}(t) :\)

\[ (m_2)^{(5)} \leq u^{(5)}(t) \leq (m_1)^{(5)}, \quad u^{(5)}(t) = \frac{\partial x}{\partial y} \]
In the same manner, we get

Definition of \( \nu(6) \):

\[
\nu(6) = \frac{a_{32}}{a_{33}}
\]

It follows

\[
-\left( (a_{33})^6 \nu(6) \right)^2 + (\sigma_2)^6 \nu(6) - (a_{32})^6 \leq \frac{dv(6)}{dt} \leq -\left( (a_{33})^6 \nu(6) \right)^2 + (\sigma_1)^6 \nu(6) - (a_{32})^6
\]

From which one obtains

Definition of \( \tilde{\nu}_1(6), (\nu_0(6)) \):

\[
\nu(6)(t) \geq \frac{(\nu_1(6) + C(6)(\tilde{\nu}_2(6)) e^{-[(a_{33})^6(\nu_1(6) - \nu_0(6))] t}}{1 + C(6)e^{-[(a_{33})^6(\nu_1(6) - \nu_0(6))] t}}
\]

\[
(C(6)) = \frac{(\nu_1(6) - \nu_0(6))}{(\nu_0(6) - \nu_2(6))}
\]

it follows \( \nu(6) \leq \nu(6)(t) \leq (\nu_1(6)) \)

In the same manner, we get

\[
\nu(6)(t) \leq \frac{(\tilde{\nu}_2(6) + C(6)(\tilde{\nu}_2(6)) e^{-[(a_{33})^6(\nu_1(6) - \nu_2(6))] t}}{1 + C(6)e^{-[(a_{33})^6(\nu_1(6) - \nu_2(6))] t}}
\]

\[
(C(6)) = \frac{(\nu_1(6) - \nu_0(6))}{(\nu_0(6) - \nu_2(6))}
\]

From which we deduce \( \nu_0(6) \leq \nu(6)(t) \leq (\tilde{\nu}_1(6)) \)

(k) If \( 0 < (\nu_1(6)) < (\nu_0(6)) = \frac{a_{32}}{a_{33}} < (\tilde{\nu}_1(6)) \) we find like in the previous case,

\[
(\nu_1(6)) \leq \frac{(\nu_1(6) + C(6)(\tilde{\nu}_2(6)) e^{-[(a_{33})^6(\nu_1(6) - \nu_0(6)) t}}}{1 + C(6)e^{-[(a_{33})^6(\nu_1(6) - \nu_2(6))] t}} \leq \nu(6)(t) \leq \frac{(\nu_1(6) + C(6)(\tilde{\nu}_2(6)) e^{-[(a_{33})^6(\nu_1(6) - \nu_2(6))] t}}{1 + C(6)e^{-[(a_{33})^6(\nu_1(6) - \nu_2(6))] t}}
\]
\[
\frac{(\varphi_1)^{(6)\epsilon} + (\varphi_2)^{(6)\epsilon}(v_2)^{(6}\epsilon) - (a_{33})^{(6)\epsilon}(v_1)^{(6}\epsilon) - (a_{33})^{(6)\epsilon}(v_2)^{(6}\epsilon)}{1 + (c)^{(6)\epsilon}e^{(a_{33})^{(6)\epsilon}(v_1)^{(6)} - (a_{33})^{(6)\epsilon}(v_2)^{(6)}} \leq (\tilde{\varphi})^{(6)\epsilon}
\]

(1) If \( 0 < (v_1)^{(6)} \leq (\tilde{\varphi})^{(6)} \leq (v_0)^{(6)} = \frac{g_{32}^{(6)}}{g_{33}^{(6)}} \), we obtain

\[
(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(v_1)^{(6)} + (c)^{(6)}(v_2)^{(6)}e^{-(a_{33})^{(6)\epsilon}(v_1)^{(6)} - (a_{33})^{(6)\epsilon}(v_2)^{(6)}}}{1 + (c)^{(6)\epsilon}e^{-(a_{33})^{(6)\epsilon}(v_1)^{(6)} - (a_{33})^{(6)\epsilon}(v_2)^{(6)}}} \leq (v_0)^{(6)}
\]

And so with the notation of the first part of condition (c), we have

**Definition of** \( v^{(6)}(t) \):

\[
(m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)}, \quad v^{(6)}(t) = \frac{g_{32}^{(6)}}{g_{33}^{(6)}}
\]

In a completely analogous way, we obtain

**Definition of** \( u^{(6)}(t) \):

\[
(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad u^{(6)}(t) = \frac{f_{32}^{(6)}}{f_{33}^{(6)}}
\]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case:**

If \( (a_{32}''^{(6)}) = (a_{33}''^{(6)}) \), then \( (\sigma_1)^{(6)} = (\sigma_2)^{(6)} \) and in this case \( (v_1)^{(6)} = (\tilde{\varphi})^{(6)} \) if in addition \( (v_0)^{(6)} = (v_1)^{(6)} \) then \( v^{(6)}(t) = (v_0)^{(6)} \) and as a consequence \( G_{32}(t) = (v_0)^{(6)}G_{33}(t) \) this also defines \( (v_0)^{(6)} \) for the special case.

Analogously if \( (b_{32}''^{(6)}) = (b_{33}''^{(6)}) \), then \( (\tau_1)^{(6)} = (\tau_2)^{(6)} \) and then

\[
(u_1)^{(6)} = (\tilde{u}_1)^{(6)}\text{ if in addition } (u_0)^{(6)} = (u_1)^{(6)} \text{ then } T_{32}(t) = (u_0)^{(6)}T_{33}(t) \text{ This is an important consequence of the relation between } (v_1)^{(6)} \text{ and } (\tilde{\varphi})^{(6)} \text{, and definition of } (u_0)^{(6)}.
\]

**Behavior of the solutions**

(1) By \( (v_1)^{(7)} > 0 \), \( (v_2)^{(7)} < 0 \) and respectively \( (u_1)^{(7)} > 0 \), \( (u_2)^{(7)} < 0 \) the roots of the equations \( a_{37}^{(7)}(v)^{(7)} + (\sigma_1)^{(7)} v - (a_{38}^{(7)}) = 0 \).
and $\left(b_{37}\right)^{(7)}(u^{(7)})^2 + \left(t_{1}\right)^{(7)}u^{(7)} - \left(b_{36}\right)^{(7)} = 0$ and

**Definition of** $(\bar{v}_{1})^{(7)}, (\bar{v}_{2})^{(7)}, (\bar{u}_{1})^{(7)}, (\bar{u}_{2})^{(7)}$:

By $(\bar{v}_{1})^{(7)} > 0, (\bar{v}_{2})^{(7)} < 0$ and respectively $(\bar{u}_{1})^{(7)} > 0, (\bar{u}_{2})^{(7)} < 0$ the roots of the equations $(\alpha_{37})^{(7)}(\nu^{(7)})^2 + (\sigma_{2})^{(7)}\nu^{(7)} - (\alpha_{36})^{(7)} = 0$

and $\left(b_{37}\right)^{(7)}(u^{(7)})^2 + \left(t_{2}\right)^{(7)}u^{(7)} - \left(b_{36}\right)^{(7)} = 0$

**Definition of** $(m_{1})^{(7)}, (m_{2})^{(7)}, (\mu_{1})^{(7)}, (\mu_{2})^{(7)}, (v_{0})^{(7)}$:

(u) If we define $(m_{1})^{(7)}, (m_{2})^{(7)}, (\mu_{1})^{(7)}, (\mu_{2})^{(7)}$ by

$(m_{2})^{(7)} = (v_{0})^{(7)}, (m_{1})^{(7)} = (v_{1})^{(7)},$ if $(v_{0})^{(7)} < (v_{1})^{(7)}$

$(m_{2})^{(7)} = (v_{1})^{(7)}, (m_{1})^{(7)} = (v_{0})^{(7)},$ if $(v_{1})^{(7)} < (v_{0})^{(7)} < (v_{1})^{(7)},$

and $\left(v_{0}\right)^{(7)} = \frac{u_{0}}{q_{36}}$

$(m_{2})^{(7)} = (v_{1})^{(7)}, (m_{1})^{(7)} = (v_{0})^{(7)},$ if $(v_{0})^{(7)} < (v_{1})^{(7)}$

and analogously

$(\mu_{2})^{(7)} = (u_{0})^{(7)}, (\mu_{1})^{(7)} = (u_{1})^{(7)},$ if $(u_{0})^{(7)} < (u_{1})^{(7)}$

$(\mu_{2})^{(7)} = (u_{1})^{(7)}, (\mu_{1})^{(7)} = (u_{0})^{(7)},$ if $(u_{1})^{(7)} < (u_{0})^{(7)} < (u_{1})^{(7)},$

and $\left(u_{0}\right)^{(7)} = \frac{u_{0}}{q_{36}}$

$(\mu_{2})^{(7)} = (u_{1})^{(7)}, (\mu_{1})^{(7)} = (u_{0})^{(7)},$ if $(u_{0})^{(7)} < (u_{1})^{(7)}$

where $(\mu_{1})^{(7)}$ is defined respectively

Then the solution of GLOBAL EQUATIONS satisfies the inequalities

$G_{36}^{0}(s_{1})^{(7)-(p_{36})^{(7)}}t \leq G_{36}^{0}(s_{1})^{(7)}t$

where $(p_{1})^{(7)}$ is defined

$1 \left((m_{2})^{(7)}\right) G_{36}^{0}(s_{1})^{(7)-(p_{36})^{(7)}}t \leq G_{37}^{0}(t) \leq 1 \left((m_{2})^{(7)}\right) G_{36}^{0}(s_{1})^{(7)}t$

$\frac{(a_{36})^{(7)}(a_{0})^{0}}{(m_{1})^{(7)}(s_{1})^{(7)-(p_{36})^{(7)}}} [e\left((s_{1})^{(7)-(p_{36})^{(7)}}t - e\left(-(s_{2})^{(7)}\right)t\right] + G_{36}^{0}e^{-(s_{2})^{(7)}t} \leq G_{38}^{0}(t) \leq$

$\frac{(a_{38})^{(7)}(a_{0})^{0}}{(m_{2})^{(7)}(s_{1})^{(7)-(a_{36})^{(7)}}} [e\left(s_{1})^{(7)}t - e\left(-(a_{36})^{(7)}\right)\right] + G_{38}^{0}e^{-(a_{36})^{(7)}t}$

$T_{36}^{0}e^{(R_{1})^{(7)}t} \leq T_{36}^{0}(t) \leq T_{36}^{0}e^{(R_{1})^{(7)}+(R_{36})^{(7)}t}$

$\frac{1}{(\mu_{2})^{(7)}} T_{36}^{0}e^{(R_{1})^{(7)}t} \leq T_{36}^{0}(t) \leq \frac{1}{(\mu_{2})^{(7)}} T_{36}^{0}e^{(R_{1})^{(7)}+(R_{36})^{(7)}t}$

130
\[
\frac{(b_{38})^7 y_{38}}{(\mu_1)^7 y_{38}} \left[ e^{(R_1^7 t) - e^{-(b_{38})^7 t}} \right] + T_{38}^0 e^{-(b_{38})^7 t} \leq T_{38}(t) \leq 432
\]
\[
\frac{(a_{38})^7 y_{38}}{(\mu_2)^7 y_{38}} \left[ e^{(R_1^7 t + (p_{38})^7 t)e^{-(b_{38})^7 t}} \right] + T_{38}^0 e^{-(b_{38})^7 t} \]

**Definition of** \((S_1^7), (S_2^7), (R_1^7), (R_2^7)\):-

Where \((S_1^7) = (a_{38})^7 (m_2^7) - (a_{36})^7\)

\((S_2^7) = (a_{38})^7 - (p_{38})^7\)

\((R_1^7) = (b_{38})^7 (\mu_2) - (b_{36})^7\)

\((R_2^7) = (b_{38})^7 - (r_{38})^7\)

From CONCATENATED GLOBAL EQUATIONS we obtain

\[
\frac{dy^7}{dt} = (a_{36})^7 - (a_{37})^7 + (a_{36})^7 (T_{37}, t) \leq (a_{37})^7 (T_{37}, t) y^7 - (a_{37})^7 y^7
\]

**Definition of** \(y^7\):-

\[
y^7 = \frac{a_{36}}{a_{37}}
\]

It follows

\[-\left( (a_{37})^7 y^7 + (a_{37})^7 y^7 - (a_{36})^7 \right) \leq \frac{dy^7}{dt} \leq\]

\[-\left( (a_{37})^7 y^7 + (a_{37})^7 y^7 - (a_{36})^7 \right)\]

From which one obtains

**Definition of** \((\bar{v}^7), (v_0^7)\):

(m) For \(0 < (v_0^7) = \frac{a_{36}}{a_{37}} < (v_1^7) < (\bar{v}_1^7)\)

\[
v^7(t) \geq \frac{(v_1^7) + (C^7)(v_2^7)e^{-\left( (a_{37})^7 (v_1^7 - v_0^7) \right)} e^{-\left( (a_{37})^7 (v_1^7 - v_0^7) \right)} e^{1+(C^7)(v_2^7)(v_1^7) - (a_{37})^7 (v_1^7 - v_0^7)} e^{1+(C^7)(v_2^7)(v_1^7) - (a_{37})^7 (v_1^7 - v_0^7)}}{1+(C^7)(v_2^7)(v_1^7) - (a_{37})^7 (v_1^7 - v_0^7)}
\]

it follows \((v_0^7) \leq v^7(t) \leq (v_1^7)\)

In the same manner, we get

\[
v^7(t) \leq \frac{(v_2^7) + (C^7)(v_2^7)e^{-\left( (a_{37})^7 (v_1^7 - v_2^7) \right)} e^{-\left( (a_{37})^7 (v_1^7 - v_2^7) \right)} e^{1+(C^7)(v_2^7)(v_1^7) - (a_{37})^7 (v_1^7 - v_2^7)} e^{1+(C^7)(v_2^7)(v_1^7) - (a_{37})^7 (v_1^7 - v_2^7)}}{1+(C^7)(v_2^7)(v_1^7) - (a_{37})^7 (v_1^7 - v_2^7)}
\]

From which we deduce \((v_0^7) \leq v^7(t) \leq (\bar{v}_1^7)\)

(n) If \(0 < (v_1^7) < (v_0^7) = \frac{a_{36}}{a_{37}} < (\bar{v}_1^7)\) we find like in the previous case,

\[
(v_1^7) \leq \frac{(v_2^7) + (C^7)(v_2^7)e^{-\left( (a_{37})^7 (v_1^7 - v_2^7) \right)} e^{-\left( (a_{37})^7 (v_1^7 - v_2^7) \right)} e^{1+(C^7)(v_2^7)(v_1^7) - (a_{37})^7 (v_1^7 - v_2^7)} e^{1+(C^7)(v_2^7)(v_1^7) - (a_{37})^7 (v_1^7 - v_2^7)}}{1+(C^7)(v_2^7)(v_1^7) - (a_{37})^7 (v_1^7 - v_2^7)} \leq \frac{(v_1^7) + (C^7)(v_2^7)e^{-\left( (a_{37})^7 (v_1^7 - v_2^7) \right)} e^{-\left( (a_{37})^7 (v_1^7 - v_2^7) \right)} e^{1+(C^7)(v_2^7)(v_1^7) - (a_{37})^7 (v_1^7 - v_2^7)} e^{1+(C^7)(v_2^7)(v_1^7) - (a_{37})^7 (v_1^7 - v_2^7)}}{1+(C^7)(v_2^7)(v_1^7) - (a_{37})^7 (v_1^7 - v_2^7)} \leq v^7(t) \leq
\]
there exists a unique decreasing function in

Where in

\( (a_i)''(0)\) and \((b_i')''(0)\) are independent on \( t \), and the conditions

\( (a_4')''(0)(a_4')''(0)(a_5')''(0) - (a_4')(0)(a_5')(0) < 0 \)

\( (a_4')''(0)(a_5')''(0) - (a_4')(0)(a_5')(0) + (a_4')(0)(p_44)(0) + (a_5')(0)(p_45)(0) + (p_44)(0)(p_45)(0) > 0 \)

\( (b_4')(0)(b_4')(0)(b_4')(0) - (b_4')(0)(b_4')(0) > 0 \)

\( (b_4')(0)(b_4')(0)(b_4')(0) - (b_4')(0)(r_41)(0)(r_41)(0) - (b_4')(0)(r_41)(0)(r_41)(0) + (r_43)(0)(r_43)(0) < 0 \)

with \( (p_40)(0), (r_41)(0) \) as defined are satisfied, then the system

\( (a_40)(0)G_{40} - [(a_4')(0)(a_4')(0)]G_{40} = 0 \)

\( (a_41)(0)G_{40} - [(a_4')(0)(a_4')(0)]G_{41} = 0 \)

\( (a_42)(0)G_{41} - [(a_4')(0)(a_4')(0)]G_{42} = 0 \)

\( (b_40)(0)T_{41} - [(b_4')(0)(b_4')(0)]T_{40} = 0 \)

\( (b_41)(0)T_{41} - [(b_4')(0)(b_4')(0)]T_{41} = 0 \)

\( (b_42)(0)T_{41} - [(b_4')(0)(b_4')(0)]T_{41} = 0 \)

has a unique positive solution, which is an equilibrium solution for the system

**Proof:**

(a) Indeed the first two equations have a nontrivial solution \( G_{40}, G_{41} \) if

\[ F(T_{41}) = (a_4')(0)(a_4')(0) - (a_4')(0)(a_4')(0) + (a_4')(0)(a_4')(0)(T_{41}) + (a_4')(0)(a_4')(0)(T_{41}) + (a_4')(0)(a_4')(0)(T_{41}) = 0 \]

**Definition and uniqueness of** \( T_{41} \):

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \( (a_i')(0)(T_{41}) \) are increasing, it follows that there exists a unique \( T_{41}^* \) for which \( f(T_{41}^*) = 0 \). With this value, we obtain from the three first equations

\[ G_{44} = \frac{(a_4')(0)(G_{41})}{[(a_4')(0)(a_4')(0)(T_{41})]} \]

\[ G_{46} = \frac{(a_4')(0)(G_{41})}{[(a_4')(0)(a_4')(0)(T_{41})]} \]

(a) By the same argument, the equations(GLOBAL) admit solutions \( G_{40}, G_{41} \) if

\[ \varphi(G_{43}) = (b_4')(0)(b_4')(0) - (b_4')(0)(b_4')(0) - \]

\[ [(b_4')(0)(b_4')(0)(G_{43}) + (b_4')(0)(b_4')(0)(G_{43})] + (b_4')(0)(b_4')(0)(G_{43}) = 0 \]

Where in \( (G_{43}), (G_{40}, G_{41}, G_{42}), G_{40}, G_{42} \) must be replaced by their values. It is easy to see that \( \varphi \) is a decreasing function in \( G_{45} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{41}^* \) such that \( \varphi(G_{43}^*) = 0 \)
Finally we obtain the unique solution

\[ G_{41}^* \] given by \( \varphi((G_{43})^*) = 0 \), \[ T_{41}^* \] given by \( f(T_{41}^*) = 0 \) and

\[ G_{42}^* = \frac{(a_{40})(8)G_{41}^*}{(a_{40})(8)+G_{42}^*(T_{41}^*)} \]

\[ T_{42}^* = \frac{(b_{40})(8)T_{41}^*}{(b_{40})(8)+T_{42}^*((G_{43})^*)} \]

\[ (b_{14})(1)T_{13} - [(b_{14})(1) - (b_{14})(1)(G)]T_{14} = 0 \]

\[ (b_{15})(1)T_{14} - [(b_{15})(1) - (b_{15})(1)(G)]T_{15} = 0 \]

has a unique positive solution, which is an equilibrium solution for the system

\[ (a_{16})(2)G_{17} - [(a_{16})(2) + (a_{16})''(2)(T_{17})]G_{16} = 0 \]

\[ (a_{17})(2)G_{16} - [(a_{17})(2) + (a_{17})''(2)(T_{17})]G_{17} = 0 \]

\[ (a_{18})(2)G_{17} - [(a_{18})(2) + (a_{18})''(2)(T_{17})]G_{18} = 0 \]

\[ (b_{16})(2)T_{17} - [(b_{16})(2) - (b_{16})''(2)(G_{19})]T_{16} = 0 \]

\[ (b_{17})(2)T_{16} - [(b_{17})(2) - (b_{17})''(2)(G_{19})]T_{17} = 0 \]

\[ (b_{18})(2)T_{17} - [(b_{18})(2) - (b_{18})''(2)(G_{19})]T_{18} = 0 \]

\[ (a_{24})(4)G_{25} - [(a_{24})(4) + (a_{24})''(4)(T_{25})]G_{24} = 0 \]

\[ (a_{25})(4)G_{24} - [(a_{25})(4) + (a_{25})''(4)(T_{25})]G_{25} = 0 \]

\[ (a_{26})(4)G_{25} - [(a_{26})(4) + (a_{26})''(4)(T_{25})]G_{26} = 0 \]

\[ (b_{24})(4)T_{25} - [(b_{24})(4) - (b_{24})''(4)(G_{27})]T_{24} = 0 \]

\[ (b_{25})(4)T_{24} - [(b_{25})(4) - (b_{25})''(4)(G_{27})]T_{25} = 0 \]

\[ (b_{26})(4)T_{25} - [(b_{26})(4) - (b_{26})''(4)(G_{27})]T_{26} = 0 \]

has a unique positive solution, which is an equilibrium solution for the system

\[ (a_{28})(5)G_{29} - [(a_{28})(5) + (a_{28})''(5)(T_{29})]G_{28} = 0 \]

\[ (a_{29})(5)G_{28} - [(a_{29})(5) + (a_{29})''(5)(T_{29})]G_{29} = 0 \]

\[ (a_{30})(5)G_{29} - [(a_{30})(5) + (a_{30})''(5)(T_{29})]G_{30} = 0 \]

\[ (b_{28})(5)T_{29} - [(b_{28})(5) - (b_{28})''(5)(G_{31})]T_{28} = 0 \]

\[ (b_{29})(5)T_{28} - [(b_{29})(5) - (b_{29})''(5)(G_{31})]T_{29} = 0 \]
has a unique positive solution, which is an equilibrium solution for the system

\((a_{32})^6 G_{32} - [(a'_{32})^6 + (a''_{32})^6(T_{33})] G_{32} = 0\)  
\((a_{33})^6 G_{33} - [(a'_{33})^6 + (a''_{33})^6(T_{33})] G_{33} = 0\)  
\((a_{34})^6 G_{34} - [(a'_{34})^6 + (a''_{34})^6(T_{33})] G_{34} = 0\)  
\((b_{32})^6 T_{32} - [(b'_{32})^6 - (b''_{32})^6(G_{35})] T_{32} = 0\)  
\((b_{33})^6 T_{33} - [(b'_{33})^6 - (b''_{33})^6(G_{35})] T_{33} = 0\)  
\((b_{34})^6 T_{34} - [(b'_{34})^6 - (b''_{34})^6(G_{35})] T_{34} = 0\)

has a unique positive solution, which is an equilibrium solution for the system

\((a_{36})^7 G_{36} - [(a'_{36})^7 + (a''_{36})^7(T_{37})] G_{36} = 0\)  
\((a_{37})^7 G_{37} - [(a'_{37})^7 + (a''_{37})^7(T_{37})] G_{37} = 0\)  
\((a_{38})^7 G_{38} - [(a'_{38})^7 + (a''_{38})^7(T_{37})] G_{38} = 0\)  
\((b_{36})^7 T_{36} - [(b'_{36})^7 - (b''_{36})^7(G_{39})] T_{36} = 0\)  
\((b_{37})^7 T_{37} - [(b'_{37})^7 - (b''_{37})^7(G_{39})] T_{37} = 0\)  
\((b_{38})^7 T_{38} - [(b'_{38})^7 - (b''_{38})^7(G_{39})] T_{38} = 0\)

has a unique positive solution, which is an equilibrium solution for the system

\((a_{36})^7 G_{36} - [(a'_{36})^7 + (a''_{36})^7(T_{37})] G_{36} = 0\)  
\((a_{37})^7 G_{37} - [(a'_{37})^7 + (a''_{37})^7(T_{37})] G_{37} = 0\)  
\((a_{38})^7 G_{38} - [(a'_{38})^7 + (a''_{38})^7(T_{37})] G_{38} = 0\)  
\((b_{36})^7 T_{36} - [(b'_{36})^7 - (b''_{36})^7(G_{39})] T_{36} = 0\)  
\((b_{37})^7 T_{37} - [(b'_{37})^7 - (b''_{37})^7(G_{39})] T_{37} = 0\)  
\((b_{38})^7 T_{38} - [(b'_{38})^7 - (b''_{38})^7(G_{39})] T_{38} = 0\)

(a) Indeed the first two equations have a nontrivial solution \(G_{36}, G_{37}\) if

\[F(T_{39}) = (a'_{36})^7(a_{37})^7 - (a_{36})^7(a_{37})^7 + (a'_{36})^7(a_{37})^7(T_{37}) + (a_{37})^7(a''_{36})^7(T_{37}) + (a''_{36})^7(T_{37})(a_{37})^7(T_{37}) = 0\]

**Definition and uniqueness of \(T_{37}\):**

After hypothesis \(f(0) < 0, f(\infty) > 0\) and the functions \((a'_{i})^7(T_{37})\) are increasing, it follows that there exists a unique \(T_{37}\) for which \(f(T_{37}) = 0\). With this value, we obtain from the three first equations

\[G_{36} = \frac{(a_{36})^7 G_{37}}{[(a'_{36})^7 + (a''_{36})^7(T_{37})]}, \quad G_{38} = \frac{(a_{38})^7 G_{37}}{[(a'_{38})^7 + (a''_{38})^7(T_{37})]}\]

(b) By the same argument, the equations admit solutions \(G_{36}, G_{37}\) if

\[\varphi(G_{39}) = (b'_{36})^7(b_{37})^7 - (b_{36})^7(b_{37})^7 - \left[(b'_{36})^7(b''_{37})^7(G_{39}) + (b'_{37})^7(b''_{36})^7(G_{39})\right] + (b''_{36})^7(G_{39})(b''_{37})^7(G_{39}) = 0\]
Where in \((G_{39})G_{36}, G_{37}, G_{39}\), \(G_{36} G_{39}\) must be replaced by their values from 96. It is easy to see that \(\varphi\) is a decreasing function in \(G_{37}\) taking into account the hypothesis \(\varphi(0) > 0, \varphi(\infty) < 0\) it follows that there exists a unique \(G_{37}^*\) such that \(\varphi(G_{37}^*) = 0\)

Finally we obtain the unique solution OF THE SYSTEM

\[ G_{37}^* = \frac{(a_{36})(^3)G_{37}}{[(a_{36})(^3)+(a_{36})(^3)(T_{37})]} , \quad G_{38}^* = \frac{(a_{38})(^3)G_{37}}{[(a_{38})(^3)+(a_{38})(^3)(T_{37})]} \]

\[ T_{36}^* = \frac{(b_{36})(^3)G_{37}}{[(b_{36})(^3)+(b_{36})(^3)(G_{39})]} , \quad T_{38}^* = \frac{(b_{38})(^3)G_{37}}{[(b_{38})(^3)+(b_{38})(^3)(G_{39})]} \]

**Definition and uniqueness of \(T_{21}\)**

After hypothesis \(f(0) < 0, f(\infty) > 0\) and the functions \((a_{i})^{^{(1)}}(T_{21})\) are increasing, it follows that there exists a unique \(T_{21}^*\) for which \(f(T_{21}^*) = 0\). With this value, we obtain from the three first equations

\[ G_{20} = \frac{(a_{20})(^2)G_{21}}{[(a_{20})(^2)+(a_{20})(^2)(T_{21})]} , \quad G_{22} = \frac{(a_{22})(^2)G_{21}}{[(a_{22})(^2)+(a_{22})(^2)(T_{21})]} \]

**Definition and uniqueness of \(T_{25}\)**

After hypothesis \(f(0) < 0, f(\infty) > 0\) and the functions \((a_{i})^{^{(4)}}(T_{25})\) are increasing, it follows that there exists a unique \(T_{25}^*\) for which \(f(T_{25}^*) = 0\). With this value, we obtain from the three first equations

\[ G_{24} = \frac{(a_{24})(^3)G_{25}}{[(a_{24})(^3)+(a_{24})(^3)(T_{25})]} , \quad G_{26} = \frac{(a_{26})(^3)G_{25}}{[(a_{26})(^3)+(a_{26})(^3)(T_{25})]} \]

**Definition and uniqueness of \(T_{29}\)**

After hypothesis \(f(0) < 0, f(\infty) > 0\) and the functions \((a_{i})^{^{(5)}}(T_{29})\) are increasing, it follows that there exists a unique \(T_{29}^*\) for which \(f(T_{29}^*) = 0\). With this value, we obtain from the three first equations

\[ G_{28} = \frac{(a_{28})(^4)G_{29}}{[(a_{28})(^4)+(a_{28})(^4)(T_{29})]} , \quad G_{30} = \frac{(a_{30})(^4)G_{29}}{[(a_{30})(^4)+(a_{30})(^4)(T_{29})]} \]

**Definition and uniqueness of \(T_{33}\)**

After hypothesis \(f(0) < 0, f(\infty) > 0\) and the functions \((a_{i})^{^{(6)}}(T_{33})\) are increasing, it follows that there exists a unique \(T_{33}^*\) for which \(f(T_{33}^*) = 0\). With this value, we obtain from the three first equations

\[ G_{32} = \frac{(a_{32})(^5)G_{33}}{[(a_{32})(^5)+(a_{32})(^5)(T_{33})]} , \quad G_{34} = \frac{(a_{34})(^5)G_{33}}{[(a_{34})(^5)+(a_{34})(^5)(T_{33})]} \]

(c) By the same argument, the equations GLOBAL admit solutions \(G_{13}, G_{14}\) if

\[ \varphi(G) = (b_{13})^{^{(1)}}(b_{14})^{^{(1)}} - (b_{13})^{^{(1)}}(b_{14})^{^{(1)}} - \]

\[ [(b_{13})^{^{(1)}}(b_{14})^{^{(1)}}(G) + (b_{14})^{^{(1)}}(b_{13})^{^{(1)}}(G)] + (b_{13})^{^{(1)}}(G)(b_{14})^{^{(1)}}(G) = 0 \]

Where in \(G(G_{13}, G_{14}, G_{15})\), \(G_{13}, G_{15}\) must be replaced by their values from 96. It is easy to see that
\( \varphi \) is a decreasing function in \( G_{14} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{14}^* \) such that \( \varphi(G^*) = 0 \)

(d) By the same argument, the equations 92, 93 admit solutions \( G_{16}, G_{17} \) if

\[
\varphi(G_{19}) = (b_{16}^{(2)}(b_{17}^{(2)} - (b_{16}^{(2)}(b_{17}^{(2)} - \\
\quad [(b_{16}^{(2)}(b_{17}^{(2)}(G_{19}) + (b_{17}^{(2)}(b_{16}^{(2)}(G_{19}) + (b_{16}^{(2)}(G_{19}) - (G_{19}) = 0
\]

Where in \( G_{19} \) \( G_{16}, G_{17}, G_{18} \), \( G_{16}, G_{18} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{17} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{14}^* \) such that \( \varphi(G_{14}^*) = 0 \)

(a) By the same argument, the concatenated equations admit solutions \( G_{20}, G_{21} \) if

\[
\varphi(G_{23}) = (b_{20}^{(3)}(b_{21}^{(3)} - (b_{20}^{(3)}(b_{22}^{(3)} - \\
\quad [(b_{20}^{(3)}(b_{21}^{(3)}(G_{23}) + (b_{22}^{(3)}(b_{20}^{(3)}(G_{23}) + (b_{20}^{(3)}(G_{23}) - (G_{23}) = 0
\]

Where in \( G_{23} \) \( G_{20}, G_{21}, G_{22} \), \( G_{20}, G_{22} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{21} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{21}^* \) such that \( \varphi(G_{21}^*) = 0 \)

(b) By the same argument, the equations of modules admit solutions \( G_{24}, G_{25} \) if

\[
\varphi(G_{27}) = (b_{24}^{(4)}(b_{25}^{(4)} - (b_{24}^{(4)}(b_{25}^{(4)} - \\
\quad [(b_{24}^{(4)}(b_{25}^{(4)}(G_{27}) + (b_{25}^{(4)}(b_{24}^{(4)}(G_{27}) + (b_{24}^{(4)}(G_{27}) - (G_{27}) = 0
\]

Where in \( G_{27} \) \( G_{24}, G_{25}, G_{26} \), \( G_{24}, G_{26} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{25} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{25}^* \) such that \( \varphi(G_{25}^*) = 0 \)

(c) By the same argument, the equations (modules) admit solutions \( G_{28}, G_{29} \) if

\[
\varphi(G_{31}) = (b_{28}^{(5)}(b_{29}^{(5)} - (b_{28}^{(5)}(b_{29}^{(5)} - \\
\quad [(b_{28}^{(5)}(b_{29}^{(5)}(G_{31}) + (b_{29}^{(5)}(b_{28}^{(5)}(G_{31}) + (b_{28}^{(5)}(G_{31}) - (G_{31}) = 0
\]

Where in \( G_{31} \) \( G_{28}, G_{29}, G_{30} \), \( G_{28}, G_{30} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{29} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{29}^* \) such that \( \varphi(G_{29}^*) = 0 \)

(d) By the same argument, the equations (modules) admit solutions \( G_{32}, G_{33} \) if

\[
\varphi(G_{35}) = (b_{32}^{(6)}(b_{33}^{(6)} - (b_{32}^{(6)}(b_{33}^{(6)} - \\
\quad [(b_{32}^{(6)}(b_{33}^{(6)}(G_{35}) + (b_{33}^{(6)}(b_{32}^{(6)}(G_{35}) + (b_{32}^{(6)}(G_{35}) - (G_{35}) = 0
\]

Where in \( G_{35} \) \( G_{32}, G_{33}, G_{34} \), \( G_{32}, G_{34} \) must be replaced by their values It is easy to see that \( \varphi \) is a decreasing function in \( G_{33} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{33}^* \) such that \( \varphi(G_{33}^*) = 0 \)

Finally we obtain the unique solution

\( G_{14}^* \) given by \( \varphi(G^*) = 0, T_{14}^* \) given by \( f(T_{14}^*) = 0 \) and
Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^* \text{ given by } \varphi((G_{19})^*) = 0$, $T_{17}^* \text{ given by } f(T_{17})^* = 0$ and

$G_{16} = \frac{(a_{16})^{(2)} G_{17}}{[(a_{16})^{(2)}+(a_{16}^T)^{(2)}(T_{17})^*]}$, $G_{18} = \frac{(a_{18})^{(2)} G_{17}}{[(a_{18})^{(2)}+(a_{18}^T)^{(2)}(T_{17})^*]}$  

$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}}{[(b_{16})^{(2)}-(b_{16}^T)^{(2)}(G_{16})^*]}$, $T_{18}^* = \frac{(b_{18})^{(2)} T_{17}}{[(b_{18})^{(2)}-(b_{18}^T)^{(2)}(G_{18})^*]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{21}^* \text{ given by } \varphi((G_{23})^*) = 0$, $T_{21}^* \text{ given by } f(T_{21})^* = 0$ and

$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a_{20})^{(3)}+(a_{20}^T)^{(3)}(T_{21})^*]}$, $G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a_{22})^{(3)}+(a_{22}^T)^{(3)}(T_{21})^*]}$  

$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}}{[(b_{20})^{(3)}-(b_{20}^T)^{(3)}(G_{20})^*]}$, $T_{22}^* = \frac{(b_{22})^{(3)} T_{21}}{[(b_{22})^{(3)}-(b_{22}^T)^{(3)}(G_{22})^*]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{25}^* \text{ given by } \varphi(G_{27}) = 0$, $T_{25}^* \text{ given by } f(T_{25})^* = 0$ and

$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a_{24})^{(4)}+(a_{24}^T)^{(4)}(T_{25})^*]}$, $G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a_{26})^{(4)}+(a_{26}^T)^{(4)}(T_{25})^*]}$  

$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}}{[(b_{24})^{(4)}-(b_{24}^T)^{(4)}(G_{24})^*]}$, $T_{26}^* = \frac{(b_{26})^{(4)} T_{25}}{[(b_{26})^{(4)}-(b_{26}^T)^{(4)}(G_{26})^*]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{29}^* \text{ given by } \varphi((G_{31})^*) = 0$, $T_{29}^* \text{ given by } f(T_{29})^* = 0$ and

$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a_{28})^{(5)}+(a_{28}^T)^{(5)}(T_{29})^*]}$, $G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a_{30})^{(5)}+(a_{30}^T)^{(5)}(T_{29})^*]}$  

$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}}{[(b_{28})^{(5)}-(b_{28}^T)^{(5)}(G_{28})^*]}$, $T_{30}^* = \frac{(b_{30})^{(5)} T_{29}}{[(b_{30})^{(5)}-(b_{30}^T)^{(5)}(G_{30})^*]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{33}^* \text{ given by } \varphi((G_{35})^*) = 0$, $T_{33}^* \text{ given by } f(T_{33})^* = 0$ and
\[
G_{32}^* = \frac{(a_{32})^6 G_{33}^*}{(a_{32})^6 (a_{32})^6 (T_{32})}, \quad G_{34}^* = \frac{(a_{34})^6 G_{33}^*}{(a_{34})^6 (a_{32})^6 (T_{32})} \\
T_{32}^* = \frac{(b_{32})^6 T_{32}}{(b_{32})^6 (b_{32})^6 (T_{32})}, \quad T_{34}^* = \frac{(b_{34})^6 T_{32}}{(b_{34})^6 (b_{32})^6 (T_{32})}
\]

Obviously, these values represent an equilibrium solution.

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions \((a''_i)^{(1)}\) and \((b''_i)^{(1)}\) Belong to \(C^{(1)}(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

**Proof:**

**Definition of** \(G_i, T_i\) :-

\[
G_i = G_i^* + G_i^+, \quad T_i = T_i^* + T_i
\]

\[
\frac{\partial (a''_i)^{(1)}}{\partial T_{14}} (T_{14}) = (q_{14})^{(1)} , \quad \frac{\partial (b''_i)^{(1)}}{\partial T_{14}} (G^*) = S_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{13}}{dt} = -((a_{13})^{(1)} + (p_{13})^{(1)}) G_{13} + (a_{13})^{(1)} G_{14} - (q_{13})^{(1)} G_{13}^* T_{14}
\]

\[
\frac{dG_{14}}{dt} = -((a_{14})^{(1)} + (p_{14})^{(1)}) G_{14} + (a_{14})^{(1)} G_{13} - (q_{14})^{(1)} G_{14}^* T_{14}
\]

\[
\frac{dG_{15}}{dt} = -((a_{15})^{(1)} + (p_{15})^{(1)}) G_{15} + (a_{15})^{(1)} G_{14} - (q_{15})^{(1)} G_{15}^* T_{14}
\]

\[
\frac{dT_{13}}{dt} = -((b_{13})^{(1)} - (r_{13})^{(1)}) T_{13} + (b_{13})^{(1)} T_{14} + \sum_{j=13}^{15} (S_{13}(j) T_{14} G_{j})
\]

\[
\frac{dT_{14}}{dt} = -((b_{14})^{(1)} - (r_{14})^{(1)}) T_{14} + (b_{14})^{(1)} T_{13} + \sum_{j=13}^{15} (S_{14}(j) T_{14} G_{j})
\]

\[
\frac{dT_{15}}{dt} = -((b_{15})^{(1)} - (r_{15})^{(1)}) T_{15} + (b_{15})^{(1)} T_{14} + \sum_{j=13}^{15} (S_{15}(j) T_{14} G_{j})
\]

If the conditions of the previous theorem are satisfied and if the functions \((a''_i)^{(2)}\) and \((b''_i)^{(2)}\) Belong to \(C^{(2)}(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

**Denote**

**Definition of** \(G_i, T_i\) :-

\[
G_i = G_i^* + G_i^+, \quad T_i = T_i^* + T_i
\]

\[
\frac{\partial (a''_i)^{(2)}}{\partial T_{17}} (T_{17}) = (q_{17})^{(2)} , \quad \frac{\partial (b''_i)^{(2)}}{\partial T_{17}} (G^*) = S_{ij}
\]

taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{16}}{dt} = -((a_{16})^{(2)} + (p_{16})^{(2)}) G_{16} + (a_{16})^{(2)} G_{17} - (q_{16})^{(2)} G_{16}^* T_{17}
\]

\[
\frac{dG_{17}}{dt} = -((a_{17})^{(2)} + (p_{17})^{(2)}) G_{17} + (a_{17})^{(2)} G_{16} - (q_{17})^{(2)} G_{17}^* T_{17}
\]

\[
\frac{dG_{18}}{dt} = -((a_{18})^{(2)} + (p_{18})^{(2)}) G_{18} + (a_{18})^{(2)} G_{17} - (q_{18})^{(2)} G_{18}^* T_{17}
\]
If the conditions of the previous theorem are satisfied and if the functions \((a''_i)\) and \((b''_i)\) Belong to \(C^2(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stabl

Denote

\[ G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i \]

\[ \frac{dG_{20}}{dt} = -((a''_{20}) + (p_{20})G_{20} + (a_{20})G_{21} - (q_{20})G_{20}^* T_{21} \]

\[ \frac{dG_{21}}{dt} = -((a''_{21}) + (p_{21})G_{21} + (a_{21})G_{22} - (q_{21})G_{21}^* T_{22} \]

\[ \frac{dG_{22}}{dt} = -((a''_{22}) + (p_{22})G_{22} + (a_{22})G_{21} - (q_{22})G_{22}^* T_{21} \]

\[ \frac{dT_{20}}{dt} = -((b''_{20}) + (r_{20})T_{20} + (b_{20})T_{21} + \sum_{j=20}^{22}(s_{20})T_{20}^2 G_{2 j} \]

\[ \frac{dT_{21}}{dt} = -((b''_{21}) + (r_{21})T_{21} + (b_{21})T_{22} + \sum_{j=20}^{22}(s_{21})T_{21}^2 G_{2 j} \]

\[ \frac{dT_{22}}{dt} = -((b''_{22}) + (r_{22})T_{22} + (b_{22})T_{21} + \sum_{j=20}^{22}(s_{22})T_{22}^2 G_{2 j} \]

If the conditions of the previous theorem are satisfied and if the functions \((a''_i)\) and \((b''_i)\) Belong to \(C^4(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stabl

Denote

\[ G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i \]

\[ \frac{dG_{24}}{dt} = -((a''_{24}) + (p_{24})G_{24} + (a_{24})G_{25} - (q_{24})G_{24}^* T_{25} \]

\[ \frac{dG_{25}}{dt} = -((a''_{25}) + (p_{25})G_{25} + (a_{25})G_{24} - (q_{25})G_{25}^* T_{25} \]

\[ \frac{dG_{26}}{dt} = -((a''_{26}) + (p_{26})G_{26} + (a_{26})G_{25} - (q_{26})G_{26}^* T_{25} \]
If the conditions of the previous theorem are satisfied and if the functions \((a''_i)\) and \((b''_i)\) Belong to \(C^{(5)}(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable

Denote

**Definition of** \(G_i, T_i :\)

\[G_i = G^\prime_i + G_i, \quad T_i = T^\prime_i + T_i\]

\[\frac{\partial (a''_i)}{\partial T_1}(T_{29}) = (q_{29})^{(5)} , \quad \frac{\partial (b''_i)}{\partial G_i} (\text{ } (G_{33})^\prime ) = s_{ij}\]

Then taking into account equations (global) and neglecting the terms of power \(2\), we obtain

\[\frac{dG_{28}}{dt} = -((a''_2)^{(5)} + (p_{28})^{(5)}) G_{28} + (a_{28})^{(5)} G_{29} - (q_{28})^{(5)} G_{28} T_{28}\]

\[\frac{dG_{29}}{dt} = -((a''_2)^{(5)} + (p_{29})^{(5)}) G_{29} + (a_{29})^{(5)} G_{29} - (q_{29})^{(5)} G_{29} T_{29}\]

\[\frac{dG_{30}}{dt} = -((a''_2)^{(5)} + (p_{30})^{(5)}) G_{30} + (a_{30})^{(5)} G_{29} - (q_{30})^{(5)} G_{30} T_{29}\]

\[\frac{dG_{28}}{dt} = -((b''_2)^{(5)} - (r_{28})^{(5)}) T_{28} + (b_{28})^{(5)} T_{28} + \sum_{j=28}^{30} \left( s_{(28)(j)} T_{28} G_j \right)\]

\[\frac{dG_{29}}{dt} = -((b''_2)^{(5)} - (r_{29})^{(5)}) T_{29} + (b_{29})^{(5)} T_{29} + \sum_{j=29}^{30} \left( s_{(29)(j)} T_{29} G_j \right)\]

\[\frac{dG_{30}}{dt} = -((b''_2)^{(5)} - (r_{30})^{(5)}) T_{30} + (b_{30})^{(5)} T_{30} + \sum_{j=29}^{30} \left( s_{(30)(j)} T_{30} G_j \right)\]

If the conditions of the previous theorem are satisfied and if the functions \((a''_i)\) and \((b''_i)\) Belong to \(C^{(6)}(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable

Denote

**Definition of** \(G_i, T_i :\)

\[G_i = G^\prime_i + G_i, \quad T_i = T^\prime_i + T_i\]

\[\frac{\partial (a''_i)}{\partial T_1}(T_{33}) = (q_{33})^{(6)} , \quad \frac{\partial (b''_i)}{\partial G_i} (\text{ } (G_{33})^\prime ) = s_{ij}\]

Then taking into account equations (global) and neglecting the terms of power \(2\), we obtain

\[\frac{dG_{32}}{dt} = -((a''_3)^{(6)} + (p_{32})^{(6)}) G_{32} + (a_{32})^{(6)} G_{33} - (q_{32})^{(6)} G_{32} T_{33}\]

\[\frac{dG_{33}}{dt} = -((a''_3)^{(6)} + (p_{33})^{(6)}) G_{33} + (a_{33})^{(6)} G_{33} - (q_{33})^{(6)} G_{33} T_{33}\]

\[\frac{dG_{34}}{dt} = -((a''_3)^{(6)} + (p_{34})^{(6)}) G_{34} + (a_{34})^{(6)} G_{33} - (q_{34})^{(6)} G_{34} T_{33}\]
Obviously, these values represent an equilibrium solution of 79, 20, 36, 22, 23. If the conditions of the previous theorem are satisfied and if the functions \((a''_r)^{(7)}\) and \((b''_l)^{(7)}\) Belong to \(C^{(7)}(\mathbb{R}_+\)) then the above equilibrium point is asymptotically stable.

**Proof:**

**Definition of** \(T_i := \):  
\[
G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i
\]

\[
\frac{\partial (a''_r)^{(7)}}{\partial T_{37}} (T_{37}) = (q_{37})^{(7)} \quad , \quad \frac{\partial (b''_l)^{(7)}}{\partial T_{37}} (G_{37})^* = s_{ij}
\]

Then taking into account equations (SOLUTIONAL) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{36}}{dt} = -((a'_{36})^{(7)} + (p_{36})^{(7)})G_{36} + (a_{36})^{(7)}G_{37} - (q_{36})^{(7)}G_{36}^* T_{37}
\]

\[
\frac{dG_{37}}{dt} = -((a'_{37})^{(7)} + (p_{37})^{(7)})G_{37} + (a_{37})^{(7)}G_{36} - (q_{37})^{(7)}G_{37}^* T_{37}
\]

\[
\frac{dG_{38}}{dt} = -((a'_{38})^{(7)} + (p_{38})^{(7)})G_{38} + (a_{38})^{(7)}G_{37} - (q_{38})^{(7)}G_{38}^* T_{37}
\]

\[
\frac{dT_{36}}{dt} = -((b''_{36})^{(7)} - (r_{36})^{(7)})T_{36} + (b_{36})^{(7)}T_{37} + \sum_{j=36}^{38}(s_{36}(j)T_{36}T_{i})
\]

\[
\frac{dT_{37}}{dt} = -((b''_{37})^{(7)} - (r_{37})^{(7)})T_{37} + (b_{37})^{(7)}T_{38} + \sum_{j=36}^{38}(s_{37}(j)T_{37}T_{i})
\]

\[
\frac{dT_{38}}{dt} = -((b''_{38})^{(7)} - (r_{38})^{(7)})T_{38} + (b_{38})^{(7)}T_{37} + \sum_{j=36}^{38}(s_{38}(j)T_{38}T_{i})
\]

If the conditions of the previous theorem are satisfied and if the functions \((a''_r)^{(8)}\) and \((b''_l)^{(8)}\) Belong to \(C^{(8)}(\mathbb{R}_+\)) then the above equilibrium point is asymptotically stable.

**Denote**

**Definition of** \(T_i := \):  
\[
G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i
\]

\[
\frac{\partial (a''_r)^{(8)}}{\partial T_{41}} (T_{41}) = (q_{41})^{(8)} \quad , \quad \frac{\partial (b''_l)^{(8)}}{\partial T_{41}} (G_{41})^* = s_{ij}
\]

Then taking into account equations CONCATENATED EQUATIONS and neglecting the terms of power 2, we obtain

\[
\frac{dG_{40}}{dt} = -((a'_{40})^{(6)} + (p_{40})^{(8)})G_{40} + (a_{40})^{(8)}G_{41} - (q_{40})^{(6)}G_{40}^* T_{41}
\]
\[
\frac{d\eta_{41}}{dt} = -(a_{41}(0) + (p_{41}(0))\eta_{41} + (a_{41}(0)\eta_{40} - (q_{41}(0)\eta_{41}^2 T_{41})
\]

\[
\frac{d\eta_{42}}{dt} = -(a_{42}(0) + (p_{42}(0))\eta_{42} + (a_{42}(0)\eta_{40} - (q_{42}(0)\eta_{42}^2 T_{41})
\]

\[
\frac{dT_{40}}{dt} = -((b_{40}(0)^2 - (r_{40}(0))T_{40} + (b_{40}(0)^2 T_{40} + \sum_{j=40}^{42}S_{(40)(j)}T_{40}\eta_{j}))
\]

\[
\frac{dT_{41}}{dt} = -((b_{41}(0)^2 - (r_{41}(0))T_{41} + (b_{41}(0)^2 T_{41} + \sum_{j=40}^{42}S_{(41)(j)}T_{41}\eta_{j}))
\]

\[
\frac{dT_{42}}{dt} = -((b_{42}(0)^2 - (r_{42}(0))T_{42} + (b_{42}(0)^2 T_{42} + \sum_{j=40}^{42}S_{(42)(j)}T_{42}\eta_{j}))
\]

The characteristic equation of this system is

\[
(\lambda)^{(1)} + (b_{13}^{(1)} - (r_{13}^{(1)}))((\lambda)^{(1)} + (a_{13}^{(1)} + (p_{13}^{(1)}))
\]

\[
[((\lambda)^{(1)} + (a_{13}^{(1)} + (p_{13}^{(1)}))((\lambda)^{(1)} + (q_{13}^{(1)}))\eta_{13}^2 + (a_{13}^{(1)}(q_{13}^{(1)}))\eta_{13}^3
\]

\[
(((\lambda)^{(1)} + (b_{13}^{(1)} - (r_{13}^{(1)}))S_{(43)(43)}T_{43} + (b_{13}^{(1)}S_{(33)(33)}T_{33}
\]

\[
+ ((\lambda)^{(1)} + (a_{14}^{(1)} + (p_{14}^{(1)}))((\lambda)^{(1)} + (a_{14}^{(1)} + (q_{14}^{(1)}))\eta_{14}^2 + (a_{14}^{(1)}(q_{14}^{(1)}))\eta_{14}^3
\]

\[
(((\lambda)^{(1)} + (b_{14}^{(1)} - (r_{14}^{(1)}))S_{(43)(43)}T_{43} + (b_{14}^{(1)}S_{(33)(33)}T_{33}
\]

\[
+ ((\lambda)^{(1)} + (a_{15}^{(1)} + (p_{15}^{(1)}))((\lambda)^{(1)} + (a_{15}^{(1)} + (q_{15}^{(1)}))\eta_{15}^2 + (a_{15}^{(1)}(q_{15}^{(1)}))\eta_{15}^3
\]

\[
(((\lambda)^{(1)} + (b_{15}^{(1)} - (r_{15}^{(1)}))S_{(43)(43)}T_{43} + (b_{15}^{(1)}S_{(33)(33)}T_{33}) = 0
\]

+ 

\[
(\lambda)^{(2)} + (b_{18}^{(2)} - (r_{18}^{(2)}))((\lambda)^{(2)} + (a_{18}^{(2)} + (p_{18}^{(2)}))
\]

\[
(((\lambda)^{(2)} + (a_{18}^{(2)} + (p_{18}^{(2)}))((\lambda)^{(2)} + (q_{18}^{(2)}))\eta_{18}^2 + (a_{18}^{(2)}(q_{18}^{(2)}))\eta_{18}^3
\]

\[
(((\lambda)^{(2)} + (b_{18}^{(2)} - (r_{18}^{(2)}))S_{(43)(43)}T_{43} + (b_{18}^{(2)}S_{(33)(33)}T_{33}
\]

\[
+ ((\lambda)^{(2)} + (a_{17}^{(2)} + (p_{17}^{(2)}))((\lambda)^{(2)} + (a_{17}^{(2)} + (q_{17}^{(2)}))\eta_{17}^2 + (a_{17}^{(2)}(q_{17}^{(2)}))\eta_{18}^2
\]

\[
(((\lambda)^{(2)} + (b_{17}^{(2)} - (r_{17}^{(2)}))S_{(43)(43)}T_{43} + (b_{17}^{(2)}S_{(33)(33)}T_{33}
\]

\[
+ ((\lambda)^{(2)} + (a_{16}^{(2)} + (p_{16}^{(2)}))((\lambda)^{(2)} + (a_{16}^{(2)} + (q_{16}^{(2)}))\eta_{16}^2 + (a_{16}^{(2)}(q_{16}^{(2)}))\eta_{17}^2
\]

\[
(((\lambda)^{(2)} + (b_{16}^{(2)} - (r_{16}^{(2)}))S_{(43)(43)}T_{43} + (b_{16}^{(2)}S_{(33)(33)}T_{33}) = 0
\]

+ 

\[
\frac{\lambda^{(2)}}{\lambda^{(2)}} + b_{16}^{(2)} - r_{16}^{(2)}((\lambda)^{(2)} + a_{16}^{(2)} + p_{16}^{(2)})
\]

\[
(((\lambda)^{(2)} + a_{16}^{(2)} + p_{16}^{(2)})((\lambda)^{(2)} + q_{16}^{(2)}))\eta_{16}^2 + (a_{16}^{(2)}(q_{16}^{(2)}))\eta_{16}^3
\]

\[
(((\lambda)^{(2)} + b_{16}^{(2)} - r_{16}^{(2)}))S_{(43)(43)}T_{43} + (b_{16}^{(2)}S_{(33)(33)}T_{33}
\]

\[
+ ((\lambda)^{(2)} + a_{17}^{(2)} + p_{17}^{(2)}))((\lambda)^{(2)} + a_{17}^{(2)} + q_{17}^{(2)}))\eta_{17}^2 + (a_{17}^{(2)}(q_{17}^{(2)}))\eta_{17}^3
\]

\[
(((\lambda)^{(2)} + b_{17}^{(2)} - r_{17}^{(2)}))S_{(43)(43)}T_{43} + (b_{17}^{(2)}S_{(33)(33)}T_{33}
\]

\[
+ ((\lambda)^{(2)} + a_{18}^{(2)} + p_{18}^{(2)}))((\lambda)^{(2)} + a_{18}^{(2)} + q_{18}^{(2)}))\eta_{18}^2 + (a_{18}^{(2)}(q_{18}^{(2)}))\eta_{18}^3
\]

\[
(((\lambda)^{(2)} + b_{18}^{(2)} - r_{18}^{(2)}))S_{(43)(43)}T_{43} + (b_{18}^{(2)}S_{(33)(33)}T_{33}) = 0
\]
+ \((\lambda)^{(2)}\)^2 + \((a_{16}^{(2)} + a_{17}^{(2)} + p_{16}^{(2)} + p_{17}^{(2)}) (\lambda)^{(2)}\)(q_{18}^{(2)}G_{18}^{(2)})

+((\lambda)^{(2)} + (a_{16}^{(2)} + p_{16}^{(2)}) (a_{18}^{(2)}(q_{17}^{(2)}G_{17}^{*} + a_{17}^{(2)}(a_{18}^{(2)}q_{16}^{(2)}G_{16}^{*})))

\((\lambda)^{(2)} + (b_{16}^{(2)} - (r_{16}^{(2)})S_{(17),(18)}T_{17}^{*} + (b_{17}^{(2)})S_{(16),(18)}T_{16}^{*})\) = 0

+ 

\((\lambda)^{(3)} + (b_{22}^{(3)} - (r_{22}^{(3)}))(\lambda)^{(3)} + (a_{22}^{(3)} + p_{22}^{(3)})\)

\[\left[\left((\lambda)^{(3)} + (a_{20}^{(3)} + p_{20}^{(3)})(q_{21}^{(3)}G_{21}^{*} + a_{21}^{(3)}q_{20}^{(3)}G_{20}^{*})\right)\]

\((\lambda)^{(3)} + (b_{20}^{(3)} - (r_{20}^{(3)}))S_{(21),(21)}T_{21}^{*} + (b_{21}^{(3)})S_{(20),(21)}T_{21}^{*}\)

\((\lambda)^{(3)} + (a_{21}^{(3)} + p_{21}^{(3)})(q_{20}^{(3)}G_{20} + a_{20}^{(3)}q_{21}^{(3)}G_{21})\)

\((\lambda)^{(3)} + (b_{20}^{(3)} - (r_{20}^{(3)}))S_{(21),(20)}T_{21}^{*} + (b_{21}^{(3)})S_{(20),(20)}T_{20}^{*}\)

\((\lambda)^{(3)} + (a_{21}^{(3)} + p_{21}^{(3)})(q_{20}^{(3)}G_{20} + a_{20}^{(3)}q_{21}^{(3)}G_{21})\)

\((\lambda)^{(3)} + (b_{20}^{(3)} + p_{20}^{(3)} - (r_{20}^{(3)})(r_{21}^{(3)})(\lambda)^{(3)})\)

\((\lambda)^{(3)} + (a_{20}^{(3)} + a_{21}^{(3)} + p_{20}^{(3)} + p_{21}^{(3)})(a_{22}^{(3)}q_{23}^{(3)}G_{23}^{*} + a_{23}^{(3)}q_{22}^{(3)}G_{22}^{*})\)

\((\lambda)^{(3)} + (b_{23}^{(3)} - (r_{23}^{(3)}))S_{(25),(25)}T_{25}^{*} + (b_{25}^{(3)})S_{(24),(25)}T_{25}^{*}\)

\((\lambda)^{(3)} + (a_{25}^{(3)} + p_{25}^{(3)})(q_{24}^{(3)}G_{24} + a_{24}^{(3)}q_{25}^{(3)}G_{25}^{*})\)

\((\lambda)^{(3)} + (b_{24}^{(3)} - (r_{24}^{(3)}))S_{(25),(24)}T_{25}^{*} + (b_{25}^{(3)})S_{(24),(24)}T_{24}^{*}\)

\((\lambda)^{(4)} + (\lambda)^{(4)} + (\lambda)^{(4)} + (\lambda)^{(4)}\)

\((\lambda)^{(4)} + (a_{26}^{(4)} + p_{26}^{(4)})(q_{27}^{(4)}G_{27}^{*} + a_{27}^{(4)}q_{26}^{(4)}G_{26}^{*})\)

\((\lambda)^{(4)} + (b_{26}^{(4)} - (r_{26}^{(4)}))S_{(28),(26)}T_{26}^{*} + (b_{28}^{(4)})S_{(26),(26)}T_{26}^{*}\)

\((\lambda)^{(4)} + (a_{26}^{(4)} + a_{27}^{(4)} + p_{26}^{(4)} + p_{27}^{(4)})(a_{28}^{(4)}q_{29}^{(4)}G_{29}^{*} + a_{29}^{(4)}a_{26}^{(4)}q_{27}^{(4)}G_{27}^{*} + a_{27}^{(4)}a_{26}^{(4)}q_{26}^{(4)}G_{26}^{*})\)

143
\[
\left( (\lambda)^{(4)} + (b_{24}^{(4)} + (r_{24})^{(4)})s_{(25), (26)}T_{25}^{(4)} + (b_{25}^{(4)} s_{(24), (26)}T_{24}^{(4)}) \right) = 0
\]

\[
+ (\lambda)^{(5)} + (b_{30}^{(5)} + (r_{30})^{(5)})( (\lambda)^{(5)} + (a_{30}^{(5)} + (p_{30})^{(5)})
\]

\[
\left[ ((\lambda)^{(5)} + (a_{20}^{(5)} + (p_{28})^{(5)} + (q_29)^{(5)} G_{29}^* + (a_{29})^{(5)}(q_{29})^{(5)}G_{28}^*)
\]

\[
\left( (\lambda)^{(5)} + (b_{28}^{(5)} + (r_{28})^{(5)} s_{(29), (29)} T_{29}^{(5)} + (b_{29}^{(5)} s_{(28), (28)} T_{28}^{(5)})
\]

\[
\left( (\lambda)^{(5)} + (a_{20}^{(5)} + (p_{28})^{(5)} + (q_29)^{(5)} G_{28}^* + (a_{29})^{(5)}(q_{29})^{(5)}G_{29}^*)
\]

\[
\left( (\lambda)^{(5)} + (b_{28}^{(5)} + (r_{28})^{(5)} s_{(29), (29)} T_{29}^{(5)} + (b_{29}^{(5)} s_{(28), (28)} T_{28}^{(5)})
\]

\[
\left( (\lambda)^{(5)} + (a_{20}^{(5)} + (p_{28})^{(5)} + (q_29)^{(5)} G_{29}^* + (a_{29})^{(5)}(q_{29})^{(5)}G_{28}^*)
\]

\[
\left( (\lambda)^{(5)} + (b_{28}^{(5)} + (r_{28})^{(5)} s_{(29), (30)} T_{29}^{(5)} + (b_{29}^{(5)} s_{(28), (30)} T_{28}^{(5)})\right) = 0
\]

\[
+ (\lambda)^{(6)} + (b_{34}^{(6)} + (r_{34})^{(6)})( (\lambda)^{(6)} + (a_{34}^{(6)} + (p_{34})^{(6)})
\]

\[
\left[ ((\lambda)^{(6)} + (a_{32}^{(6)} + (p_{32})^{(6)} + (q_33)^{(6)} G_{33}^* + (a_{33})^{(6)}(q_{32})^{(6)}G_{32}^*)
\]

\[
\left( (\lambda)^{(6)} + (b_{32}^{(6)} + (r_{32})^{(6)} s_{(33), (33)} T_{33}^{(6)} + (b_{33}^{(6)} s_{(32), (33)} T_{33}^{(6)})
\]

\[
\left( (\lambda)^{(6)} + (a_{32}^{(6)} + (p_{32})^{(6)} + (q_33)^{(6)} G_{32}^* + (a_{33})^{(6)}(q_{32})^{(6)}G_{32}^*)
\]

\[
\left( (\lambda)^{(6)} + (b_{32}^{(6)} + (r_{32})^{(6)} s_{(33), (32)} T_{32}^{(6)} + (b_{33}^{(6)} s_{(32), (32)} T_{32}^{(6)})
\]

\[
\left( (\lambda)^{(6)} + (a_{32}^{(6)} + (p_{32})^{(6)} + (q_33)^{(6)} G_{32}^* + (a_{33})^{(6)}(q_{32})^{(6)}G_{32}^*)
\]

\[
\left( (\lambda)^{(6)} + (b_{32}^{(6)} + (r_{32})^{(6)} s_{(33), (34)} T_{33}^{(6)} + (b_{33}^{(6)} s_{(32), (34)} T_{32}^{(6)})\right) = 0
\]

\[
+ \ldots
\]
\[
((\lambda)^{7} + (b^{38}_{38})^{7} - (r^{38}_{38})^{7})((\lambda)^{7} + (a^{38}_{38})^{7} + (p^{38}_{38})^{7})

\]
\[
\left[ ((\lambda)^{7} + (a^{38}_{38})^{7} + (p^{38}_{38})^{7})q_{38}^{7}G_{38}^{7} + (a^{38}_{37})^{7}(q_{38}^{7})G_{38}^{7} \right]
\]
\[
((\lambda)^{7} + (b^{38}_{36})^{7} - (r^{38}_{36})^{7})S_{(37),(37)}^{7}T_{37}^{7} + (b^{38}_{37})^{7}S_{(36),(37)}^{7}T_{37}^{7}
\]
\[
+ \left( ((\lambda)^{7} + (a^{38}_{37})^{7} + (p^{38}_{37})^{7})q_{38}^{7}G_{38}^{7} + (a^{38}_{38})^{7}(q_{38}^{7})G_{38}^{7} \right)
\]
\[
((\lambda)^{7} + (b^{38}_{36})^{7} - (r^{38}_{36})^{7})S_{(37),(36)}^{7}T_{37}^{7} + (b^{38}_{37})^{7}S_{(36),(36)}^{7}T_{36}^{7}
\]
\[
((\lambda)^{7})^{2} + ((a^{38}_{38})^{7} + (a^{38}_{37})^{7} + (p^{38}_{38})^{7} + (p^{38}_{37})^{7})(\lambda)^{7}
\]
\[
((\lambda)^{7})^{2} + ((b^{38}_{38})^{7} + (b^{38}_{37})^{7} - (r^{38}_{38})^{7} + (r^{38}_{37})^{7})(\lambda)^{7}
\]
\[
+ \left( ((\lambda)^{7} + (a^{38}_{37})^{7} + (p^{38}_{36})^{7} + (p^{38}_{37})^{7})(\lambda)^{7} \right)(q_{38}^{7})G_{38}^{7}
\]
\[
+ \left( (\lambda)^{7} + (a^{38}_{38})^{7} + (p^{38}_{36})^{7} \right)((a^{38}_{38})^{7}(q_{38}^{7})G_{38}^{7} + (a^{38}_{37})^{7}(a^{38}_{38})^{7}(q_{38}^{7})G_{38}^{7}
\]
\[
\left( ((\lambda)^{7} + (b^{38}_{36})^{7} - (r^{38}_{36})^{7})S_{(37),(38)}^{7}T_{37}^{7} + (b^{38}_{37})^{7}S_{(36),(38)}^{7}T_{36}^{7} \right) = 0
\]

SECTION 2 :

Schödinger’s Wave Equation, Relativistic Theories And Werner Heisenberg Matrix- An Accolytish Representation Apophthegmatic Aneurysm And Atrophied Asseveration

Preface- In quantum mechanics and quantum field theory, a Schrödinger field, named after Erwin Schrödinger, is a quantum field, which obeys the Schrödinger equation. While any situation described by a Schrödinger field can also be described by a many-body Schrödinger equation for identical particles, the field theory is more suitable for situations where the particle number changes. A Schrödinger field is also the classical limit of a quantum Schrödinger field, a classical wave which satisfies the Schrödinger equation. Unlike the quantum mechanical wavefunction, if there are interactions between the particles the equation will be nonlinear. These nonlinear equations describe the classical wave limit of a system of interacting identical particles. The path integral of a Schrödinger field is also known as a coherent state path integral, because the field itself is an annihilation operator whose eigenstates can be thought of as coherent states of the harmonic oscillations of the field modes. Schrödinger fields are useful for describing Bose-Einstein condensation, the Bogolyubov-de Gennes equation of superconductivity, super fluidity, and many-body theory in general. They are also a useful alternative formalism for nonrelativistic quantum mechanics. A Schrödinger field is the nonrelativistic limit of a Klein-Gordon field. Dirac equation rips open glittering façade of into four coupled linear first-order partial differential equations for the four quantities that make up the field. These matrices, and the form of the field, have a deep mathematical significance. Dirac's main objective and primary aim in casting this equation was to explain the behavior of the relativistically moving electron, and so to allow the atom to be treated in a manner consistent with relativity. Corrections introduced this way might have bearing on the problem of atomic spectra. Up until that time, attempts to make the old quantum theory of the atom compatible with the theory of relativity by discretizing the angular momentum of the electron's orbit had failed – and the new quantum mechanics of Heisenberg, Pauli, Jordan, Schrödinger, and Dirac himself had not developed sufficiently to treat this problem. Although Dirac's original intentions were satisfied, his equation had far deeper implications.
for the structure of matter, and introduced new mathematical classes of objects that are now essential elements of fundamental physics. The Dirac theory, while providing a wealth of information that is accurately confirmed by experiments, nevertheless introduces a new physical paradigm that appears at first difficult to interpret and even dissymmetrical paradoxical. Sometimes, it might even appear that there is deployed conceptual significance and attributed conceptual implications to the very equation itself. But is figment of one’s imagination and product of fecund prognostication. Some of these issues of interpretation must be regarded as open questions. The Dirac theory brilliantly answered some of the outstanding issues in physics at the time it was put forward, while posing others that are still the subject of debate. Many of these issues were resolved in modern quantum field theory by considering the Dirac equation not as a relativistic description of quantum mechanics but merely as another relativistic field equation, on the same footing as the Klein–Gordon equation or Maxwell’s equations, in which ψ is not interpreted as a wave function but rather as a fermion field, similar to the Klein–Gordon scalar field or electromagnetic field. Nevertheless, considering Dirac’s equation as a relativistic version of Schrödinger’s equation is extremely computationally useful, and raises important issues like Newton’s Second law, the Schrödinger equation can be mathematically transformed into other formulations such as Werner Heisenberg’s matrix, and Richard Feynman’s path integral formulation. Also like Newton’s Second law, the Schrödinger equation describes time in a way that is inconvenient for relativistic theories, a problem that is not as severe in matrix mechanics and completely absent in the path integral formulation.

INTRODUCTION:

Quantum action principle

In ordinary quantum mechanics, the Hamiltonian is the infinitesimal generator of time-translations. This means that the state at a slightly later time is related to the state at the current time by acting with the Hamiltonian operator (multiplied by the negative imaginary unit, −i). For states with a definite energy, this is a statement of the De Broglie relation between frequency and energy, and the general relation is consistent with that plus the superposition principle. But the Hamiltonian in classical mechanics is derived from a Lagrangian, which is a more fundamental quantity considering special relativity. The Hamiltonian tells you how to march forward in time, but the notion of time are different in different reference frames. So the Hamiltonian is different in different frames, and this type of symmetry is not apparent in the original formulation of quantum mechanics. The Hamiltonian is a function of the position and momentum at one time, and it tells you the position and momentum a little later. The Lagrangian is a function of the position now and the position a little later (or, equivalently for infinitesimal time separations, it is a function of the position and velocity). The relation between the two is by a Legendre transform, and the condition that determines the classical equations is that the Action is a minimum.

In quantum mechanics, the Legendre transform is hard to interpret, because the motion is not over a definite trajectory. So what does the Legendre transform mean? In classical mechanics, with discretization in time,

\[ \epsilon H = p(q(t + \epsilon) - q(t)) - \epsilon L \]

And \[ p = \frac{\partial L}{\partial \dot{q}} \]

Where the partial derivative with respect to q holds q(t + ε) fixed. The inverse Legendre transform is:

\[ \epsilon L = p\epsilon \dot{q} - \epsilon H \]
Where \( \dot{q} = \frac{\partial H}{\partial p} \)

And the partial derivative now is with respect to \( p \) at fixed \( q \).

In quantum mechanics, the state is a superposition of different states with different values of \( q \), or different values of \( p \), and the quantities \( p \) and \( q \) can be interpreted as noncommuting operators. The operator \( p \) is only definite on states that are indefinite with respect to \( q \). So consider two states separated in time and act with the operator corresponding to the Lagrangian:

\[
\exp\left[i\left(p(q(t)+\epsilon) - q(t)\right) - \epsilon H(p,q)\right]
\]

If the multiplications implicit in this formula are reinterpreted as matrix multiplications, what does this mean?

It can be given a meaning as follows: The first factor is \( e^{-ipq(t)} \)

If this is interpreted as doing a matrix multiplication, the sum over all states integrates over all \( q(t) \), and so it takes the Fourier transform in \( q(t) \), to change basis to \( p(t) \). That is the action on the Hilbert space – change basis to \( p \) at time \( t \).

Next comes: \( e^{-i\epsilon H(p,q)} \)

Or evolve an infinitesimal time into the future.

Finally, the last factor in this interpretation is \( e^{ipq(t+\epsilon)} \)

Which means change basis back to \( q \) at a later time?

This is not very different from just ordinary time evolution: the \( H \) factor contains all the dynamical information – it pushes the state forward in time. The first part and the last part are just doing Fourier transforms to change to a pure \( q \) basis from an intermediate \( p \) basis.

Another way of saying this is that since the Hamiltonian is naturally a function of \( p \) and \( q \), exponentiation this quantity and changing basis from \( p \) to \( q \) at each step allows the matrix element of \( H \) to be expressed as a simple function along each path. This function is the quantum analog of the classical action. This observation is due to Paul Dirac.

Dirac further noted that one could square the time-evolution operator in the \( S \) representation \( e^{i\omega=\epsilon S} \)

And this gives the time evolution operator between time \( t \) and time \( t + 2\epsilon \). While in the \( H \) representation the quantity that is being summed over the intermediate states is an obscure matrix element, in the \( S \) representation it is reinterpreted as a quantity associated to the path. In the limit that one takes a large power of this operator, one reconstructs the full quantum evolution between two states, the early one with a fixed value of \( q(0) \) and the later one with a fixed value of \( q(t) \). The result is a sum over paths with a phase which is the quantum action.

Feynman’s interpretation

Feynman’s work did not provide a precise prescription to calculate the sum over paths, and he did not show that one could recover the Schrödinger equation or the canonical commutation relations from this rule. This was done by Feynman.

Feynman showed that Dirac’s quantum action was, for most cases of interest, simply equal to the classical action, appropriately discretized. This means that the classical action is the phase acquired by
quantum evolution between two fixed endpoints. He proposed to recover all of quantum mechanics from the following postulates:

The probability for an event is given by the squared length of a complex number called the "probability amplitude".

The probability amplitude is given by adding together the contributions of all the histories in configuration space.

The contribution of a history to the amplitude is proportional to $C^{iS/\hbar}$. While $S$ is the action of that history, given by the time integral of the Lagrangian along the corresponding path.

In order to find the overall probability amplitude for a given process, then, one adds up, or integrates, the amplitude of postulate 3 over the space of all possible histories of the system in between the initial and final states, including histories that are absurd by classical standards. In calculating the amplitude for a single particle to go from one place to another in a given time, it would be correct to include histories in which the particle describes elaborate curlicues, histories in which the particle shoots off into outer space and flies back again, and so forth. The path integral assigns all of these histories amplitudes of equal magnitude but with varying phase, or argument of the complex number. The contributions that are wildly different from the classical history are suppressed only by the interference of similar, canceling histories.

Feynman showed that this formulation of quantum mechanics is equivalent to the canonical approach to quantum mechanics, when the Hamiltonian is quadratic in the momentum. Amplitude computed according to Feynman’s principles will also obey the Schrödinger equation for the Hamiltonian corresponding to the given action.

Classical action principles are puzzling because of their seemingly teleological quality: given a set of initial and final conditions one is able to find a unique path connecting them, as if the system somehow knows where it's going to end up and how it's going to get there. The path integral explains why this works in terms of quantum superposition. The system doesn't have to know in advance where it's going or what path it'll take: the path integral simply calculates the sum of the probability amplitudes for every possible path to any possible endpoint. After a long enough time, interference effects guarantee that only the contributions from the stationary points of the action give histories with appreciable probabilities.

Concrete formulation

Feynman’s postulates can be interpreted as follows:

**Time-slicing definition**

For a particle in a smooth potential, the path integral is approximated by zig-zag paths, which in one dimension is a product of ordinary integrals. For the motion of the particle from position $x_a$ at time $t_a$ to $x_b$ at time $t_b$, the time sequence

$$t_a = t_0 < t_1 < \ldots < t_{n-1} < t_n < t_b = t_{n+1}$$

can be divided up into $n + 1$ little segments $t_j - t_{j-1}$, where $j = 1, \ldots, n + 1$, of fixed duration

$$\epsilon = \Delta t = \frac{t_b - t_a}{n + 1}.$$ 

This process is called time-slicing.

An approximation for the path integral can be computed as proportional to

$$\int_{-\infty}^{\infty} dx_1 \ldots \int_{-\infty}^{\infty} dx_n \exp \left( \frac{i}{\hbar} \int_{t_a}^{t_b} \mathcal{L}(x(t), v(t), t) \, dt \right)$$

where $\mathcal{L}(x, v, t)$ is the Lagrangian of the 1d-system with position variable $x(t)$ and velocity $v = \dot{x}(t)$ considered (see below), and $dx_j$ corresponds to the position at the $j$-th time step, if the time integral is approximated by a sum of $n$ terms.
In the limit $n \to \infty$, this becomes a functional integral, which - apart from a nonessential factor - is directly the product of the probability amplitudes $\langle x_{a1}, \dot{t}_{a1} | x_{b1}, \dot{t}_{b1} \rangle$ - more precisely, since one must work with a continuous spectrum, the respective densities - to find the quantum mechanical particle at $t_a$ in the initial state $x_a$ and at $t_b$ in the final state $x_b$.

Actually $\mathcal{L}$ is the classical Lagrangian of the one-dimensional system considered, also

$$\mathcal{L}(x, \dot{x}, t) = p \cdot \dot{x} - \mathcal{H}(x, p, t),$$

where $\mathcal{H}$ is the Hamiltonian,

and the above-mentioned "zigzagging" corresponds to the appearance of the terms:

$$\exp \left( \frac{i}{\hbar} \sum_{j=1}^{n} \mathcal{L} \left( \dot{x}_j, \frac{x_j - x_{j-1}}{\epsilon}, j \right) \right)$$

In the Riemannian sum approximating the time integral, which are finally integrated over $x_j$ to $x_{j+1}$ with the integration measure $dx_1...dx_n\delta$, is an arbitrary value of the interval corresponding to $j$, e.g. its center, $(x_j + x_{j-1})/2$.

Thus, in contrast to classical mechanics, not only does the stationary path contribute, but actually all virtual paths between the initial and the final point also contribute.

Feynman's time-sliced approximation does not, however, exist for the most important quantum-mechanical path integrals of atoms, due to the singularity of the Coulomb potential $e^2/r$ at the origin. Only after replacing the time $t$ by another path-dependent pseudo-time parameter

$$s = \int \frac{dt}{r(t)}$$

the singularity is removed and a time-sliced approximation exists, that is exactly integrable, since it can be made harmonic by a simple coordinate transformation, as discovered in 1979 by Ismail Hakkı Duru and Hagen Kleinert. The combination of a path-dependent time transformation and a coordinate transformation is an important tool to solve many path integrals and is called generically the Duru-Kleinert transformation.

NOTATION :

$G_{13}$ : Category one of Schrodinger’s Wave Equation (It is to be reiterated that we are classifying the universally applicable Schrödinger’s Wave equation based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Schrödinger’s equation could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Wave Equation itself.)

$G_{14}$ : Category two of Schrodinger's Wave Equation (It is to be reiterated that we are classifying the universally applicable Schrödinger’s Wave equation based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Schrödinger’s equation could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Wave Equation itself.)

$G_{15}$ Category Three of Schrodinger’s Wave Equation (It is to be reiterated that we are classifying the universally applicable Schrödinger’s Wave equation based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer
scrolls represent the totalistic transactions, Schrödinger’s equation could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Wave Equation itself.

\( T_{13} \) : Category one of Werner –Heisenberg matrix (It is to be reiterated that we are classifying the universally applicable Werner –Heisenberg matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Werner Heisenberg Matrix itself).

\( T_{14} \) : Category Two of Werner –Heisenberg matrix (It is to be reiterated that we are classifying the universally applicable Werner –Heisenberg matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Werner Heisenberg Matrix itself).

\( T_{15} \) : Category Three of Werner –Heisenberg matrix (It is to be reiterated that we are classifying the universally applicable Werner –Heisenberg matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Werner Heisenberg Matrix itself).

\( G_{16} \) : Category one of Richard Feynman's path integral formulation like Werner Heisenberg Matrix (It is to be reiterated that we are classifying the universally applicable Richard Feynman's path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself).

\( G_{17} \) : Category Two of Richard Feynman's path integral formulation like Werner Heisenberg Matrix (It is to be reiterated that we are classifying the universally applicable Richard Feynman's path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself).

\( G_{18} \) : Category Three of Richard Feynman's path integral formulation like Werner Heisenberg Matrix (It
is to be reiterated that we are classifying the universally applicable Richard Feynman's path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself).

$T_{16}$ : Category one of Relativistic Theories vis-a-vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix (It is to be reiterated that we are classifying the universally applicable Relativistic Theories vis-a-vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, penhance, predilection, proclivities, propensities, Solutional behaviour conceptual implications thereof, deployed contextual importance therein, anticipatory correlation and consubstationary causal association parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Relativistic Theories Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Relativistic and concomitant and corresponding classificatory categorization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself).

$T_{17}$ : Category two of Relativistic Theories vis-a-vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix (It is to be reiterated that we are classifying the universally applicable Relativistic Theories vis-a-vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, penhance, predilection, proclivities, propensities, Solutional behaviour conceptual implications thereof, deployed contextual importance therein, anticipatory correlation and consubstationary causal association parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Relativistic Theories Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Relativistic and concomitant and corresponding classificatory categorization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself).

$T_{18}$ : Category one of Relativistic Theories vis-a-vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix (It is to be reiterated that we are classifying the universally applicable Relativistic Theories vis-a-vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, penhance, predilection, proclivities, propensities, Solutional behaviour conceptual implications thereof, deployed contextual importance therein, anticipatory correlation and consubstationary causal association parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Relativistic Theories Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Relativistic and concomitant and corresponding classificatory categorization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself in the foregoing
\((a_{13}^{(1)}, a_{14}^{(1)}, a_{15}^{(1)}, b_{13}^{(1)}, b_{14}^{(1)}, b_{15}^{(1)}, a_{16}^{(2)}, a_{17}^{(2)}, a_{18}^{(2)}
(\cdot)_{(2)}\): are Accentuation coefficients
\((a_{13}^{(1)}, a_{14}^{(1)}, a_{15}^{(1)}, b_{13}^{(1)}, b_{14}^{(1)}, b_{15}^{(1)}, a_{16}^{(2)}, a_{17}^{(2)}, a_{18}^{(2)},
(b_{16}^{(2)}, b_{17}^{(2)}, b_{18}^{(2)}))\) are Dissipation coefficients

**Schrödinger’s Wave Equation And Werner Heisenberg Matrix**

**Module Numbered One Governing Equations:**

The differential system of this model is now

\[
\frac{dG_{13}}{dt} = (a_{13}^{(1)})G_{14} - \left[ (a_{13}'^{(1)}) + (a_{13}''^{(1)})(T_{14}, t) \right]G_{13} \quad 565
\]

\[
\frac{dG_{14}}{dt} = (a_{14}^{(1)})G_{13} - \left[ (a_{14}'^{(1)}) + (a_{14}''^{(1)})(T_{14}, t) \right]G_{14} \quad 566
\]

\[
\frac{dG_{15}}{dt} = (a_{15}^{(1)})G_{14} - \left[ (a_{15}'^{(1)}) + (a_{15}''^{(1)})(T_{14}, t) \right]G_{15} \quad 567
\]

\[
\frac{dT_{13}}{dt} = (b_{13}^{(1)})T_{14} - \left[ (b_{13}'^{(1)}) - (b_{13}''^{(1)})(G_t) \right]T_{13} \quad 568
\]

\[
\frac{dT_{14}}{dt} = (b_{14}^{(1)})T_{13} - \left[ (b_{14}'^{(1)}) - (b_{14}''^{(1)})(G_t) \right]T_{14} \quad 569
\]

\[
\frac{dT_{15}}{dt} = (b_{15}^{(1)})T_{14} - \left[ (b_{15}'^{(1)}) - (b_{15}''^{(1)})(G_t) \right]T_{15} \quad 570
\]

\[+(a_{13}''^{(1)})(T_{14}, t) = \text{First augmentation factor} \]

\[-(b_{13}''^{(1)})(G_t) = \text{First detritions factor} \]

**Feynman Path Integral Formulation And Relativistic Theories**

**Module Numbered Two**

**Governing Equations:**

The differential system of this model is now

\[
\frac{dG_{16}}{dt} = (a_{16}^{(2)})G_{17} - \left[ (a_{16}'^{(2)}) + (a_{16}''^{(2)})(T_{17}, t) \right]G_{16} \quad 571
\]

\[
\frac{dG_{17}}{dt} = (a_{17}^{(2)})G_{16} - \left[ (a_{17}'^{(2)}) + (a_{17}''^{(2)})(T_{17}, t) \right]G_{17} \quad 572
\]

\[
\frac{dG_{18}}{dt} = (a_{18}^{(2)})G_{17} - \left[ (a_{18}'^{(2)}) + (a_{18}''^{(2)})(T_{17}, t) \right]G_{18} \quad 573
\]

\[
\frac{dT_{16}}{dt} = (b_{16}^{(2)})T_{17} - \left[ (b_{16}'^{(2)}) - (b_{16}''^{(2)})(G_{19}, t) \right]T_{16} \quad 574
\]

\[
\frac{dT_{17}}{dt} = (b_{17}^{(2)})T_{16} - \left[ (b_{17}'^{(2)}) - (b_{17}''^{(2)})(G_{19}, t) \right]T_{17} \quad 575
\]

\[
\frac{dT_{18}}{dt} = (b_{18}^{(2)})T_{17} - \left[ (b_{18}'^{(2)}) - (b_{18}''^{(2)})(G_{19}, t) \right]T_{18} \quad 576
\]

\[+(a_{16}''^{(2)})(T_{17}, t) = \text{First augmentation factor} \]

\[-(b_{16}''^{(2)})(G_{19}, t) = \text{First detritions factor} \]

**Concatenated Equations For The Holistic Model**
\[
\begin{align*}
\frac{dg_{13}}{dt} &= (a_{13})^{(1)}g_{14} - \left[ (a'_{13})^{(1)} \right. + (a''_{13})^{(1)}(T_{14}, t) + (a''')^{(2,2)}(T_{17}, t) \left. \right] g_{13} \\
\frac{dg_{14}}{dt} &= (a_{14})^{(1)}g_{13} - \left[ (a'_{14})^{(1)} \right. + (a''_{14})^{(1)}(T_{14}, t) + (a''')^{(2,2)}(T_{17}, t) \left. \right] g_{14} \\
\frac{dg_{15}}{dt} &= (a_{15})^{(1)}g_{14} - \left[ (a'_{15})^{(1)} \right. + (a''_{15})^{(1)}(T_{14}, t) + (a''')^{(2,2)}(T_{17}, t) \left. \right] g_{15}
\end{align*}
\]

Where \(a''_{13}^{(1)}(T_{14}, t), (a''_{14}^{(1)}(T_{14}, t), (a''_{15}^{(1)}(T_{14}, t)\) are first augmentation coefficients for category 1, 2 and 3.

\[
\begin{align*}
+ (a''_{16}^{(2,2)}(T_{17}, t), (a''_{17}^{(2,2)}(T_{17}, t), (a''_{18}^{(2,2)}(T_{17}, t)\) are second augmentation coefficients for category 1, 2 and 3.
\]

\[
\begin{align*}
\frac{dt_{13}}{dt} &= (b_{13})^{(1)}t_{14} - \left[ (b'_{13})^{(1)} \right. - (b''_{13})^{(1)}(G, t) + (b''')^{(2,2)}(G_{19}, t) \left. \right] t_{13} \\
\frac{dt_{14}}{dt} &= (b_{14})^{(1)}t_{13} - \left[ (b'_{14})^{(1)} \right. - (b''_{14})^{(1)}(G, t) + (b''')^{(2,2)}(G_{19}, t) \left. \right] t_{14} \\
\frac{dt_{15}}{dt} &= (b_{15})^{(1)}t_{14} - \left[ (b'_{15})^{(1)} \right. - (b''_{15})^{(1)}(G, t) + (b''')^{(2,2)}(G_{19}, t) \left. \right] t_{15}
\end{align*}
\]

Where \(- (b''_{13})^{(1)}(G, t), -(b''_{14})^{(1)}(G, t), -(b''_{15})^{(1)}(G, t)\) are first detritions coefficients for category 1, 2 and 3.

\[
\begin{align*}
+ (b''_{16})^{(2,2)}(G_{19}, t), +(b''_{17})^{(2,2)}(G_{19}, t), +(b''_{18})^{(2,2)}(G_{19}, t)\) are second augmentation coefficients for category 1, 2 and 3.
\]

\[
\begin{align*}
\frac{dg_{16}}{dt} &= (a_{16})^{(2)}g_{17} - \left[ (a'_{16})^{(2)} \right. + (a''_{16})^{(2)}(T_{17}, t) + (a''')^{(1,1)}(T_{14}, t) \left. \right] g_{16} \\
\frac{dg_{17}}{dt} &= (a_{17})^{(2)}g_{16} - \left[ (a'_{17})^{(2)} \right. + (a''_{17})^{(2)}(T_{17}, t) + (a''')^{(1,1)}(T_{14}, t) \left. \right] g_{17} \\
\frac{dg_{18}}{dt} &= (a_{18})^{(2)}g_{17} - \left[ (a'_{18})^{(2)} \right. + (a''_{18})^{(2)}(T_{17}, t) + (a''')^{(1,1)}(T_{14}, t) \left. \right] g_{18}
\end{align*}
\]

Where \(+ (a''_{16})^{(2)}(T_{17}, t), (a''_{17})^{(2)}(T_{17}, t), (a''_{18})^{(2)}(T_{17}, t)\) are first augmentation coefficients for category 1, 2 and 3.

\[
\begin{align*}
+ (a''_{13})^{(1,1)}(T_{14}, t), (a''_{14})^{(1,1)}(T_{14}, t), (a''_{15})^{(1,1)}(T_{14}, t)\) are second detrition coefficients for category 1, 2 and 3.
\]

\[
\begin{align*}
\frac{dt_{16}}{dt} &= (b_{16})^{(2)}t_{17} - \left[ (b'_{16})^{(2)} \right. - (b''_{16})^{(2)}(G_{19}, t) - (b''')^{(1,1)}(G, t) \left. \right] t_{16} \\
\frac{dt_{17}}{dt} &= (b_{17})^{(2)}t_{16} - \left[ (b'_{17})^{(2)} \right. - (b''_{17})^{(2)}(G_{19}, t) - (b''')^{(1,1)}(G, t) \left. \right] t_{17} \\
\frac{dt_{18}}{dt} &= (b_{18})^{(2)}t_{17} - \left[ (b'_{18})^{(2)} \right. - (b''_{18})^{(2)}(G_{19}, t) - (b''')^{(1,1)}(G, t) \left. \right] t_{18}
\end{align*}
\]

Where \(- (b''_{16})^{(2)}(G_{19}, t), -(b''_{17})^{(2)}(G_{19}, t), -(b''_{18})^{(2)}(G_{19}, t)\) are first detritions coefficients for category 1, 2 and 3.

\[
\begin{align*}
-(b''_{13})^{(1,1)}(G, t), -(b''_{14})^{(1,1)}(G, t), -(b''_{15})^{(1,1)}(G, t)\) are second detrition coefficients for category 1, 2 and 3.
\]

Where we suppose

\[
(0) \quad (a_{1})^{(1)}, (a_{2})^{(1)}, (a'_{13})^{(1)}, (b_{1})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)} > 0.
\]
\[ i, j = 13, 14, 15 \]

(P) The functions \((a_i^{(1)}), (b_i^{(1)})\) are positive continuous increasing and bounded.

**Definition of** \((p_i^{(1)}), (r_i^{(1)})\):

\[
(a_i^{(1)})(T_{14}, t) \leq (p_i^{(1)}) \leq (\hat{A}_{13})^{(1)}
\]
\[
(b_i^{(1)})(G, t) \leq (r_i^{(1)}) \leq (b_i^{(1)}) \leq (\hat{B}_{13})^{(1)}
\]

(Q) \[
\lim_{t \to \infty}(a_i^{(1)})(T_{14}, t) = (p_i^{(1)})
\]
\[
\lim_{G \to \infty}(b_i^{(1)})(G, t) = (r_i^{(1)})
\]

**Definition of** \((\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}\):

Where \((\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i^{(1)}), (r_i^{(1)})\) are positive constants and \(i = 13, 14, 15\).

They satisfy Lipschitz condition:

\[
|(a_i^{(1)})(T_{14}^{(1)} - T_{14}^{(1)})(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14}^{(1)} - T_{14}^{(1)}|e^{-(\hat{h}_{13})^{(1)}t}
\]
\[
|(b_i^{(1)})(G', t) - (b_i^{(1)})(G, t)| < (\hat{k}_{13})^{(1)}|G - G'|e^{-(\hat{h}_{13})^{(1)}t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i^{(1)})(T_{14}^{(1)}), (a_i^{(1)})(T_{14}^{(1)}), (a_i^{(1)})(T_{14}^{(1)}), (a_i^{(1)})(T_{14}^{(1)})\) and \((T_{14}^{(1)}), (T_{14}^{(1)}), (T_{14}^{(1)})\) are points belonging to the interval \([[\hat{k}_{13})^{(1)}, (\hat{h}_{13})^{(1)}]\). It is to be noted that \((a_i^{(1)})(T_{14}^{(1)})\) is uniformly continuous. In the eventual terms of the fact, if \((\hat{h}_{13})^{(1)} = 1\) then the function \((a_i^{(1)})(T_{14}^{(1)})\), the first augmentation coefficient attributable to terrestrial organisms, would be uniformly continuous.

**Definition of** \((\hat{h}_{13})^{(1)}, (\hat{k}_{13})^{(1)}\):

\[
\frac{(a_i^{(1)})}{(\hat{h}_{13})^{(1)}}, \frac{(b_i^{(1)})}{\hat{k}_{13})^{(1)}} < 1
\]

**Definition of** \((\hat{B}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}\):

\(S\) There exists two constants \((\hat{P}_{13})^{(1)}\) and \((\hat{Q}_{13})^{(1)}\) which together with \((\hat{h}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}\) and \((\hat{B}_{13})^{(1)}\) and the constants \((a_i^{(1)}), (a_i^{(1)}), (b_i^{(1)}), (b_i^{(1)}), (p_i^{(1)}), (r_i^{(1)})\) satisfy the inequalities

\[
\frac{1}{(\hat{h}_{13})^{(1)}}[(a_i^{(1)})(1) + (a_i^{(1)})(1) + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1
\]
\[
\frac{1}{(\hat{h}_{13})^{(1)}}[(b_i^{(1)})(1) + (b_i^{(1)})(1) + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1
\]

Where we suppose

(T) \((a_i^{(2)}), (a_i^{(2)}), (a_i^{(2)}), (b_i^{(2)}), (b_i^{(2)}), (b_i^{(2)}), (b_i^{(2)}) > 0\), \(i, j = 16, 17, 18\)

(U) The functions \((a_i^{(2)}), (b_i^{(2)}), (b_i^{(2)})\) are positive continuous increasing and bounded.

**Definition of** \((p_i^{(2)}), (r_i^{(2)})\);
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\[(a_i^0)'(T_{17}, t) \leq (p_i)'(T_{17}) \leq (\hat{A}_{16})'(2)\]
\[(b_i^0)'(G, t) \leq (r_i)'(t) \leq (\hat{B}_{16})'(2)\]

(V) \[\lim_{T_{17} \to 0}(a_i^0)'(T_{17}, t) = (p_i)'\]
\[\lim_{G \to 0}(b_i^0)'((G_{19}), t) = (r_i)\]

**Definition of** \((\hat{A}_{16})'(2), (\hat{B}_{16})'(2)\):

\[\text{Where} \quad (\hat{A}_{16})'(2), (\hat{B}_{16})'(2), (p_i)'(2), (r_i)'(2)\] are positive constants and \(i = 16, 17, 18\).

They satisfy the Lipschitz condition:

\[|[(a_i^0)'(T_{17}, t) - (a_i^0)'(T_{17}, t)] \leq (\hat{k}_{16})'(2)|T_{17} - T_{17}'|e^{-(\hat{A}_{16})'(2)t}\]
\[|[(b_i^0)'((G_{19}), t) - (b_i^0)'((G_{19}), t)] < (\hat{G}_{19})'(2)||G_{19} - (G_{19})'||e^{-(\hat{A}_{16})'(2)t}\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i^0)'(T_{17}, t)\) and \((a_i^0)'(T_{17}, t)\) which together with \((\hat{A}_{16})'(2), (\hat{B}_{16})'(2)\) and the constants \(a_i^0(2), a_i^0(2), b_i(2), b_i(2), p_i(2), r_i(2), i = 16, 17, 18,\) satisfy the inequalities

\[\frac{1}{(A_{16})'(2)}[a_i'(2) + a_i'(2) + (\hat{A}_{16})'(2) + (\hat{B}_{16})'(2) < 1\]
\[\frac{1}{(M_{16})'(2)}[b_i'(2) + b_i'(2) + (\hat{B}_{16})'(2) + (\hat{G}_{16})'(2) + (\hat{K}_{16})'(2) < 1\]

**Theorem 1:** If the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions

\[\text{Definition of } G_i(0), T_i(0) :\]
\[G_i(t) \leq (\hat{P}_{13})''(\hat{R}_{13})'t, \quad G_i(0) = G_i^0 > 0\]
\[T_i(t) \leq (\hat{Q}_{13})''(\hat{R}_{13})'t, \quad T_i(0) = T_i^0 > 0\]
**Theorem 1:** if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions.

**Definition of:** \( G_i(0), T_i(0) \)

\[ G_i(t) \leq (\hat{P}_{16})^{(2)}e^{(R_{16})^{(2)}t}, \quad G_i(0) = G^0_i > 0 \]

\[ T_i(t) \leq (\hat{Q}_{16})^{(2)}e^{(R_{16})^{(2)}t}, \quad T_i(0) = T^0_i > 0 \]

**PROOF:**

Consider operator \( \mathcal{A}^{(1)} \) defined on the space of sextuples of continuous functions \( G_i, T_i : \mathbb{R}_+ \to \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G^0_i, \quad T_i(0) = T^0_i, \quad G_i \leq (\hat{P}_{13})^{(1)}, T_i \leq (\hat{Q}_{13})^{(1)}, \]

\[ 0 \leq G_i(t) - G^0_i \leq (\hat{P}_{13})^{(1)}e^{(R_{13})^{(1)}t} \]

\[ 0 \leq T_i(t) - T^0_i \leq (\hat{Q}_{13})^{(1)}e^{(R_{13})^{(1)}t} \]

By

\[ \bar{G}_{13}(t) = G^0_{13} + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{13}) - \left( (a_{13}')^{(1)} + (a_{13}'')^{(1)} \right) T_{14}(s_{13}), s_{13} \right] G_{15}(s_{13}) \] \[ d s_{13} \]

\[ \bar{G}_{14}(t) = G^0_{14} + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{13}) - \left( (a_{14}')^{(1)} + (a_{14}'')^{(1)} \right) T_{13}(s_{13}), s_{13} \right] G_{14}(s_{13}) \] \[ d s_{13} \]

\[ \bar{G}_{15}(t) = G^0_{15} + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{13}) - \left( (a_{15}')^{(1)} + (a_{15}'')^{(1)} \right) T_{14}(s_{13}), s_{13} \right] G_{15}(s_{13}) \] \[ d s_{13} \]

\[ \bar{T}_{13}(t) = T^0_{13} + \int_0^t \left( (b_{13})^{(1)} T_{14}(s_{13}) - \left( (b_{13}')^{(1)} - (b_{13}'')^{(1)} \right) G(s_{13}), s_{13} \right) T_{13}(s_{13}) \] \[ d s_{13} \]

\[ \bar{T}_{14}(t) = T^0_{14} + \int_0^t \left( (b_{14})^{(1)} T_{13}(s_{13}) - \left( (b_{14}')^{(1)} - (b_{14}'')^{(1)} \right) G(s_{13}), s_{13} \right) T_{14}(s_{13}) \] \[ d s_{13} \]

\[ \bar{T}_{15}(t) = T^0_{15} + \int_0^t \left( (b_{15})^{(1)} T_{14}(s_{13}) - \left( (b_{15}')^{(1)} - (b_{15}'')^{(1)} \right) G(s_{13}), s_{13} \right) T_{15}(s_{13}) \] \[ d s_{13} \]

Where \( s_{13} \) is the integrand that is integrated over an interval \((0, t)\)

**PROOF:**

Consider operator \( \mathcal{A}^{(2)} \) defined on the space of sextuples of continuous functions \( G_i, T_i : \mathbb{R}_+ \to \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G^0_i, \quad T_i(0) = T^0_i, \quad G_i \leq (\hat{P}_{16})^{(2)}, T_i \leq (\hat{Q}_{16})^{(2)}, \]

\[ 0 \leq G_i(t) - G^0_i \leq (\hat{P}_{16})^{(2)}e^{(R_{16})^{(2)}t} \]

\[ 0 \leq T_i(t) - T^0_i \leq (\hat{Q}_{16})^{(2)}e^{(R_{16})^{(2)}t} \]

By

\[ \bar{G}_{16}(t) = G^0_{16} + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{16}) - \left( (a_{16}')^{(2)} + (a_{16}'')^{(2)} \right) T_{17}(s_{16}), s_{16} \right] G_{16}(s_{16}) \] \[ d s_{16} \]

\[ \bar{G}_{17}(t) = G^0_{17} + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{16}) - \left( (a_{17}')^{(2)} + (a_{17}'')^{(2)} \right) T_{16}(s_{16}), s_{16} \right] G_{17}(s_{16}) \] \[ d s_{16} \]

\[ \bar{G}_{18}(t) = G^0_{18} + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{16}) - \left( (a_{18}')^{(2)} + (a_{18}'')^{(2)} \right) T_{17}(s_{16}), s_{16} \right] G_{18}(s_{16}) \] \[ d s_{16} \]
\[ \tilde{T}_{16}(t) = T_{16}^0 + \int_0^t \left[ (b_{16})^{(2)}(T_{17}(s_{16}) - \left( b_{16}^{(2)} - (b_{16})^{(2)}(G(s_{16}), s_{16}) \right) T_{16}(s_{16}) \right] ds_{16} \]

\[ \tilde{T}_{17}(t) = T_{17}^0 + \int_0^t \left[ (b_{17})^{(2)}(T_{16}(s_{16}) - \left( b_{17}^{(2)} - (b_{17})^{(2)}(G(s_{16}), s_{16}) \right) T_{17}(s_{16}) \right] ds_{16} \]

\[ \tilde{T}_{18}(t) = T_{18}^0 + \int_0^t \left[ (b_{18})^{(2)}(T_{17}(s_{16}) - \left( b_{18}^{(2)} - (b_{18})^{(2)}(G(s_{16}), s_{16}) \right) T_{18}(s_{16}) \right] ds_{16} \]

Where \( s_{16} \) is the integrand that is integrated over an interval \((0, t)\)

(e) The operator \( \mathcal{A}^{(1)} \) maps the space of functions satisfying 34,35,36 into itself. Indeed it is obvious that

\[ G_{13}(t) \leq G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} \left( G_{14}^0 + \left( \tilde{\mathcal{P}}_{13} \right)^{(1)} e^{(\tilde{\mathcal{R}}_{13})^{(1)} t} \right) \right] ds_{13} =
\]

\[ (1 + (a_{13})^{(1)} t) G_{14}^0 + \frac{(a_{13})^{(1)} \left( \tilde{\mathcal{P}}_{13} \right)^{(1)} e^{(\tilde{\mathcal{R}}_{13})^{(1)} t} - 1}{G_{14}^0} \]

From which it follows that

\[ (G_{13}(t) - G_{13}^0) e^{-(\tilde{\mathcal{R}}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)} \left( \tilde{\mathcal{P}}_{13} \right)^{(1)} + G_{14}^0 e^{\left( \frac{\left( \tilde{\mathcal{P}}_{13} \right)^{(1)} + G_{14}^0}{G_{14}^0} \right)} + \left( \tilde{\mathcal{P}}_{13} \right)^{(1)}}{G_{14}^0} \]

\( (G_{13}^0) \) is as defined in the statement of theorem 1

Analogous inequalities hold also for \( G_{14}, G_{15}, T_{13}, T_{14}, T_{15} \)

(f) The operator \( \mathcal{A}^{(2)} \) maps the space of functions satisfying 34,35,36 into itself. Indeed it is obvious that

\[ G_{16}(t) \leq G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} \left( G_{17}^0 + \left( \tilde{\mathcal{P}}_{16} \right)^{(6)} e^{(\tilde{\mathcal{R}}_{16})^{(2)} t} \right) \right] ds_{16} =
\]

\[ (1 + (a_{16})^{(2)} t) G_{17}^0 + \frac{(a_{16})^{(2)} \left( \tilde{\mathcal{P}}_{16} \right)^{(2)} e^{(\tilde{\mathcal{R}}_{16})^{(2)} t} - 1}{G_{17}^0} \]

From which it follows that

\[ (G_{16}(t) - G_{16}^0) e^{-(\tilde{\mathcal{R}}_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)} \left( \tilde{\mathcal{P}}_{16} \right)^{(2)} + G_{17}^0 e^{\left( \frac{\left( \tilde{\mathcal{P}}_{16} \right)^{(2)} + G_{17}^0}{G_{17}^0} \right)} + \left( \tilde{\mathcal{P}}_{16} \right)^{(2)}}{G_{17}^0} \]

Analogous inequalities hold also for \( G_{17}, G_{18}, T_{16}, T_{17}, T_{18} \)

It is now sufficient to take \( \frac{(a_{13})^{(1)}}{(\tilde{\mathcal{R}}_{13})^{(1)}} \), \( \frac{(b_{13})^{(1)}}{(\tilde{\mathcal{R}}_{13})^{(1)}} \) large to have

\[ \left( \tilde{\mathcal{P}}_{13} \right)^{(1)} \text{ and } \left( \tilde{\mathcal{Q}}_{13} \right)^{(1)} \text{ large to have} \]

\[ \frac{(a_{13})^{(1)}}{(\tilde{\mathcal{R}}_{13})^{(1)}} \left[ \left( \tilde{\mathcal{P}}_{13} \right)^{(1)} + \left( \tilde{\mathcal{P}}_{13} \right)^{(1)} + G_{14}^0 e^{\left( \frac{\left( \tilde{\mathcal{P}}_{13} \right)^{(1)} + G_{14}^0}{G_{14}^0} \right)} \right] \leq \left( \tilde{\mathcal{P}}_{13} \right)^{(1)} \]

\[ \frac{(b_{13})^{(1)}}{(\tilde{\mathcal{R}}_{13})^{(1)}} \left[ \left( \tilde{\mathcal{Q}}_{13} \right)^{(1)} + T_{14}^0 e^{\left( \frac{\left( \tilde{\mathcal{Q}}_{13} \right)^{(1)} + T_{14}^0}{T_{14}^0} \right)} \right] \leq \left( \tilde{\mathcal{Q}}_{13} \right)^{(1)} \]
In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying 34,35,36 into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d \left( (G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)}) \right) =$$

$$\sup \{ \max_{t \in \mathbb{R}_+} \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-\left( R_{13} \right)^{(1)} t} , \max_{t \in \mathbb{R}_+} \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-\left( R_{13} \right)^{(1)} t} \}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $\left( \tilde{G}, \tilde{T} \right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left| \tilde{G}_i^{(1)} - \tilde{G}_i^{(2)} \right| \leq \int_0^1 (a_{13})^{(1)} \left| G_{14}^{(1)} - G_{14}^{(2)} \right| e^{-\left( R_{13} \right)^{(1)} s_{13}} e^{\left( R_{13} \right)^{(1)} s_{13}} d s_{13} +$$

$$\int_0^1 ((a_{13}')^{(1)} G_{13}^{(1)} - G_{13}^{(2)}) e^{-\left( R_{13} \right)^{(1)} s_{13}} e^{\left( R_{13} \right)^{(1)} s_{13}} +$$

$$(a_{13}'')^{(1)} (T_{14}^{(1)}, s_{13}) \left| G_{13}^{(1)} - G_{13}^{(2)} \right| e^{-\left( R_{13} \right)^{(1)} s_{13}} e^{\left( R_{13} \right)^{(1)} s_{13}} +$$

$$G_{13}^{(1)} (a_{13}'')^{(1)} (T_{14}^{(1)}, s_{13}) - (a_{13}'')^{(1)} (T_{14}^{(2)}, s_{13}) \left| e^{-\left( R_{13} \right)^{(1)} s_{13}} e^{\left( R_{13} \right)^{(1)} s_{13}} d s_{13}$$

Where $s_{13}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 25,26,27,28 and 29 it follows

$$\left| G^{(1)} - G^{(2)} \right| e^{-\left( R_{13} \right)^{(1)} t} \leq \frac{1}{\left( R_{13} \right)^{(1)}} \left( (a_{13})^{(1)} + (a_{13}')^{(1)} + (\lambda_{13})^{(1)} + (\mu_{13})^{(1)} (\kappa_{13})^{(1)} \right) d \left( (G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}) \right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis (34,35,36) the result follows

Remark 1: The fact that we supposed $(a_{13}'')^{(1)}$ and $(b_{13}'')^{(1)}$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\tilde{G}_{13})^{(1)} e^{\left( R_{13} \right)^{(1)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_{13}'')^{(1)}$ and $(b_{13}'')^{(1)}$, $i = 13, 14, 15$ depend only on $T_{14}$ and respectively on $G$ (and not on $t$) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any $t$ where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_{13}^{(1)} e^{-\int_0^t \left( (a_j^{(1)}) - (a_j^{(1)}) (T_{14}(s_{13}), s_{13})) ds_{13} \right) \geq 0}$$

$$T_i(t) \geq T_{13}^{(1)} e^{-\left( b_j^{(1)} \right)} > 0 \text{ for } t > 0$$

Definition of $(\mathcal{M}_{13})^{(1)}$, $(\mathcal{M}_{12})^{(1)}$ and $(\mathcal{M}_{13})^{(1)}$: $\mathcal{M}_{13}$

Remark 3: if $G_{13}$ is bounded, the same property have also $G_{14}$ and $G_{15}$.
\[ G_{13} < (M_{13})^{(1)} \text{ it follows } \frac{dG_{13}}{dt} \leq ( (M_{13})^{(1)} ) - (a_{14})^{(1)} G_{14} \text{ and by integrating} \\
G_{14} \leq ( (M_{13})^{(1)} )_{2} = G_{14}^{0} + 2(a_{14})^{(1)}( (M_{13})^{(1)} )_{1}/(a_{14})^{(1)} \\
\]

In the same way, one can obtain
\[ G_{15} \leq ( (M_{13})^{(1)} )_{3} = G_{15}^{0} + 2(a_{15})^{(1)}( (M_{13})^{(1)} )_{2}/(a_{15})^{(1)} \]

If \( G_{14} \) or \( G_{15} \) is bounded, the same property follows for \( G_{13} \), \( G_{15} \) and \( G_{13} \), \( G_{14} \) respectively.

**Remark 4:** If \( G_{13} \) is bounded, from below, the same property holds for \( G_{14} \) and \( G_{15} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{14} \) is bounded from below.

**Remark 5:** If \( T_{13} \) is bounded from below and \( \lim_{t \to \infty} ((b_{13}')^{(1)} (G(t), t)) = (b_{14}')^{(1)} \) then \( T_{14} \to \infty \).

**Definition of** \((m)^{(1)}\) and \( \varepsilon_{1} \):

Indeed let \( t_{1} \) be so that for \( t > t_{1} \)

\[(b_{14})^{(1)} - (b_{15}')^{(1)} (G(t), t) < \varepsilon_{1}, T_{13}(t) > (m)^{(1)} \]

Then \( \frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_{1}T_{14} \) which leads to

\[ T_{14} \geq \frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_{1}} (1 - e^{-\varepsilon_{1}t}) + T_{13}e^{-\varepsilon_{1}t} \]

If we take \( t \) such that \( e^{-\varepsilon_{1}t} = \frac{1}{2} \) it results

\[ T_{14} \geq \frac{(a_{14})^{(1)}(m)^{(1)}}{2} \times t = log \frac{2}{\varepsilon_{1}} \]

By taking now \( \varepsilon_{1} \) sufficiently small one sees that \( T_{14} \) is unbounded. The same property holds for \( T_{15} \) if \( \lim_{t \to \infty} ((b_{15}')^{(1)} (G(t), t)) = (b_{13}')^{(1)} \)

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

It is now sufficient to take \( \frac{(a_{1})^{(2)}}{(M_{16})^{(2)}} \cdot \frac{(b_{1})^{(2)}}{(M_{16})^{(2)}} < 1 \) and to choose

\( \rho_{16}^{(2)} \) and \( \rho_{16}^{(2)} \) large to have

\[ \frac{(a_{1})^{(2)}}{(M_{16})^{(2)}} (\rho_{16}^{(2)} + (\rho_{16}^{(2)} + \rho_{16}^{(2)} e^{-\frac{(\rho_{16}^{(2)} + \rho_{16}^{(2)})}{\rho_{16}^{(2)}}} ) \leq (\rho_{16}^{(2)})^{(2)} \]

\[ \frac{(b_{1})^{(2)}}{(M_{16})^{(2)}} ((\rho_{16}^{(2)} + \rho_{16}^{(2)} ) e^{-\frac{(\rho_{16}^{(2)} + \rho_{16}^{(2)})}{\rho_{16}^{(2)}}} ) \leq (\rho_{16}^{(2)})^{(2)} \]

In order that the operator \( \mathcal{A}^{(2)} \) transforms the space of sextuples of functions \( G_{i}, T_{i} \) satisfying 34,35,36 into itself

The operator \( \mathcal{A}^{(2)} \) is a contraction with respect to the metric

\[ d \left( (G_{19})^{(1)} (T_{19})^{(1)}, (G_{19})^{(2)} (T_{19})^{(2)} \right) = \]

\[ \max_{t \in \mathbb{R}^{+}} \left| G_{i}^{(1)}(t) - G_{i}^{(2)}(t) \right| e^{-\left(\frac{(Q_{16})^{(2)}}{T_{i}^{(2)}}\right)} \max_{t \in \mathbb{R}^{+}} \left| T_{i}^{(1)}(t) - T_{i}^{(2)}(t) \right| e^{-\left(\frac{(Q_{16})^{(2)}}{T_{i}^{(2)}}\right)} \]

Indeed if we denote
**Definition of** $\vec{G}_{19}, \vec{T}_{19}$: 

$\vec{G}_{19}, \vec{T}_{19} = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results

$$|\vec{G}_{16}^{(1)} - \vec{G}_{1}^{(2)}| \leq \int_{0}^{T} (a_{16}^{(2)}) |G_{17}^{(1)} - G_{17}^{(2)}| e^{-\vec{R}_{16}^{(2)}x_{16}(s)} e^{(\vec{R}_{16}^{(2)})x_{16}(s)} ds_{16} +$$

$$\int_{0}^{T} (a_{16}^{(2)}) |G_{16}^{(1)} - G_{16}^{(2)}| e^{-\vec{R}_{16}^{(2)}x_{16}(s)} e^{(\vec{R}_{16}^{(2)})x_{16}(s)} +$$

$$\int_{0}^{T} (a_{16}^{(2)}) (T_{17}^{(1)}, s_{16}) |G_{16}^{(1)} - G_{16}^{(2)}| e^{-\vec{R}_{16}^{(2)}x_{16}(s)} e^{(\vec{R}_{16}^{(2)})x_{16}(s)} +$$

$$\int_{0}^{T} (a_{16}^{(2)}) (T_{17}^{(1)}, s_{16}) (a_{16}^{(2)}) (T_{17}^{(2)}, s_{16}) |e^{-\vec{R}_{16}^{(2)}x_{16}(s)} e^{(\vec{R}_{16}^{(2)})x_{16}(s)}| ds_{16}$$

Where $s_{16}$ represents integrand that is integrated over the interval $[0, T]$.

From the hypotheses on $25, 26, 27, 28$ and $29$ it follows

$$|((G_{19})^{(1)} - (G_{19})^{(2)}) e^{-\vec{R}_{16}^{(2)}x_{16}(s)} e^{(\vec{R}_{16}^{(2)})x_{16}(s)}| \leq \frac{1}{(\vec{M}_{16})^{(2)}} (a_{16}^{(2)} + (a_{16}^{(2)}) + (\vec{A}_{16}^{(2)}) + (\vec{P}_{16}^{(2)})(\vec{k}_{16}^{(2)})) d \left( ((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}) \right)$$

And analogous inequalities for $G_{1}$ and $T_{1}$. Taking into account the hypothesis $(34, 35, 36)$ the result follows

**Remark 1:** The fact that we supposed $(a_{16}^{(2)})$ and $(b_{16}^{(2)})$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\vec{P}_{16}^{(2)} e^{(\vec{R}_{16}^{(2)})x_{16}(s)} e^{(\vec{R}_{16}^{(2)})x_{16}(s)})$ and $(\vec{Q}_{16}^{(2)} e^{(\vec{M}_{16}^{(2)})})$ respectively of $\mathbb{R}_{+}$.

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_{16}^{(2)})$ and $(b_{16}^{(2)})$, $i = 16, 17, 18$ depend only on $T_{17}$ and respectively on $(G_{19})$ and not on $t$ and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_{1}(t) = 0$ and $T_{1}(t) = 0$.

From 19 to 24 it results

$$G_{1}(t) \geq G_{1}^{0} \left[ \int_{0}^{t} (a_{16}^{(2)} - (a_{16}^{(2)})(T_{17}(s_{16}), x_{16}) ds_{16} \right] \geq 0$$

$$T_{1}(t) \geq T_{1}^{0} e^{-(b_{16}^{(2)} t)} > 0 \text{ for } t > 0$$

**Definition of** $(\vec{M}_{16}^{(2)})^{(2)}$, $(\vec{M}_{16}^{(2)})^{(2)}$, and $(\vec{M}_{16}^{(2)})^{(2)}$

**Remark 3:** If $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$, indeed if

$$G_{16} \leq (\vec{M}_{16}^{(2)}) \text{ it follows } \frac{dG_{17}}{dt} \leq ((\vec{M}_{16}^{(2)}) - (a_{17}^{(2)}) G_{17} \text{ and by integrating }$$

$$G_{17} \leq ((\vec{M}_{16}^{(2)})_{2} = G_{17}^{0} + 2(a_{17}^{(2)}((\vec{M}_{16}^{(2)})_{1}) / (a_{17}^{(2)})$$

In the same way, one can obtain

$$G_{18} \leq ((\vec{M}_{16}^{(2)})_{3} = G_{18}^{0} + 2(a_{18}^{(2)}((\vec{M}_{16}^{(2)})_{2}) / (a_{18}^{(2)})$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 4:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below.
Remark 5: If $T_{16}$ is bounded from below and $\lim_{t \to \infty} ((b^{(2)}_i) ((G_{19}(t), t)) = (b^{(2)}_{17})$ then $T_{17} \to \infty$.

Definition of $(m)^{(2)}$ and $\varepsilon_2$:

Indeed let $t_2$ be so that for $t > t_2$

$$(b^{(2)}_{17}) - (b^{(2)}_i) ((G_{19}(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a^{(2)}_{17}) (m)^{(2)} - \varepsilon_2 T_{17}$ which leads to

$$T_{17} \geq \left( \frac{(a^{(2)}_{17}) (m)^{(2)}}{\varepsilon_2} \right) (1 - e^{-\varepsilon_2 t}) + T_{17}^1 e^{-\varepsilon_2 t}$$

If we take $t$ such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$$T_{17} \geq \left( \frac{(a^{(2)}_{17}) (m)^{(2)}}{2 \varepsilon_2} \right)$$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Behavior of the solutions of equation 37 to 42

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$$-(\sigma_2)^{(1)} \leq -(a^{(1)}_{13}) + (a^{(1)}_{14}) - (a^{(1)}_{13}) (T_{14}, t) + (a^{(1)}_{14}) (T_{14}, t) \leq -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \leq -(b^{(1)}_{13}) + (b^{(1)}_{14}) - (b^{(1)}_{13}) (G, t) - (b^{(1)}_{14}) (G, t) \leq -(\tau_1)^{(1)}$$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, (v^{(1)}), (u^{(1)})$:

By $(v_1)^{(1)} > 0, (v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations $(a^{(1)}_{14}) (v^{(1)})^2 + (\sigma_1)^{(1)} v^{(1)} - (a^{(1)}_{13}) = 0$ and $(b^{(1)}_{14}) (u^{(1)})^2 + (\tau_1)^{(1)} u^{(1)} - (b^{(1)}_{13}) = 0$

Definition of $(\bar{v}_1)^{(1)}, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$:

By $(v_1)^{(1)} > 0, (v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations

$$(a^{(1)}_{14}) (v^{(1)})^2 + (\sigma_1)^{(1)} v^{(1)} - (a^{(1)}_{13}) = 0$$

and

$$(b^{(1)}_{14}) (u^{(1)})^2 + (\tau_1)^{(1)} u^{(1)} - (b^{(1)}_{13}) = 0$$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$ by

$$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$$

and

$$(m_2)^{(1)} = (v_0)^{(1)}$$

and analogously
(\mu_2^{(1)}) = (u_0^{(1)}, (\mu_1^{(1)}) = (u_1^{(1)}) \text{ if } (u_0^{(1)}) < (u_1^{(1)})

\mu_2^{(1)} = (u_1^{(1)}), (\mu_1^{(1)}) = (\bar{u}_1^{(1)}) \text{ if } (u_1^{(1)}) < (u_0^{(1)}) < (\bar{u}_1^{(1)})

and \left(\begin{array}{c}
\text{If } (u_0^{(1)}) = \frac{\gamma_0}{\gamma_1}
\end{array}\right)

(\mu_2^{(1)}) = (u_1^{(1)}, (\mu_1^{(1)}) = (u_0^{(1)}) \text{ if } (\bar{u}_1^{(1)}) < (u_0^{(1)}) \text{ where } (u_1^{(1)}), (\bar{u}_1^{(1)})

Then the solution of Global equations satisfies the inequalities

\frac{1}{(m_1^{(1)})} G_{13}^{0} e^{((S_1^{(1)}) - (P_{13}^{(1)}))t} \leq G_{13}^{0} e^{(S_1^{(1)})t} \leq \frac{1}{(m_2^{(1)})} G_{13}^{0} e^{(S_1^{(1)})t}

where \((p_j^{(1)})\) is defined by equation 25

\left(\begin{array}{c}
(a_{15})^{(1)} G_{13}^{0} e^{((S_1^{(1)}) - (P_{13}^{(1)}))t - e^{(S_1^{(1)})t}} \leq G_{13}^{0} e^{(S_1^{(1)})t} \leq G_{13}^{0} e^{(S_1^{(1)})t}
\end{array}\right)

\frac{1}{(m_1^{(1)})} T_{13}^{0} e^{((R_1^{(1)}))t} \leq T_{13}^{0} e^{((R_1^{(1)}) + (R_{13}^{(1)}))t}

\frac{1}{(m_2^{(1)})} \frac{1}{(m_2^{(1)})} T_{13}^{0} e^{((R_1^{(1)}) + (R_{13}^{(1)}))t} \leq T_{13}^{0} e^{((R_1^{(1)}) + (R_{13}^{(1)}))t}

\left(\begin{array}{c}
(b_{15})^{(1)} T_{13}^{0} e^{((R_1^{(1)}) - (b_{13}^{(1)}))t} \leq T_{13}^{0} e^{((R_1^{(1)}) - (b_{13}^{(1)}))t}
\end{array}\right)

\left(\begin{array}{c}
(a_{15})^{(1)} T_{13}^{0} e^{((R_1^{(1)}) + (R_{13}^{(1)}))t} \leq T_{13}^{0} e^{((R_1^{(1)}) + (R_{13}^{(1)}))t}
\end{array}\right)

Definition of \((S_1^{(1)}), (S_2^{(1)}), (R_1^{(1)}), (R_2^{(1)})\):

Where \((S_1^{(1)}) = (a_{13})^{(1)} (m_2^{(1)}) - (a_{13}^{(1)})\)

\((S_2^{(1)}) = (a_{13})^{(1)} - (p_{13}^{(1)})\)

\((R_1^{(1)}) = (b_{13})^{(1)} (\mu_2^{(1)}) - (b_{13}^{(1)})\)

\((R_2^{(1)}) = (b_{13}^{(1)}) - (r_{13}^{(1)})\)

Behavior of the solutions

Theorem 2: If we denote and define

Definition of \((\sigma_1^{(2)}), (\sigma_2^{(2)}), (\tau_1^{(2)}), (\tau_2^{(2)})\):

\((y) \quad \sigma_1^{(2)}, (\sigma_2^{(2)}), (\tau_1^{(2)}), (\tau_2^{(2)}) \text{ four constants satisfying} \quad -(\sigma_2^{(2)}) \leq -(a_{16}^{(2)}) + (a_{17}^{(2)}) - (a_{18}^{(2)})(T_{17}, t) + (a_{17}^{(2)})(T_{17}, t) \leq -(\sigma_1^{(2)})\)

\left(\begin{array}{c}
-(\sigma_2^{(2)}) \leq -(a_{16}^{(2)}) + (a_{17}^{(2)}) - (a_{18}^{(2)})(T_{17}, t) + (a_{17}^{(2)})(T_{17}, t) \leq -(\sigma_1^{(2)})
\end{array}\right)

\left(\begin{array}{c}
-(\tau_2^{(2)}) \leq -(b_{16}^{(2)}) + (b_{17}^{(2)}) - (b_{18}^{(2)})(G_{19}, t) - (b_{17}^{(2)})(G_{19}, t) \leq -(\tau_1^{(2)})
\end{array}\right)
Definition of \((v_1)^{(2)}, (v_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}\):

By \((v_1)^{(2)} > 0, (v_2)^{(2)} < 0\) and respectively \((u_1)^{(2)} > 0, (u_2)^{(2)} < 0\) the roots of the equations

\[\left(a_{17}\right)^{(2)}(v^{(2)})^2 + \left(a_{16}\right)^{(2)} = 0\]

and \((b_{17})^2(u^{(2)})^2 + (b_{16})^2 = 0\) and

Definition of \((\tilde{v}_1)^{(2)}, (\tilde{v}_2)^{(2)}, (\tilde{u}_1)^{(2)}, (\tilde{u}_2)^{(2)}\):

By \((\tilde{v}_1)^{(2)} > 0, (\tilde{v}_2)^{(2)} < 0\) and respectively \((\tilde{u}_1)^{(2)} > 0, (\tilde{u}_2)^{(2)} < 0\) the roots of the equations

\[\left(a_{17}\right)^{(2)}(v^{(2)})^2 + \left(a_{16}\right)^{(2)} = 0\]

and \((b_{17})^2(u^{(2)})^2 + (b_{16})^2 = 0\)

Definition of \((m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}\):

\[(m_1)^{(2)} = (v_1)^{(2)}, (m_2)^{(2)} = (v_0)^{(2)}, \text{ if } (v_1)^{(2)} < (v_0)^{(2})\]

and

\[(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}, \text{ if } (v_1)^{(2)} < (v_0)^{(2}) < (\tilde{v}_1)^{(2)},\]

and analogously

\[(\mu_1)^{(2)} = (u_1)^{(2)}, (\mu_2)^{(2)} = (u_0)^{(2)}, \text{ if } (u_1)^{(2)} < (u_0)^{(2})\]

and

\[(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \text{ if } (u_1)^{(2)} < (u_0)^{(2}) \text{ or } (\tilde{u}_1)^{(2)},\]

Then the solution satisfies the inequalities

\[G_{16}^0e^{(S_1)^{(2)}-(p_{16})^2t} \leq G_{16}(t) \leq G_{16}^0e^{(S_1)^{(2)}t}\]

\((p_1)^{(2)}\) is defined by equation

\[\frac{1}{(m_1)^{(2)}}G_{16}^0e^{(S_1)^{(2)}-(p_{16})^2t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}}G_{16}^0e^{(S_1)^{(2)}t}\]

\[\left(\frac{(a_{18})^{(2)}G_{16}}{(m_1)^{(2)}(S_1)^{(2)}-(p_{16})^2-(S_2)^{(2)}t)}\right)e^{(S_2)^{(2)}t} \leq G_{18}(t) \leq \left(\frac{(a_{18})^{(2)}G_{16}}{(m_2)^{(2)}(S_1)^{(2)}-(a_{18})^{(2)}t}\right)e^{(S_2)^{(2)}t}\]

\[T_{16}^0e^{(R_1)^2t} \leq T_{16}(t) \leq T_{16}^0e^{(R_1)^2+(r_{16})^2t}\]

\[\frac{1}{(m_1)^{(2)}T_{16}^0e^{(R_1)^2t}} \leq T_{16}(t) \leq \frac{1}{(m_2)^{(2)}T_{16}^0e^{(R_1)^2+(r_{16})^2t}}\]
\[
\frac{(b_{18})^2 \gamma_{16}}{(\mu_1)^2 ((R_1)^2 - (b_{16})^2)} [e^{(R_1)^2 t} - e^{-(b_{16})^2 t}] + T_{18} e^{-(b_{16})^2 t} \leq T_{18}(t) \leq 689
\]

\[
\frac{\gamma_{16}}{(\mu_2)^2 (R_1)^2 + (R_2)^2} \left[ e^{((R_1)^2 + (R_2)^2) t} - e^{-(R_2)^2 t} \right] + T_{16} e^{-(R_2)^2 t}
\]

**Definition of** \((S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}\).\] 690

Where \((S_1)^{(2)} = (a_{16})^2 (m_2)^2 - (a'_{16})^2\)

\((S_2)^{(2)} = (a_{16})^2 - (p_{18})^2\)

\((R_1)^{(2)} = (b_{16})^2 (\mu_2)^2 - (b'_{16})^2\)

\((R_2)^{(2)} = (b_{16})^2 - (r_{16})^2\)

**Proof:** We obtain

\[
\frac{d\psi^{(1)}}{dt} = (a_{18})^{(1)} - \left[ (a'_{18})^{(1)} - (a''_{18})^{(1)} + (a''_{18})^{(1)} (T_{14}, t) \right] - (a''_{18})^{(1)} (T_{14}, t) \psi^{(1)} - (a_{18})^{(1)} \psi^{(1)}
\]

**Definition of** \((\bar{v})^{(1)}, (\bar{v}_0)^{(1)}\).\] 693

It follows

\[-(a_{18})^{(1)} \psi^{(1)}{^2} + (\sigma_1)^{(1)} \psi^{(1)} - (a_{18})^{(1)} \leq \frac{d\psi^{(1)}}{dt} \leq -(a_{18})^{(1)} \psi^{(1)}{^2} + (\sigma_1)^{(1)} \psi^{(1)} - (a_{18})^{(1)}\]

From which one obtains

**Definition of** \((\bar{v})^{(1)}, (\bar{v}_0)^{(1)}\).\] 694

(o) For \(0 < (\bar{v}_0)^{(1)} = \frac{\psi^{(1)}}{\psi^{(1)}} < (v_1)^{(1)} < (\bar{v})^{(1)}\)

\[v^{(1)}(t) \geq \frac{(v_1)^{(1)} + (\bar{v})^{(1)}}{1 + (\bar{v})^{(1)}} - \frac{(a_{18})^{(1)} ((\bar{v}_0)^{(1)} - (v_0)^{(1)}))}{1 + (\bar{v})^{(1)}} \]

it follows \((\bar{v}_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}\)

In the same manner, we get

\[v^{(1)}(t) \leq \frac{(v_1)^{(1)} + (\bar{v})^{(1)}}{(\bar{v}_0)^{(1)} - (v_0)^{(1)}} - \frac{(a_{18})^{(1)} ((v_0)^{(1)} - (v_0)^{(1)}))}{1 + (\bar{v})^{(1)}} \]

From which we deduce \((\bar{v}_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}\)

(p) If \(0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{\psi^{(1)}}{\psi^{(1)}} < (\bar{v})^{(1)}\) we find like in the previous case,

\[(v_2)^{(1)} \leq \frac{(v_1)^{(1)} + (\bar{v})^{(1)}}{1 + (\bar{v})^{(1)}} - \frac{(a_{18})^{(1)} ((v_0)^{(1)} - (v_0)^{(1)}))}{1 + (\bar{v})^{(1)}} \leq v^{(1)}(t) \leq \]

\[\frac{(v_2)^{(1)} + (\bar{v})^{(1)}}{(\bar{v}_0)^{(1)} - (v_0)^{(1)}} - \frac{(a_{18})^{(1)} ((v_0)^{(1)} - (v_0)^{(1)}))}{1 + (\bar{v})^{(1)}} \leq (\bar{v})^{(1)}\]

(q) If \(0 < (v_1)^{(1)} \leq (\bar{v}_1)^{(1)} \leq (v_0)^{(1)} = \frac{\psi^{(1)}}{\psi^{(1)}}\), we obtain
(v_2)^{(1)} \leq v^{(1)}(t) \leq \frac{(v_2)^{(1)}(C)^{(1)}(v_2)^{(1)}e^{[-(a_{14})^{(1)}(v_1^{(1)}-v_2^{(1)})t]} + (v_1)^{(1)}(C)^{(1)}(v_2)^{(1)}e^{[-(a_{14})^{(1)}(v_2^{(1)}-v_2^{(1)})t]}}{1+(C)^{(1)}e^{[-(a_{14})^{(1)}(v_2^{(1)}-v_2^{(1)})t]}} \leq (v_0)^{(1)}

And so with the notation of the first part of condition (c), we have

**Definition of** \( v^{(1)}(t) \) :

\[
(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad v^{(1)}(t) = \frac{g_{13}(t)}{g_{14}(t)}
\]

In a completely analogous way, we obtain

**Definition of** \( u^{(1)}(t) \) :

\[
(m_2)^{(1)} \leq u^{(1)}(t) \leq (m_1)^{(1)}, \quad u^{(1)}(t) = \frac{r_{13}(t)}{r_{14}(t)}
\]

Now, using this result we get easily the result stated in the theorem.

**Particular case** :

If \( a_{13}^{(1)} = a_{14}^{(1)} \), then \( \sigma_1^{(1)} = \sigma_2^{(1)} \) and in this case \( (v_1)^{(1)} = (\bar{v}_1)^{(1)} \) if in addition \( (v_0)^{(1)} = (v_1)^{(1)} \) then \( v^{(1)}(t) = (v_0)^{(1)} \) and as a consequence \( G_{13}(t) = (v_0)^{(1)}G_{14}(t) \) this also defines \( (v_0)^{(1)} \) for the special case

Analogously if \( b_{13}^{(1)} = b_{14}^{(1)}, \text{then } (\tau_1)^{(1)} = (\tau_2)^{(1)} \) and then

\[
(u_1)^{(1)} = (\bar{u}_1)^{(1)} \text{if in addition } (u_0)^{(1)} = (u_1)^{(1)} \text{then } T_{13}(t) = (u_0)^{(1)}T_{14}(t) \text{ This is an important consequence of the relation between } (v_1)^{(1)} \text{ and } (\bar{v}_1)^{(1)} \text{ and definition of } (u_0)^{(1)}.
\]

**PROOF** : we obtain

\[
\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left( (a_{16})^{(2)} - (a_{17})^{(2)} + (a_{16})^{(2)}(T_{17}, t) - (a_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)} \right.
\]

**Definition of** \( v^{(2)} \) :

\[
\frac{v^{(2)}}{g_{14}}
\]

It follows

\[
-(a_{17})^{(2)}v^{(2)} + (\sigma_2^{(2)}v^{(2)} - (a_{16})^{(2)}) \leq \frac{dv^{(2)}}{dt} \leq -(a_{17})^{(2)}v^{(2)} + (\sigma_1^{(2)}v^{(2)} - (a_{16})^{(2)})
\]

From which one obtains

**Definition of** \( \bar{v}_1^{(2)} \), \( v_0^{(2)} \) :

\[
(r) \text{ For } 0 < (v_0)^{(2)} = \frac{g_0}{g_{17}} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}
\]

\[
v^{(2)}(t) \geq \frac{(v_1)^{(2)}(C)^{(2)}(v_2)^{(2)}e^{[-(a_{17})^{(2)}(v_1^{(2)}-v_2^{(2)})t]} + (v_2)^{(2)}(C)^{(2)}(v_2)^{(2)}e^{[-(a_{17})^{(2)}(v_2^{(2)}-v_2^{(2)})t]}}{1+e^{[-(a_{17})^{(2)}(v_2^{(2)}-v_2^{(2)})t]}} \leq (C)^{(2)}
\]

it follows \( (v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)} \)

In the same manner, we get

\[
v^{(2)}(t) \leq \frac{(v_1)^{(2)}(C)^{(2)}(v_2)^{(2)}e^{[-(a_{17})^{(2)}(v_1^{(2)}-v_2^{(2)})t]} + (v_2)^{(2)}(C)^{(2)}(v_2)^{(2)}e^{[-(a_{17})^{(2)}(v_2^{(2)}-v_2^{(2)})t]}}{1+e^{[-(a_{17})^{(2)}(v_2^{(2)}-v_2^{(2)})t]}} \leq (C)^{(2)}
\]
From which we deduce:

\[ (v_0)^{(2)}(t) \leq (\bar{v}_1)^{(2)}(t) \leq (v_1)^{(2)}(t) \leq (v_2)^{(2)}(t) \]

(s) If \( 0 < (v_1)^{(2)}(t) \leq (v_0)^{(2)} = \frac{g_1^2}{G_{17}} \leq (\bar{v}_1)^{(2)}(t) \), we find like in the previous case,

\[ (v_2)^{(2)} \leq \frac{(v_3)^{(2)} + (C(\bar{C})^{(2)})_{2e} \left[ \left( q_{17}^{(2)} \right)^2 \left( v_1^{(2)} \right)^2 - \left( v_2^{(2)} \right)^2 \right] t}{1 + (C(\bar{C})^{(2)})_{2e} \left[ \left( q_{17}^{(2)} \right)^2 \left( v_1^{(2)} \right)^2 - \left( v_2^{(2)} \right)^2 \right] t} \leq (v(t)) \leq (v_0)^{(2)} \]

(t) If \( 0 < (v_1)^{(2)}(t) \leq (\bar{v}_1)^{(2)}(t) \leq (v_0)^{(2)} = \frac{g_2^2}{G_{17}} \), we obtain

\[ (v_2)^{(2)} \leq \frac{(v_3)^{(2)} + (C(\bar{C})^{(2)})_{2e} \left[ \left( q_{17}^{(2)} \right)^2 \left( v_1^{(2)} \right)^2 - \left( v_2^{(2)} \right)^2 \right] t}{1 + (C(\bar{C})^{(2)})_{2e} \left[ \left( q_{17}^{(2)} \right)^2 \left( v_1^{(2)} \right)^2 - \left( v_2^{(2)} \right)^2 \right] t} \leq (v_0)^{(2)} \]

And so with the notation of the first part of condition (c), we have

**Definition of** \( v^{(2)}(t) \) :

\[ (m_{12})^{(2)} \leq v^{(2)}(t) \leq (m_{12})^{(2)}, \quad v^{(2)}(t) = \frac{g_{16}(t)}{g_{17}(t)} \]

In a completely analogous way, we obtain

**Definition of** \( u^{(2)}(t) \) :

\[ (\mu_{12})^{(2)} \leq u^{(2)}(t) \leq (\mu_{12})^{(2)}, \quad u^{(2)}(t) = \frac{\tau_{16}(t)}{\tau_{17}(t)} \]

Now, using this result and replacing it in Global equations we get easily the result stated in the theorem.

**Particular case:**

If \((a_{16}^\prime)^{(2)} = (a_{17}^\prime)^{(2)}\), then \((\sigma_1)^{(2)} = (\sigma_2)^{(2)}\) and in this case \((v_1)^{(2)} = (\bar{v}_1)^{(2)}\) if in addition \((v_0)^{(2)} = (v_1)^{(2)}\) then \(v^{(2)}(t) = (v_0)^{(2)}\) and as a consequence \(G_{16}(t) = (v_0)^{(2)}G_{17}(t)\)

Analogously if \((b_{16}^\prime)^{(2)} = (b_{17}^\prime)^{(2)}\), then \((\tau_1)^{(2)} = (\tau_2)^{(2)}\) and then

\(u^{(2)}(t) = (u_2)^{(2)}\) if in addition \((u_{10})^{(2)} = (u_{12})^{(2)}\) then \(T_{16}(t) = (u_{10})^{(2)}T_{17}(t)\) This is an important consequence of the relation between \((v_1)^{(2)}\) and \((\bar{v}_1)^{(2)}\)

We can prove the following

**Theorem 3:** If \((a_{16}^\prime)^{(1)}\) and \((b_{16}^\prime)^{(1)}\) are independent on \(t\), and the conditions (with the notations 25,26,27,28)

\[ (a_{13}^\prime)^{(1)}(a_{14}^\prime)^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0 \]

\[ (a_{13}^\prime)^{(1)}(a_{14}^\prime)^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a_{14}^\prime)^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0 \]

\[ (b_{13}^\prime)^{(1)}(b_{14}^\prime)^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0 \]

\[ (b_{13}^\prime)^{(1)}(b_{14}^\prime)^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b_{13})^{(1)}(r_{14})^{(1)} - (b_{14}^\prime)^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0 \]

with \((p_{13})^{(1)}, (r_{14})^{(1)}\) as defined by equation are satisfied, then the system
**Theorem 3:** If \((a_i^{(2)})_{(2)}\) and \((b_j^{(2)})_{(2)}\) are independent on \(t\), and the conditions (with the notations 25,26,27,28)

\[
(a_{16}^{(2)})^{(2)}(a_{17}^{(2)})^{(2)} - (a_{16}^{(2)})^{(2)}(a_{17}^{(2)})^{(2)} < 0
\]

\[
(a_{16}^{(2)})^{(2)}(a_{17}^{(2)})^{(2)} - (a_{16}^{(2)})^{(2)}(a_{17}^{(2)})^{(2)} + (a_{16}^{(2)})^{(2)}(p_{16}^{(2)})^{(2)} + (a_{17}^{(2)})^{(2)}(p_{17}^{(2)})^{(2)} + (p_{16}^{(2)})^{(2)}(p_{17}^{(2)})^{(2)} > 0
\]

\[
(b_{16}^{(2)})^{(2)}(b_{17}^{(2)})^{(2)} - (b_{16}^{(2)})^{(2)}(b_{17}^{(2)})^{(2)} > 0,
\]

\[
(b_{16}^{(2)})^{(2)}(b_{17}^{(2)})^{(2)} - (b_{16}^{(2)})^{(2)}(b_{17}^{(2)})^{(2)} - (b_{16}^{(2)})(r_{17}^{(2)})^{(2)} + (b_{17}^{(2)})(r_{17}^{(2)})^{(2)} + (r_{16}^{(2)})(r_{17}^{(2)})^{(2)} < 0
\]

with \((p_{16}^{(2)}), (r_{17}^{(2)})\) as defined by equation are satisfied, then the system

\[
\begin{align*}
(a_{13}^{(1)})G_{14} & - [(a_{13}^{(1)})^{(1)}(a_{13}^{(1)})^{(1)}(T_{14})] G_{13} = 0 \quad \text{(1)} \\
(a_{14}^{(1)})G_{13} & - [(a_{14}^{(1)})^{(1)}(a_{14}^{(1)})^{(1)}(T_{14})] G_{14} = 0 \quad \text{(2)} \\
(a_{15}^{(1)})G_{14} & - [(a_{15}^{(1)})^{(1)}(a_{15}^{(1)})^{(1)}(T_{14})] G_{15} = 0 \quad \text{(3)} \\
(b_{13}^{(1)})T_{14} & - [(b_{13}^{(1)})^{(1)} - (b_{13}^{(1)})^{(1)}(G)] T_{13} = 0 \quad \text{(4)} \\
(b_{14}^{(1)})T_{13} & - [(b_{14}^{(1)})^{(1)} - (b_{14}^{(1)})^{(1)}(G)] T_{14} = 0 \quad \text{(5)} \\
(b_{15}^{(1)})T_{14} & - [(b_{15}^{(1)})^{(1)} - (b_{15}^{(1)})^{(1)}(G)] T_{15} = 0 \quad \text{(6)}
\end{align*}
\]

has a unique positive solution, which is an equilibrium solution for the system

\[
\begin{align*}
(a_{16}^{(2)})G_{17} & - [(a_{16}^{(2)})^{(2)} + (a_{16}^{(2)})^{(2)}(T_{17})] G_{16} = 0 \quad \text{(1)} \\
(a_{17}^{(2)})G_{16} & - [(a_{17}^{(2)})^{(2)} + (a_{17}^{(2)})^{(2)}(T_{17})] G_{17} = 0 \quad \text{(2)} \\
(a_{18}^{(2)})G_{17} & - [(a_{18}^{(2)})^{(2)} + (a_{18}^{(2)})^{(2)}(T_{17})] G_{18} = 0 \quad \text{(3)} \\
(b_{16}^{(2)})T_{17} & - [(b_{16}^{(2)})^{(2)} - (b_{16}^{(2)})^{(2)}(G_{19})] T_{16} = 0 \quad \text{(4)} \\
(b_{17}^{(2)})T_{16} & - [(b_{17}^{(2)})^{(2)} - (b_{17}^{(2)})^{(2)}(G_{19})] T_{17} = 0 \quad \text{(5)} \\
(b_{18}^{(2)})T_{17} & - [(b_{18}^{(2)})^{(2)} - (b_{18}^{(2)})^{(2)}(G_{19})] T_{18} = 0 \quad \text{(6)}
\end{align*}
\]

has a unique positive solution, which is an equilibrium solution

**Proof:**

(a) Indeed the first two equations have a nontrivial solution \(G_{13}, G_{14}\) if

\[
F(T) = (a_{13}^{(1)}(a_{14}^{(1)})^{(1)}(1) - (a_{13}^{(1)}(a_{14}^{(1)})^{(1)}(1) + (a_{13}^{(1)}(a_{14}^{(1)})^{(1)}(1)T_{14}) + (a_{14}^{(1)}(a_{13}^{(1)}(1)(T_{14}) + (a_{13}^{(1)}(T_{14})(a_{14}^{(1)}))^{(1)}(T_{14}) = 0
\]

**Proof:**

(a) Indeed the first two equations have a nontrivial solution \(G_{16}, G_{17}\) if

\[
F(T_{19}) = (a_{16}^{(2)}(a_{17}^{(2)})^{(2)}(2) - (a_{16}^{(2)}(a_{17}^{(2)})^{(2)} + (a_{16}^{(2)}(a_{17}^{(2)})^{(2)}(T_{17}) + (a_{17}^{(2)}(a_{16}^{(2)})(T_{17}) + (a_{16}^{(2)}(T_{17}))(a_{17}^{(2)}))^{(2)}(T_{17}) = 0
\]

**Definition and uniqueness of** \(T_{14}\)**:**

After hypothesis \(f(0) < 0, f(\infty) > 0\) and the functions \((a_i^{(1)})(T_{14})\) being increasing, it follows
that there exists a unique $T_{*}^{14}$ for which $f(T_{*}^{14}) = 0$. With this value, we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[a_{13}^{(1)} + (a_{13}^{(1)})^{(1)}(G_{14})]}, \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[a_{15}^{(1)} + (a_{15}^{(1)})^{(1)}(G_{14})]}$$

**Definition and uniqueness of** $T_{*}^{17}$ **:**

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_{i}^{(2)})^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T_{*}^{17}$ for which $f(T_{*}^{17}) = 0$. With this value, we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[a_{16}^{(2)} + (a_{16}^{(2)})^{(2)}(T_{17})]}, \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[a_{18}^{(2)} + (a_{18}^{(2)})^{(2)}(T_{17})]}$$

(g) By the same argument, the equations admit solutions $G_{15}, G_{14}$ if

$$\varphi(G) = (b_{13}^{(1)}(1)(b_{14}^{(1)}(1) - (b_{13}^{(1)}(1)) + (b_{14}^{(1)}(1))(G) + (b_{13}^{(1)}(1))(G)(b_{14}^{(1)}(1))(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi(G^{*}) = 0$

(h) By the same argument, the equations admit solutions $G_{16}, G_{17}$ if

$$\varphi(G_{19}) = (b_{16}^{(2)}(2)(b_{17}^{(2)}(2) - (b_{16}^{(2)}(2))(G_{19}^{(2)}(2))$$

$$[b_{16}^{(2)}(2)(b_{17}^{(2)}(2)(G_{19}) + (b_{16}^{(2)}(2)(b_{16}^{(2)}(2)(G_{19}) + (b_{16}^{(2)}(2)(G_{19})(b_{17}^{(2)}(2)(G_{19}) = 0$$

Where in $(G_{19}), G_{16}, G_{17}, G_{18}, G_{16}, G_{19}$ must be replaced by their values. It is easy to see that $\varphi$ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi(G_{19}^{*}) = 0$

Finally we obtain the unique solution

$$G_{14}^{*} \text{ given by } \varphi(G^{*}) = 0, T_{*}^{14} \text{ given by } f(T_{*}^{14}) = 0 \text{ and}$$

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}}{[a_{13}^{(1)} + (a_{13}^{(1)})^{(1)}(G_{14})]}, \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}}{[a_{15}^{(1)} + (a_{15}^{(1)})^{(1)}(G_{14})]}$$

$$T_{*}^{14} = \frac{(b_{13}^{(1)}(1))^{(1)}G_{14}}{[b_{13}^{(1)}(1)(G_{14}^{*})]}, \quad T_{*}^{15} = \frac{(b_{14}^{(1)}(1))^{(1)}G_{14}}{[b_{14}^{(1)}(1)(G_{14}^{*})]}$$

Obviously, these values represent an equilibrium solution of the global equations

Finally we obtain the unique solution

$$G_{17} \text{ given by } \varphi(G_{19}^{*}) = 0, T_{*}^{17} \text{ given by } f(T_{*}^{17}) = 0 \text{ and}$$

$$G_{16}^{*} = \frac{(a_{16})^{(2)}G_{17}}{[a_{16}^{(2)} + (a_{16}^{(2)})^{(2)}(T_{17})]}, \quad G_{18}^{*} = \frac{(a_{18})^{(2)}G_{17}}{[a_{18}^{(2)} + (a_{18}^{(2)})^{(2)}(T_{17})]}$$

$$T_{*}^{16} = \frac{(b_{16}^{(2)}(2))T_{17}^{*}}{[b_{16}^{(2)}(2)(G_{19}^{*})]}, \quad T_{*}^{18} = \frac{(b_{18}^{(2)}(2))T_{17}^{*}}{[b_{18}^{(2)}(2)(G_{19}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations
ASYMPTOTIC STABILITY ANALYSIS

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions 
\((a''_i)^{(1)}\) and \((b''_i)^{(1)}\) Belong to \(C^1(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of \(G_i, T_i: \)

\[ G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i \]

\[ \frac{\partial (a''_i)^{(1)}}{\partial t_{14}}(T_i^*) = (q_{14})^{(1)} \quad \frac{\partial (b''_i)^{(1)}}{\partial G_j}(G^*) = s_{ij} \]

Neglecting the terms of power 2, we obtain from the global equations

\[ \frac{dG_{i1}}{dt} = -((a'_{i1})^{(1)} + (p_{13})^{(1)})G_{i1} + (a_{i1})^{(1)}G_{i4} - (q_{13})^{(1)}G_{13}^*T_{14} \]

\[ \frac{dG_{i4}}{dt} = -((a'_{i4})^{(1)} + (p_{14})^{(1)})G_{i4} + (a_{i4})^{(1)}G_{i14} - (q_{14})^{(1)}G_{14}^*T_{14} \]

\[ \frac{dG_{15}}{dt} = -((a'_{i5})^{(1)} + (p_{15})^{(1)})G_{i15} + (a_{i5})^{(1)}G_{i14} - (q_{15})^{(1)}G_{15}^*T_{14} \]

\[ \frac{dT_{13}}{dt} = -((b'_{13})^{(1)} - (r_{13})^{(1)})T_{13} + (b_{13})^{(1)}T_{14} + \sum_{j=13}^{15} (s_{13}(j)T_{14}G_j) \]

\[ \frac{dT_{14}}{dt} = -((b'_{14})^{(1)} - (r_{14})^{(1)})T_{14} + (b_{14})^{(1)}T_{13} + \sum_{j=13}^{15} (s_{14}(j)T_{14}G_j) \]

\[ \frac{dT_{15}}{dt} = -((b'_{15})^{(1)} - (r_{15})^{(1)})T_{15} + (b_{15})^{(1)}T_{14} + \sum_{j=13}^{15} (s_{15}(j)T_{14}G_j) \]

ASYMPTOTIC STABILITY ANALYSIS

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions 
\((a''_i)^{(2)}\) and \((b''_i)^{(2)}\) Belong to \(C^2(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of \(G_i, T_i: \)

\[ G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i \]

\[ \frac{\partial (a''_i)^{(2)}}{\partial t_{17}}(T_i^*) = (q_{17})^{(2)} \quad \frac{\partial (b''_i)^{(2)}}{\partial G_j}(G_i^*) = s_{ij} \]

taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

\[ \frac{dG_{16}}{dt} = -((a'_{i1})^{(2)} + (p_{i16})^{(2)})G_{16} + (a_{i1})^{(2)}G_{17} - (q_{16})^{(2)}G_{16}^*T_{17} \]

\[ \frac{dG_{17}}{dt} = -((a'_{i1})^{(2)} + (p_{i17})^{(2)})G_{17} + (a_{i1})^{(2)}G_{16} - (q_{17})^{(2)}G_{17}^*T_{17} \]

\[ \frac{dG_{18}}{dt} = -((a'_{i1})^{(2)} + (p_{i18})^{(2)})G_{18} + (a_{i1})^{(2)}G_{17} - (q_{18})^{(2)}G_{18}^*T_{17} \]

\[ \frac{dT_{16}}{dt} = -((b'_{i1})^{(2)} - (r_{i16})^{(2)})T_{16} + (b_{i1})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{i16}(j)T_{17}G_j) \]

169
\[
\frac{dT}{dt} = -((b_{17}^{(2)} - r_{17}^{(2)})T_{17} + (b_{17}^{(2)}T_{16} + \sum_{j=16}^{18} (s_{1(17)(j)}T_{17}G_j)) \\
\frac{dt}{dt} = -((b_{18}^{(2)} - r_{18}^{(2)})T_{18} + (b_{18}^{(2)}T_{17} + \sum_{j=18}^{19} (s_{1(18)(j)}T_{18}G_j))
\]

The characteristic equation of this system is

\[
\begin{align*}
((\lambda)^{(1)} + (b_{17}^{(1)} - r_{17}^{(1)}) & ((\lambda)^{(1)} + (a_{17}^{(1)} + (p_{17}^{(1)})) \\
&([((\lambda)^{(1)} + (a_{17}^{(1)} + (p_{17}^{(1)}))(q_{14}^{(1)}G_{14}^{(1)} + (a_{14}^{(1)}(q_{13}^{(1)}G_{13}^{(1)})) \\
&((\lambda)^{(1)} + (b_{13}^{(1)} - r_{13}^{(1)})S_{13(14)}T_{14}^{(1)} + (b_{14}^{(1)}S_{13(14)}T_{14}^{(1)} \\
&+ ((\lambda)^{(1)} + (a_{14}^{(1)} + (p_{14}^{(1)}))(q_{13}^{(1)}G_{13}^{(1)} + (a_{13}^{(1)}(q_{14}^{(1)}G_{14}^{(1)})) \\
&((\lambda)^{(1)} + (b_{13}^{(1)} - r_{13}^{(1)})S_{13(14)}T_{14}^{(1)} + (b_{14}^{(1)}S_{13(14)}T_{14}^{(1)} \\
&((\lambda)^{(1)} + (a_{13}^{(1)} + (p_{13}^{(1)})) + (p_{14}^{(1)})) (\lambda)^{(1)} \\
&((\lambda)^{(1)} + (b_{13}^{(1)} + (b_{14}^{(1)} - r_{12}^{(1)} + (r_{12}^{(1)})) (\lambda)^{(1)} \\
&+ ((\lambda)^{(1)} + (a_{13}^{(1)} + (a_{14}^{(1)} + (p_{13}^{(1)} + (p_{14}^{(1)})) (\lambda)^{(1)})(q_{15}^{(1)}G_{15} \\
&+(a_{13}^{(1)}(q_{14}^{(1)}G_{14}^{(1)} + (a_{14}^{(1)}(a_{13}^{(1)}(q_{13}^{(1)}G_{13}^{(1)})) \\
&((\lambda)^{1} + (b_{13}^{(1)} - r_{13}^{(1)})S_{13(14)}(15)T_{15}^{(1)} + (b_{14}^{(1)}S_{13(14)}T_{15}^{(1)} \\
&((\lambda)^{(2)} + (b_{16}^{(2)} - r_{16}^{(2)})(((\lambda)^{(2)} + (a_{16}^{(2)} + (p_{16}^{(2)})) \\
&([(\lambda)^{(2)} + (a_{16}^{(2)} + (p_{16}^{(2)}))(q_{17}^{(2)}G_{17}^{(2)} + (a_{17}^{(2)}q_{16}^{(2)}G_{16}^{(2)})) \\
&((\lambda)^{(2)} + (b_{16}^{(2)} - r_{16}^{(2)})S_{17(16)}T_{17}^{(2)} + (b_{17}^{(2)}S_{16(17)}T_{17}^{(2)} \\
&+ ((\lambda)^{(2)} + (a_{17}^{(2)} + (p_{17}^{(2)})))(q_{16}^{(2)}G_{16}^{(2)} + (a_{16}^{(2)}(q_{17}^{(2)}G_{17}^{(2)})) \\
&((\lambda)^{(2)} + (b_{17}^{(2)} - r_{17}^{(2)})(S_{16(17)}T_{16}^{(2)} + (b_{17}^{(2)}S_{16(17)}T_{16}^{(2)} \\
&((\lambda)^{(2)} + (a_{16}^{(2)} + (a_{17}^{(2)} + (p_{16}^{(2)} + (p_{17}^{(2)})) (\lambda)^{(2)} \\
&((\lambda)^{(2)} + (b_{16}^{(2)} + (b_{17}^{(2)} - r_{16}^{(2)} + (r_{17}^{(2)})(\lambda)^{(2)} \\
&+ ((\lambda)^{(2)} + (a_{16}^{(2)} + (a_{17}^{(2)} + (p_{16}^{(2)} + (p_{17}^{(2)})) (\lambda)^{(2)})(q_{18}^{(2)}G_{18} \\
&+(a_{17}^{(2)})(q_{17}^{(2)}G_{17}^{(2)} + (a_{17}^{(2)}(a_{16}^{(2)}(q_{16}^{(2)}G_{15} \\
&((\lambda)^{(2)} + (b_{18}^{(2)} - r_{18}^{(2)})S_{17(18)}T_{17}^{(2)} + (b_{17}^{(2)}S_{16(18)}T_{17}^{(2)})) = 0
\]

And as one sees, all the coefficients are positive. It follows that all the roots have negative real
part, and this proves the theorem.

Section 3: Platonic World-Physical World And Mental World: A Aletheia Akrasia Model

We give a model based on Penrose's analysis. For details please refer The Large, The Small and The Human Mind by Roger Penrose

NOTATION :

$G_{13}$ : Category One Of Platonic World (No One Has Seen Entire World Or Universe. It Is Based On This Concept That Classification Is Done. Everything Needs To Be Explored)

$G_{14}$ : Category Two Of Platonic World(Or The Part Thereof)

$G_{15}$ : Category Three Of Platonic World

$T_{13}$ : Human Mind Corresponding To The Above Classification Category One

$T_{14}$ : Human Mind Corresponding To The Above Classification Category two

$T_{15}$ : Human Mind Corresponding To The Above Classification Category three

$G_{16}$ : Category One Of Physical World

$G_{17}$ : Category Two Of Physical World

$G_{18}$ : Category Three Of Physical World

$T_{16}$ : Individuals With Mind Corresponding To Category One In The Foregoing

$T_{17}$ : Individuals With Mind Corresponding To Category two In The Foregoing

$T_{18}$ : Individuals With Mind Corresponding To Category Three In The Foregoing

$G_{20}$ : Category One Of Mental World

$G_{21}$ : Category Two Of Mental World

$G_{22}$ : Category Three Of Mental World

$T_{20}$ : Individuals With Mind Corresponding To Category One In The Foregoing

$T_{21}$ : Individuals With Mind Corresponding To Category One In The Foregoing

$T_{22}$ : Individuals With Mind Corresponding To Category One In The Foregoing

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}, (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$

$(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}; (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$ are Accentuation coefficients

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}, (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$

$(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}; (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$ are Dissipation coefficients

Platonic World And Corresponding Human Mind(Individual Consciousness)

Governing Equations:

The differential system of this model is now
First augmentation factor

\[-(b'_{16}^{(2)}(G_{19}, t)) = \text{ First detritions factor}\]

**Physical World And Concomitant Human Mind (Individual Consciousness Thereof)**

**Governing Equations:**

The differential system of this model is now

\[
\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)]G_{16}
\]

\[
\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)]G_{17}
\]

\[
\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)]G_{18}
\]

\[
\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19}), t)]T_{16}
\]

\[
\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}), t)]T_{17}
\]

\[
\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}), t)]T_{18}
\]

\[+(a''_{16})^{(2)}(T_{17}, t)) = \text{ First augmentation factor}\]

\[-(b''_{16})^{(2)}((G_{19}), t)) = \text{ First detritions factor}\]

**Mental World And The Individual Consciousness Thereof (Note The Variation From Individual To Individual Which Depends on Their Penchance , Predilection, Proclivities And Propensities, And Their Usage Of Sense Organs)**

**Governing Equations:**

The differential system of this model is now

\[
\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)]G_{20}
\]

\[
\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)]G_{21}
\]

\[
\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)]G_{22}
\]

\[
\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}((G_{23}), t)]T_{20}
\]
\[
\frac{dT_{21}}{dt} = (b_{21})^{(3)}(T_{20}) - \left[ (b_{21}^{(3)} - (b_{21}^{(3)})(G_{23}, t) \right] T_{21}
\]

\[
\frac{dT_{22}}{dt} = (b_{22})^{(3)}(T_{21}) - \left[ (b_{22}^{(3)} - (b_{22}^{(3)})(G_{23}, t) \right] T_{22}
\]

\[+(a_{20}^{(3)}(T_{21}, t) = \text{First augmentation factor}
\]

\[-(b_{20}^{(3)}(G_{23}, t) = \text{First detrition factor}
\]

**First Module Concatenation:**

\[
\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[ (a_{13}^{(1)} + (a_{13}^{(2,2)})(T_{14}, t) + (a_{13}^{(3,3)})(T_{21}, t) \right] G_{13}
\]

\[
\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[ (a_{14}^{(1)} + (a_{14}^{(2,2)})(T_{14}, t) + (a_{14}^{(3,3)})(T_{21}, t) \right] G_{14}
\]

\[
\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[ (a_{15}^{(1)} + (a_{15}^{(2,2)})(T_{14}, t) + (a_{15}^{(3,3)})(T_{21}, t) \right] G_{15}
\]

Where \(a_{13}^{(1)}(T_{14}, t)\), \(a_{14}^{(1)}(T_{14}, t)\), \(a_{15}^{(1)}(T_{14}, t)\) are first augmentation coefficients for category 1, 2 and 3

\[+(a_{10}^{(2,2)})(T_{17}, t)\], \(+(a_{11}^{(2,2)})(T_{17}, t)\), \(+(a_{12}^{(2,2)})(T_{17}, t)\) are second augmentation coefficient for category 1, 2 and 3

\[+(a_{20}^{(3,3)})(T_{21}, t)\], \(+(a_{21}^{(3,3)})(T_{21}, t)\), \(+(a_{22}^{(3,3)})(T_{21}, t)\) are third augmentation coefficient for category 1, 2 and 3

\[+(a_{30}^{(3,3)})(T_{23}, t)\], \(+(a_{31}^{(3,3)})(T_{23}, t)\), \(+(a_{32}^{(3,3)})(T_{23}, t)\) are fourth augmentation coefficient for category 1, 2 and 3

\[+(b_{13}^{(1)}(G, t))\], \(+(b_{14}^{(1)}(G, t))\), \(+(b_{15}^{(1)}(G, t))\) are first detrition coefficients for category 1, 2 and 3

\[+(b_{20}^{(2,2)})(G_{19}, t)\], \(+(b_{21}^{(2,2)})(G_{19}, t)\), \(+(b_{22}^{(2,2)})(G_{19}, t)\) are second detrition coefficients for category 1, 2 and 3

\[+(b_{30}^{(3,3)})(G_{23}, t)\], \(+(b_{31}^{(3,3)})(G_{23}, t)\), \(+(b_{32}^{(3,3)})(G_{23}, t)\) are third detrition coefficients for category 1, 2 and 3

\[+(a_{16}^{(2)})(G_{17}, t)\], \(+(a_{17}^{(2)})(G_{17}, t)\), \(+(a_{18}^{(2)})(G_{17}, t)\) are first augmentation coefficients for category 1, 2 and 3

\[+(a_{13}^{(1,1)})(T_{14}, t)\], \(+(a_{14}^{(1,1)})(T_{14}, t)\), \(+(a_{15}^{(1,1)})(T_{14}, t)\) are second augmentation coefficients for category 1, 2 and 3
coefficient for category 1, 2 and 3

\[ + (a_{20}^u)^{(3,3,3)}(G_{21}, t) \]
\[ + (a_{21}^u)^{(3,3,3)}(T_{21}, t) \]
\[ + (a_{22}^u)^{(3,3,3)}(T_{21}, t) \]

are third augmentation coefficient for category 1, 2 and 3

Second Module Concatenation:

\[ \frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b_{16})^{(2)}(2)(G_{19}, t) - (b_{17})^{(2)}(1,1,1)(G, t) - (b_{20})^{(3,3,3)}(G_{23}, t)] T_{16} \]
\[ \frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b_{17})^{(2)}(2)(G_{19}, t) - (b_{14})^{(1,1,1)}(G, t) - (b_{21})^{(3,3,3)}(G_{23}, t)] T_{17} \]
\[ \frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - [(b_{18})^{(2)}(2)(G_{19}, t) - (b_{15})^{(1,1,1)}(G, t) - (b_{22})^{(3,3,3)}(G_{23}, t)] T_{18} \]

where

\[ (b_{16})^{(2)}(2)(G_{19}, t) \]
\[ (b_{17})^{(2)}(2)(G_{19}, t) \]
\[ (b_{18})^{(2)}(2)(G_{19}, t) \]

are first detrition coefficients for category 1, 2 and 3

\[ (b_{16})^{(1,1,1)}(G, t) \]
\[ (b_{17})^{(1,1,1)}(G, t) \]
\[ (b_{18})^{(1,1,1)}(G, t) \]

are second detrition coefficients for category 1, 2 and 3

\[ (b_{20})^{(3,3,3)}(G_{23}, t) \]
\[ (b_{21})^{(3,3,3)}(G_{23}, t) \]
\[ (b_{22})^{(3,3,3)}(G_{23}, t) \]

are third detrition coefficients for category 1, 2 and 3

Third Module Concatenation:

\[ \frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - [(a_{20})^{(3)}(3)(G_{21}, t) + (a_{20})^{(2,2,2)}(T_{21}, t) + (a_{20})^{(2,2,2)}(T_{17}, t) + (a_{20})^{(1,1,1,1)}(T_{14}, t)] G_{20} \]
\[ \frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - [(a_{21})^{(3)}(3)(G_{21}, t) + (a_{21})^{(2,2,2)}(T_{21}, t) + (a_{21})^{(2,2,2)}(T_{17}, t) + (a_{21})^{(1,1,1,1)}(T_{14}, t)] G_{21} \]
\[ \frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - [(a_{22})^{(3)}(3)(G_{21}, t) + (a_{22})^{(2,2,2)}(T_{21}, t) + (a_{22})^{(2,2,2)}(T_{17}, t) + (a_{22})^{(1,1,1,1)}(T_{14}, t)] G_{22} \]

\[ +(a_{20})^{(2,2,2)}(T_{21}, t) \]
\[ +(a_{20})^{(2,2,2)}(T_{17}, t) \]
\[ +(a_{20})^{(2,2,2)}(T_{17}, t) \]

are first augmentation coefficients for category 1, 2 and 3

\[ +(a_{20})^{(2,2,2)}(T_{21}, t) \]
\[ +(a_{20})^{(2,2,2)}(T_{17}, t) \]
\[ +(a_{20})^{(2,2,2)}(T_{17}, t) \]

are second augmentation coefficients for category 1, 2 and 3

\[ +(a_{20})^{(1,1,1,1)}(T_{14}, t) \]
\[ +(a_{20})^{(1,1,1,1)}(T_{14}, t) \]
\[ +(a_{20})^{(1,1,1,1)}(T_{14}, t) \]

are third augmentation coefficients for category 1, 2 and 3

\[ \frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - [(b_{20})^{(3)}(3)(G_{23}, t) - (b_{20})^{(2,2,2)}(G_{19}, t) - (b_{20})^{(1,1,1,1)}(G, t)] T_{20} \]
\[ \frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - [(b_{21})^{(3)}(3)(G_{23}, t) - (b_{21})^{(2,2,2)}(G_{19}, t) - (b_{21})^{(1,1,1,1)}(G, t)] T_{21} \]
\[ \frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - [(b_{22})^{(3)}(3)(G_{23}, t) - (b_{22})^{(2,2,2)}(G_{19}, t) - (b_{22})^{(1,1,1,1)}(G, t)] T_{22} \]

where

\[ (b_{20})^{(3)}(3)(G_{23}, t) \]
\[ (b_{21})^{(3)}(3)(G_{23}, t) \]
\[ (b_{22})^{(3)}(3)(G_{23}, t) \]

are first detrition coefficients for category 1, 2 and 3

\[ (b_{20})^{(2,2,2)}(G_{19}, t) \]
\[ (b_{21})^{(2,2,2)}(G_{19}, t) \]
\[ (b_{22})^{(2,2,2)}(G_{19}, t) \]

are second detrition coefficients for category 1, 2 and 3
\[-(b_{13}^{(1,1,1)}(G,t)), -(b_{14}^{(1,1,1)}(G,t)), -(b_{15}^{(1,1,1)}(G,t))\] are third detritions coefficients for category 1, 2 and 3.

Where we suppose

\[ (X) \quad (a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \]

\[ i, j = 13, 14, 15 \]

\[ (Y) \quad \text{The functions } (a_i'')^{(1)}, (b_i'')^{(1)} \text{ are positive continuous increasing and bounded.} \]

Definition of \((p_i)^{(1)}, (r_i)^{(1)}):\]

\[ (a_i'')^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)} \]

\[ (b_i'')^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)} \]

\[ (Z) \quad \lim_{T_{14} \to 0}(a_i'')^{(1)}(T_{14}, t) = (p_0)^{(1)} \]

\[ \lim_{G \to 0}(b_i'')^{(1)}(G, t) = (r_0)^{(1)} \]

Definition of \((\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}):\]

Where \([(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_0)^{(1)}, (r_0)^{(1)}]\]

and \([i = 13, 14, 15]\)

They satisfy Lipschitz condition:

\[ |(a_i'')^{(1)}(T_{14}, t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t} \]

\[ |(b_i'')^{(1)}(G, t) - (b_i'')^{(1)}(G, t)| \leq (\hat{k}_{13})^{(1)}|G - G'|e^{-(\hat{M}_{13})^{(1)}t} \]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i'')^{(1)}(T_{14}, t) \text{ and } (a_i'')^{(1)}(T_{14}, t), (T_{14}', t) \text{ and } (T_{14}, t)\) are points belonging to the interval \([k_{13}]^{(1)}, (M_{13})^{(1)}\). It is to be noted that \((a_i'')^{(1)}(T_{14}, t)\) is uniformly continuous. In the eventuality of the fact, if \((M_{13})^{(1)} = 1\) then the function \((a_i'')^{(1)}(T_{14}, t)\), the first augmentation coefficient would be absolutely continuous.

Definition of \((\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}):\]

\[ (AA) \quad (\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}\] are positive constants

\[ \frac{(a_i)^{(2)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1 \]

Definition of \((\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}):\]

\[ (BB) \quad \text{There exist two constants } (\hat{P}_{13})^{(1)} \text{ and } (\hat{Q}_{13})^{(1)} \text{ which together with } (\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)} \text{ and } (\hat{B}_{13})^{(1)} \text{ and the constants } (a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_0)^{(1)}, (r_0)^{(1)}, i = 13, 14, 15,\]

satisfy the inequalities

\[ \frac{1}{(\hat{M}_{13})^{(1)}}[\ (a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1 \]

\[ \frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1 \]
Where we suppose

(CC) \((a_i)_{(2)}, (a_i')_{(2)}, (a_i'')_{(2)}, (b_i)_{(2)}, (b_i')_{(2)}, (b_i'')_{(2)} > 0, \quad i, j = 16, 17, 18\)

(DD) The functions \((a_i'')_{(2)}, (b_i'')_{(2)}\) are positive continuous increasing and bounded.

**Definition of** \((p_l)_{(2)}, (r_l)_{(2)}:\)

\((a_i'')(T_{17}, t) \leq (p_l)_{(2)} \leq (A_{16})_{(2)}\) \(795\)

\((b_i'')(G_{19}, t) \leq (r_l)_{(2)} \leq (B_{16})_{(2)}\) \(796\)

(EE) \(\lim_{t \to \infty}(a_i'')(T_{17}, t) = (p_l)_{(2)}\)

\(\lim_{t \to \infty}(b_i'')(G_{19}, t) = (r_l)_{(2)}\) \(798\)

**Definition of** \((\hat{A}_{16})_{(2)}, (\hat{B}_{16})_{(2)}:\)

Where \([\hat{A}_{16})_{(2)}, (\hat{B}_{16})_{(2)}(p_l)_{(2)}, (r_l)_{(2)}\] are positive constants and \(l = 16, 17, 18\)

They satisfy Lipschitz condition:

\(|(a_i'')(T_{17}, t) - (a_i'')(T_{17}, t)| \leq (\hat{k}_{16})_{(2)}|T_{17} - T_{17}'|e^{-(\hat{K}_{16})_{(2)}t}\) \(801\)

\(|(b_i'')(G_{19}, t) - (b_i'')(G_{19}, t)| \leq (\hat{k}_{16})_{(2)}||G_{19} - (G_{19}')||e^{-(\hat{K}_{16})_{(2)}t}\) \(802\)

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i'')(T_{17}, t)\) and \((a_i'')(T_{17}, t)\). \(T_{17}, t\) and \(T_{17}, t\) are points belonging to the interval \([\hat{A}_{16})_{(2)}, (\hat{B}_{16})_{(2)}\] .

It is to be noted that \((a_i'')(T_{17}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{16})_{(2)} = 1\) then the function \((a_i'')(T_{17}, t)\), the SECOND augmentation coefficient would be absolutely continuous.

**Definition of** \((\hat{M}_{16})_{(2)}, (\hat{k}_{16})_{(2)}:\)

\((\hat{M}_{16})_{(2)}, (\hat{k}_{16})_{(2)}\), are positive constants

\(\frac{(a_l)_{(2)}}{(\hat{M}_{16})_{(2)}} = \frac{(b_l)_{(2)}}{(\hat{M}_{16})_{(2)}} < 1\) \(803\)

**Definition of** \((\hat{P}_{13})_{(2)}, (\hat{Q}_{13})_{(2)}:\)

There exists two constants \((\hat{P}_{16})_{(2)}(\hat{Q}_{16})_{(2)}\) which together with \((\hat{M}_{16})_{(2)}, (\hat{k}_{16})_{(2)}, (\hat{A}_{16})_{(2)}\) and \((\hat{B}_{16})_{(2)}\) and the constants \((a_l)_{(2)}, (a_l')_{(2)}, (b_l)_{(2)}, (b_l')_{(2)}, (p_l)_{(2)}, (r_l)_{(2)}, l = 16, 17, 18,\)

satisfy the inequalities

\(\frac{1}{(\hat{M}_{16})_{(2)}}[ (a_l)_{(2)} + (a_l')_{(2)} + (\hat{A}_{16})_{(2)} + (\hat{P}_{16})_{(2)}(\hat{k}_{16})_{(2)}] < 1\) \(804\)

\(\frac{1}{(\hat{M}_{16})_{(2)}}[ (b_l)_{(2)} + (b_l')_{(2)} + (\hat{B}_{16})_{(2)} + (\hat{Q}_{16})_{(2)}(\hat{k}_{16})_{(2)}] < 1\) \(805\)

Where we suppose

(GG) \((a_j)_{(3)}, (a_j')_{(3)}, (a_j'')_{(3)}, (b_j)_{(3)}, (b_j')_{(3)}, (b_j'')_{(3)} > 0, \quad i, j = 20, 21, 22\)

\(806\)
The functions \((a''_i(t), b''_i(t))\) are positive continuous increasing and bounded.

**Definition of** \((p_i)(t), (r_i)(t)\):

\[
(a''_i(t)T_{21}, t) \leq (p_i)(t) \leq (\hat{A}_{20}(t))
\]
\[
(b''_i(t)G_{23}, t) \leq (r_i)(t) \leq (\hat{B}_{20}(t))
\]
\[
\lim_{T_{21} \to 0}(a''_i(t)T_{21}, t) = (p_i)(t)
\]
\[
\lim_{G_{23} \to 0}(b''_i(t)G_{23}, t) = (r_i)(t)
\]

**Definition of** \((\hat{A}_{20}, \hat{B}_{20})\):

Where \((\hat{A}_{20}, \hat{B}_{20})\) are positive constants and \(i = 20, 21, 22\)

They satisfy Lipschitz condition:

\[
|a''_i(T_{21}, t) - a''_i(T_{21}, t)| \leq |\hat{k}_{20}|T_{21} - T_{21}|e^{-(\hat{A}_{20})t}
\]
\[
|b''_i(G_{23}, t) - b''_i(G_{23}, t)| < |\hat{g}_{20}|G_{23} - G_{23}|e^{-(\hat{B}_{20})t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a''_i(T_{21}, t)\) and \((b''_i(G_{23}, t)\). And \((T_{21}, t)\) and \((G_{23}, t)\) are points belonging to the interval \([\hat{k}_{20}, \hat{B}_{20}]\).

It is to be noted that if \((M_{20}) = 1\) then the function \((a''_i(T_{21}, t)\), the THIRD augmentation coefficient would be absolutely continuous.

**Definition of** \((\hat{M}_{20}, \hat{G}_{20})\):

\((\hat{M}_{20}, \hat{G}_{20})\) are positive constants

There exists two constants \((\hat{A}_{20}, \hat{B}_{20})\) which together with \((\hat{M}_{20}, \hat{G}_{20})\) satisfy the inequalities

\[
\frac{1}{(\hat{M}_{20})^3} [a_i(t) + (a_i)(t) + (\hat{A}_{20})(t)] < 1
\]
\[
\frac{1}{(\hat{G}_{20})^3} [b_i(t) + (b_i)(t) + (\hat{B}_{20})(t)] < 1
\]

**Theorem 1:** If the conditions (A)-(E) are fulfilled, there exists a solution satisfying the conditions

**Definition of** \(G_i(0), T_i(0)\):

\[
G_i(t) \leq (\hat{P}_{13})e^{(\hat{A}_{13})t}, \quad G_i(0) = G_i^0 > 0
\]
\[
T_i(t) \leq (\hat{Q}_{13})e^{(\hat{A}_{13})t}, \quad T_i(0) = T_i^0 > 0
\]

**Theorem 2:** If the conditions (A)-(E) are fulfilled, there exists a solution satisfying the conditions
Definition of \(G_i(0), T_i(0)\)

\[ G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{R}_{16})^{(2)} t}, \quad G_i(0) = G^0_i > 0 \]

\[ T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{R}_{16})^{(2)} t}, \quad T_i(0) = T^0_i > 0 \]

if the conditions (A)–(E) above are fulfilled, there exists a solution satisfying the conditions

\[ G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{R}_{20})^{(3)} t}, \quad G_i(0) = G^0_i > 0 \]

\[ T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{R}_{20})^{(3)} t}, \quad T_i(0) = T^0_i > 0 \]

Proof:

Consider operator \(A^{(2)}\) defined on the space of sextuples of continuous functions \(G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+\) which satisfy

\[ G_i(0) = G^0_i, \quad T_i(0) = T^0_i, \quad G^0_i \leq (\hat{P}_{16})^{(2)} e^{(\hat{R}_{16})^{(2)} t} \leq (\hat{Q}_{16})^{(2)}, \]

\[ 0 \leq G_{i}(t) - G^0_i \leq (\hat{P}_{16})^{(2)} e^{(\hat{R}_{16})^{(2)} t} \]

\[ 0 \leq T_{i}(t) - T^0_i \leq (\hat{Q}_{16})^{(2)} e^{(\hat{R}_{16})^{(2)} t} \]

By

\[ G_{13}(t) = G^0_{13} + \int_0^t [(a_{13})^{(1)} G_{14}(s_{13}) - (a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{13}), s_{13})] G_{13}(s_{13}) ds_{13} \]

\[ G_{14}(t) = G^0_{14} + \int_0^t [(a_{14})^{(1)} G_{13}(s_{13}) - (a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{13}), s_{13})] G_{14}(s_{13}) ds_{13} \]

\[ G_{15}(t) = G^0_{15} + \int_0^t [(a_{15})^{(1)} G_{14}(s_{13}) - (a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{13}), s_{13})] G_{15}(s_{13}) ds_{13} \]

\[ T_{13}(t) = T^0_{13} + \int_0^t [(b_{13})^{(1)} T_{14}(s_{13}) - (b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{13}), s_{13})] T_{13}(s_{13}) ds_{13} \]

\[ T_{14}(t) = T^0_{14} + \int_0^t [(b_{14})^{(1)} T_{13}(s_{13}) - (b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{13}), s_{13})] T_{14}(s_{13}) ds_{13} \]

\[ T_{15}(t) = T^0_{15} + \int_0^t [(b_{15})^{(1)} T_{14}(s_{13}) - (b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{13}), s_{13})] T_{15}(s_{13}) ds_{13} \]

Where \(s_{13}\) is the integrand that is integrated over an interval \((0, t)\)

Proof:

Consider operator \(A^{(2)}\) defined on the space of sextuples of continuous functions \(G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+\) which satisfy

\[ G_i(0) = G^0_i, \quad T_i(0) = T^0_i, \quad G^0_i \leq (\hat{P}_{16})^{(2)} e^{(\hat{R}_{16})^{(2)} t} \leq (\hat{Q}_{16})^{(2)}, \]

\[ 0 \leq G_i(t) - G^0_i \leq (\hat{P}_{16})^{(2)} e^{(\hat{R}_{16})^{(2)} t} \]

\[ 0 \leq T_i(t) - T^0_i \leq (\hat{Q}_{16})^{(2)} e^{(\hat{R}_{16})^{(2)} t} \]

By

\[ G_{16}(t) = G^0_{16} + \int_0^t [(a_{16})^{(2)} G_{17}(s_{16}) - (a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{16}), s_{16))] G_{16}(s_{16}) ds_{16} \]
\[ \tilde{G}_{17}(t) = G_{17}^{0} + \int_{0}^{t} \left[ (a_{17}^{(2)}G_{16}(s_{16}) - (a_{17}^{(2)} + a_{17}^{(2)})(T_{17}(s_{16}), s_{16})) \right] ds_{16} \]

\[ \tilde{G}_{18}(t) = G_{18}^{0} + \int_{0}^{t} \left[ (a_{18}^{(2)}G_{17}(s_{16}) - (a_{18}^{(2)} + a_{18}^{(2)})(T_{18}(s_{16}), s_{16})) \right] ds_{16} \]

\[ \tilde{T}_{14}(t) = T_{14}^{0} + \int_{0}^{t} \left[ (b_{14}^{(2)}T_{17}(s_{16}) - ((b_{14}^{(2)} - b_{14}^{(2)})(G(s_{16}), s_{16})) \right] ds_{16} \]

\[ \tilde{T}_{17}(t) = T_{17}^{0} + \int_{0}^{t} \left[ (b_{17}^{(2)}T_{18}(s_{16}) - ((b_{17}^{(2)} - b_{17}^{(2)})(G(s_{16}), s_{16})) \right] ds_{16} \]

\[ \tilde{T}_{18}(t) = T_{18}^{0} + \int_{0}^{t} \left[ (b_{18}^{(2)}T_{19}(s_{16}) - ((b_{18}^{(2)} - b_{18}^{(2)})(G(s_{16}), s_{16})) \right] ds_{16} \]

Where \( s_{16} \) is the integrand that is integrated over an interval \( (0, t) \)

**Proof:**

Consider operator \( \mathcal{A}^{(3)} \) defined on the space of sextuples of continuous functions \( G_{i}, T_{i}: \mathbb{R}^{+} \rightarrow \mathbb{R}^{+} \) which satisfy

\[ G_{i}(0) = G_{i}^{0}, \quad T_{i}(0) = T_{i}^{0}, \quad G_{i}^{0} \leq (\bar{P}_{20})^{(3)}, \quad T_{i}^{0} \leq (\bar{Q}_{20})^{(3)} \]

\[ 0 \leq G_{i}(t) - G_{i}^{0} \leq (\bar{P}_{20})^{(3)}e^{(\bar{R}_{20})^{(3)}t} \]

\[ 0 \leq T_{i}(t) - T_{i}^{0} \leq (\bar{Q}_{20})^{(3)}e^{(\bar{R}_{20})^{(3)}t} \]

By

\[ \tilde{G}_{20}(t) = G_{20}^{0} + \int_{0}^{t} \left[ (a_{20}^{(3)}G_{21}(s_{20}) - (a_{20}^{(3)} + a_{20}^{(3)})(T_{21}(s_{20}), s_{20})) \right] ds_{20} \]

\[ \tilde{G}_{21}(t) = G_{21}^{0} + \int_{0}^{t} \left[ (a_{21}^{(3)}G_{20}(s_{20}) - (a_{21}^{(3)} + a_{21}^{(3)})(T_{21}(s_{20}), s_{20})) \right] ds_{20} \]

\[ \tilde{G}_{22}(t) = G_{22}^{0} + \int_{0}^{t} \left[ (a_{22}^{(3)}G_{21}(s_{20}) - (a_{22}^{(3)} + a_{22}^{(3)})(T_{21}(s_{20}), s_{20})) \right] ds_{20} \]

\[ \tilde{T}_{20}(t) = T_{20}^{0} + \int_{0}^{t} \left[ (b_{20}^{(3)}T_{21}(s_{20}) - (b_{20}^{(3)} - b_{20}^{(3)})(G(s_{20}), s_{20})) \right] ds_{20} \]

\[ \tilde{T}_{21}(t) = T_{21}^{0} + \int_{0}^{t} \left[ (b_{21}^{(3)}T_{20}(s_{20}) - (b_{21}^{(3)} - b_{21}^{(3)})(G(s_{20}), s_{20})) \right] ds_{20} \]

\[ \tilde{T}_{22}(t) = T_{22}^{0} + \int_{0}^{t} \left[ (b_{22}^{(3)}T_{21}(s_{20}) - (b_{22}^{(3)} - b_{22}^{(3)})(G(s_{20}), s_{20})) \right] ds_{20} \]

Where \( s_{20} \) is the integrand that is integrated over an interval \( (0, t) \)

(i) The operator \( \mathcal{A}^{(1)} \) maps the space of functions satisfying into itself. Indeed it is obvious that

\[ G_{13}(t) \leq G_{13}^{0} + \int_{0}^{t} \left[ (a_{13}^{(1)}G_{14}^{0} + (\bar{P}_{13})^{(1)}e^{(\bar{R}_{13})^{(1)}s_{13}}) \right] ds_{13} =

\[ (1 + (a_{13}^{(1)})t)G_{14}^{0} + \left[ \frac{a_{13}^{(1)}(\bar{P}_{13})^{(1)}}{(M_{13})^{(1)}} \right] e^{(\bar{R}_{13})^{(1)}t} - 1 \]

From which it follows that

\[ (G_{13}^{0} - G_{13}^{0})e^{-(\bar{R}_{13})^{(1)}t} \leq \left[ \frac{a_{13}^{(1)}(\bar{P}_{13})^{(1)}}{(M_{13})^{(1)}} \right] \left[ (\bar{P}_{13})^{(1)} + G_{14}^{0}e^{\left( -\frac{\bar{P}_{13}^{(1)}+G_{14}^{0}}{G_{13}^{0}} \right)} \right] \]

\( (G_{13}^{0}) \) is as defined in the statement of theorem 1

179
Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

(j) The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying equations into itself. Indeed, it is obvious that

$$G_{16}(t) \leq G_{16}^{0} + \int_{0}^{t} \left[ (a_{16})^{(2)}(G_{17}^{0} + (\hat{P}_{16})^{(6)}e^{(\hat{R}_{16})^{(2)}t \lambda_{16}}) \right] \, ds_{(16)} =$$

$$\left( 1 + (a_{16})^{(2)}t \right) G_{17}^{0} + \frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(M_{16})^{(2)}} \left( e^{(\hat{R}_{16})^{(2)}t} - 1 \right)$$

From which it follows that

$$(G_{16}(t) - G_{16}^{0})e^{-(\hat{R}_{16})^{(2)}t} \leq \frac{(a_{16})^{(2)}}{(M_{16})^{(2)}} \left[ (\hat{P}_{16})^{(2)} + G_{17}^{0} e^{\left(-\frac{(\hat{P}_{16})^{(2)}e^{\hat{R}_{16}}}{G_{17}^{0}}\right)} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

(k) The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying 34, 35, 36 into itself. Indeed, it is obvious that

$$G_{20}(t) \leq G_{20}^{0} + \int_{0}^{t} \left[ (a_{20})^{(3)}(G_{21}^{0} + (\hat{P}_{20})^{(3)}e^{(\hat{R}_{20})^{(3)}t \lambda_{20}}) \right] \, ds_{(20)} =$$

$$\left( 1 + (a_{20})^{(3)}t \right) G_{21}^{0} + \frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(M_{20})^{(3)}} \left( e^{(\hat{R}_{20})^{(3)}t} - 1 \right)$$

From which it follows that

$$(G_{20}(t) - G_{20}^{0})e^{-(\hat{R}_{20})^{(3)}t} \leq \frac{(a_{20})^{(3)}}{(M_{20})^{(3)}} \left[ (\hat{P}_{20})^{(3)} + G_{21}^{0} e^{\left(-\frac{(\hat{P}_{20})^{(3)}e^{\hat{R}_{20}}}{G_{21}^{0}}\right)} + (\hat{P}_{20})^{(3)} \right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

It is now sufficient to take $\frac{(a_{1})^{(1)}}{(M_{13})^{(1)}} \cdot \frac{(b_{1})^{(1)}}{(M_{13})^{(3)}} < 1$ and to choose

$$(\hat{P}_{13})^{(1)} \text{ and } (\hat{Q}_{13})^{(1)}$$

large to have

$$\frac{(a_{1})^{(1)}}{(M_{13})^{(1)}} \left[ (\hat{P}_{13})^{(1)} + \left( (\hat{P}_{13})^{(1)} + G_{j}^{0} \right) e^{-\left(\frac{(\hat{P}_{13})^{(1)}e^{\hat{R}_{13}}}{G_{j}^{0}}\right)} \right] \leq (\hat{P}_{13})^{(1)}$$

$$\frac{(b_{1})^{(1)}}{(M_{13})^{(1)}} \left[ (\hat{Q}_{13})^{(1)} + T_{j}^{0} \right] e^{-\left(\frac{(\hat{Q}_{13})^{(1)}e^{\hat{T}_{j}}}{T_{j}^{0}}\right)} \leq (\hat{Q}_{13})^{(1)}$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_{i}, T_{i}$ satisfying 34, 35, 36 into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d \left( (G_{i}^{(1)}, T^{(1)}), (G_{i}^{(2)}, T^{(2)}) \right) =$$

$$\sup_{t \in \mathbb{R}^+} \left[ \left| G_{i}^{(1)}(t) - G_{i}^{(2)}(t) \right| e^{-((\hat{Q}_{13})^{(1)}t)_{e^{\hat{R}_{13}}}} \cdot \sup_{t \in \mathbb{R}^+} \left| T^{(1)}(t) - T^{(2)}(t) \right| e^{-((\hat{R}_{13})^{(1)}t)_{e^{\hat{R}_{13}}}} \right]$$

Indeed, if we denote...
Definition of \( \tilde{\mathcal{G}}, \mathcal{T} : (\tilde{\mathcal{G}}, \mathcal{T}) = \mathcal{A}^{(1)}(G,T) \)

It results

\[
\left| \tilde{G}^{(1)}_{13} - \tilde{G}^{(2)}_{i} \right| \leq \int_{0}^{t}(a^{(1)}_{13})_{1} |G^{(1)}_{14} - G^{(2)}_{14}| e^{-\tilde{\mathcal{G}}_{15}(1)_{3}(s_{13})} e^{(\tilde{\mathcal{G}}_{15})_{1}(s_{13})} ds_{13} + \\
\int_{0}^{t}(a^{(1)}_{13})_{1} |G^{(1)}_{13} - G^{(2)}_{13}| e^{-\tilde{\mathcal{G}}_{15}(1)_{3}(s_{13})} e^{-(\tilde{\mathcal{G}}_{15})_{1}(s_{13})} + \\
(a^{(1)}_{13})_{1}(T^{(1)}_{i}, s_{i}(13)) |G^{(1)}_{13} - G^{(2)}_{13}| e^{-\tilde{\mathcal{G}}_{15}(1)_{3}(s_{13})} e^{(\tilde{\mathcal{G}}_{15})_{1}(s_{13})} + \\
G^{(2)}_{i}(a^{(1)}_{13})_{1}(T^{(1)}_{i}, s_{i}(13)) - (a^{(1)}_{i})_{1}(T^{(2)}_{i}, s_{i}(13)) | e^{-\tilde{\mathcal{G}}_{15}(1)_{3}(s_{13})} e^{(\tilde{\mathcal{G}}_{15})_{1}(s_{13})} ds_{13}
\]

Where \( s_{i}(13) \) represents integrand that is integrated over the interval \([0,t] \)

From the hypotheses on \( 25, 26, 27, 28 \) and \( 29 \) it follows

\[
\left| G^{(1)} - G^{(2)} \right| e^{-\tilde{\mathcal{G}}_{15}(1)_{3}t} \leq \frac{1}{(\mathcal{M}_{13})^{(1)}} (a^{(1)}_{13}) + (a^{(1)}_{13}) + (\mathcal{A}_{13})^{(1)} + (\tilde{\mathcal{G}}_{13})^{(1)} d \left( G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)} \right)
\]

And analogous inequalities for \( G_{i} \) and \( T_{i} \). Taking into account the hypothesis \( 34, 35, 36 \) the result follows

**Remark 1:** The fact that we supposed \( (a^{(1)}_{13})_{1} \) and \( (b^{(1)}_{13})_{1} \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( (\tilde{\mathcal{G}}_{13})^{(1)} e^{(\mathcal{M}_{13})^{(1)}}t \) and \( (\tilde{\mathcal{G}}_{13})^{(1)} e^{(\mathcal{M}_{13})^{(1)}}t \) respectively of \( \mathbb{R}_{+} \).

If instead of proving the existence of the solution on \( \mathbb{R}_{+} \), we have to prove it only on a compact then it suffices to consider that \( (a^{(1)}_{13})_{1} \) and \( (b^{(1)}_{13})_{1} \), \( i = 13, 14, 15 \) depend only on \( T^{(1)}_{i} \) and respectively on \( G(\text{not on } t) \) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_{i}(t) = 0 \) and \( T_{i}(t) = 0 \)

From 19 to 24 it results

\[
G_{i}(t) \geq G^{(1)}_{i} e^{-\int_{0}^{t} \left( (a^{(1)}_{13})_{1} - (a^{(1)}_{13})_{1}(T^{(1)}_{i}, s_{i}(13))) ds_{13} \right)} \geq 0 \\
T_{i}(t) \geq T^{(1)}_{i} e^{-\int_{0}^{t} \left( (b^{(1)}_{13})_{1} \right)} > 0 \text{ for } t > 0
\]

**Definition of** \( ((\mathcal{M}_{13})^{(1)})_{1}, ((\mathcal{M}_{13})^{(1)})_{2}, \text{ and } ((\mathcal{M}_{13})^{(1)})_{3} \) : 

**Remark 3:** If \( G_{13} \) is bounded, the same property have also \( G_{14} \) and \( G_{15} \). indeed if

\[
G_{13} \leq ((\mathcal{M}_{13})^{(1)})_{1} \text{ it follows } \frac{dG_{14}}{dt} \leq ((\mathcal{M}_{13})^{(1)})_{1} - (a^{(1)}_{14}) G_{14} \text{ and by integrating } \\
G_{14} \leq ((\mathcal{M}_{13})^{(1)})_{2} = G^{(1)}_{14} + 2(a^{(1)}_{14}) ((\mathcal{M}_{13})^{(1)})_{1}/(a^{(1)}_{14})^{(1)}
\]

In the same way , one can obtain

\[
G_{15} \leq ((\mathcal{M}_{13})^{(1)})_{3} = G^{(1)}_{15} + 2(a^{(1)}_{15}) ((\mathcal{M}_{13})^{(1)})_{2}/(a^{(1)}_{15})^{(1)}
\]

If \( G_{14} \) or \( G_{15} \) is bounded, the same property follows for \( G_{13}, G_{15} \) and \( G_{13}, G_{14} \) respectively.

**Remark 4:** If \( G_{13} \) is bounded, from below, the same property holds for \( G_{14} \) and \( G_{15} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{14} \) is bounded from below.
Remark 5: If $T_{13}$ is bounded from below and $\lim_{t \to \infty} ((b_1^B)^{(1)}(G(t), t)) = (b_1^B)^{(1)}$ then $T_{14} \to \infty$.

Definition of $(m)^{(1)}$ and $\varepsilon_1$:

Indeed let $t_1$ be so that for $t > t_1$

$$(b_1^B)^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq \frac{(a_{14})^{(1)}(m)^{(1)} - \varepsilon_1}{T_{14}}$ which leads to

$$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)} - \varepsilon_1}{T_{14}}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$$

If we take $t$ such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)\frac{1}{2}$$

By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded. The same property holds for $T_{15}$ if $\lim_{t \to \infty} (b_{15}^B)^{(1)}(G(t), t) = (b_{15}^B)^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

It is now sufficient to take $\left(\frac{a_1^{(2)}}{(M_{16})^{(2)}}\right), \left(\frac{(b_j)^{(2)}}{(M_{16})^{(2)}}\right) < 1$ and to choose

$$\left(\frac{a_1^{(2)}}{(M_{16})^{(2)}}\right), \left(\frac{(b_j)^{(2)}}{(M_{16})^{(2)}}\right)$$

large to have

$$\frac{(a_j)^{(2)}}{(M_{16})^{(2)}} \left[\left(\hat{P}_{16}(t)\right)^{(2)} + \left(\hat{Q}_{16}(t)\right)^{(2)}\right] - \left(\frac{(a_j)^{(2)} + (b_j)^{(2)}}{(M_{16})^{(2)}}\right)$$

$$\leq \left(\frac{(a_j)^{(2)} + (b_j)^{(2)}}{(M_{16})^{(2)}}\right)$$

In order that the operator $\mathcal{A}_{(2)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying 34,35,36 into itself

The operator $\mathcal{A}_{(2)}$ is a contraction with respect to the metric

$$d \left(\left((G_{19})^{(1)}, (T_{19})^{(1)}\right), \left((G_{19})^{(2)}, (T_{19})^{(2)}\right)\right) = \sup_{\mathbb{R}^6} \max_{t \in \mathbb{R^6}} \left|G_1^{(1)}(t) - G_1^{(2)}(t)\right| e^{-((M_{16})^{(2)})}, \max_{t \in \mathbb{R^6}} \left|T_1^{(1)}(t) - T_1^{(2)}(t)\right| e^{-((M_{16})^{(2)})}$$

Indeed if we denote

$$\bar{G}_{16}, \bar{T}_{19} : \left(\bar{G}_{19}, \bar{T}_{19}\right) = \mathcal{A}_{(2)}(G_{19}, T_{19})$$

It results

$$\left|\bar{G}_{16}^{(1)} - \bar{G}_{16}^{(2)}\right| \leq \int_t^t (a_{16})^{(2)} \left|G_{17}^{(1)} - G_{17}^{(2)}\right| e^{-((M_{16})^{(2)})} e^{(M_{16})^{(2)}} dS_{(16)} +$$

$$\int_t^t (a_{16})^{(2)} \left(G_{16}^{(1)} - G_{16}^{(2)}\right) e^{-((M_{16})^{(2)})} e^{(M_{16})^{(2)}} dS_{(16)} +$$

$$\int_t^t (a_{16})^{(2)} \left(T_{17}^{(1)}, S_{(16)}\right) \left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-((M_{16})^{(2)})} e^{(M_{16})^{(2)}} dS_{(16)} +$$

$$\int_t^t (a_{16})^{(2)} \left((T_{17}^{(1)}, S_{(16)}) - (a_{16})^{(2)} (T_{17}^{(2)}, S_{(16)})\right) e^{-((M_{16})^{(2)})} e^{(M_{16})^{(2)}} dS_{(16)}$$

182
Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 25,26,27,28 and 29 it follows

$$\left| (G_{19})^{(1)} - (G_{19})^{(2)} \right| e^{-\hat{M}_{16}^{(2)} t} \leq \frac{1}{(\hat{M}_{16})^{(2)}} (a_{16}^{(2)} + a_{16}^{(1)}) + (\hat{P}_{16}^{(2)}(\hat{K}_{16}^{(2)}) d \left( ((G_{19})^{(1)}, (T_{19})^{(1)}); (G_{19})^{(2)}, (T_{19})^{(2)}) \right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis (34,35,36) the result follows

**Remark 1:** The fact that we supposed $(a_{16}''^{(2)})$ and $(b_{16}''^{(2)})$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{16}^{(2)}) e^{(\hat{M}_{16})^{(2)} t}$ and $(\hat{Q}_{16}^{(2)}) e^{(\hat{M}_{16})^{(2)} t}$, respectively of $\mathbb{R}_+$. If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_{17}''^{(2)})$ and $(b_{17}''^{(2)})$, $i = 16,17,18$ depend only on $T_{17}$ and respectively on $(G_{19})$ and not on $t$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ and $T_i(t) = 0$. From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t (a_{17}''^{(2)}(s_{(16)})_x(t_{(16)})) ds_{(16)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-\int_0^t (b_{17}''^{(2)}(s_{(16)})) ds_{(16)}} \geq 0$$

**Definition of** $(\hat{M}_{16}^{(2)})_1$, $(\hat{M}_{16}^{(2)})_2$ and $(\hat{M}_{16}^{(2)})_3$:

**Remark 3:** If $G_16$ is bounded, the same property have also $G_{17}$ and $G_{18}$. Indeed if $G_{16} \leq (\hat{M}_{16}^{(2)})_1$ it follows $\frac{dG_{16}}{dt} \leq (\hat{M}_{16}^{(2)})_1 - (a_{17}''^{(2)}) G_{17}$ and by integrating

$$G_{17} \leq G_{17}^0 + 2(a_{17}''^{(2)}(\hat{M}_{16}^{(2)})_1)/a_{17}''^{(2)}$$

In the same way, one can obtain

$$G_{18} \leq G_{18}^0 + 2(a_{18}''^{(2)}(\hat{M}_{16}^{(2)})_1)/a_{18}''^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{19}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 4:** If $G_16$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below.

**Remark 5:** If $T_{16}$ is bounded from below and $\lim_{t \to \infty} ((b_{17}''^{(2)}((G_{19})(t), t)) = (b_{17}''^{(2)})$ then $T_{17} \to \infty$, $T_{16} \to \infty$

**Definition of** $(m)^{(2)}$ and $\varepsilon_{2}$:

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17}''^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16} \to \infty$$

Then $\frac{dT_{17}}{dt} \geq (a_{17}''^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to

183
If we take $\varepsilon_2$ such that $e^{-\varepsilon_2 t}$ it results

$$T_{17} \geq \left(\frac{(a_1)^{(2)}(m)^{(2)})}{\varepsilon_2}\right) (1 - e^{-\varepsilon_2 t}) + T_{17} e^{-\varepsilon_2 t}$$

If we take $t$ such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$$T_{17} \geq \left(\frac{(a_1)^{(2)}(m)^{(2)})}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$$

By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. The same property holds for $T_{18}$ if $\lim_{t \to m} (b_{1b}^{(2)}) ((G_{10})(t), t) = (b_{1b}^{(2)})$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

It is now sufficient to take $\left(\frac{(a_1)^{(3)}}{(M_{20})^{(3)}}\right), \left(\frac{(b_1)^{(3)}}{(M_{20})^{(3)}}\right) < 1$ and to choose

$$(\tilde{P}_{20})^{(3)}$$ and $$(\tilde{Q}_{20})^{(3)}$$ large to have

$$\left(\frac{(a_1)^{(3)}}{(M_{20})^{(3)}}\right) \left(\tilde{P}_{20}\right)^{(3)} + \left((\tilde{P}_{20})^{(3)} + G_j^0\right) e^{-\left(\frac{(\tilde{P}_{20})^{(3)}+G_j^0}{\sigma_j^0}\right)} \leq (\tilde{P}_{20})^{(3)}$$

$$\left(\frac{(b_1)^{(3)}}{(M_{20})^{(3)}}\right) \left((\tilde{Q}_{20})^{(3)} + T_j^0\right) e^{-\left(\frac{(\tilde{Q}_{20})^{(3)}+T_j^0}{\eta_j^0}\right)} + (\tilde{Q}_{20})^{(3)} \leq (\tilde{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying 34,35,36 into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric

$$d \left(\left((G_{23})^{(1)}, (T_{23})^{(1)}\right), \left((G_{23})^{(2)}, (T_{23})^{(2)}\right)\right) =$$

$$\sup_{t \in \mathbb{R}^+} \max \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-\left(\frac{(M_{20})^{(3)}}{\tau_{20}}\right)t}, \max \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-\left(\frac{(\tilde{M}_{20})^{(3)}}{\tau_{20}}\right)t}$$

Indeed if we denote

Definition of $\mathcal{G}_{23}, T_{23}: \left(\mathcal{G}_{23}, \mathcal{T}_{23}\right) = \mathcal{A}^{(3)}(\left(G_{23}, T_{23}\right))$

It results

$$|\mathcal{G}_{20}^{(1)} - \mathcal{G}_{20}^{(2)}| \leq \int_0^t (a_2)^{(3)}(a_2)^{(3)} |G_2^{(1)} - G_2^{(2)}| e^{-\left(\frac{(M_{20})^{(3)}}{\tau_{20}}\right)t} e^{\left(\frac{(\tilde{M}_{20})^{(3)}}{\tau_{20}}\right)t} dS_{20} +$$

$$\int_0^t (a_2')^{(3)}(a_2')^{(3)} |G_2^{(1)} - G_2^{(2)}| e^{-\left(\frac{(M_{20})^{(3)}}{\tau_{20}}\right)t} e^{\left(\frac{(\tilde{M}_{20})^{(3)}}{\tau_{20}}\right)t} dS_{20} +$$

$$\int_0^t (a_2'')^{(3)}(T_2)^{(1)}(S_{20}) |G_2^{(1)} - G_2^{(2)}| e^{-\left(\frac{(M_{20})^{(3)}}{\tau_{20}}\right)t} e^{\left(\frac{(\tilde{M}_{20})^{(3)}}{\tau_{20}}\right)t} dS_{20} +$$

$$\int_0^t (a_2''')^{(3)}(T_2)^{(2)}(S_{20}) |G_2^{(1)} - G_2^{(2)}| e^{-\left(\frac{(M_{20})^{(3)}}{\tau_{20}}\right)t} e^{\left(\frac{(\tilde{M}_{20})^{(3)}}{\tau_{20}}\right)t} dS_{20} +$$

Where $S_{20}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 25,26,27,28 and 29 it follows

$$|\mathcal{G}^{(1)} - \mathcal{G}^{(2)}| e^{-\left(\frac{M_{20}}{\tau_{20}}\right)t} \leq$$

$$\frac{1}{\left(M_{20})^{(3)}\right)} \left((a_{20})^{(3)} + (a_{20}')^{(3)} + (a_{20}'')^{(3)} + (P_{20})^{(3)}(K_{20})^{(3)}\right) d \left(\left((G_{23})^{(1)}, (T_{23})^{(1)}; (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis (34,35,36) the result

184
follows

**Remark 1:** The fact that we supposed \((a_{20}')(3)\) and \((b_{20}')(3)\) depending also on \(t\) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((P_{20})(3)e^{(M_{20})(3)t}\) and \((Q_{20})(3)e^{(M_{20})(3)t}\) respectively of \(\mathbb{R}_+\).

If instead of proving the existence of the solution on \(\mathbb{R}_+\), we have to prove it only on a compact then it suffices to consider that \((a_{i1}')(3)\) and \((b_{i1}')(3)\), \(i = 20, 21, 22\) depend only on \(T_{21}\) and respectively on \((G_{23})\) and not on \(t\) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \(t\) where

\[
\int_{19}^{24} [\ldots] \int_{20}^{24} [\ldots] \int_{20}^{24} [\ldots] \int_{20}^{24} [\ldots] \int_{20}^{24} [\ldots]
\]

From 19 to 24 it results

\[
G_i(t) \geq G_i^0 e^{-\int_0^t [(a_i')(3) - \xi_2(t_{20})^3]} dt \geq 0
\]

\[
T_i(t) \geq T_i^0 e^{-\int_0^t [(b_i')(3)dt]} > 0 \text{ for } t > 0
\]

**Definition of** \((M_{20})_1\), \((M_{20})_2\) and \((M_{20})_3\):

**Remark 3:** if \(G_{20}\) is bounded, the same property have also \(G_{21}\) and \(G_{22}\). Indeed if

\[
G_{20} = (M_{20})_1 \leq (M_{20})_2 = G_{21} + 2(a_{21})_1(M_{20})_2/(a_{21})_3
\]

In the same way, one can obtain

\[
G_{22} \leq (M_{20})_3 = G_{22} + 2(a_{22})_1(M_{20})_2/(a_{22})_3
\]

If \(G_{21}\) or \(G_{22}\) is bounded, the same property follows for \(G_{20}\), \(G_{22}\) and \(G_{20}\), \(G_{21}\), respectively.

**Remark 4:** If \(G_{20}\) is bounded, from below, the same property holds for \(G_{21}\) and \(G_{22}\). The proof is analogous with the preceding one. An analogous property is true if \(G_{21}\) is bounded from below.

**Remark 5:** If \(T_{20}\) is bounded from below and \(\lim_{t \to \infty} (b_{i1}')(3) (G_{23})_1(t) = (b_{i1})_3\) then \(T_{21} \to \infty\).

**Definition of** \((m)(3)\) and \(\epsilon_3\):

Indeed let \(t_3\) be so that for \(t > t_3\)

\[
(b_{21})_1'(3) - (b_{i1})'(3) (G_{23})_1(t) < \epsilon_3, T_{20}(t) > (m)(3)
\]

Then \(t_{21} > (a_{21})(3)(m)(3) - \epsilon_3 T_{21}\) which leads to

\[
T_{21} \geq \left(\frac{(a_{21})(3)(m)(3)}{\epsilon_3}\right) (1 - e^{-\epsilon_3 t}) + T_{21}^0 e^{-\epsilon_3 t}
\]

If we take \(t\) such that \(e^{-\epsilon_3 t} = \frac{1}{2}\) it results

\[
T_{21} \geq \left(\frac{(a_{21})(3)(m)(3)}{2\epsilon_3}\right) \cdot t = \log \frac{2}{\epsilon_3}
\]

By taking now \(\epsilon_3\) sufficiently small one sees that \(T_{21}\) is unbounded. The same property holds for \(T_{22}\) if \(\lim_{t \to \infty} (b_{i2}')(3) (G_{23})(t) = (b_{i2})_3\).

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42.
**Behavior of the solutions of equation 37 to 42**

**Theorem 2:** If we denote and define

**Definition of \((\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}\):**

\((bb) \quad (\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}\) four constants satisfying

\[-(\sigma_1)^{(1)} \leq -(a_{14}^{(1)} + (a_{13}^{(1)})^2 + (a_{12}^{(1)})^2)(T_{14}, t) \leq -(\sigma_1)^{(1)}\]

\[-(\tau_2)^{(1)} \leq -(b_{13}^{(1)} + (b_{14}^{(1)})^2 + (b_{12}^{(1)})^2)(G, t) \leq -(\tau_1)^{(1)}\]

**Definition of \((v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, (v)^{(1)}, (u)^{(1)}\):**

\((cc) By \quad (v_1)^{(1)} > 0, (v_2)^{(1)} < 0 \) and respectively \((u_1)^{(1)} > 0, (u_2)^{(1)} < 0\) the roots of the equations \((a_{14}^{(1)})(v)^{(1)} + (\sigma_1)^{(1)}(v)^{(1)} - (a_{13}^{(1)})^2 = 0 \) and \((b_{14}^{(1)})(u)^{(1)} + (\tau_1)^{(1)}(u)^{(1)} - (b_{13}^{(1)})^2 = 0\)

**Definition of \((\bar{v}_1)^{(1)}, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}\):**

By \((\bar{v}_1)^{(1)} > 0, (\bar{v}_2)^{(1)} < 0 \) and respectively \((\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0\) the roots of the equations \((a_{14}^{(1)})(v)^{(1)} + (\sigma_2)^{(1)}(v)^{(1)} - (a_{13}^{(1)})^2 = 0 \) and \((b_{14}^{(1)})(u)^{(1)} + (\tau_2)^{(1)}(u)^{(1)} - (b_{13}^{(1)})^2 = 0\)

**Definition of \((m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}\):**

\((dd) If we define \((m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}\) by

\[(m_1)^{(1)} = (v_0)^{(1)}, (m_2)^{(1)} = (v_1)^{(1)}, if (v_0)^{(1)} < (v_1)^{(1)}\]

\[(m_2)^{(1)} = (v_1)^{(1)}, (m_2)^{(1)} = (\bar{v}_1)^{(1)}, if (v_1)^{(1)} < (\bar{v}_1)^{(1)}\]

\[and \quad (v_0)^{(1)} = \frac{\gamma_0}{\gamma_{14}}\]

\[(m_2)^{(1)} = (v_1)^{(1)}, (m_2)^{(1)} = (v_0)^{(1)}, if (\bar{v}_1)^{(1)} < (v_0)^{(1)}\]

\[and \quad \mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \mu_2)^{(1)} = (u_0)^{(1)}, if (u_0)^{(1)} < (u_1)^{(1)}\]

\[and \quad (u_0)^{(1)} = \frac{\gamma_0}{\gamma_{14}}\]

\[(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_2)^{(1)} = (u_0)^{(1)}, if (\bar{u}_1)^{(1)} < (u_0)^{(1)}\] are defined by 59 and 61 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

\[G_{13}^{0}e^{((S_1)^{(1)} - (P_{13})^{(1)})t} \leq G_{13}(t) \leq G_{13}^{0}e^{(S_1)^{(1)}t}\]

where \((P_{13})^{(1)}\) is defined by equation 25

\[\frac{1}{(m_1)^{(1)}G_{13}^{0}e^{((S_{13})^{(1)} - (P_{13})^{(1)})t}} \leq G_{14}(t) \leq \frac{1}{(m_1)^{(1)}G_{13}^{0}e^{(S_{13})^{(1)}t}}\]

\[\left(\frac{a_{14}^{(1)}G_{13}^{0}e^{((S_{13})^{(1)} - (P_{13})^{(1)})t}}{(m_1)^{(1)}(S_{13})^{(1)} - (P_{13})^{(1)} - (S_{2})^{(1)}t)} - e^{((S_{1})^{(1)} - (P_{13})^{(1)})t} - e^{-(S_{2})^{(1)}t}\right) + G_{15}^{0}e^{-(S_{2})^{(1)}t} \leq G_{15}(t) \leq 893\]
\[
\frac{(a_{13})^{(1)}(b_{13})^{(1)}}{(m_2)^{(1)}} \left[ e^{(R_1)^{(1)}t} - e^{-(a_{13})^{(1)}t} \right] + G_{15}^0 e^{-(a_{13})^{(1)}t} \]
\]
\[
\left( T_{13}^0 e^{(R_1)^{(1)}t} - T_{13} \right) (t) \leq T_{13}^0 e^{(R_1)^{(1)}t} (r_{13})^{(1)} t 
\]
\[
\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} (t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} (r_{13})^{(1)} t 
\]
\[
\frac{(a_{13})^{(1)}(b_{13})^{(1)}}{(m_2)^{(1)}} \left[ e^{(R_1)^{(1)}t} - e^{-(b_{13})^{(1)}t} \right] + T_{15}^0 e^{-(b_{13})^{(1)}t} \leq T_{15}^0 (t) \leq 
\]
\[
\frac{(a_{13})^{(1)}(b_{13})^{(1)}}{(m_2)^{(1)}} \left[ e^{((R_1)^{(1)}t + (r_{13})^{(1)} t) - e^{-(b_{13})^{(1)}t}} \right] + T_{15}^0 e^{-(b_{13})^{(1)}t} 
\]

**Definition of \((S_1)^{(1)}(S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}\):**

Where \((S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a_{13})^{(1)}\)
\((S_2)^{(1)} = (a_{13})^{(1)} - (p_{13})^{(1)}\)
\((R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b_{13})^{(1)}\)
\((R_2)^{(1)} = (b_{13})^{(1)} - (r_{13})^{(1)}\)

If we denote and define

**Definition of \((\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}\):**

(\(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}\) four constants satisfying
\[-(\sigma_1)^{(2)} \leq -(a'_{13})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (\mu'_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)}\]
\[-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{16}), t) - (\mu'_{17})^{(2)}((G_{16}), t) \leq -(\tau_1)^{(2)}\]

**Definition of \((v_1)^{(2)}, (v_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}\):**

By \((v_1)^{(2)} > 0, (v_2)^{(2)} < 0\) and respectively \((u_1)^{(2)} > 0, (u_2)^{(2)} < 0\) the roots
\[(t_{17})^{(2)}(v^{(2)})^{(2)} + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0\]
\[(\mu'_{17})^{(2)}(u^{(2)})^{(2)} + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0\]

**Definition of \((\tilde{v}_1)^{(2)}, (\tilde{v}_2)^{(2)}, (\tilde{u}_1)^{(2)}, (\tilde{u}_2)^{(2)}\):**

By \((\tilde{v}_1)^{(2)} > 0, (\tilde{v}_2)^{(2)} < 0\) and respectively \((\tilde{u}_1)^{(2)} > 0, (\tilde{u}_2)^{(2)} < 0\) the roots
\[(a_{17})^{(2)}(v^{(2)})^{(2)} + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0\]
\[(b_{17})^{(2)}(u^{(2)})^{(2)} + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0\]

**Definition of \((m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}\):**

If we define \((m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}\) by
\[(m_2)^{(2)} = (v_0)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}, if \ (v_0)^{(2)} < (v_1)^{(2)}\]
\[(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}, if \ (v_1)^{(2)} < (v_0)^{(2)} < (v_1)^{(2)},\]
If we denote and define

\[
(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}, \quad \text{if} \quad (v_1)^{(2)} < (v_0)^{(2)}
\]

and analogously

\[
(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \quad \text{if} \quad (u_0)^{(2)} < (u_1)^{(2)}
\]

\[
(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \quad \text{if} \quad (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}
\]

and

\[
(u_0)^{(2)} = \frac{\gamma_0}{\gamma_1}
\]

\[
(\mu_2)^{(2)} = (u_2)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \quad \text{if} \quad (u_2)^{(2)} < (u_0)^{(2)}
\]

Then the solution of 19, 20, 21, 22, 23 and 24 satisfies the inequalities

\[
G_1(t) \leq G_1^0 e^{(S_1)^{(2)}t} \leq G_1^0 e^{(S_1^1)^{(2)}t}
\]

\[
(p_1)^{(2)} = \frac{1}{(m_1)^{(2)} \gamma_1} G_1^0 e^{((S_1)^{(2)} - (P_{16})^{(2)}t)} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)} \gamma_1} G_1^0 e^{((S_1)^{(2)} - (P_{16})^{(2)}t)}
\]

\[
T_{16} e^{(R_2)^{(2)}t} \leq T_{16}(t) \leq T_{16} e^{((R_1)^{(2)} + (R_1)^{(2)}t)}
\]

\[
\frac{1}{(\mu_2)^{(2)}} T_{16} e^{((R_1)^{(2)}t)} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16} e^{((R_1)^{(2)} + (R_1)^{(2)}t)}
\]

\[
\frac{1}{(\mu_2)^{(2)}(R_2)^{(2)} - (R_2)^{(2)}} e^{(R_2)^{(2)}t - e^{(R_2)^{(2)}t}} + T_{16} e^{-(R_2)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}(R_2)^{(2)} + (R_2)^{(2)}t)}
\]

Definition of \((S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}\):

- \((S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} + (a_{16})^{(2)}\)
- \((S_2)^{(2)} = (a_{16})^{(2)} - (P_{16})^{(2)}\)
- \((R_1)^{(2)} = (b_{16})^{(2)}(\mu_1)^{(2)} + (b_{16})^{(2)}\)
- \((R_2)^{(2)} = (b_{16})^{(2)} - (P_{16})^{(2)}\)

If we denote and define

Definition of \((\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}\):

\(-\sigma_3)^{(3)} \leq -a_{20}^{(3)} + a_{21}^{(3)} - a_{20}^{(3)}(T_{21}, t) + (a_{21}^{(3)}(T_{21}, t) \leq -\sigma_3)^{(3)}\)

\(-\tau_2)^{(3)} \leq -b_{20}^{(3)} + b_{21}^{(3)} - b_{20}^{(3)}(G, t) - (b_{21}^{(3)}(G, t) \leq -\tau_2)^{(3)}\)
Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$:

(ii) By $(v_1)^{(3)} > 0, (v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}(v)^{(3)^2} + (a_{20})^{(3)}v^{(3)} = 0$

and $(b_{21})^{(3)}(u)^{(3)^2} + (b_{20})^{(3)}u^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0, (\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}(v)^{(3)^2} + (a_{20})^{(3)}v^{(3)} = 0$

and $(b_{21})^{(3)}(u)^{(3)^2} + (b_{20})^{(3)}u^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:

(j) If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, if $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, if $(v_1)^{(3)} < (\bar{v}_1)^{(3)}$

and

$(v_0)^{(3)} = \frac{e_0}{e_2}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, if $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, if $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, if $(u_1)^{(3)} < (\bar{u}_1)^{(3)}$ and

$(u_0)^{(3)} = \frac{e_0}{e_2}$

Then the solution of 19, 20, 21, 22, 23 and 24 satisfies the inequalities

$G_0 e^{((s_1)^{(3)} - (p_0)^{(3)})t} \leq G_2 e^{((s_1)^{(3)} + (r_0)^{(3)})t}$

$(p_i)^{(3)}$ is defined by equation 25

$\frac{1}{(m_2)^{(3)}} G_0 e^{((s_1)^{(3)} - (p_0)^{(3)})t} \leq G_2 (t) \leq \frac{1}{(m_2)^{(3)}} G_0 e^{((s_1)^{(3)} + (r_0)^{(3)})t}$

$G_0 e^{((s_1)^{(3)} - (p_0)^{(3)})t} - e^{-(s_2)^{(3)}t} + G_0 e^{((s_1)^{(3)} - (p_0)^{(3)})t} \leq G_2 (t)$

$T_2 e^{((r_1)^{(3)} - (r_0)^{(3)})t} \leq T_2 e^{((r_1)^{(3)} + (r_0)^{(3)})t}$

$\frac{1}{(\mu_2)^{(3)}} T_0 e^{((r_1)^{(3)} - (r_0)^{(3)})t} \leq T_2 (t) \leq \frac{1}{(\mu_2)^{(3)}} T_0 e^{((r_1)^{(3)} + (r_0)^{(3)})t}$

$T_0 e^{((r_1)^{(3)} - (r_0)^{(3)})t} - e^{-(b_2)^{(3)}t} + T_0 e^{((r_1)^{(3)} - (r_0)^{(3)})t} \leq T_2 (t)$

$T_2 e^{-(b_2)^{(3)}t}$
Definition of $(S_1)^{(3)}$, $(S_2)^{(3)}$, $(R_1)^{(3)}$, $(R_2)^{(3)}$:

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (t_{22})^{(3)}$

Proof: we obtain

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left( (a'_{13})^{(1)} - (a''_{13})^{(1)} + (a'''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:

$$v^{(1)} = \frac{a_{13}}{a_{14}}$$

It follows

$$- (a_{14})^{(1)}(v^{(1)})^2 + (a_{14})^{(1)}v^{(1)} - (a_{13})^{(1)} \leq \frac{dv^{(1)}}{dt} \leq - (a_{14})^{(1)}(v^{(1)})^2 + (a_{14})^{(1)}v^{(1)} - (a_{13})^{(1)}$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}$, $(v_0)^{(1)}$:

(u) For $0 < \frac{v_0^{(1)}}{\bar{v}_1^{(1)}} = \frac{a_{14}^{(1)}}{a_{14}^{(1)}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \geq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{-[a_{14}]^{(1)}[(v_2)^{(1)} - (v_0)^{(1)}]^2]}}{1 + (C)^{(1)}e^{-[a_{14}]^{(1)}[(v_2)^{(1)} - (v_0)^{(1)}]^2]}}$$

$$\text{and} \quad (C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}$$

From which $v^{(1)}(t) \leq v^{(1)}(t) \leq (v_1)^{(1)}$

In the same manner, we get

$$v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{-[a_{14}]^{(1)}[(v_2)^{(1)} - (v_0)^{(1)}]^2]}}{1 + (C)^{(1)}e^{-[a_{14}]^{(1)}[(v_2)^{(1)} - (v_0)^{(1)}]^2]}}$$

$$\text{and} \quad (C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}$$

From which we deduce $(v_0)^{(1)} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}$

(v) If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{a_{14}^{(1)}}{a_{14}^{(1)}} < (\bar{v}_1)^{(1)}$ we find like in the previous case,

$$v^{(1)}(t) \leq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{-[a_{14}]^{(1)}[(v_2)^{(1)} - (v_0)^{(1)}]^2]}}{1 + (C)^{(1)}e^{-[a_{14}]^{(1)}[(v_2)^{(1)} - (v_0)^{(1)}]^2]}}$$

From which we deduce $(v_0)^{(1)} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}$

(w) If $0 < (v_0)^{(1)} \leq (\bar{v}_1)^{(1)} \leq \frac{(v_0)^{(1)}}{\bar{v}_1^{(1)}} = \frac{a_{14}^{(1)}}{a_{14}^{(1)}}$, we obtain

$$v^{(1)}(t) \leq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{-[a_{14}]^{(1)}[(v_2)^{(1)} - (v_0)^{(1)}]^2]}}{1 + (C)^{(1)}e^{-[a_{14}]^{(1)}[(v_2)^{(1)} - (v_0)^{(1)}]^2]}} \leq (v_0)^{(1)}$$

And so with the notation of the first part of condition (c), we have
Definition of $v^{(1)}(t)$:

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad v^{(1)}(t) = \frac{g_{13}(t)}{g_{14}(t)}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad u^{(1)}(t) = \frac{r_{23}(t)}{r_{14}(t)}$$

Now, using this result and replacing it in 19, 20, 21, 22, 23, and 24 we get easily the result stated in the theorem.

Particular case:

If $(a_{16})^{(1)} = (a_{14})^{(1)}$, then $(\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\nu_2)^{(1)}$ if in addition $(v_0)^{(1)} = (v_1)^{(1)}$ then $v^{(1)}(t) = (v_0)^{(1)}$ and as a consequence $G_{13}(t) = (v_0)^{(1)}G_{14}(t)$ this also defines $(v_0)^{(1)}$ for the special case

Analogously if $(b_{13})^{(1)} = (b_{14})^{(1)}$, then $(\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u})^{(1)}$ if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(v_1)^{(1)}$ and $(\nu_2)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof: From 19, 20, 21, 22, 23, 24 we obtain

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{14})^{(2)} -(a_{17})^{(2)} + (a_{16})^{(2)}(T_{17}, t) - (a_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}\right)$$

Definition of $v^{(2)}$:

$$v^{(2)} = \frac{G_{16}}{G_{17}}$$

It follows

$$-\left((a_{17})^{(2)}v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} \leq \frac{dv^{(2)}}{dt} \leq -\left((a_{17})^{(2)}v^{(2)}\right)^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)}$$

From which one obtains

Definition of $(\bar{v})^{(2)}, (v_0)^{(2)}$:

(x) For $0 < (v_0)^{(2)} = \frac{g_{14}}{g_{17}} \leq (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (v_2)^{(2)}[(v_2)^{(2)}(v_1)^{(2)} - (v_0)^{(2)})^2]}{1 + (\sigma_1)^{(2)}[(v_2)^{(2)}(v_1)^{(2)} - (v_0)^{(2)})^2]}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner, we get

$$v^{(2)}(t) \leq \frac{(v_2)^{(2)} + (v_2)^{(2)}[(v_2)^{(2)}(v_1)^{(2)} - (v_0)^{(2)})]}{1 + (v_0)^{(2)}[(v_2)^{(2)}(v_1)^{(2)} - (v_0)^{(2)})]}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$

(y) If $0 < (v_3)^{(2)} < (v_0)^{(2)} = \frac{g_{14}}{g_{17}} < (\bar{v}_1)^{(2)}$ we find like in the previous case,
\[
(v_2)(t) \leq \frac{(v_1)(t) + (v_3)(t) + (v_4)(t)e^{-(a_{12})(h)(v_2)(t)^2}}{1 + e^{(a_{12})(h)(v_2)(t)^2}} \leq (v_3)(t)
\]

\[
(\bar{v}_1)(t) \leq (v_1)(t) \leq \frac{((v_3)(t) + (v_4)(t) + (v_5)(t)e^{-(a_{12})(h)(v_2)(t)^2})}{1 + e^{(a_{12})(h)(v_2)(t)^2}} \leq (v_3)(t)
\]

(z) If \(0 < (v_1)(t) \leq (\bar{v}_1)(t) \leq (v_6)(t) = \frac{g_{16}}{g_{17}}\), we obtain

\[
(v_1)(t) \leq (v_3)(t) \leq \frac{(v_3)(t) + (v_4)(t) + (v_5)(t)e^{-(a_{12})(h)(v_2)(t)^2})}{1 + e^{(a_{12})(h)(v_2)(t)^2}} \leq (v_3)(t)
\]

And so with the notation of the first part of condition (c), we have

**Definition of** \(v(2)(t)\) :

\[
(m_2)(t) \leq (v(2)(t) \leq (m_1)(t), \quad v(2)(t) = \frac{g_{16}(t)}{g_{17}(t)}
\]

In a completely analogous way, we obtain

**Definition of** \(u(2)(t)\) :

\[
(\mu_2)(t) \leq (u(2)(t) \leq (\mu_1)(t), \quad u(2)(t) = \frac{t_{16}(t)}{t_{17}(t)}
\]

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case** :

If \((a_{16}(2) = (a_{17}(2)\), then \((\sigma_1)(2) = (\sigma_2)(2)\) and in this case \((v_1)(2) = (\bar{v}_1)(2)\) if in addition \((v_0)(2) = (v_1)(2)\) then \((v(2)(t) = (v_0)(2)\) and as a consequence \(G_{16}(t) = (v_0)(2)G_{17}(t)\)

Analogously if \((b_{16}(2) = (b_{17}(2)\), then \((t_1)(2) = (t_2)(2)\) and then

\((u(1)(2) = (\bar{u}_1)(2)\) if in addition \((u_0)(2) = (u_1)(2)\) then \(T_{16}(t) = (u_0)(2)T_{17}(t)\) This is an important consequence of the relation between \((v(2)(2)\) and \((\bar{v}_1)(2)\)

**Proof**: From 19,20,21,22,23,24 we obtain

\[
\frac{dv(3)}{dt} = (a_{20})(3) - (a_{20}(3) - (a_{21}(3) + (a_{20}(3)(T_{21}(t), t) - (a_{21}(3)(T_{23}(t)v(3) - (a_{21}(3)v(3)
\]

**Definition of** \(v(3)\) :

\[
v(3) = \frac{g_{20}}{g_{21}}
\]

It follows

\[
-((a_{21}(3)v(3))^2 + (a_{21}(3)v(3) - (a_{21}(3))^2) \leq \frac{dv(3)}{dt} \leq -((a_{21}(3)v(3))^2 + (a_{21}(3)v(3) - (a_{21}(3))^2)
\]

From which one obtains

(a) For \(0 < (v_0)(3) = \frac{g_{20}}{g_{21}} < (v_1)(3) < (\bar{v}_1)(3)\)
We can prove the following:

Analogously if

Particular case:

Definition of

In a completely analogous way, we obtain

Definition of \( \bar{v}_1^{(3)} \):

From which we deduce \( v_0^{(3)} \leq v^{(3)}(t) \leq (\bar{v}_1)^{(3)} \)

(b) If \( 0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{g_2}{g_1} < (\bar{v}_1)^{(3)} \) we find like in the previous case,

\[
(v_1)^{(3)} \leq \frac{(v_1)^{(3)} + (\bar{C})^{(3)}(v_2)^{(3)})e^{-[\sigma_2^{(3)}((v_1)^{(3)}-v_2)^{(3)})]t}}{1 + (\bar{C})^{(3)}e^{-[\sigma_2^{(3)}((v_1)^{(3)}-v_2)^{(3)})]t}} \leq v^{(3)}(t) \leq \frac{(v_2)^{(3)} + (\bar{C})^{(3)}(v_1)^{(3)})e^{-[\sigma_2^{(3)}((v_1)^{(3)}-v_2)^{(3)})]t}}{1 + (\bar{C})^{(3)}e^{-[\sigma_2^{(3)}((v_1)^{(3)}-v_2)^{(3)})]t}} \leq (\bar{v}_1)^{(3)}
\]

(c) If \( 0 < (v_1)^{(3)} \leq (\bar{v}_1)^{(3)} \leq (v_0)^{(3)} = \frac{g_2}{g_1} \), we obtain

\[
(v_1)^{(3)} \leq v^{(3)}(t) \leq \frac{(v_1)^{(3)} + (\bar{C})^{(3)}(v_2)^{(3)})e^{-[\sigma_2^{(3)}((v_1)^{(3)}-v_2)^{(3)})]t}}{1 + (\bar{C})^{(3)}e^{-[\sigma_2^{(3)}((v_1)^{(3)}-v_2)^{(3)})]t}} \leq (v_0)^{(3)}
\]

And so with the notation of the first part of condition (c), we have

Definition of \( v^{(3)}(t) \):

\[
(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}, \quad v^{(3)}(t) = \frac{g_2(t)}{g_1(t)}
\]

In a completely analogous way, we obtain

Definition of \( u^{(3)}(t) \):

\[
(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad u^{(3)}(t) = \frac{g_2(t)}{g_1(t)}
\]

Now, using this result and replacing it in 19, 20, 21, 22, 23, and 24 we get easily the result stated in the theorem.

Particular case:

If \( (a_0)^{(3)} = (a_1)^{(3)} \), then \( (\sigma_0)^{(3)} = (\sigma_1)^{(3)} \) and in this case \( (v_1)^{(3)} = (\bar{v}_1)^{(3)} \) if in addition \( (v_0)^{(3)} = (v_1)^{(3)} \) then \( v^{(3)}(t) = (v_0)^{(3)} \) and as a consequence \( G_{20}(t) = (v_0)^{(3)}G_{21}(t) \)

Analogously if \( (b_0)^{(3)} = (b_1)^{(3)} \), then \( (\tau_0)^{(3)} = (\tau_1)^{(3)} \) and then

\[
(u_1)^{(3)} = (\bar{u}_1)^{(3)} \text{ if in addition } (u_0)^{(3)} = (u_1)^{(3)} \text{ then } T_{20}(t) = (u_0)^{(3)}T_{21}(t) \text{ This is an important consequence of the relation between } (v_1)^{(3)} \text{ and } (\bar{v}_1)^{(3)}
\]

We can prove the following:

954
If \((a')^{(1)}\) and \((b')^{(1)}\) are independent on \(t\), and the conditions (with the notations 25, 26, 27, 28)

\[(a_{13}^{(1)}(a_{14}^{(1)} - (a_{13}^{(1)}(a_{14}^{(1)}) < 0)
\]

\[(a_{13}^{(1)}(a_{14}^{(1)} - (a_{13}^{(1)}(a_{14}^{(1)}) + (a_{13}^{(1)}(p_{13}^{(1)} + (a_{14}^{(1)}(p_{14}^{(1)} + (p_{13}^{(1)}(p_{14}^{(1)} > 0)
\]

\[(b_{13}^{(1)}(b_{14}^{(1)} - (b_{13}^{(1)}(b_{14}^{(1)}) > 0,
\]

\[(b_{13}^{(1)}(b_{14}^{(1)} - (b_{13}^{(1)}(b_{14}^{(1)}) - (b_{13}^{(1)}(r_{14}^{(1)} - (b_{14}^{(1)}(r_{14}^{(1)}) + (r_{13}^{(1)}(r_{14}^{(1)} < 0
\]

with \((p_{13}^{(1)}), (r_{14}^{(1)})\) as defined by equation 25 are satisfied, then the system

If \((a_{i}^{(2)})\) and \((b_{i}^{(2)})\) are independent on \(t\), and the conditions (with the notations 25, 26, 27, 28)

\[(a_{16}^{(2)}(a_{17}^{(2)} - (a_{16}^{(2)}(a_{17}^{(2)} < 0
\]

\[(a_{16}^{(2)}(a_{17}^{(2)} - (a_{16}^{(2)}(a_{17}^{(2)}) + (a_{16}^{(2)}(p_{16}^{(2)} + (a_{17}^{(2)}(p_{17}^{(2)} + (p_{16}^{(2)}(p_{17}^{(2)} > 0
\]

\[(b_{16}^{(2)}(b_{17}^{(2)} - (b_{16}^{(2)}(b_{17}^{(2)}) > 0,
\]

\[(b_{16}^{(2)}(b_{17}^{(2)} - (b_{16}^{(2)}(b_{17}^{(2)}) - (b_{16}^{(2)}(r_{17}^{(2)} - (b_{17}^{(2)}(r_{17}^{(2)}) + (r_{16}^{(2)}(r_{17}^{(2)} < 0
\]

with \((p_{16}^{(2)}), (r_{17}^{(2)})\) as defined by equation 25 are satisfied, then the system

If \((a_{i}^{(3)})\) and \((b_{i}^{(3)})\) are independent on \(t\), and the conditions (with the notations 25, 26, 27, 28)

\[(a_{20}^{(3)}(a_{21}^{(3)} - (a_{20}^{(3)}(a_{21}^{(3)} < 0
\]

\[(a_{20}^{(3)}(a_{21}^{(3)} - (a_{20}^{(3)}(a_{21}^{(3)}) + (a_{20}^{(3)}(p_{20}^{(3)} + (a_{21}^{(3)}(p_{21}^{(3)} + (p_{20}^{(3)}(p_{21}^{(3)} > 0
\]

\[(b_{20}^{(3)}(b_{21}^{(3)} - (b_{20}^{(3)}(b_{21}^{(3)}) > 0,
\]

\[(b_{20}^{(3)}(b_{21}^{(3)} - (b_{20}^{(3)}(b_{21}^{(3)}) - (b_{20}^{(3)}(r_{21}^{(3)} - (b_{21}^{(3)}(r_{21}^{(3)}) + (r_{20}^{(3)}(r_{21}^{(3)} < 0
\]

with \((p_{20}^{(3)}), (r_{21}^{(3)})\) as defined by equation 25 are satisfied, then the system

\[(a_{13}^{(1)}G_{14} - [(a_{13}^{(1)} + (a_{15}^{(1)}(T_{13}^{4})]G_{13} = 0
\]

\[(a_{14}^{(1)}G_{13} - [(a_{14}^{(1)} + (a_{16}^{(1)}(T_{14}^{4})]G_{14} = 0
\]

\[(a_{15}^{(1)}G_{14} - [(a_{15}^{(1)} + (a_{17}^{(1)}(T_{14}^{4})]G_{15} = 0
\]

\[(b_{13}^{(1)}T_{14} - [(b_{13}^{(1)} - (b_{13}^{(1)}(G) T_{13} = 0
\]

\[(b_{14}^{(1)}T_{13} - [(b_{14}^{(1)} - (b_{14}^{(1)}(G) T_{14} = 0
\]

\[(b_{15}^{(1)}T_{14} - [(b_{15}^{(1)} - (b_{15}^{(1)}(G) T_{15} = 0
\]

has a unique positive solution, which is an equilibrium solution for the system (19 to 24)

\[(a_{16}^{(2)}G_{17} - [(a_{16}^{(2)} + (a_{18}^{(2)}(T_{17}^{4})]G_{16} = 0
\]

\[(a_{17}^{(2)}G_{16} - [(a_{17}^{(2)} + (a_{19}^{(2)}(T_{17}^{4})]G_{17} = 0
\]

\[(a_{18}^{(2)}G_{17} - [(a_{18}^{(2)} + (a_{20}^{(2)}(T_{17}^{4})]G_{18} = 0
\]

\[(b_{16}^{(2)}T_{17} - [(b_{16}^{(2)} - (b_{18}^{(2)}(G_{19}) T_{16} = 0
\]
(b_{17})^{(2)}T_{16} - [(b_{17})^{(2)} - (b_{17}^{''})^{(2)}(G_{19})]T_{17} = 0 \hspace{1cm} 970
(b_{18})^{(2)}T_{17} - [(b_{18})^{(2)} - (b_{18}^{''})^{(2)}(G_{19})]T_{18} = 0 \hspace{1cm} 971

has a unique positive solution, which is an equilibrium solution for (19 to 24)

(a_{20})^{(3)}G_{21} - [(a_{20})^{(3)} + (a_{20}^{''})^{(3)}(T_{21})]G_{20} = 0 \hspace{1cm} 972
(a_{21})^{(3)}G_{20} - [(a_{21})^{(3)} + (a_{21}^{''})^{(3)}(T_{21})]G_{21} = 0 \hspace{1cm} 973
(a_{22})^{(3)}G_{21} - [(a_{22})^{(3)} + (a_{22}^{''})^{(3)}(T_{21})]G_{22} = 0 \hspace{1cm} 974
(b_{20})^{(3)}T_{21} - [(b_{20})^{(3)} - (b_{20}^{''})^{(3)}(G_{23})]T_{20} = 0 \hspace{1cm} 975
(b_{21})^{(3)}T_{20} - [(b_{21})^{(3)} - (b_{21}^{''})^{(3)}(G_{23})]T_{21} = 0 \hspace{1cm} 976
(b_{22})^{(3)}T_{21} - [(b_{22})^{(3)} - (b_{22}^{''})^{(3)}(G_{23})]T_{22} = 0 \hspace{1cm} 977

has a unique positive solution, which is an equilibrium solution for (19 to 24)

Proof:

(a) Indeed the first two equations have a nontrivial solution \( G_{13}, G_{14} \) if
\[
F(T) = (a_{13}^{(1)}(a_{14}^{(1)}) - (a_{13}^{(1)}(a_{14}^{(1)}) + (a_{13}^{(1)}(a_{14}^{(1)}(T_{14}) + (a_{13}^{(1)}(a_{14}^{(1)}(T_{14}) = 0
\]

(b) Indeed the first two equations have a nontrivial solution \( G_{16}, G_{17} \) if
\[
F(T_{19}) = (a_{16}^{(2)}(a_{17}^{(2)}) - (a_{16}^{(2)}(a_{17}^{(2)}) + (a_{16}^{(2)}(a_{17}^{(2)}(T_{17}) + (a_{17}^{(2)}(a_{16}^{(2)}(T_{17}) = 0
\]

(a) Indeed the first two equations have a nontrivial solution \( G_{20}, G_{21} \) if
\[
F(T_{23}) = (a_{20}^{(3)}(a_{21}^{(3)}) - (a_{20}^{(3)}(a_{21}^{(3)}) + (a_{20}^{(3)}(a_{21}^{(3)}(T_{21}) + (a_{21}^{(3)}(a_{20}^{(3)}(T_{21}) + (a_{21}^{(3)}(a_{20}^{(3)}(T_{21}) = 0
\]

Definition and uniqueness of \( T_{14} \):

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \((a_{i}^{''})^{(1)}(T_{14})\) being increasing, it follows that there exists a unique \( T_{14}^{*} \) for which \( f(T_{14}^{*}) = 0 \). With this value, we obtain from the three first equations
\[
G_{13} = \frac{(a_{13}^{(1)})G_{14}}{[a_{13}^{(1)}(a_{14}^{(1)}(T_{14})}, \hspace{0.5cm} G_{15} = \frac{(a_{15}^{(1)})G_{14}}{[a_{15}^{(1)}(a_{14}^{(1)}(T_{14})}
\]

Definition and uniqueness of \( T_{17} \):

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \((a_{i}^{''})^{(2)}(T_{17})\) being increasing, it follows that there exists a unique \( T_{17}^{*} \) for which \( f(T_{17}^{*}) = 0 \). With this value, we obtain from the three first equations
\[
G_{16} = \frac{(a_{16}^{(2)})G_{17}}{[a_{16}^{(2)}(a_{17}^{(2)}(T_{17})}, \hspace{0.5cm} G_{18} = \frac{(a_{18}^{(2)})G_{17}}{[a_{18}^{(2)}(a_{17}^{(2)}(T_{17})}
\]

Definition and uniqueness of \( T_{21} \):

195
After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \((a_i')^{(1)}(T_{21})\) being increasing, it follows that there exists a unique \( T_{21}^* \) for which \( f(T_{21}) = 0 \). With this value, we obtain from the three first equations

\[
G_{20} = \frac{(a_{20})^{(3)}G_{21}}{(a_{20})^{(2)} + (a_{10})^{(2)}(T_{21})}, \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{(a_{22})^{(2)} + (a_{20})^{(2)}(T_{21})}
\]

(l) By the same argument, the equations 92, 93 admit solutions \( G_{13}, G_{14} \) if

\[
\varphi(G) = (b_{13})^{(1)}(b_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - \left[(b_{13})^{(1)}(b_{14})^{(1)}(G) + (b_{14})^{(1)}(b_{13})^{(1)}(G)\right] + (b_{13})^{(1)}(G) = 0
\]

Where in \( G(G_{13}, G_{14}, G_{15}) \), \( G_{13}, G_{15} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{14} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{14}^* \) such that \( \varphi(G^*) = 0 \)

(m) By the same argument, the equations 92, 93 admit solutions \( G_{16}, G_{17} \) if

\[
\varphi(G_{19}) = (b_{16})^{(2)}(b_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - \left[(b_{16})^{(2)}(b_{17})^{(2)}(G_{19}) + (b_{17})^{(2)}(b_{16})^{(2)}(G_{19})\right] + (b_{16})^{(2)}(G_{19}) = 0
\]

Where in \( G_{16}(G_{16}, G_{17}, G_{19}, G_{16}, G_{18}) \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{17} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{14}^* \) such that \( \varphi(G_{16}^*) = 0 \)

(n) By the same argument, the equations 92, 93 admit solutions \( G_{20}, G_{21} \) if

\[
\varphi(G_{23}) = (b_{20})^{(3)}(b_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - \left[(b_{20})^{(3)}(b_{21})^{(3)}(G_{23}) + (b_{21})^{(3)}(b_{20})^{(3)}(G_{23})\right] + (b_{20})^{(3)}(G_{23}) = 0
\]

Where in \( G_{23}(G_{20}, G_{21}, G_{22}) \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{23} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{21}^* \) such that \( \varphi(G_{23}^*) = 0 \)

Finally we obtain the unique solution of 89 to 94

\[
G_{14}^* \quad \text{given by \( \varphi(G^*) = 0, T_{14}^* \) given by \( f(T_{14}^*) = 0 \) and}
\[
G_{13}^* = \frac{(a_{13})^{(2)}G_{14}^*}{(a_{13})^{(1)} + (a_{13})^{(1)}(T_{14}^*)}, \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{(a_{15})^{(1)} + (a_{15})^{(1)}(T_{14}^*)}
\]

\[
T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{(b_{13})^{(1)} - (b_{13})^{(1)}(G^*)}, \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{(b_{15})^{(1)} - (b_{15})^{(1)}(G^*)}
\]

Obviously, these values represent an equilibrium solution of 19, 20, 21, 22, 23, 24

Finally we obtain the unique solution of 89 to 94

\[
G_{17}^* \quad \text{given by \( \varphi(G_{19})^* = 0, T_{17}^* \) given by \( f(T_{17}^*) = 0 \) and}
\[
G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{(a_{16})^{(2)} + (a_{16})^{(2)}(T_{17}^*)}, \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{(a_{18})^{(2)} + (a_{18})^{(2)}(T_{17}^*)}
\]

196
\[ T_{i_{16}} = \frac{(b_{14}\delta T_{i_{16}})T_{i_{16}}}{[b_{14}\delta T_{i_{16}} - (b_{14})^2[(G_{10})^2]]}, \quad T_{i_{18}} = \frac{(b_{18}\delta T_{i_{18}})T_{i_{18}}}{[b_{18}\delta T_{i_{18}} - (b_{18})^2[(G_{10})^2]]} \]

Obviously, these values represent an equilibrium solution of 19,20,21,22,23,24

Finally we obtain the unique solution of 89 to 94

\[ G_{21} \] given by \( \varphi((G_{23})^*) = 0 \), \( T_{21}^* \) given by \( f(T_{21}^*) = 0 \) and

\[ G_{20} = \frac{(a_{20}\delta T_{i_{20}})G_{20}^*}{[a_{20}\delta T_{i_{20}} + (a_{20})^2(T_{i_{20}})]}, \quad G_{22} = \frac{(a_{22}\delta T_{i_{22}})G_{22}^*}{[a_{22}\delta T_{i_{22}} + (a_{22})^2(T_{i_{22}})]} \]

\[ T_{20} = \frac{(b_{20}\delta T_{i_{20}})T_{i_{20}}}{[b_{20}\delta T_{i_{20}} - (b_{20})^2(T_{i_{20}})]}, \quad T_{22}^* = \frac{(b_{22}\delta T_{i_{22}})T_{i_{22}}^*}{[b_{22}\delta T_{i_{22}} - (b_{22})^2(T_{i_{22}}^*)]} \]

Obviously, these values represent an equilibrium solution of 19,20,21,22,23,24

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions \( (a_i')^{(1)} \) and \( (b_i')^{(2)} \) Belong to \( C^1(\mathbb{R}_+) \) then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** \( G_i, T_i := \)

\[ G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i \]

\[ \frac{\partial G_i^{(1)}}{\partial r_{14}}(T_{i_{14}}) = (q_{14})^{(1)}(G^*), \quad \frac{\partial G_i^{(2)}}{\partial r_{14}}(G^*) = s_{ij} \]

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

\[ \frac{dg_{13}}{dt} = -(a_{13})^{(1)}(1 + p_{13})^{(1)}G_{13} + (a_{13})^{(1)}G_{14} - (q_{13})^{(1)}G_{13}^* T_{14} \]

\[ \frac{dg_{14}}{dt} = -(a_{14})^{(1)}(1 + p_{14})^{(1)}G_{14} + (a_{14})^{(1)}G_{13} - (q_{14})^{(1)}G_{14}^* T_{14} \]

\[ \frac{dg_{15}}{dt} = -(a_{15})^{(1)}(1 + p_{15})^{(1)}G_{15} + (a_{15})^{(1)}G_{14} - (q_{15})^{(1)}G_{15}^* T_{14} \]

\[ \frac{dt_{13}}{dt} = -(b_{13}^*)^{(1)}(1 - r_{13})^{(1)}T_{13} + (b_{13})^{(1)}T_{14} + \sum_{j=1}^{15} (s_{13}(j)T_{14} G_j) \]

\[ \frac{dt_{14}}{dt} = -(b_{14}^*)^{(1)}(1 - r_{14})^{(1)}T_{14} + (b_{14})^{(1)}T_{13} + \sum_{j=1}^{15} (s_{14}(j)T_{14} G_j) \]

\[ \frac{dt_{15}}{dt} = -(b_{15}^*)^{(1)}(1 - r_{15})^{(1)}T_{15} + (b_{15})^{(1)}T_{14} + \sum_{j=1}^{15} (s_{15}(j)T_{14} G_j) \]

If the conditions of the previous theorem are satisfied and if the functions \( (a_i')^{(2)} \) and \( (b_i')^{(2)} \) Belong to \( C^2(\mathbb{R}_+) \) then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** \( G_i, T_i := \)

\[ G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i \]
taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

$$\frac{dG_{16}}{dt} = -((a'_{16})^{(2)} + (p_{16})^{(2)})G_{16} + (a_{16})^{(2)}G_{16} - (q_{16})^{(2)}G_{16}^* T_{17}$$

$$\frac{dG_{17}}{dt} = -((a'_{17})^{(2)} + (p_{17})^{(2)})G_{17} + (a_{17})^{(2)}G_{16} - (q_{17})^{(2)}G_{17}^* T_{17}$$

$$\frac{dG_{18}}{dt} = -((a'_{18})^{(2)} + (p_{18})^{(2)})G_{18} + (a_{18})^{(2)}G_{17} - (q_{18})^{(2)}G_{18}^* T_{17}$$

$$\frac{dT_{16}}{dt} = -((b'_{16})^{(2)} - (r_{16})^{(2)})T_{16} + (b_{16})^{(2)}T_{17} + \sum_{j=16}^{18}( s_{(16)(j)} T_{16} G_j )$$

$$\frac{dT_{17}}{dt} = -((b'_{17})^{(2)} - (r_{17})^{(2)})T_{17} + (b_{17})^{(2)}T_{16} + \sum_{j=16}^{18}( s_{(17)(j)} T_{17} G_j )$$

$$\frac{dT_{18}}{dt} = -((b'_{18})^{(2)} - (r_{18})^{(2)})T_{18} + (b_{18})^{(2)}T_{17} + \sum_{j=16}^{18}( s_{(18)(j)} T_{18} G_j )$$

If the conditions of the previous theorem are satisfied and if the functions \((a_i')^{(3)}\) and \((b_i')^{(3)}\) Belong to \(C^{(3)}(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** \(G_i, T_i :\)

\[G_i = G_i^* + G_i \quad , T_i = T_i^* + T_i\]

\[\frac{\partial (a''_{21})^{(3)}}{\partial T_{21}} (T_{21}^*) = (q_{21})^{(3)} , \quad \frac{\partial (b''_{21})^{(3)}}{\partial G_j} ((G_{21})^*) = s_{ij}\]

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

$$\frac{dG_{20}}{dt} = -((a''_{20})^{(3)} + (p_{20})^{(3)})G_{20} + (a_{20})^{(3)}G_{21} - (q_{20})^{(3)}G_{20}^* T_{21}$$

$$\frac{dG_{21}}{dt} = -((a''_{21})^{(3)} + (p_{21})^{(3)})G_{21} + (a_{21})^{(3)}G_{20} - (q_{21})^{(3)}G_{21}^* T_{21}$$

$$\frac{dG_{22}}{dt} = -((a''_{22})^{(3)} + (p_{22})^{(3)})G_{22} + (a_{22})^{(3)}G_{21} - (q_{22})^{(3)}G_{22}^* T_{21}$$

$$\frac{dT_{20}}{dt} = -((b''_{20})^{(3)} - (r_{20})^{(3)})T_{20} + (b_{20})^{(3)}T_{21} + \sum_{j=20}^{22}( s_{(20)(j)} T_{20} G_j )$$

$$\frac{dT_{21}}{dt} = -((b''_{21})^{(3)} - (r_{21})^{(3)})T_{21} + (b_{21})^{(3)}T_{20} + \sum_{j=20}^{22}( s_{(21)(j)} T_{21} G_j )$$

$$\frac{dT_{22}}{dt} = -((b''_{22})^{(3)} - (r_{22})^{(3)})T_{22} + (b_{22})^{(3)}T_{21} + \sum_{j=20}^{22}( s_{(22)(j)} T_{22} G_j )$$

The characteristic equation of this system is

\[
((\lambda)^{(1)} + (b_{13})^{(1)} - (r_{15})^{(1)})(((\lambda)^{(1)} + (a_{15})^{(1)}) + (p_{15})^{(1)})
\]

\[\left[ (\lambda)^{(1)} + (a_{13})^{(1)} + (p_{13})^{(1)}(q_{14})^{(1)}G_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G_{13} \right] \]

198
\[
\begin{align*}
&\left(\left(\lambda^{(1)} + (b^{(1)}_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T_{14}^{(1)} + (b_{14})^{(1)}s_{(13),(14)}T_{14}^{*}\right) \\
&+ \left(\left(\lambda^{(1)} + (a^{(1)}_{14})^{(1)} + (p_{14})^{(1)}\right)q_{13}^{(1)}G_{13}^{*} + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^{*}\right) \\
&\left(\left(\lambda^{(1)} + (b^{(1)}_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T_{14}^{(1)} + (b_{14})^{(1)}s_{(13),(13)}T_{13}^{(1)}\right) \\
&\left(\left(\lambda^{(1)}\right)^{2} + \left( (a_{13}^{(1)})^{(1)} + (a_{14}^{(1)})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)\lambda^{(1)}\right) \\
&\left(\left(\lambda^{(1)}\right)^{2} + \left( (b_{13}^{(1)})^{(1)} + (b_{14}^{(1)})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)\lambda^{(1)}\right) \\
&+ \left(\left(\lambda^{(1)}\right)^{2} + \left( (a_{13}^{(1)})^{(1)} + (a_{14}^{(1)})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)\lambda^{(1)}\right) q_{13}^{(1)}G_{15}^{*} \\
&\left(\left(\lambda^{(1)}\right)^{2} + \left( (a_{13}^{(1)})^{(1)} + (a_{14}^{(1)})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)\lambda^{(1)}\right) \left( (a_{15})^{(1)}(q_{14})^{(1)}G_{14}^{*} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^{*}\right) \\
&\left(\left(\lambda^{(1)} + (b^{(1)}_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T_{14}^{(1)} + (b_{14})^{(1)}s_{(13),(15)}T_{13}^{*}\right) = 0 \\
&+ \left(\left(\lambda^{(2)} + (b^{(2)}_{16})^{(2)} - (r_{18})^{(2)}\right)\left(\left(\lambda^{(2)} + (a^{(2)}_{18})^{(2)} + (p_{18})^{(2)}\right)\right)\right. \\
&\left(\left(\lambda^{(2)} + (a^{(2)}_{16})^{(2)} + (p_{16})^{(2)}\right)q_{17}^{(2)}G_{17}^{*} + (a_{17})^{(2)}(q_{18})^{(2)}G_{18}\right) \\
&\left(\left(\lambda^{(2)} + (b^{(2)}_{16})^{(2)} - (r_{18})^{(2)}\right)s_{(17),(17)}T_{17}^{(2)} + (b_{17})^{(2)}s_{(16),(17)}T_{17}^{*}\right) \\
&\left(\left(\lambda^{(2)} + (a^{(2)}_{16})^{(2)} + (p_{16})^{(2)}\right)s_{(17),(16)}T_{17}^{(2)} + (b_{17})^{(2)}s_{(16),(16)}T_{16}^{*}\right) \\
&\left(\left(\lambda^{(2)}\right)^{2} + \left( (a_{16}^{(2)})^{(2)} + (a_{17}^{(2)})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)\lambda^{(2)}\right) \\
&\left(\left(\lambda^{(2)}\right)^{2} + \left( (b_{16}^{(2)})^{(2)} + (b_{17}^{(2)})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)\lambda^{(2)}\right) \\
&+ \left(\left(\lambda^{(2)}\right)^{2} + \left( (a_{16}^{(2)})^{(2)} + (a_{17}^{(2)})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)\lambda^{(2)}\right)q_{18}^{(2)}G_{18}^{*} \\
&+ \left(\left(\lambda^{(2)}\right)^{2} + \left( (a_{16}^{(2)})^{(2)} + (a_{17}^{(2)})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)\lambda^{(2)}\right) \left( (a_{18})^{(2)}(q_{17})^{(2)}G_{17}^{*} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{18})^{(2)}G_{18}^{*}\right) \\
&\left(\left(\lambda^{(2)} + (b^{(2)}_{18})^{(2)} - (r_{18})^{(2)}\right)s_{(17),(18)}T_{17}^{(2)} + (b_{17})^{(2)}s_{(16),(18)}T_{18}^{*}\right) = 0 \\
&+ \left(\left(\lambda^{(3)} + (b^{(3)}_{22})^{(3)} - (r_{22})^{(3)}\right)\left(\left(\lambda^{(3)} + (a^{(3)}_{22})^{(3)} + (p_{22})^{(3)}\right)\right.\right. \\
&\left(\left(\lambda^{(3)} + (a^{(3)}_{20})^{(3)} + (p_{20})^{(3)}\right)q_{21}^{(3)}G_{21}^{*} + (a_{21})^{(3)}(q_{20})^{(3)}G_{20}^{*}\right) \\
&\left(\left(\lambda^{(3)} + (b^{(3)}_{20})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(21)}T_{21}^{(3)} + (b_{21})^{(3)}s_{(20),(21)}T_{21}^{*}\right) \\
&+ \left(\left(\lambda^{(3)} + (a^{(3)}_{21})^{(3)} + (p_{21})^{(3)}\right)(q_{20})^{(3)}G_{20}^{*} + (a_{20})^{(3)}(q_{21})^{(3)}G_{21}^{*}\right)
\end{align*}
\]
\[
\begin{align*}
(s(21,20)T_{21}^{*} + (b_{21})^{(3)}s(20,20)T_{20}) + ((\lambda)^{(3)})^{2} + (a_{20}^{(3)} + (a_{21}^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}(\lambda)^{(3)})) \\
((\lambda)^{(3)})^{2} + (b_{20}^{(3)} + (b_{21}^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)}(\lambda)^{(3)})) \\
+ ((\lambda)^{(3)})^{2} + (a_{20}^{(3)} + (a_{21}^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}(\lambda)^{(3)})) (q_{22})^{(3)}G_{22} \\
+ (a_{22}^{(3)} + (p_{20})^{(3)}) ((a_{22})^{(3)}(q_{21})^{(3)}G_{21}^{*} + (a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G_{20}^{*}) \\
((\lambda)^{(3)} + (b_{20}^{(3)} - (r_{20})^{(3)}s(21,22)T_{21}^{*} + (b_{21})^{(3)}s(20,22)T_{20}^{*}) = 0
\end{align*}
\]

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

**SECTION 4:**

**Time Dependent Schrodinger's Equation: Variant Virtuosities And Concomitant Complementarities And Singularities**

**Abstract:** Each of these three rows is a wave function which satisfies the time-dependent Schrödinger equation for a harmonic. Left: The real part (blue) and imaginary part (red) of the wave function. Right: The Schrödinger equation for a harmonic. Left: The real part (blue) and imaginary part (red) of the wave function which satisfies the time-dependent Schrödinger equation. The bottom row an example of a state with a wave function that satisfies the time-dependent Schrödinger equation.

The evolution of the quantum field is:

\[
\phi \sim \int [d^{3}p] e^{-i(p2/(2m)\cdot t - p \cdot x})
\]

The Hamiltonian is: \( H \sim \partial_{t} \phi \sim (1/2m) \partial_{x} \phi \sim \int [d^{3}p] e^{-i(p2/(2m)\cdot t - p \cdot x}) \)

The state verifies the Schrödinger equation: \( H |\psi(t)\rangle \sim \partial_{t} \psi = -i \partial_{x} \psi \sim -V2 \psi / (2m) \)

This state verifies the Schrödinger equation:

When the Hamiltonian operator acts on the wave function \( \Psi \), the result might be proportional to the same wave function \( \Psi \). If it is, then \( \Psi \) is a stationary state, and the proportionality constant, \( E \), is the

\( (\lambda)^{(3)} + (b_{20}^{(3)} - (r_{20})^{(3)}s(21,22)T_{21}^{*} + (b_{21})^{(3)}s(20,22)T_{20}^{*}) = 0 \)
energy of the state $\Psi$. The time-independent Schrödinger equation is discussed further below. In linear algebra terminology, this equation is an eigenvalue equation. As before, the most famous manifestation is the non-relativistic Schrödinger equation for a single particle moving in an electric field (but not a magnetic field):

The Schrödinger equation predicts that if certain properties of a system are measured, the result may be quantized, meaning that only specific discrete values can occur. One example is energy quantization: the energy of an electron in an atom is always one of the quantized energy levels, a fact discovered via atomic spectroscopy. (Energy quantization is discussed below.) Another example is quantization of angular momentum. This was an assumption in the earlier Bohr model of the atom, but it is a prediction of the Schrödinger equation. Not every measurement gives a quantized result in quantum mechanics. For example, position, momentum, time, and (in some situations) energy can have any value across a continuous range. The overall form of the equation is not unusual or unexpected. The terms of the nonrelativistic Schrödinger equation can be interpreted as: 

$$\text{Total energy} = \text{kinetic energy} + \text{potential energy},$$

In this respect, it is just the same as in classical physics. For example, a frictionless roller coaster has constant total energy; therefore it travels slower (low kinetic energy) when it is high off the ground (high gravitational potential energy) and vice versa.

**INTRODUCTION:**

An evanescent wave is a near-field standing wave with an intensity that exhibits exponential decay with distance from the boundary at which the wave was formed. Evanescent waves are a general property of wave-equations, and can in principle occur in any context to which a wave-equation applies. They are formed at the boundary between two media with different wave motion properties, and are most intense within one third of a wavelength from the surface of formation. In particular, evanescent waves can occur in the contexts of optics and other forms of electromagnetic radiation, acoustics, quantum mechanics, and “waves on strings”.

**Evanescent wave applications**

In optics and acoustics, evanescent waves are formed when waves traveling in a medium undergo total internal reflection at its boundary because they strike it at an angle greater than the so-called critical angle. The physical explanation for the existence of the evanescent wave is that the electric and magnetic fields (or pressure gradients, in the case of acoustical waves) cannot be discontinuous at a boundary, as would be the case if there was no evanescent wave field. In quantum mechanics, the physical explanation is exactly analogous—the Schrödinger wavefunction representing particle motion normal to the boundary cannot be discontinuous at the boundary.

Electromagnetic evanescent waves have been used to exert optical radiation pressure on small particles to trap them for experimentation, or to cool them to very low temperatures, and to illuminate very small objects such as biological cells for microscopy (as in the total internal reflection fluorescence microscope). The evanescent wave from an optical fiber can be used in a gas sensor, and evanescent waves figure in the infrared spectroscopy technique known as attenuated total reflectance.

In electrical engineering, evanescent waves are found in the near-field region within one third of a wavelength of any radio antenna. During normal operation, an antenna emits electromagnetic fields into the surrounding near-field region, and a portion of the field energy is reabsorbed, while the remainder is radiated as EM waves.

In quantum mechanics, the evanescent-wave solutions of the Schrödinger equation give rise to the phenomenon of wave-mechanical tunneling. In microscopy, systems that capture the information contained in evanescent waves can be used to create super-resolution images. Matter radiates both propagating and evanescent electromagnetic waves. Conventional optical systems capture only the information in the propagating waves and hence are subject to the diffraction limit. Systems that capture the information contained in evanescent waves, such as the super.
lens and near field scanning optical microscopy, can overcome the diffraction limit; however these systems are then limited by the system’s ability to accurately capture the evanescent waves. The limitation on their resolution is given by

\[ k \propto \frac{1}{d} \ln \frac{d}{\delta}, \]

Where \( k \) is the maximum wave vector that can be resolved, \( d \) is the distance between the object and the sensor, and \( \delta \) is a measure of the quality of the sensor. More generally, practical applications of evanescent waves can be classified in the following way:

Those in which the energy associated with the wave is used to excite some other phenomenon within the region of space where the original traveling wave becomes evanescent (for example, as in the total internal reflection fluorescence microscope) Those in which the evanescent wave couples two media in which traveling waves are allowed, and hence permits the transfer of energy or a particle between the media (depending on the wave equation in use), even though no traveling-wave solutions are allowed in the region of space between the two media. An example of this is so-called wave and is known generally as evanescent wave coupling. For example, consider total internal reflection in two dimensions, with the interface between the media lying on the x axis, the normal along y, and the polarization along z. One might naively expect that for angles leading to total internal reflection, the solution would consist of an incident wave and a reflected wave, with no transmitted wave at all, but there is no such solution that obeys Maxwell’s equations. Maxwell’s equations in a dielectric medium impose a boundary condition of continuity for the components of the fields \( E_{x}, H_{y}, D_{y}, \) and \( B_{y} \). For the polarization considered in this example, the conditions on \( E_{y} \) and \( B_{y} \) are satisfied if the reflected wave has the same amplitude as the incident one, because these components of the incident and reflected waves are equal but out of phase. Their \( H_{x} \) components, however, superimpose constructively, so there can be no solution without a non-vanishing transmitted wave. The transmitted wave cannot, however, be a sinusoidal wave, since it would then transport energy away from the boundary, but since the incident and reflected waves have equal energy, this would violate conservation of energy. We therefore conclude that the transmitted wave must be a non-vanishing solution to Maxwell’s equations that is not a traveling wave, and the only such solutions in a dielectric are those that decay exponentially: evanescent waves.

Mathematically, evanescent waves can be characterized by a wave vector where one or more of the vector’s components have an imaginary value. Because the vector has imaginary components, it may have a magnitude that is less than its real components. If the angle of incidence exceeds the critical angle, then the wave vector of the transmitted wave has the form

\[ k = k_{y} \hat{y} + k_{z} \hat{z} = i\alpha \hat{y} + \beta \hat{z}, \]

Which represents an evanescent wave because the \( y \) component is imaginary? (Here \( \alpha \) and \( \beta \) are real and \( i \) represents the imaginary unit.)

For example, if the polarization is perpendicular to the plane of incidence, then the electric field of any of the waves (incident, reflected, or transmitted) can be expressed as

\[ E(r, t) = \text{Re} \left\{ E(r) e^{i\omega t} \right\} \hat{z} \]

Where \( \hat{z} \) is the unit vector in the z direction.

Substituting the evanescent form of the wave vector \( k \) (as given above), we find for the transmitted wave:

\[ E(r) = E_{0} e^{-i(\alpha y + \beta z)} = E_{0} e^{\alpha y - i\beta z} \]

Where \( \alpha \) is the attenuation constant and \( \beta \) is the propagation constant.

**Evanescent-wave coupling**

In optics, evanescent-wave coupling is a process by which electromagnetic waves are transmitted
from one medium to another by means of the evanescent, exponentially decaying electromagnetic field.

plot of 1/e-penetration depth of the evanescent wave against angle of incidence in units of wavelength for different refraction indices

Coupling is usually accomplished by placing two or more electromagnetic elements such as optical waveguides close together so that the evanescent field generated by one element does not decay much before it reaches the other element. With waveguides, if the receiving waveguide can support modes of the appropriate frequency, the evanescent field gives rise to propagating-wave modes, thereby connecting (or coupling) the wave from one waveguide to the next.

Evanescent-wave coupling is fundamentally identical to near field interaction in electromagnetic field theory. Depending on the impedance of the radiating source element, the evanescent wave is either predominantly electric (capacitive) or magnetic (inductive), unlike in the far field where these components of the wave eventually reach the ratio of the impedance of free space and the wave propagates radioactively. The evanescent wave coupling takes place in the non-radiative field near each medium and as such is always associated with matter; i.e., with the induced currents and charges within a partially reflecting surface. This coupling is directly analogous to the coupling between the primary and secondary coils of a transformer, or between the two plates of a capacitor. Mathematically, the process is the same as that of quantum tunneling, except with electromagnetic waves instead of quantum-mechanical wavefunctions.

Evanescent wave coupling is commonly used in photonic and nanophotonics devices as waveguide sensors. Evanescent wave coupling is used to excite, for example, dielectric microsphere resonators. A typical application is resonant energy transfer, useful, for instance, for charging electronic gadgets without wires. A particular implementation of this is WiTricity; the same idea is also used in some Tesla coils. Evanescent coupling, as near field interaction, is one of the concerns in electromagnetic compatibility. Evanescent wave coupling plays a major role in the theoretical explanation of extraordinary optical transmission.

The Time-Dependent Schrödinger Equation

We are now ready to consider the time-dependent Schrödinger equation. Although we were able to derive the single-particle time-independent Schrödinger equation starting from the classical wave equation and the de Broglie relation, the time-dependent Schrödinger equation cannot be derived using elementary methods and is generally given as a postulate of quantum mechanics. It is possible to show that the time-dependent equation is at least reasonable if not derivable, but the arguments are rather involved (cf. Merzbacher [2], Section 3.2; Levine [3], Section 1.4).

The single-particle three-dimensional time-dependent Schrödinger equation is

\[ i\hbar \frac{\partial \psi(r, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(r, t) + V(r) \psi(r, t) \]  \hspace{1cm} 21

Where \( V \) is assumed to be a real function and represents the potential energy of the system (a complex function \( \bar{V} \) will act as a source or sink for probability, as shown in Merzbacher [2], problem 4.1). *Wave Mechanics* is the branch of quantum mechanics with equation (21) as its
dynamical law. Note that equation (21) does not yet account for spin or relativistic effects.

Of course the time-dependent equation can be used to derive the time-independent equation. If we write the wavefunction as a product of spatial and temporal terms, \( \psi(r, t) = \psi(r)f(t) \), then equation (21) becomes

\[
\psi(r) i\hbar \frac{df(t)}{dt} = f(t) \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(r) \right] \psi(r)
\]

or

\[
\frac{i\hbar}{f(t)} \frac{df}{dt} = \frac{1}{\psi(r)} \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(r) \right] \psi(r)
\]

Since the left-hand side is a function of \( t \) only and the right hand side is a function of \( r \) only, the two sides must equal a constant. If we tentatively designate this constant \( E \) (since the right-hand side clearly must have the dimensions of energy), then we extract two ordinary differential equations, namely

\[
\frac{1}{f(t)} \frac{df(t)}{dt} = -\frac{iE}{\hbar}
\]

and

\[
-\frac{\hbar^2}{2m} \nabla^2 \psi(r) + V(r)\psi(r) = E\psi(r)
\]

The latter equation is once again the time-independent Schrödinger equation. The former equation is easily solved to yield

\[
f(t) = e^{-iEt/\hbar}
\]

The Hamiltonian in equation (25) is a Hermitian operator, and the eigenvalues of a Hermitian operator must be real, so \( E \) is real. This means that the solutions \( f(t) \) are purely oscillatory, since \( f(t) \) never changes in magnitude (recall Euler's formula \( e^{\pm i\theta} = \cos \theta \pm i \sin \theta \)). Thus if

\[
\psi(r, t) = \psi(r)e^{-iEt/\hbar}
\]

then the total wave function \( \psi(r, t) \) differs from \( \psi(r) \) only by a phase factor of constant magnitude. There are some interesting consequences of this. First of all, the quantity \( |\psi(r, t)|^2 \) is time independent, as we can easily show:

\[
|\psi[r, t]|^2 = \psi^*(r, t)\psi(r, t) = e^{iEt/\hbar}\psi^*(r)e^{-iEt/\hbar}\psi(r) = \psi^*(r)\psi(r)
\]

Secondly, the expectation value for any time-independent operator is also time-independent, if \( \psi(r, t) \) satisfies equation (27). By the same
reasoning applied above,

\[ <A> = \int \psi^*(r,t) \hat{A} \psi(r,t) = \int \psi^*(r) \hat{A} \psi(r) \]

For these reasons, wave functions of the form (27) are called stationary states. The state \( \psi(x,t) \) is "stationary," but the particle it describes is not!

Of course equation represents a particular solution to equation (21). The general solution to equation (21) will be a linear combination of these particular solutions, i.e.

\[ \psi(x,t) = \sum c_k e^{-iE_k t/\hbar} \psi_k(x) \]

**FORMULATION OF THE PROBLEM:**

\[ E\Psi = \hat{H}\Psi \]

**NOTATION:**

\( G_{48} \) : Category one of the term on the LHS of the Time dependent Schrödinger's Wave Equation, (Such a classification is based on the characteristics of the systems under investigation to which the Wave Equation is applied).

\( G_{49} \) : Category two of the term on the LHS of the Time dependent Schrödinger's Wave Equation, (Such a classification is based on the characteristics of the systems under investigation to which the Wave Equation is applied).

\( G_{50} \) : Category two of the term on the LHS of the Time dependent Schrödinger’s Wave Equation, (Such a classification is based on the characteristics of the systems under investigation to which the Wave Equation is applied).

\( (a_{48})^{(10)}, (a_{49})^{(10)}, (a_{50})^{(10)} \) : Accentuation coefficients

\( (a'_{48})^{(10)}, (a'_{49})^{(10)}, (a'_{50})^{(10)} \) : Dissipation coefficients

**FORMULATION OF THE SYSTEM:**

In the light of the assumptions stated in the foregoing, we infer the following:

(a) The growth speed in category 1 is the sum of a accentuation term \((a_{48})^{(10)}G_{49}\) and a dissipation term \(-(a'_{48})^{(10)}G_{48}\), the amount of dissipation

(b) The growth speed in category 2 is the sum of two parts \((a_{49})^{(10)}G_{48}\) and \(-(a'_{49})^{(10)}G_{49}\) the inflow from the category 1.

(c) The growth speed in category 3 is equivalent to \((a_{50})^{(10)}G_{49}\) and \(-(a'_{50})^{(10)}G_{50}\) dissipation ascribed only to depletion phenomenon.

**GOVERNING EQUATIONS:**

The differential equations governing the above system can be written in the following form

\[ \frac{dG_{48}}{dt} = (a_{48})^{(10)}G_{49} - (a'_{48})^{(10)}G_{48} \]

205
We can rewrite equation in the following form

\[
\frac{dG_{48}}{(a_{48})^{(10)}G_{48}-(a'_{48})^{(10)}G_{48}} = dt
\]

\[
\frac{dG_{49}}{(a_{49})^{(10)}G_{49}-(a'_{49})^{(10)}G_{49}} = dt
\]

Or we write a single equation as

\[
\frac{dG_{48}}{(a_{48})^{(10)}G_{48}-(a'_{48})^{(10)}G_{48}} = \frac{dG_{49}}{(a_{49})^{(10)}G_{49}-(a'_{49})^{(10)}G_{49}} = \frac{dG_{50}}{(a_{50})^{(10)}G_{50}-(a'_{50})^{(10)}G_{50}} = dt
\]

The equality of the ratios in equation remains unchanged in the event of multiplication of numerator and denominator by a constant factor.

For constant multiples \(\alpha, \beta, \gamma\) all positive we can write equation as

\[
\frac{\alpha dG_{48}}{\alpha(a_{48})^{(10)}G_{48}-(a'_{48})^{(10)}G_{48}} = \frac{\beta dG_{49}}{\beta(a_{49})^{(10)}G_{49}-(a'_{49})^{(10)}G_{49}} = \frac{\gamma dG_{50}}{\gamma(a_{50})^{(10)}G_{50}-(a'_{50})^{(10)}G_{50}} = dt
\]

The general solution of the system can be written in the form

\[\alpha t G_1 + \beta t G_2 + \gamma t G_3 = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + C_3 e^{\lambda_3 t}\] where \(i = 48, 49, 50\) and \(C_{48}, C_{49}, C_{50}\) are arbitrary constant coefficients.

**STABILITY ANALYSIS:**

Supposing \(G_i(0) = G_i^0(0) > 0\), and denoting by \(\lambda_i\) the characteristic roots of the system, it easily results that

1. If \((a_{48})^{(10)}(a'_{49})^{(10)} - (a_{48})^{(10)}(a_{49})^{(10)} > 0\) all the components of the solution, i.e all the three parts of the tend to zero, and the solution is stable with respect to the initial data.

2. If \((a_{49})^{(10)}(a'_{49})^{(10)} - (a_{48})^{(10)}(a_{49})^{(10)} < 0\) and

\[\lambda_{49} + (a_{48})^{(10)}G_{48}^0 - (a_{48})^{(10)}G_{49}^0 \neq 0, (\lambda_{49} < 0),\] the first two components of the solution tend to infinity as \(t \to \infty\), and \(G_{50} \to 0\), i.e. The category 1 and category 2 parts grows to infinity, whereas the third part category 3 tends to zero.

3. If \((a'_{49})^{(10)}(a_{49})^{(10)} - (a_{48})^{(10)}(a_{49})^{(10)} < 0\) and

\[(\lambda_{49} + (a_{48})^{(10)}G_{48}^0 - (a_{48})^{(10)}G_{49}^0 = 0\] Then all the three parts tend to zero, but the solution is
not stable i.e. at a small variation of the initial values of $G_i$, the corresponding solution tends to infinity.

From the above stability analysis we infer the following:

1. The adjustment process is stable in the sense that the system converges to equilibrium.
2. The approach to equilibrium is a steady one, and there exists progressively diminishing oscillations around the equilibrium point.
3. Conditions 1 and 2 are independent of the size and direction of initial disturbance.
4. The actual shape of the time path is determined by efficiency parameter, the strength of the response of the portfolio in question, and the initial disturbance.
5. Result 3 warns us that we need to make an exhaustive study of the behavior of any case in which generalization derived from the model do not hold.
6. Growth studies as the one in the extant context are related to the systemic growth paths with full employment of resources that are available in question.
7. Some authors Nober F J, Agee, Winfree were interested in such questions, whether growing system could produce full employment of all factors, whether or not there was a full employment natural rate growth path and perpetual oscillations around it. It is to be noted some systems pose extremely difficult stability problems. As an instance, one can quote example of pockets of open cells and drizzle in complex networks in marine stratocumulus. Other examples are clustering and synchronization of lightning flashes adjunct to thunderstorms, coupled studies of microphysics and aqueous chemistry.

**RHS OF TIME DEPENDENT SCHRODINGERS'S EQUATION:** $E\Psi = \hat{H}\Psi$

a) The speed of growth is linear function of the category 2 at the time of reckoning. As before the accentuation coefficient that characterizes the speed of growth in category 1 is the proportionality factor between category 1 and category 2.

b) The dissipation coefficient in the growth model is attributable to aging and depletion phenomenon in systems under investigation.

c) Inflow into category 2 is only from category 1. The balance of terrestrial organism sector in category 3 is because of transfer from category 2.

**NOTATION:**

- $T_{48}$: Category one of the term on RHS.
- $T_{49}$: Category two of the term on RHS.
- $T_{50}$: Category three of the term on RHS.

- $(b_{48})^{(1)}$, $(b_{49})^{(10)}$, $(b_{50})^{(10)}$: Accentuation coefficients.
- $(b'_{48})^{(1)}$, $(b'_{49})^{(10)}$, $(b'_{50})^{(10)}$: Dissipation coefficients.

**FORMULATION OF THE SYSTEM:**

Under the above assumptions, we derive the following:
a) The growth speed in category 1 is the sum of two parts:
   1. A term \((b_{48})^{(10)}T_{49}\)
   2. A term \(-(b'_{48})^{(10)}T_{48}\)
   3. The growth speed in category 2 is the sum of two parts:
      1. A term \((b_{49})^{(10)}T_{48}\) constitutive of the amount of inflow from the category 1
      2. A term \(-(b'_{49})^{(10)}T_{49}\) the dissipation factor.

b) The growth speed under category 3 is attributable to inflow from category 2.

\[ E\Psi = \hat{H}\Psi \]

THE TIME DEPENDENT SCHRODINGER’S WAVE EQUATION:

GOVERNING EQUATIONS:

Following are the differential equations that govern the growth in the terrestrial organisms portfolio

\[ \frac{dT_{48}}{dt} = (b_{48})^{(10)}T_{49} - (b'_{48})^{(10)}T_{48} \]  

\[ \frac{dT_{49}}{dt} = (b_{49})^{(10)}T_{48} - (b'_{49})^{(10)}T_{49} \]  

\[ \frac{dT_{50}}{dt} = (b_{50})^{(10)}T_{49} - (b'_{50})^{(10)}T_{50} \]  

\((b_{i})^{(10)} > 0 \), \( i = 48, 49, 50 \)  

\((b'_{i})^{(10)} > 0 \), \( i = 48, 49, 50 \)  

\((b_{49})^{(10)} < (b'_{49})^{(10)} \)  

\((b_{50})^{(10)} < (b'_{50})^{(10)} \)  

Following the same procedure outlined in the previous section, the general solution of the governing equations is

\[ aT_{i} + bT_{i} + \gamma_{i}T_{i} = C_{i}e^{\lambda_{i}t}, \( i = 48, 49, 50 \) \]

where \( C_{48}, C_{49}, C_{50} \) are arbitrary constant coefficients and \( a_{48}, a_{49}, a_{50}, \gamma_{48}, \gamma_{49}, \gamma_{50} \) corresponding multipliers to the characteristic roots of the terrestrial organism system

THE TIME DEPENDENT SCHRODINGER’S WAVE EQUATION: DUAL SYSTEM ANALYSIS

\[ E\Psi = \hat{H}\Psi \]

We will denote

1) By \( T_{i}(t), i = 48, 49, 50 \), the three parts of the RHS OF TDSE(Time Dependent Schrödinger’s equation)  
2) By \( (a_{i})^{(10)}(T_{49}, t) \) \( (T_{49} \geq 0, t \geq 0) \),  
3) By \( (b'_{i})^{(10)}(G_{48}, G_{49}, G_{50}, t) = -(b''_{i})^{(10)}((G_{51}), t) \),

THE TIME DEPENDENT SCHRODINGER’S WAVE EQUATION: DUAL SYSTEM ANALYSIS
GOVERNING EQUATIONS:

The differential system of this model is now

\[
\frac{dG_i}{dt} = (a_{i48})^{(10)} G_{i49} - [(a'_{i48})^{(10)} + (a''_{i48})^{(10)}(T_{49}, t)] G_{i48} \\
\frac{dG_{i49}}{dt} = (a_{i49})^{(10)} G_{i48} - [(a'_{i49})^{(10)} + (a''_{i49})^{(10)}(T_{49}, t)] G_{i49} \\
\frac{dG_{i50}}{dt} = (a_{i50})^{(10)} G_{i49} - [(a'_{i50})^{(10)} + (a''_{i50})^{(10)}(T_{49}, t)] G_{i50} \\
\frac{dT_{48}}{dt} = (b_{i48})^{(10)} T_{i49} - [(b'_{i48})^{(10)} - (b''_{i48})^{(10)}((G_{i51}), t)] T_{i48} \\
\frac{dT_{i49}}{dt} = (b_{i49})^{(10)} T_{i48} - [(b'_{i49})^{(10)} - (b''_{i49})^{(10)}((G_{i51}), t)] T_{i49} \\
\frac{dT_{i50}}{dt} = (b_{i50})^{(10)} T_{i49} - [(b'_{i50})^{(10)} - (b''_{i50})^{(10)}((G_{i51}), t)] T_{i50} \\
+ (a''_{i48})^{(10)}(T_{49}, t) = \text{First augmentation factor} \\
- (b''_{i48})^{(10)}(G_{i51}, t) = \text{First dettritions factor}
\]

Where we suppose

\[(a_i)^{(10)}, (a'_i)^{(10)}, (a''_i)^{(10)}, (b_i)^{(10)}, (b'_i)^{(10)}, (b''_i)^{(10)} > 0, \quad i, j = 48, 49, 50\]

The functions \((a''_i)^{(10)}, (b''_i)^{(10)}\) are positive continuous increasing and bounded.

**Definition of \((p_i)^{(10)}, (r_i)^{(10)}:\)**

\[
(a''_i)^{(10)}(T_{49}, t) \leq (p_i)^{(10)} \leq (\hat{A}_{48})^{(10)} \\
(b''_i)^{(10)}(G_{51}, t) \leq (r_i)^{(10)} \leq (b'_i)^{(10)} \leq (\hat{B}_{48})^{(10)}
\]

**\((II)\)** \[
\lim_{T_{49} \to \infty} (a''_i)^{(10)}(T_{49}, t) = (p_i)^{(10)} \\
\lim_{G_{51} \to \infty} (b''_i)^{(10)}(G_{51}, t) = (r_i)^{(10)}
\]

**Definition of \((\hat{A}_{48})^{(10)}, (\hat{B}_{48})^{(10)}:\)**

Where \((\hat{A}_{48})^{(10)}, (\hat{B}_{48})^{(10)}, (p_i)^{(10)}, (r_i)^{(10)}\) are positive constants and \(t = 48, 49, 50\)

They satisfy Lipschitz condition:

\[
| (a''_i)^{(10)}(T_{49}, t) - (a''_i)^{(10)}(T_{49}, t) | \leq (\hat{k}_{48})^{(10)}|T_{49} - T'_{49}|e^{-(\hat{A}_{48})^{(10)} t} \\
| (b''_i)^{(10)}((G_{51}), t) - (b''_i)^{(10)}((G_{51}), t) | < (\hat{k}_{48})^{(10)}|((G_{51}) - (G_{51}'))|e^{-(\hat{B}_{48})^{(10)} t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a''_i)^{(10)}(T_{49}, t)\) and \((b''_i)^{(10)}(T_{49}, t)\). \((T_{49}, t)\) and \((T_{49}, t)\) are points belonging to the interval \([\hat{k}_{48}]^{(10)}, \hat{M}_{48})^{(10)}\). It is to be noted that \((a''_i)^{(10)}(T_{49}, t)\) is uniformly continuous. In the eventuality of the fact, that if \(\hat{M}_{48})^{(10)} = 10\) then the function \((a''_i)^{(10)}(T_{49}, t)\), the first
augmentation coefficient, would be absolutely continuous.

**Definition of** \((\tilde{M}_{48})^{(10)}, (\tilde{k}_{48})^{(10)}\): \(^{1049}\)

\((\tilde{M}_{48})^{(10)}, (\tilde{k}_{48})^{(10)}\), are positive constants

\[
\frac{(a_i)^{(10)}}{(M_{48})^{(10)}} \cdot \frac{(b_j)^{(10)}}{(M_{48})^{(10)}} < 1
\]

**Definition of** \((\tilde{P}_{48})^{(10)}, (\tilde{Q}_{48})^{(10)}\):

\(^{1050}\)

(KK) There exists two constants \((\tilde{P}_{48})^{(10)}\) and \((\tilde{Q}_{48})^{(10)}\) which together with \((\tilde{M}_{48})^{(10)}, (\tilde{k}_{48})^{(10)}, (\tilde{A}_{48})^{(10)}\) and \((\tilde{B}_{48})^{(10)}\) and the constants \((a_i)^{(10)}, (a'_i)^{(10)}, (b_i)^{(10)}, (b'_i)^{(10)}, (p_i)^{(10)}, (r_i)^{(10)}, i = 48, 49, 50,\) satisfy the inequalities

\[
\frac{1}{(\tilde{M}_{48})^{(10)}} \left[ (a_i)^{(10)} + (a'_i)^{(10)} + (\tilde{A}_{48})^{(10)} + (\tilde{P}_{48})^{(10)} (\tilde{k}_{48})^{(10)} \right] < 1
\]

\[
\frac{1}{(\tilde{M}_{48})^{(10)}} \left[ (b_i)^{(10)} + (b'_i)^{(10)} + (\tilde{B}_{48})^{(10)} + (\tilde{Q}_{48})^{(10)} (\tilde{k}_{48})^{(10)} \right] < 1
\]

**Theorem 1:** if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions

**Definition of** \(G_i(0), T'_i(0)\):

\(^{1051}\)

\[
G_i(t) \leq (\tilde{P}_{48})^{(10)} e^{(\tilde{A}_{48})^{(10)} t}, \quad G_i(0) = G^0_i > 0
\]

\[
T_i(t) \leq (\tilde{Q}_{48})^{(10)} e^{(\tilde{A}_{48})^{(10)} t}, \quad T_i(0) = T^0_i > 0
\]

**Proof:**

Consider operator \(A^{(10)}\) defined on the space of sextuples of continuous functions \(G_i, T_i: \mathbb{R}^+ \rightarrow \mathbb{R}^+\) which satisfy

\[
G_i(0) = G^0_i, T_i(0) = T^0_i, G_i \leq (\tilde{P}_{48})^{(10)}, T_i \leq (\tilde{Q}_{48})^{(10)}, \quad 0 \leq G_i(t) - G^0_i \leq (\tilde{P}_{48})^{(10)} e^{(\tilde{A}_{48})^{(10)} t}
\]

\[
0 \leq T_i(t) - T^0_i \leq (\tilde{Q}_{48})^{(10)} e^{(\tilde{A}_{48})^{(10)} t}
\]

By

\[
\tilde{G}_{48}(t) = G^0_{48} + \int_0^t \left[ (a_{48})^{(10)} G_{48} (s_{48}) - \left( (a'_{48})^{(10)} + \alpha_{48}^{(10)} \right) (T_{48}(s_{48}), s_{48}) \right] G_{48} (s_{48}) ds_{48}
\]

\[
\tilde{G}_{49}(t) = G^0_{49} + \int_0^t \left[ (a_{49})^{(10)} G_{48} (s_{48}) - \left( (a'_{49})^{(10)} + \alpha_{49}^{(10)} \right) (T_{49}(s_{48}), s_{48}) \right] G_{49} (s_{48}) ds_{48}
\]

\[
\tilde{G}_{50}(t) = G^0_{50} + \int_0^t \left[ (a_{50})^{(10)} G_{49} (s_{48}) - \left( (a'_{50})^{(10)} + \alpha_{50}^{(10)} \right) (T_{50}(s_{48}), s_{48}) \right] G_{50} (s_{48}) ds_{48}
\]
\[\tilde{T}_{48}(t) = T_{48}^0 + \int_0^t \left[ (b_{48})^{(10)} \mathcal{T}_{48}(s(48)) - \left( (b_{48}')(10) - (b_{48}^0)'(10) \mathcal{G}(s(48),s(48)) \right) \mathcal{T}_{48}(s(48)) \right] ds(48) \]

\[\tilde{T}_{49}(t) = T_{49}^0 + \int_0^t \left[ (b_{49})^{(10)} \mathcal{T}_{49}(s(49)) - \left( (b_{49}')(10) - (b_{49}^0)'(10) \mathcal{G}(s(49),s(49)) \right) \mathcal{T}_{49}(s(49)) \right] ds(49) \]

\[\tilde{T}_{50}(t) = T_{50}^0 + \int_0^t \left[ (b_{50})^{(10)} \mathcal{T}_{50}(s(50)) - \left( (b_{50}')(10) - (b_{50}^0)'(10) \mathcal{G}(s(50),s(50)) \right) \mathcal{T}_{50}(s(50)) \right] ds(50) \]

Where \(s(n)\) is the integrand that is integrated over an interval \((0, t)\)

\((o)\) The operator \(\mathcal{A}^{(10)}\) maps the space of functions satisfying 34, 35, 36 into itself. Indeed it is obvious that

\[G_{48}(t) \leq G_{48}^0 + \int_0^t \left[ (a_{48})^{(10)} \left( \mathcal{G}_{48}^0 + \left( \mathcal{P}_{48} \right)^{(10)} e^{\left( \mathcal{R}_{48} \right)^{(10)} s(48)} \right) \right] ds(48) = \]

\[
(1 + (a_{48}^{(10)} t) G_{49}^0 + \left( \frac{(a_{48})^{(10)} (P_{48})^{(10)}}{(M_{48})^{(10)}} \right) e^{\left( \mathcal{R}_{48} \right)^{(10)} t} - 1)
\]

From which it follows that

\[\left( G_{48}(t) - G_{48}^0 \right) e^{-\left( \mathcal{R}_{48} \right)^{(10)} t} \leq \left( \frac{(a_{48})^{(10)}}{(M_{48})^{(10)}} \right) \left[ \left( \mathcal{P}_{48} \right)^{(10)} + G_{49}^0 \left( \frac{(P_{48}^0)^{(10)} + G_{48}^0}{c_{48}^0} \right) + \left( \mathcal{P}_{48} \right)^{(10)} \right]
\]

\((G_{i}^{(10)})\) is as defined in the statement of theorem

Analogous inequalities hold also for \(G_{49}, G_{50}, T_{48}, T_{49}, T_{50}\)

It is now sufficient to take \(\left( \frac{(a_{48})^{(10)}}{(M_{48})^{(10)}} \right), \left( \frac{(b_{48})^{(10)}}{(M_{48})^{(10)}} \right) < 1\) and to choose

\((\mathcal{P}_{48}^{(10)})\) and \((\mathcal{Q}_{48}^{(10)})\) large to have

\[
\left( \frac{(a_{48})^{(10)}}{(M_{48})^{(10)}} \right) \left[ \left( \mathcal{P}_{48}^{(10)} \right) + \left( \mathcal{P}_{48}^{(10)} + G_{49}^0 \right) e^{-\left( \mathcal{P}_{48}^{(10)} + G_{49}^0 / c_{48}^0 \right)} \right] \leq \left( \mathcal{P}_{48}^{(10)} \right)
\]

\[
\left( \frac{(b_{48})^{(10)}}{(M_{48})^{(10)}} \right) \left[ \left( \mathcal{Q}_{48}^{(10)} \right) + T_{48}^0 e^{-\left( \mathcal{Q}_{48}^{(10)} + T_{48}^0 / T_{48} \right)} \right] \leq \left( \mathcal{Q}_{48}^{(10)} \right)
\]

In order that the operator \(\mathcal{A}^{(10)}\) transforms the space of sextuples of functions \(G_{i}, T_{i}\) satisfying 34, 35, 36 into itself

The operator \(\mathcal{A}^{(10)}\) is a contraction with respect to the metric

\[d \left( \left( G_{51}^{(10)}, (T_{51})^{(10)} \right), \left( (G_{51})^{(2)}, (T_{51})^{(2)} \right) \right) = \]

\[
\sup_{t \in [0,c_{k}]} \left\{ G_{i}^{(10)}(t) - G_{i}^{(2)}(t) e^{-\left( \mathcal{R}_{48} \right)^{(10)} t}, \max_{t \in [0,c_{k}]} \left| T_{i}^{(10)}(t) - T_{i}^{(2)}(t) \right| e^{-\left( \mathcal{R}_{48} \right)^{(10)} t} \right\}
\]

Indeed if we denote

**Definition of** \(\mathcal{A}^{(10)}((G_{51}), (T_{51}))\) : \(\left( (G_{51}), (T_{51}) \right) = \mathcal{A}^{(10)}((G_{51}), (T_{51}))\)

It results
\[
\begin{align*}
&\left|G_{49}^{(1)} - G_{49}^{(2)}\right| \leq \int_0^t \left|a_{49}^{(1)} \right| e^{-\left(\mathcal{M}_{48}^{(10)}\right) \cdot s(48)} d\mathcal{S}(48) + \\
&\int_0^t \left(\left|a_{49}^{(1)}\right| - \left|a_{49}^{(2)}\right|\right) e^{-\left(\mathcal{M}_{48}^{(10)}\right) \cdot s(48)} d\mathcal{S}(48) + \\
&(a_{49}''^{(10)})^2 (T_{49}^{(1)} + S(48)) \left|G_{49}^{(1)} - G_{49}^{(2)}\right| e^{-\left(\mathcal{M}_{48}^{(10)}\right) \cdot s(48)} d\mathcal{S}(48) + \\
&(a_{49}''^{(10)})^2 \left((T_{49}^{(1)} + S(48)) e^{-\left(\mathcal{M}_{48}^{(10)}\right) \cdot s(48)} d\mathcal{S}(48)\right) + \\
&G_{49}^{(2)} \left((a_{49}''^{(10)})^2 (T_{49}^{(1)} + S(48)) - (a_{49}''^{(10)})^2 (T_{49}^{(2)} + S(48))\right) e^{-\left(\mathcal{M}_{48}^{(10)}\right) \cdot s(48)} d\mathcal{S}(48)
\end{align*}
\]

Where \(s(48)\) represents integrand that is integrated over the interval \([0, t]\).

From the hypotheses on 25, 26, 27, 28 and 29 it follows

\[
\left|G_{51}^{(1)} - G_{51}^{(2)}\right| e^{-\left(\mathcal{M}_{48}^{(10)}\right) \cdot t} \leq \frac{1}{\left(\mathcal{P}_{48}^{(10)}\right) \left(\mathcal{Q}_{48}^{(10)}\right) d \left(\left(G_{51}^{(1)}, (T_{51}^{(1)}), (G_{52}^{(2)})^{(2)}, (T_{51}^{(2)})^{(2)}\right)\right)}
\]

And analogous inequalities for \(G_t\) and \(T_t\). Taking into account the hypothesis \((34, 35, 36)\) the result follows

**Remark 1:** The fact that we supposed \((a_{49}''^{(10)})\) and \((b_{49}''^{(10)})\) depending also on \(t\) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\mathcal{P}_{48}^{(10)}) e^{-\left(\mathcal{M}_{48}^{(10)}\right) \cdot t}\) and \((\mathcal{Q}_{48}^{(10)}) e^{-\left(\mathcal{M}_{48}^{(10)}\right) \cdot t}\) respectively of \(\mathbb{R}_+\).

If instead of proving the existence of the solution on \(\mathbb{R}_+\), we have to prove it only on a compact then it suffices to consider that \((a_{49}''^{(10)}(t), (b_{49}''^{(10)}(t), i = 48, 49, 50)\) depend only on \(T_{49}\) and respectively on \((G_{51})(\text{not on } t)\) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \(t\) where \(G_t(t) = 0\) and \(T_t(t) = 0\).

From 19 to 24 it results

\[
G_t(t) \geq G_t^0 e^{-\int_0^t \left|a_{49}''(t)\right| (T_{49} + S(48)) d\mathcal{S}(48)} \geq 0
\]

\[
T_t(t) \geq T_t^0 e^{-\left|b_{49}''(t)\right| t} > 0 \text{ for } t > 0
\]

**Definition of** \((\mathcal{M}_{48}^{(10)}), (\mathcal{M}_{48}^{(10)}), (\mathcal{M}_{48}^{(10)}))\) :

**Remark 3:** If \(G_{48}\) is bounded, the same property have also \(G_{49}\) and \(G_{50}\). Indeed if \(G_{49} < (\mathcal{M}_{48}^{(10)})\) it follows \(\frac{dG_{49}}{dt} \leq (\mathcal{M}_{48}^{(10)}) - (a_{49}''(t))G_{49}\) and by integrating

\[
G_{49} \leq (\mathcal{M}_{48}^{(10)}) + 2a_{49}''(t)(\mathcal{M}_{48}^{(10)}) / (a_{49}''(t))
\]

In the same way, one can obtain

\[
G_{50} \leq (\mathcal{M}_{48}^{(10)}) + 2a_{50}''(t)(\mathcal{M}_{48}^{(10)}) / (a_{50}''(t))
\]
If \( G_{49} \) or \( G_{50} \) is bounded, the same property follows for \( G_{48}, G_{50} \) and \( G_{49}, G_{49} \) respectively.

**Remark 4:** If \( G_{48} \) is bounded, from below, the same property holds for \( G_{49} \) and \( G_{50} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{49} \) is bounded from below.

\[
\text{Remark 5: If } G_{49} \text{ is bounded from below and } \lim_{t \to \infty} (b''_{49})(G_{51})(t, t) = (b'_{49})(10) \text{ then } T_{49} \to \infty.
\]

**Definition of** \((m)^{(10)}\) and \(\varepsilon_{10} :\)

Indeed let \( t_{10} \) be so that for \( t > t_{10} \)

\[
(b_{49})^{(10)} - (b')_{49}^{(10)}(G_{51})(t, t) < \varepsilon_{10}, T_{49}(t) > (m)^{(10)}
\]

Then \( \frac{dT_{49}}{dt} \geq (a_{49})^{(10)}(m)^{(10)} - \varepsilon_{10} T_{49} \) which leads to

\[
T_{49} \geq \left( \frac{(a_{49})^{(10)}(m)^{(10)}}{\varepsilon_{10}} \right) (1 - e^{-\varepsilon_{10}t}) + T_{49}^0 e^{-\varepsilon_{10}t} \text{ If we take } t \text{ such that } e^{-\varepsilon_{10}t} = \frac{1}{2} \text{ it results}
\]

\[
T_{49} \geq \left( \frac{(a_{49})^{(10)}(m)^{(10)}}{2 \varepsilon_{10}} \right), t = \log \frac{2}{\varepsilon_{10}} \text{ By taking now } \varepsilon_{10} \text{ sufficiently small one sees that } T_{49} \text{ is unbounded. The same property holds for } T_{50} \text{ if } \lim_{t \to \infty} (b''_{50})(G_{51})(t, t) = (b'_{50})(10)
\]

We now state a more precise theorem about the behaviors at infinity of the solutions of equations.

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** \((\sigma_1)^{(10)}, (\sigma_2)^{(10)}, (\tau_1)^{(10)}, (\tau_2)^{(10)} :\)

\[
\sigma_1)^{(10)}, (\sigma_2)^{(10)}, (\tau_1)^{(10)}, (\tau_2)^{(10)} \text{ four constants satisfying}
\]

\[
-(\sigma_2)^{(10)} \leq -(a_{49}''^{(10)} + (a_{49}''^{(10)}(T_{49}, t) + (a_{49}''^{(10)}(T_{49}, t) \leq -(\sigma_1)^{(10)}
\]

\[
-(\tau_2)^{(10)} \leq -(b_{49}''^{(10)} + (b_{49}''^{(10)} - (b'_{49})^{(10)}((G_{51}), t) - (b''_{49})^{(10)}((G_{51}), t) \leq -(\tau_1)^{(10)}
\]

**Definition of** \((v_1)^{(10)}, (v_2)^{(10)}, (u_1)^{(10)}, (u_2)^{(10)}, v^{(10)}, u^{(10)} :\)

(i) By \( (v_1)^{(10)} > 0, (v_2)^{(10)} < 0 \) and respectively \( (u_1)^{(10)} > 0, (u_2)^{(10)} < 0 \) the roots of the equations \( a_{49}^{(10)}v^{(10)} + (\sigma_1)^{(10)}v^{(10)} - (a_{49}^{(10)}u^{(10)})^2 = 0 \) and \( b_{49}^{(10)}u^{(10)} - (b_{49}^{(10)}u^{(10)} = 0 \)

**Definition of** \((\tilde{v}_1)^{(10)}, (\tilde{v}_2)^{(10)}, (\tilde{u}_1)^{(10)}, (\tilde{u}_2)^{(10)} :\)

By \( (\tilde{v}_1)^{(10)} > 0, (\tilde{v}_2)^{(10)} < 0 \) and respectively \( (\tilde{u}_1)^{(10)} > 0, (\tilde{u}_2)^{(10)} < 0 \) the roots of the equations \( a_{49}^{(10)}(v^{(10)})^2 + (\sigma_2)^{(10)}v^{(10)} - (a_{49}^{(10)}u^{(10)} = 0 \) and \( b_{49}^{(10)}(u^{(10)})^2 + (\tau_2)^{(10)}u^{(10)} - (b_{49}^{(10)}u^{(10)} = 0 \)

**Definition of** \((m_1)^{(10)}, (m_2)^{(10)}, (\mu_1)^{(10)}, (\mu_2)^{(10)}, v_0)^{(10)} :)\)
(mm) If we define \((m_1)^{(10)}, (m_2)^{(10)}, (\mu_1)^{(10)}, (\mu_2)^{(10)}\) by

\[
(m_2)^{(10)} = (v_0)^{(10)}, (m_1)^{(10)} = (v_1)^{(10)}, \text{ if } (v_0)^{(10)} < (v_1)^{(10)}
\]

\[
(m_2)^{(10)} = (v_1)^{(10)}, (m_1)^{(10)} = (\bar{v}_1)^{(10)}, \text{ if } (v_1)^{(10)} < (v_0)^{(10)} < (\bar{v}_1)^{(10)},
\]

and

\[
(v_0)^{(10)} = \frac{g_{48}}{g_{46}}
\]

\[
(m_2)^{(10)} = (v_2)^{(10)}, (m_1)^{(10)} = (v_0)^{(10)}, \text{ if } (v_2)^{(10)} < (v_0)^{(10)}
\]

and analogously

\[
(\mu_2)^{(10)} = (u_0)^{(10)}, (\mu_1)^{(10)} = (u_1)^{(10)}, \text{ if } (u_0)^{(10)} < (u_1)^{(10)}
\]

\[
(\mu_2)^{(10)} = (u_1)^{(10)}, (\mu_1)^{(10)} = (\bar{u}_1)^{(10)}, \text{ if } (u_1)^{(10)} < (u_0)^{(10)} < (\bar{u}_1)^{(10)},
\]

and

\[
(u_0)^{(10)} = \frac{\varphi_{48}}{\varphi_{46}}
\]

\[
(\mu_2)^{(10)} = (u_2)^{(10)}, (\mu_1)^{(10)} = (u_0)^{(10)}, \text{ if } (u_2)^{(10)} < (u_0)^{(10)}
\]

where \((u_2)^{(10)}, (\bar{u}_2)^{(10)}\) are defined by 59 and 61 respectively.

Then the solution of 19, 20, 21, 22, 23 and 24 satisfies the inequalities

\[
G_{48}^0 e^{((S_2)^{(10)} - (p_{48})^{(10)})t} \leq G_{48}^0(t) \leq G_{48}^0 e^{((S_1)^{(10)})t}
\]

where \((p_2)^{(10)}\) is defined by equation 25

\[
\frac{1}{(m_{10})^{(10)}} G_{48}^0 e^{((S_2)^{(10)} - (p_{48})^{(10)})t} \leq G_{48}^0(t) \leq \frac{1}{(m_{20})^{(10)}} G_{48}^0 e^{((S_1)^{(10)})t}
\]

\[
\left(\frac{d_2}{(m_{10})^{(10)}(S_2)^{(10)} - (p_{48})^{(10)} - (S_1)^{(10)}}\right) e^{((S_2)^{(10)} - (p_{48})^{(10)})t - e^{((S_1)^{(10)})t}} + G_{50}^0 e^{-(S_2)^{(10)}t} \leq G_{50}^0(t) \leq
\]

\[
\left(\frac{d_2}{(m_{20})^{(10)}(S_2)^{(10)} - (p_{50})^{(10)} - (S_1)^{(10)}}\right) e^{e^{((S_1)^{(10)})t} - e^{-(S_2)^{(10)}t}} + G_{50}^0 e^{-(a_{50})^{(10)}t}
\]

\[
T_{48}^0 e^{(R_2)^{(10)}t} \leq T_{48}^0(t) \leq T_{48}^0 e^{((R_2)^{(10)} + (r_{48})^{(10)})t}
\]

\[
\frac{1}{(\mu_{10})^{(10)}} T_{48}^0 e^{((R_1)^{(10)})t} \leq T_{48}^0(t) \leq \frac{1}{(\mu_{20})^{(10)}} T_{48}^0 e^{((R_2)^{(10)} + (r_{48})^{(10)})t}
\]

\[
\left(\frac{b_2}{(\mu_{10})^{(10)}(R_1)^{(10)} - (b_{48})^{(10)}}\right) e^{e^{((R_1)^{(10)})t} - e^{-(b_{50})^{(10)}t}} + T_{50}^0 e^{-(b_{50})^{(10)}t} \leq T_{50}^0(t) \leq
\]

\[
\left(\frac{b_2}{(\mu_{20})^{(10)}(R_1)^{(10)} + (r_{48})^{(10)} + (R_2)^{(10)}t} + T_{50}^0 e^{-(R_2)^{(10)}t}
\]

**Definition of \((S_1)^{(10)}, (S_2)^{(10)}, (R_1)^{(10)}, (R_2)^{(10)}\):**

Where

\[
(S_1)^{(10)} = (a_{48})^{(10)}(m_2)^{(10)} - (a_{48})^{(10)}
\]

\[
(S_2)^{(10)} = (a_{50})^{(10)} - (p_{50})^{(10)}
\]

\[
(R_1)^{(10)} = (b_{48})^{(10)}(\mu_2)^{(10)} - (b_{48})^{(10)}
\]

\[
(R_2)^{(10)} = (b_{50})^{(10)} - (r_{50})^{(10)}
\]

214
Proof: From 19, 20, 21, 22, 23, 24 we obtain
\[
\frac{dv^{(10)}}{dt} = (a_{49})^{(10)} - (a_{49}')^{(10)} - (a_{49})^{(10)(T_{49}, t)} - (a_{49})^{(10)(T_{49}, t)}v^{(10)} - (a_{49})^{(10)}v^{(10)}
\]

Definition of $v^{(10)}$:
\[
v^{(10)} = \frac{\tilde{a}_{49}}{a_{49}}
\]

It follows
\[
-(a_{49})^{(10)}\left(v^{(10)}\right)^2 + (\sigma_2)^{(10)}v^{(10)} - (a_{49})^{(10)} \leq \frac{dv^{(10)}}{dt} \leq -(a_{49})^{(10)}\left(v^{(10)}\right)^2 + (\sigma_2)^{(10)}v^{(10)} - (a_{49})^{(10)}
\]

From which one obtains

Definition of $(\tilde{v}_2)(10), (v_0)(10)$:

(d) For $0 < \frac{\tilde{a}_{49}}{a_{49}} < (v_1)(10) < (\tilde{v}_1)(10)$
\[
v^{(10)}(t) \geq \frac{(v_1)(10)(C)(10)(v_2)(10)e^{-a_{49}(10)((v_1)(10) - (v_0)(10))t}}{1 + (C)(10)e^{[-a_{49}(10)((v_1)(10) - (v_0)(10))t]}}
\]

From which we deduce $(v_0)(10) \leq v^{(10)}(t) \leq (\tilde{v}_1)(10)$

(e) If $0 < (v_1)(10) < (v_0)(10) = \frac{\tilde{a}_{49}}{a_{49}} < (\tilde{v}_1)(10)$ we find like in the previous case,
\[
(v_1)(10) \leq \frac{(v_1)(10)(C)(10)(v_2)(10)e^{-a_{49}(10)((v_1)(10) - (v_0)(10))t}}{1 + (C)(10)e^{[-a_{49}(10)((v_1)(10) - (v_0)(10))t]}} \leq v^{(10)}(t) \leq \frac{v^{(10)}(10)(C)(10)(v_2)(10)e^{-a_{49}(10)((v_1)(10) - (v_2)(10))t}}{1 + (C)(10)e^{[-a_{49}(10)((v_1)(10) - (v_2)(10))t]}}
\]

(f) If $0 < (v_1)(10) \leq (\tilde{v}_1)(10) \leq \frac{\tilde{a}_{49}}{a_{49}}$, we obtain
\[
(v_1)(10) \leq v^{(10)}(t) \leq \frac{(v_1)(10)(C)(10)(v_2)(10)e^{-a_{49}(10)((v_1)(10) - (v_0)(10))t}}{1 + (C)(10)e^{[-a_{49}(10)((v_1)(10) - (v_0)(10))t]}} \leq (v_0)(10)
\]

And so with the notation of the first part of condition (c), we have
Definition of $v^{(10)}(t) :=$

$$(m_2)^{(10)} \leq v^{(10)}(t) \leq (m_1)^{(10)}, \quad v^{(10)}(t) = \frac{g_{48}(t)}{g_{49}(t)}$$

In a completely analogous way, we obtain

Definition of $u^{(10)}(t) :=$

$$(\mu_2)^{(10)} \leq u^{(10)}(t) \leq (\mu_1)^{(10)}, \quad u^{(10)}(t) = \frac{r_{48}(t)}{r_{49}(t)}$$

Now, using this result and replacing it in 19, 20, 21, 22, 23, and 24 we get easily the result stated in the theorem.

Particular case:

If $$(v_0)^{(10)} = (v_1)^{(10)} = (v_2)^{(10)}$$ and in this case $$(v_1)^{(10)} = (\bar{v}_1)^{(10)}$$ if in addition $$(v_0)^{(10)} = (v_1)^{(10)}$$ then $$v^{(10)}(t) = (v_0)^{(10)}$$ and as a consequence $$g_{48}(t) = (v_0)^{(10)}g_{49}(t)$$ this also defines $$(v_0)^{(10)}$$ for the special case.

Analogously, if $$(b_0''')^{(11)} = (b_1''')^{(11)}$$ then $$(\tau_1)^{(10)} = (\tau_2)^{(10)}$$ and then

$$(u_1)^{(10)} = (\bar{u}_1)^{(10)}$$ if in addition $$(u_0)^{(10)} = (u_1)^{(10)}$$ then $$T_{48}(t) = (u_0)^{(10)}T_{49}(t)$$ This is an important consequence of the relation between $$(v_1)^{(10)}$$ and $$(\bar{v}_1)^{(10)}$$, and definition of $$(u_0)^{(10)}$$.

4. STATIONARY SOLUTIONS AND STABILITY

Stationary solutions and stability curve representative of the variation of systemic parameters LHS AND RHS variation curve lies below the tangent at $$(G_{51}) = G_0$$ for $$(G_{51}) < G_0$$ and above the tangent for $$(G_{51}) > G_0$$. Wherever such a situation occurs the point $$G_0$$ is called the "point of inflexion". In this case, the tangent has a positive slope that simply means the rate of change of oxygen consumption due to cellular respiration is greater than zero. Above factor shows that it is possible, to draw a curve that has a point of inflexion at a point where the tangent (slope of the curve) is horizontal.

Stationary value:

In all the cases $$(G_{51}) = G_0$$, $$(G_{51}) < G_0$$, $$(G_{51}) > G_0$$ the condition that the rate of change of LHS OF TDSE is maximum or minimum holds. When this condition holds we have stationary value. We now infer that:

1. A necessary and sufficient condition for there to be stationary value of $$(G_{51})$$ is that the rate of change of LHS OF TDSE at $$G_0$$ is zero.
2. A sufficient condition for the stationary value at $$G_0$$, to be maximum is that the acceleration of the LHS OF TDSE is less than zero.
3. A sufficient condition for the stationary value at $$G_0$$, be minimum is that acceleration of LHS OF TDSE is greater than zero.

We can prove the following

Theorem 3: If $$(a''')^{(10)}$$ and $$(b''')^{(10)}$$ are independent on $$t$$, and the conditions
(a) Indeed the first two equations have a nontrivial solution \( G_{49}, G_{48} \) if

\[
(a_{48}')(10)(a_{49}')(10) - (a_{48})(10)(a_{49})(10) < 0
\]

\[
(a_{48})(10)(a_{49})(10) - (a_{48})(10)(a_{49})(10) + (a_{48})(10)(p_{48})(10) + (a_{49})(10)(p_{49})(10) + (p_{48})(10)(p_{49})(10) > 0
\]

\[
b_{48}'(10)(b_{49}'(10) - (b_{48})(10)(b_{49})(10) > 0,
\]

\[
b_{48}'(10)(b_{49}'(10) - (b_{48})(10)(b_{49})(10) - (b_{48})'(10)(r_{49})(10) - (b_{49})'(10)(r_{49})(10) + (r_{49})'(10)(r_{49})(10) < 0
\]

with \((p_{48})(10), (r_{49})(10)\) as defined by equation are satisfied, then the system

\[
(a_{48})(10)G_{49} - [(a_{48})'(10) + (a_{48})''(10)(T_{49})]G_{48} = 0
\]

\[
(a_{49})(10)G_{48} - [(a_{49})'(10) + (a_{49})''(10)(T_{49})]G_{49} = 0
\]

\[
(a_{50})(10)G_{49} - [(a_{50})'(10) + (a_{50})''(10)(T_{49})]G_{50} = 0
\]

\[
b_{48}(10)T_{49} - [(b_{48})'(10) - (b_{48})''(10)(G_{51})]T_{48} = 0
\]

\[
b_{49}(10)T_{48} - [(b_{49})'(10) - (b_{49})''(10)(G_{51})]T_{49} = 0
\]

\[
b_{50}(10)T_{49} - [(b_{50})'(10) - (b_{50})''(10)(G_{51})]T_{50} = 0
\]

has a unique positive solution, which is an equilibrium solution for the system.

**Proof:**

(a) Indeed the first two equations have a nontrivial solution \( G_{49}, G_{48} \) if

\[
F(T_{49}) = (a_{48})(10)(a_{49})(10) - (a_{48})(10)(a_{49})(10) + (a_{48})(10)(a_{49})(10)(T_{49}) + (a_{49})(10)(a_{48})(10)(T_{49}) + (a_{48})(10)(T_{49})(a_{49})''(10)(T_{49}) = 0
\]

**Definition and uniqueness of** \( T_{49} \):

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \( (a_i')^{(10)}(T_{49}) \) are increasing, it follows that there exists a unique \( T_{49} \) for which \( F(T_{49}) = 0 \). With this value, we obtain from the three first equations

\[
G_{48} = \frac{(a_{48})(10)G_{49}}{[(a_{48})'(10) + (a_{48})''(10)(T_{49})]} , \quad G_{50} = \frac{(a_{50})(10)G_{49}}{[(a_{50})'(10) + (a_{50})''(10)(T_{49})]}
\]

\[(p) By the same argument, the equations 92, 93 admit solutions \( G_{49}, G_{48} \) if

\[
\varphi(G_{51}) = (b_{48})(10)(b_{49})(10) - (b_{48})(10)(b_{49})(10) -
\]

\[
[(b_{48})(10)(b_{49})(10)(G_{51}) + (b_{49})(10)(b_{50})(10)(G_{51})] + (b_{48})(10)(G_{51})(b_{49})(10)(G_{51}) = 0
\]

217
Where in \((G_{51})(G_{49}, G_{50}), G_{49}, G_{50}\) must be replaced by their values It is easy to see that \(\varphi\) is a decreasing function in \(G_{49}\) taking into account the hypothesis \(\varphi(0) > 0, \varphi(\infty) < 0\) it follows that there exists a unique \(G_{49}\) such that \(\varphi((G_{51})') = 0\)

Finally we obtain the unique solution

\[
G_{49}^* = \frac{(a_{49})(10)G_{49}}{(a_{49})(10)+(a_{49})(10)(T_{49})}, \quad G_{50}^* = \frac{(a_{50})(10)G_{49}}{(a_{50})(10)+(a_{50})(10)(T_{49})}
\]

\[
T_{49}^* = \frac{(b_{49})(10)G_{49}}{(b_{49})(10)-(b_{49})(10)((G_{51})')}, \quad T_{50}^* = \frac{(b_{50})(10)G_{49}}{(b_{50})(10)-(b_{50})(10)((G_{51})')}
\]

Obviously, these values represent an equilibrium solution global equations

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions \((a_i'')(10)\) and \((b_i'')(10)\) Belong to \(C^{(10)}(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** \(G_i, T_i :=\)

\[
G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i
\]

\[
\frac{\partial(a_i'')(10)}{\partial T_{49}} (T_{49}) = (q_{49})^{(10)}, \quad \frac{\partial(b_i'')(10)}{\partial G_j} - (G_{52})^* = s_{ij}
\]

Then taking into account equations and neglecting the terms of power 2, we obtain from global equations

\[
\frac{dG_{48}}{dt} = -((a_{48})^{(10)} + (p_{48})^{(10)})G_{48} + (a_{48})^{(10)}G_{49} - (q_{48})^{(10)}G_{48}T_{49}
\]

\[
\frac{dG_{49}}{dt} = -((a_{49})^{(10)} + (p_{49})^{(10)})G_{49} + (a_{49})^{(10)}G_{48} - (q_{49})^{(10)}G_{49}T_{49}
\]

\[
\frac{dG_{50}}{dt} = -((a_{50})^{(10)} + (p_{50})^{(10)})G_{50} + (a_{50})^{(10)}G_{49} - (q_{50})^{(10)}G_{50}T_{49}
\]

\[
\frac{dT_{48}}{dt} = -((b_{48})^{(10)} - (r_{48})^{(10)})T_{48} + (b_{48})^{(10)}T_{49} + \sum_{j=48}^{50} (s_{(48)(j)} T_{48}^* G_j)
\]

\[
\frac{dT_{49}}{dt} = -((b_{49})^{(10)} - (r_{49})^{(10)})T_{49} + (b_{49})^{(10)}T_{48} + \sum_{j=48}^{50} (s_{(49)(j)} T_{49}^* G_j)
\]

\[
\frac{dT_{50}}{dt} = -((b_{50})^{(10)} - (r_{50})^{(10)})T_{50} + (b_{50})^{(10)}T_{49} + \sum_{j=48}^{50} (s_{(50)(j)} T_{50}^* G_j)
\]

The characteristic equation of this system is

\[
((\lambda)^{(10)} + (b_{50})^{(10)} - (r_{50})^{(10)})(((\lambda)^{(10)} + (a_{50})^{(10)} + (p_{50})^{(10)})
\]

\[
[(((\lambda)^{(10)} + (a_{48})^{(10)} + (p_{48})^{(10)})(q_{48})^{(10)}G_{49} + (a_{49})^{(10)}(q_{48})^{(10)}G_{48})]
\]
Affective Consciousness and Quantum Field Part One- Portmanteau Porte-cochere and contrariness of identities models

 Preface: A Core emotional feeling in animals and humans has been studied by various authors. Some of them are Panksepp. The position advanced in this investigation is that the bedrock of emotional feelings is contained within the evolved emotional action apparatus of mammalian brains. This dual-aspect monism approach to brain-mind functions, which asserts that emotional feelings may reflect the neurodynamics of brain systems that generate instinctual emotional behaviors, saves us from various conceptual conundrums. In coarse form, primary process affective consciousness seems to be fundamentally an unconditional "gift of nature" rather than an acquired skill, even though those systems facilitate skill acquisition via various felt reinforcements. Affective consciousness, being a comparatively intrinsic function of the brain, shared homogenously by all mammalian species, should be the easiest variant of consciousness to study in animals. This is not to deny that some secondary processes (e.g., awareness of feelings in the generation of behavioral choices) cannot be evaluated in animals with sufficiently clever behavioral learning procedures, as with place-preference procedures and the analysis of changes in learned behaviors after one has induced reevaluation of incentives. Rather, the claim is that a direct neuroscientific study of primary process emotional/affective states is best achieved through the study of the intrinsic ("instinctual"), albeit experientially refined, emotional action tendencies of other animals. In this view, core emotional feelings may reflect the neurodynamics attractor landscapes of a variety of extended trans-diencphalic,
limbic emotional action systems-including SEEKING, FEAR, RAGE, LUST, CARE, PANIC, and PLAY.

Through a study of these brain systems, the neural infrastructure of human and animal affective consciousness may be revealed. Emotional feelings are instantiated in large-scale neurodynamics that can be most effectively monitored via the ethological analysis of emotional action tendencies and the accompanying brain neurochemical/electrical changes. The intrinsic coherence of such emotional responses is demonstrated by the fact that they can be provoked by electrical and chemical stimulation of specific brain zones—effects that are affectively laden. For substantive progress in this emerging research arena, animal brain researchers need to discuss affective brain functions more openly. Secondary awareness processes, because of their more conditional, contextually situated nature, are more difficult to understand in any neuroscientific detail. In other words, the information-processing brain functions, critical for cognitive consciousness, are harder to study in other animals than the more homologous emotional/motivational affective state functions of the brain.

Form of discrete and stochastic pulse trains and point processes, the “macroscopic” activity of large assemblies of neurons appears to be spatially coherent and highly structured in phase and amplitude (Freeman 1996, 2000). Motivated by such an experimental situation, Ricciardi and Umezawa (1967) formulated the quantum model of the brain as a many-body physics problem, by using the formalism of QFT with a spontaneous breakdown of symmetry, successfully tested in condensed matter experiments. In fact, such formalism provides the only available theoretical tool capable of describing long-range correlations in many-body systems. In classical physics, long-range correlations are usually explained as the result of classical processes causally propagating along different pathways. However, such a view is not supported by many experimental observations in condensed matter physics. In contrast, such observations are fully described and predicted by quantum field theory (Itzykson and Zuber 1980, Umezawa 1993). The hypothesis by Ricciardi and Umezawa (1967) was, then, that the QFT approach could also be applied to describe the observation that the brain presents almost simultaneous responses in several regions to some external stimuli. As a matter of fact, the understanding of such correlations in terms of modern biochemical and electrochemical processes is still lacking, which suggests that these responses could not be explained in terms of single neuron activity (Pribram 1971, 1991).

In QFT the dynamics (i.e. the Lagrangian) is in general invariant under some group G of continuous transformations. This means that the dynamical evolution of the system is constrained by the conservation laws of some observables depending on G. For example, invariance under the time translation group implies energy conservation, and similarly one gets the conservation of momentum, of electric charge, etc. as consequences of the dynamical invariance under corresponding continuous transformation groups. The invariance of the dynamics, thus, characterizes physical states of the system by assigning conserved quantities to them. Spontaneous breakdown of symmetry occurs when the minimum energy state (the ground state or vacuum) of the system is not invariant under the full group G, but less than one of its subgroups. Then it can be shown (Itzykson and Zuber 1980).
Umezawa 1993) that collective modes, the so-called Nambu-Goldstone (NG) boson modes, are dynamically generated. At this point, let us mention that particles may also be represented as wave excitations or modes. Referring to NG modes, thus, emphasizes wave-like behavior rather than particle-like behavior. Propagating over the whole system, these modes are the carriers of the ordering information in terms of long-range correlations: order manifests itself as a global property dynamically generated. The long-range correlation modes are responsible for maintaining the ordered pattern: they are coherently condensed in the ground state. In the case of crystals, for Quantum Field Theory of Mind/Brain States 61example, they keep the atoms trapped at their lattice sites. Long-range correlations thus form a sort of net, extending all over the system, which traps its components in an ordered pattern. This explains the macroscopic collective behavior of the system as a “whole”. The macroscopic observable specifying the ordered state of a system is called the order parameter. For example, the density (or a quantity proportional to it) is the order parameter for a crystal, the magnetization for a ferromagnetic, etc. Properties like the stiffness of a crystal, electrical conductivity, etc., depend on the value assumed by the order parameter under specific boundary conditions. A most difficult problem in neuroscience is how conscious mind emerges from brain activities. To make headway on this, we may need to focus more on the vast neuronal contexts that unconditionally enable phenomenal experience rather than on the specific contents of consciousness. From this perspective, consciousness was initially built on fundamental survival concerns of organisms. Psychologically, such concerns may have been first instantiated in the glimmers of affective feelings – basic, internally felt neurodynamics reflecting intrinsic survival values that are experienced but not necessarily reflected upon. Unfortunately, affective experience has been profoundly neglected in consciousness studies. It is commonly assumed that consciousness cannot be scientifically studied without linguistic reports of subjective experiences. That premise arbitrarily limits consciousness studies to humans. (See JAAK PANKSEPP)

A neuro-evolutionary view suggests that primary-process consciousness emerged long before organisms had enough brain matter to speak or to cognitively reflect on their experiences. In any event, current knowledge supports the conjecture that primary-process affective experience emerged in brain evolution much earlier than the cognitive processes that allow us to think and talk about our internal experiences (i.e., secondary and tertiary forms of consciousness).

INTRODUCTION:

QUANTUM FIELD:

(1) With the search for the Higgs boson, the last missing piece of the Standard Model of particle physics, apparently reaching its long-anticipated-and-finally-successful conclusion, anticipation of the next set of discoveries is growing. Repeatedly, it is reported that the theorists joked that, with the exception of the actual CERN experimentalists present, all of us know that the Higgs has now been discovered with a mass of 125 GeV/c². Super symmetry – a theory that posits that for every known particle there is another (or more than one) yet-to-be-discovered partner particle – is the leading candidate for physics Beyond the Standard Model.

(2) It is central to string theory (a.k.a. super-string theory), required(e) for gauge coupling unification (see below), useful for solving the Higgs Fine Tuning Problem

(3) This gives us the leading candidate for dark matter = the Lightest Supersymmetric Particle (LSP). A main rationale for super symmetry evaporates on closer inspection. So what is Beyond the Standard Model (BSM) physics, but still people are convinced that Higgs is the ultimate neplus ultra. At least since the discovery of the W and Z particles at CERN in 1983, physicists have been pretty much convinced that the Standard Model (SM) that emerged from the late 1960s and early 1970s is the correct model of fundamental physics. At least at energies below the so-called weak-scale – a few hundred GeV – or maybe a few times that. But particle theorists variously hoped/expected/knew that at higher energies the Standard Model was not the whole story and a more fundamental theory would need to be found.

(4) According to Relativity, our lives are world lines in space time.

(5) Space time does not happen, it always exists.
(6) **It is our brain that shows us a movie of matter evolving in time.**

(7) All spacetime events are **conscious;** they are conscious of **other** spacetime events.

(8) The "experience" of a spacetime event is **static,** a frozen region of spacetime events.

(9) All the subjective features of the "psycho space" of an observer can be completely derived from the objective features of the region of spacetime that the observer is connected to.

(10) Special circuits in our brain create the impression of a time flow, of a time travel through the region of spacetime events connected to the brain.

(11) Memory of an event is re-experiencing that spacetime event, which is fixed in spacetime.

(12) We don't store an event; we only keep a link to it.

(13) Conscious memory is not in the brain but in is in spacetime.

(14) The inner life of a system is (=) its spacetime history.

There are two types of reasons to doubt the completeness of the Standard Model – aesthetic (philosophical) and mathematical. Aesthetic problem number one, physicists adore simplicity. Zero and one are our favorite numbers. Two can be suffered. After two comes "too many", although identical copies (twins, triplets,) may receive special dispensation. The Standard Model has too many too-man’s: three fundamental forces (a.k.a. gauge groups); way too many fundamental fermions (particles that make up matter)– three generations each with at least 5 representations (groups) of them — plus three sets of gauge bosons and the set of particles of which the Higgs boson is a member. It also has far too many (more than 20) independent parameters. Aesthetic problem number two — for no apparent reason the weak scale is much (as in about 10^{16} times) smaller than what we believe to be the fundamental energy scale of physics – the Planck scale (about 10^{19} GeV), a scale set by the strength of gravity (the one fundamental force not included in the Standard Model). This is known as the (Weak) Hierarchy Problem – and can also be understood in terms of the absolutely enormous strength of the three Standard Model forces compared to that of gravity between pairs of fundamental particles separated by appropriately microscopic scales. It is however the technical problem that has carried the most weight in convincing people that there must be physics beyond the Standard Model. It is the story we tell our children — quantum mechanics makes the Standard Model unstable. Quantum mechanics teaches us that, as a particle such as a Higgs boson travels along, it can emit and reabsorb another particle. This process represents a “loop contribution” to the mass of the Higgs boson, so-called because a pictorial representation of the process – Feynman diagrams – depicts these processes as loops attached to the traveling Higgs boson. Unfortunately, when you add up the loop contributions to the mass of the Higgs boson from all possible particles with all possible energies and momenta, they appear to be infinite or at least proportional to the maximum possible momentum that can be carried. For technical reasons these are called quadratic divergences and are widely derided. For the actual Higgs boson mass to be finite, there must apparently be subtle and precise cancellation of the loop contributions against the underlying “tree” (loop-free) mass. This Higgs Fine-Tuning Problem, so the lore tells us, must be remedied. Following points are of seminal and perennial importance:

(1) Potential features of quantum computation could explain enigmatic aspects of consciousness.

(2) The Penrose—Hameroff model (orchestrated objective reduction: 'Orch OR') suggests that quantum superposition and a form of quantum computation occur in microtubules—cylindrical protein lattices of the cell cytoskeleton within the brain's neurons.

(3) Microtubules couple to and regulate neural–level synaptic functions.

(4) They may be ideal quantum computers because of dynamical lattice structure, quantum–level subunit states and intermittent isolation from environmental interactions.

(5) In addition to its biological setting, the Orch OR proposal differs in an essential way from technologically envisioned quantum computers in which collapse, or reduction to classical output states, is caused by environmental decoherence (hence introducing randomness). In the Orch OR proposal, reduction of microtubule quantum superposition to classical output states occurs by an objective factor.

(6) Roger Penrose's quantum gravity threshold stems quintessentially from instability in Planck–scale
separations (superpositions) in spacetime geometry.

(7) Output states following Penrose's objective reduction are neither totally deterministic nor random, but influenced by a non–computable factor ingrained in fundamental spacetime.

(8) Taking a modern pan–psychist view in which protoconscious experience and Platonic values are embedded in Planck–scale spin networks, the Orch OR model portrays poignantly and in a functionally topological manner with abstract problematic structure consciousness as brain activities linked to fundamental ripples in spacetime geometry. (Stuart Hameroff, MD)

(9) Protein conformational dynamics and pharmacological evidence suggest that protein conformational states—fundamental information units ('bits') in biological systems—are governed by quantum events.

BSM physics is the proposed remedy. Super symmetry cancels the loop of every known particle against the loop of an as-yet-to-be-discovered partner particle. Technicolor eliminates the Higgs boson—replacing it by a composite of new particles called techni-quarks. If there are large extra dimensions then the largest momentum that can circulate in a loop is actually only a little larger than the weak scale. Clearly BSM physics is not just desirable but essential. Recently, however, my colleague Bryan Lynn suggested, and together with Katie Freese and Dmitry Podolsky, he and I explained, how the Standard Model actually comes up with a remedy all on its own. The Higgs boson is one member of a set of quadruplets in the Standard Model. At energies below the weak scale, its three siblings get eaten—they get incorporated into the W and Z bosons. According to a famous theorem due to MIT’s Jeffrey Goldstone (hence “Goldstone’s Theorem”), the masses of the three siblings must be exactly zero. In particular, the quadratic ally divergent contribution to their masses are zero.

Although this doesn’t force the mass of the Higgs boson to be zero (a good thing, since it seems likely to be about 125 GeV/c²), it does mean that the quadratic divergences in the Higgs mass that have worried us for decades are not a problem of the Standard Model after all. Now, not everybody buys our argument. Some of them prefer to focus on the aesthetic challenge of the Weak Hierarchy Problem, while others argue that we have no choice but to add quantum gravity to the Standard Model, inevitably resurrecting the Higgs Fine Tuning Problem. We would counter that the absence of a Higgs Fine Tuning Problem in the Standard Model is a virtue that, absent any hard evidence for BSM physics, preserving the Standard Model’s Goldstone miracle should be taken as a requirement of any proposed BSM theories. The implication is clear. If there is no problem, there may be no need for a solution. Beyond the Standard Model Physics isn’t ruled out by the absence of a Higgs Fine Tuning Problem in the Standard Model, but it does mean that the Standard Model may well be the whole story, or at least the whole story at the energies that the LHC can command. In short, don’t be surprised if the Higgs is the last new particle discovered by the LHC. Theorists may hunger for physics beyond the Standard Model, but nature may be quite content without it, thank you very much.

For decades now physicists have contemplated the idea of an entire shadow world of elementary particles, called super symmetry. It would elegantly solve mysteries that the current Standard Model of particle physics leaves unexplained, such as what cosmic dark matter is. Now some are starting to wonder. The most powerful collider in history, the Large Hadron Collider (LHC), has yet to see any new phenomena that would betray an unseen level of reality. Although the search has only just begun, it has made some theorists ask what physics might be like if super symmetry is not true after all. “Wherever we look, we see nothing—that is, we see no deviations from the Standard Model,” says Giacomo Polesello of Italy’s National Institute of Nuclear Physics in Pavia. Polesello is a leading member of the 3,000-strong international collaboration that built and operates ATLAS, one of two cathedral-size general-purpose detectors on the LHC ring. The other such detector, CMS, has seen nothing, either, according to an update presented at a conference in the Italian Alps in March. Theorists introduced super symmetry in the 1960s to connect the two basic types of particles seen in nature, called fermions and bosons. Roughly speaking, fermions are the constituents of matter (the electron being the quintessential example), whereas bosons are the carriers of the fundamental forces (the photon in the case of electromagnetism). Super symmetry would give every known boson a heavy “super partner” that is a fermion and every known fermion a heavy partner that is a boson. “It is the next step up toward the ultimate view of the world, where we make everything symmetric and beautiful,” says Michael Peskin, a theorist at SLAC National Accelerator Laboratory.

Following are the tit bits from the article by Sir Jogesh C. Pati, Professor at Princeton University.
Recently a new class of composite Higgs models have been developed which give rise to naturally light Higgs bosons without super symmetry.

Based on the chiral symmetries of “theory space,” involving replicated gauge groups and appropriate gauge symmetry breaking patterns, these models allow the scale of the underlying strong dynamics giving rise to the composite particles to be as large as of order 10 TeV, without any fine tuning to prevent large corrections to Higgs boson mass(es) of order 100 GeV.

Prof. Pati showed that the size of flavor violating interactions arising generically from underlying flavor dynamics constrains the scale of the Higgs boson compositeness to be greater than of order 75 TeV, implying that significant fine-tuning is required.

Without fine-tuning, the low-energy structure of the composite Higgs model alone is not sufficient to eliminate potential problems with flavor-changing neutral currents or excessive CP violation; solving those problems requires additional information or assumptions about the symmetries of the underlying flavor or strong dynamics.

Weaker, but more model-independent, bounds which arise from limits on weak isospin violation. In the presence of massless chiral fermions, a 0 term in can be rotated away by an appropriate chiral transformation of the fermion fields, because due to the chiral anomaly, this transformation induces a contribution to the fermion path integral measure proportional to the 0 term Lagrangian.

The gluons have the same coupling to the right and left handed quarks, and a chiral rotation leaves the mass matrix variant.

The SU(2)L fields (however, are coupled to the left handed components of the fermions only, thus both the left and right handed components can be rotated with the same angle, rotating away the 0 term) without altering the mass matrix.

Bose-Fermi symmetry play a crucial role in all recent attempts at higher unification, which include the ideas of(i) the conventional approach to grand unification; (ii) the preonic approach; and (iii) Superstrings.

The concept of massless particles with two states of polarization (these are the photons with spin 1) and the concept that their number is not conserved; that they obey a new statistics.

Connection between spin and statistics which asserts that particles of integer spins (i.e, spin 0, 1, 2,) obey what is now called the Bose-Einstein statistics while those of half-integer spins (i.e.,spin-1/2, 3/2, 5/2, . . . ) obey the Fermi-Dirac statistics or the exclusion principle. This in turn allows us to classify all known elementary particles as either bosons or fermions which respectively obey the Bose-Einstein and the Fermi Dirac statistics.

(Concept of symmetry relates) a symmetry that relates fermions to bosons

Principle of local gauge invariance dictates the existence of the photon and the gluons as well as that of the associated forces of quantum electro- and chromodynamics.

The principle of general coordinate invariance proposed by Einstein leads to the familiar gravitational “forces” as a consequence of the curvature of space-time

Principle of general coordinate invariance dictates the existence of spin-2 graviton.

(Coordinate invariance and curvature of space and time)in turn help preserve the masslessness of these “gauge” particles despite quantum corrections, at least in perturbation theory.

Spin-1/2 particles (i.e., electrons) are at least needed since they obey the exclusion principle which is relevant to the explanation of chemistry and in turn to the biology of life.

Now, once spin-1/2 particles are introduced into the Lagrangian, they have the good feature that their masses remain protected against arbitrarily large quantum corrections through the so-called chiral symmetry

Chiral symmetry guarantees that the quantum corrections to the masses of spin-1/2 fermions in perturbation theory either vanish or are bounded by a logarithmic cutoff (symbolizing short-
distance physics) depending upon whether their “bare” mass-terms are zero or non-zero.

19. In particular, the pions are the carriers of the nuclear force.
20. Higgs bosons induce spontaneous breaking of the electroweak symmetry and give masses to W and Z bosons.
21. Mass of the pion controlled by the relevant chiral symmetry breaking parameter which is determined by (a) the finite “bare” masses of the up and down 3quarks and (b) the QCD scale-parameter.
22. Compositeness of Higgs Boson violates the flavor-changing neutral current processes and oblique electroweak parameters.
23. Elementaryness of Higgs Boson leads to (i) What if any is an a priori rationale for its existence, and (ii) how can one protect its mass against large quantum corrections?
24. Higgs boson couples to the gauge bosons and also possesses quartic self couplings, one obtains corrections in one loop to its mass which are proportional to αiΛ (Λ is the cutoff characterizing short-distance physics and α’s are coupling parameters.
25. Fine tuning by some 24 orders of magnitude (or higher) in the choice of the bare mass of the Higgs boson cancels the large quantum corrections.
26. “Super symmetry” transforms spin-1/2 fermions into spin-0 bosons and vice versa.
27. Bose fermi Symmetry transforms Bosons in to fermions.
28. The power of super symmetry arises because its generator(s) Q transforms a spin-0 (or spin-1) boson into a spin-1/2 fermion and thereby changes the spin of the particle by 1/2 unit as well as its statistics.
29. Generators of Lie algebras, associated with the familiar symmetries such as isospin and SU (3) transform as Lorentz-scalars – i.e., as spin-0 bosonic operators; and which can thus transform a particle of a given spin into another of the same spin, only.
30. Super symmetry unites fermions and bosons (as members of a supermultiplet and thereby provides the rationale for the existence of spin-0 matter, on a par with that of spin-1/2 matter.)
31. Super symmetry permits a non-trivial marriage of space-time (Poincaré) symmetries with internal symmetries in accord with relativity.
32. Lopuszanski and Sohnius the graded Lie algebra associated with super symmetry is the only framework within which such a marriage of space and time can be achieved (consistent with relativistic quantum field theory. For example, SU(2)-isospin symmetry together with super symmetry groups the spin-1/2 doublet of (u, d)-quarks with the spin-0 doublet (‘u,’d)-squarks to make a super-bidoublet).

The monumental collider at CERN near Geneva should have the oomph to produce those super particles. Currently the LHC is smashing protons with energy of four trillion electron volts (TeV) apiece, up from 3.5 TeV last year. This energy is divided among the quarks and gluons that make up the protons, so the collision can generate new particles with the equivalent of about 1 TeV of mass. But despite the high expectations (and energies), so far nature has not cooperated. LHC physicists have been searching for signs of particles new to science and have seen none. If super particles exist, they must be even heavier than many physicists had hoped. “To put it bluntly,” Polesello says, “the situation is that we have ruled out a number of ‘easy’ models that should have showed up right away.” His colleague Ian Hinchcliffe of Lawrence Berkeley National Laboratory echoes him: “If you look at the range of masses and particles that have been excluded, it’s quite impressive.” Many are still hopeful. “There are still very viable ways of building super symmetry models,” Peskin says. Expecting to see new physics after just a year of data taking was unrealistic, says Joseph Lykken, a theorist on the CMS team. What has theorists on edge, however, is that for super symmetry to solve the problems for which it was invented in the first place, at least a few of the super particles should not be too heavy. To constitute dark matter, for example, they need to weigh no more than a few tenths of 1 TeV. Another reason most physicists want some super particles to be light lies in the Higgs boson, another major target of the LHC. All elementary particles that have mass are supposed to get it through their interaction with this boson and, secondarily, with a halo of fleeting “virtual particles.” In most cases, the symmetries of the Standard Model ensure that these virtual particles cancel one another out, so they contribute only modestly to mass. The exception, ironically, is the Higgs itself. Calculations based on the Standard
Model yield the paradoxical result that the boson’s mass should be infinite. Super partners would solve this mystery by providing greater scope for cancellations. A Higgs mass of around 0.125 TeV, as suggested by preliminary results announced in December 2011, would be right in the range where super symmetry predicts it should be. But in that case, the super particles would need to have a fairly low mass.

If the Higgs boson with mass 125 GeV is fermiophobic, as suggested by the most recent LHC results, then the MSSM is excluded. The minimal supersymmetric fermiophobic Higgs scenario can naturally be formulated in the context of the NMSSM that admits Z_3 discrete symmetries. In the fermiophobic NMSSM, the SUSY naturalness criteria are relaxed by a factor N_c y_t^4/g^4 ~ 25, removing the little hierarchy problem and allowing particle masses to be naturally of order 2-3 TeV. This scale motivates wino or Higginson dark matter. The SUSY flavour and CP problems as well as the constraints on s-particle and Higgs boson masses from b -> s\gamma, B_s -> \mu\nu\nu\nu() and direct LHC searches are relaxed in fermiophobic NMSSM. The price to pay is that a new, yet unknown, mechanism must be introduced to generate fermion masses. We show that in the fermiophobic NMSSM the radiative Higgs boson branching to \gamma\gamma, \gamma Z can be modified compared to the fermiophobic and ordinary standard model predictions, and fit present collider data better. Suppression of dark matter scattering off nuclei explains the absence of signal in XENON100.

A SOCIOLOGICAL INTRUSION:

Change is a potentially produces and transforms as an emotional event as people anticipate or experience its outcomes and processes. Emotional aspects of organizational change, promote acceptance of change or resistance to it. Cognitive, affective and behavioral responses to organizational change, indicates outcomes, scale, temporal issues and justice

Emotional intelligence, disposition, previous experience of change, and change and stress outside the workplace affect those in the employee’s perceptions of the leaders/managers/agents (their leadership ability, emotional intelligence and trustworthiness); and those in the employee’s perception of the organization (its culture and change context)

Cognitive appraisal theory takes the position that emotion derives from cognition as people contemplate the importance of events (such as organizational change) to their wellbeing and consider how they will cope.

Social constructionism was used as a theoretical platform because it combines the individual experience of emotions during change with the social forces that help shape them.

The Higgs Field is the fundamental quantum arena, quantitative form of means and ends that gives birth to all mass in the universe. In the finding of Higgs Boson, there exists an inferential thought and diversity and essential predications, predicational anteriority, character constitution of primordial exactitude and ontological resonance of the formation of the world itself.

The particle Higgs Boson

Higgs Quantum Field
Mass of the matter
Quantum Field Theory
Electro-Weak Unification and Broken Symmetry

(1) Grand Unification Theory Super symmetry
(2) Quantum Gravity and Super gravity
(3) The Heterotic String
(4) Unified Quantum Field Theories
(5) Principle of spontaneously broken symmetry, locates (eb) deeply hidden symmetries of nature at fundamental space-time scales.
(6) Principle of broken symmetry explains the emergence of diverse forces from an initially unified field.
(7) Profound symmetry principle called super symmetry, which has the potential of unifying force fields.
and matter fields in the context of a single field.

SECTION 5:

AFFECTIVE CONSCIOUSNESS AND QUANTUM FIELD - A PISCATORIAL PIRATISH-BUBONIC BUCANEER MODEL - PART ONE

PREFACE: The connection between statistical mechanics and quantum field theory (See Barry M. McCoy) are elucidated and expatiated more often than not. The general principles, axiomatic predications and postulation alcovishness, conception, and creation relating statistical mechanics and the path integral formulation of quantum field theory are intertwined in such a way that they cannot be disentangled. Theory of the Ising model for $H=0$, where both the homogeneous and randomly inhomogeneous models are treated and the scaling theory and the relation with Freehold determinants and Painlevé equations bears ample testimony and infallible observatory and impeccable demonstration to the intricate links that the two field possess. Ising model with $H\neq 0$, where the relation with gauge theory is used to discuss the phenomenon of confinement. We conclude in the last lecture with a discussion of quantum spin diffusion in one dimensional chains and a presentation of the chiral Potts model which illustrates the physical effects that can occur when the Euclidean and Minkowski regions are not connected by an analytic continuation.

Nonequilibrium Statistical Mechanics with Quantum Field has been studied and investigated by Takafumi Kita where a conception, creation, process oriented and system oriented a concise and self-contained introduction to nonequilibrium statistical mechanics with quantum field theory by considering an ensemble of interacting identical bosons or fermions as an example. Matsubara formalism of equilibrium statistical mechanics such as Feynman diagrams, the proper self-energy, and Dyson's equation are widely used in the subject matter in question. The author states that the aim is three pronged: (i) to explain the fundamentals of nonequilibrium quantum field theory as simple as possible on the basis of the knowledge of the equilibrium counterpart; (ii) to elucidate the hierarchy in describing nonequilibrium systems from Dyson's equation on the Keldysh contour to the Navier-Stokes equation in fluid mechanics via quantum transport equations and the Boltzmann equation; (iii) to derive an expression of nonequilibrium entropy that evolves with time. Nonequilibrium Green's function and the self-energy uniquely on the round-trip Keldysh contour, thereby avoiding possible confusions that may arise from defining multiple Green's functions at the very beginning. Feynman rules for the perturbation expansion are presented. Self-consistent perturbation expansion with the Luttinger-Ward thermodynamic functional, i.e., Baym's Phi-derivable approximation, which has a crucial property for nonequilibrium systems of obeying various conservation laws automatically. Two-particle correlations can be calculated within the Phi-derivable approximation, i.e., an issue of how to handle the "Bogoliubov-Born-Green-Kirkwood-Yvons (BBGY) hierarchy". In the extant and existential model we converge some of the salient features of the Quantum Field Theory, Statistical Mechanics, Yang Baxter Equations, Bose fermion Equivalence, Rational conformal field theory to present a holistic and consummate scenario of the asymmetric freedom and translations in space time. We intend to extend the model for virtual photons and mass energy momentum tensor.

HIGGS OLIO PODRADA AND POT POURRI:

BECAUSE Higgs particles interact most strongly with other high-mass particles, it is hard to make them directly in the collisions of lightweights like electrons. Instead, we reach the Higgs particle indirectly, through virtual Z or W bosons or pairs of top quarks, which then decay into Higgs.

In diagrams like those shown below, only the particles with free ends extending backwards exist for a
noticeable time in the past, and only the particles with free ends extending forwards exist for a noticeable time in the future. The lines with no free ends have only a very fleeting existence and cannot be observed - they are said to be virtual particles.

In part (A) of the diagram, we see how an electron and a positron create a virtual Z boson, which then emits a Higgs boson and becomes real. This is the process LEP experimenters hope to see. At the Tevatron, instead of electrons, experimenters will use quarks and antiquarks (B) found within their colliding protons and antiprotons, and produce W bosons in place of Zs. (C) Alternatively at the Tevatron and especially at the LHC, gluons - again found within colliding protons - should make pairs of virtual top quarks that will annihilate to form Higgs particles. This process is my own contribution to the Higgs particle cookbook.

The Mysteries of Mass

Physicists are hunting for an elusive particle that would reveal the presence of a new kind of field that permeates all of reality. Finding that Higgs field will give us a more complete understanding about how the universe works. Most people think they know what mass is, but they understand only part of the story. For instance, an elephant is clearly bulkier and weighs more than an ant. Even in the absence of gravity, the elephant would have greater mass--it would be harder to push and set in motion. Obviously the elephant is more massive because it is made of many more atoms than the ant is, but what determines the masses of the individual atoms? What about the elementary particles that makes up the atoms--what determines their masses? Indeed, why do they even have mass?

We see that the problem of mass has two independent aspects. First, we need to learn how mass arises at all. It turns out mass results from at least three different mechanisms, which I will describe below. A key player in physicists' tentative theories about mass is a new kind of field that permeates all of reality, called the Higgs field. Elementary particle masses are thought to come about from the interaction with the Higgs field. If the Higgs field exists, theory demands that it have an associated particle, the Higgs boson. Using particle accelerators, scientists are now hunting for the Higgs.

The second aspect is that scientists want to know why different species of elementary particles have their specific quantities of mass. Their intrinsic masses span at least 11 orders of magnitude, but we do not yet know why that should be so. For comparison, an elephant and the smallest of ants differ by about 11 orders of magnitude of mass.

What Is Mass?

Isaac Newton presented the earliest scientific definition of mass in 1687 in his landmark Principia: "The quantity of matter is the measure of the same, arising from its density and bulk conjointly." That very basic definition was good enough for Newton and other scientists for more than 200 years. They understood that science should proceed first by describing how things work and later by understanding why. In recent years, however, the why of mass has become a research topic in physics. Understanding the meaning and origins of mass will complete and extend the Standard Model of particle physics, the well-established theory that describes the known elementary particles and their interactions. It will also resolve mysteries such as dark matter, which makes up about 25 percent of the universe.

Why is the Higgs field present throughout the universe? What is the Higgs field?

The foundation of our modern understanding of mass is far more intricate than Newton's definition and is based on the Standard Model. At the heart of the Standard Model is a mathematical function called a
Lagrangian, which represents how the various particles interact? From that function, by following rules known as relativistic quantum theory, physicists can calculate the behavior of the elementary particles, including how they come together to form compound particles, such as protons. For both the elementary particles and the compound ones, we can then calculate how they will respond to forces, and for a force F, we can write Newton's equation F = ma, which relates the force, the mass and the resulting acceleration. The Lagrangian tells us what to use for m here, and that is what is meant by the mass of the particle.

But mass, as we ordinarily understand it, shows up in more than just F = ma. For example, Einstein's special relativity theory predicts those massless particles in a vacuum travel at the speed of light and those particles with mass travel more slowly, in a way that can be calculated if we know their mass. The laws of gravity predict that gravity acts on mass and energy as well, in a precise manner. The quantity m deduced from the Lagrangian for each particle behaves correctly in all those ways, just as we expect for a given mass. Fundamental particles have an intrinsic mass known as their rest mass (those with zero rest mass are called massless). For a compound particle, the constituents' rest mass and also their kinetic energy of motion and potential energy of interactions contribute to the particle's total mass.

Energy and mass are related, as described by Einstein's famous equation, E = mc² (energy equals mass times the speed of light squared). An example of energy contributing to mass occurs in the most familiar kind of matter in the universe—the protons and neutrons that make up atomic nuclei in stars, planets, people and all that we see. These particles amount to 4 to 5 percent of the mass-energy of the universe. The Standard Model tells us that protons and neutrons are composed of elementary particles called quarks that are bound together by massless particles called gluons. Although the constituents are whirling around inside each proton, from outside we see a proton as a coherent object with an intrinsic mass, which is given by adding up the masses and energies of its constituents. The Standard Model lets us calculate that nearly all the mass of protons and neutrons is from the kinetic energy of their constituent quarks and gluons (the remainder is from the quarks' rest mass). Thus, about 4 to 5 percent of the entire universe—almost all the familiar matter around us—comes from the energy of motion of quarks and gluons in protons and neutrons.

The Higgs Mechanism

Unlike protons and neutrons, truly elementary particles—such as quarks and electrons—are not made up of smaller pieces. The explanation of how they acquire their rest masses gets to the very heart of the problem of the origin of mass. As I noted above, the account proposed by contemporary theoretical physics is that fundamental particle masses arise from interactions with the Higgs field. But why is the Higgs field present throughout the universe? Why isn't its strength essentially zero on cosmic scales, like the electromagnetic field? What is the Higgs field? The Higgs field is a quantum field. That may sound mysterious, but the fact is that all elementary particles arise as quanta of a corresponding quantum field. The electromagnetic field is also a quantum field (its corresponding elementary particle is the photon). So in this respect, the Higgs field is no more enigmatic than electrons and light. The Higgs field does, however, differ from all other quantum fields in three crucial ways. The first difference is somewhat technical. All fields have a property called spin, an intrinsic quantity of angular momentum that is carried by each of their particles. Particles such as electrons have spin 1/2 and most particles associated with a force, such as the photon, have spin 1. The Higgs boson (the particle of the Higgs field) has spin 0. Having 0 spin enables the Higgs field to appear in the Lagrangian in different ways than the other particles do, which in turn allows—and leads to—it's other two distinguishing features. The second unique property of the Higgs field explains how and why it has nonzero strength throughout the universe. Any system, including a universe, will tumble into its lowest energy state, like a ball bouncing down to the bottom of a valley. For the familiar fields, such as the electromagnetic fields that give us radio broadcasts, the lowest energy state is the one in which the fields have zero value (that is, the fields vanish)—if any nonzero field is introduced, the energy stored in the fields
increases the net energy of the system. But for the Higgs field, the energy of the universe is lower if the field is not zero but instead has a constant nonzero value. In terms of the valley metaphor, for ordinary fields the valley floor is at the location of zero fields; for the Higgs, the valley has a hillock at its center (at zero fields) and the lowest point of the valley forms a circle around the hillock. The universe, like a ball, comes to rest somewhere on this circular trench, which corresponds to a nonzero value of the field. That is, in its natural, lowest energy state, the universe is permeated throughout by a nonzero Higgs field.

The final distinguishing characteristic of the Higgs field is the form of its interactions with the other particles. Particles that interact with the Higgs field behave as if they have mass, proportional to the strength of the field times the strength of the interaction. The masses arise from the terms in the Lagrangian that have the particles interacting with the Higgs field.

Our understanding of all this is not yet complete, however, and we are not sure how many kinds of Higgs fields there are. Although the Standard Model requires only one Higgs field to generate all the elementary particle masses, physicists know that the Standard Model must be superseded by a more complete theory. Leading contenders are extensions of the Standard Model known as Supersymmetric Standard Models (SSMs). In these models, each Standard Model particle has a so-called super partner (as yet undetected) with closely related properties [see "The Dawn of Physics beyond the Standard Model," by Gordon Kane; Scientific American, June 2003]. With the Supersymmetric Standard Model, at least two different kinds of Higgs fields are needed. Interactions with those two fields give mass to the Standard Model particles. They also give some (but not all) mass to the super partners. The two Higgs fields give rise to five species of Higgs boson: three that are electrically neutral and two that are charged. The masses of particles called neutrinos, which are tiny compared with other particle masses, could arise rather indirectly from these interactions or from yet a third kind of Higgs field.

Theorists have several reasons for expecting the SSM picture of the Higgs interaction to be correct. First, without the Higgs mechanism, the W and Z bosons that mediate the weak force would be massless, just like the photon (which they are related to), and the weak interaction would be as strong as the electromagnetic one. Theory holds that the Higgs mechanism confers mass to the W and Z in a very special manner. Predictions of that approach (such as the ratio of the W and Z masses) have been confirmed experimentally.

Second, essentially all other aspects of the Standard Model have been well tested, and with such a detailed, interlocking theory it is difficult to change one part (such as the Higgs) without affecting the rest. For example, the analysis of precision measurements of W and Z boson properties led to the accurate prediction of the top quark mass before the top quark had been directly produced. Changing the Higgs mechanism would spoil that and other successful predictions.

Third, the Standard Model Higgs mechanism works very well for giving mass to all the Standard Model particles, W and Z bosons, as well as quarks and leptons; the alternative proposals usually do not. Next, unlike the other theories, the SSM provides a framework to unify our understanding of the forces of nature. Finally, the SSM can explain why the energy "valley" for the universe has the shape needed by the Higgs mechanism. In the basic Standard Model the shape of the valley has to be put in as a postulate, but in the SSM that shape can be derived mathematically.

Testing the Theory

Naturally, physicists want to carry out direct tests of the idea that mass arises from the interactions with the different Higgs fields. We can test three key features. First, we can look for the signature particles called Higgs bosons. These quanta must exist, or else the explanation is not right. Physicists are
currently looking for Higgs bosons at the Tevatron Collider at Fermi National Accelerator Laboratory in Batavia, Ill. Second, once they are detected we can observe how Higgs bosons interact with other particles. The very same terms in the Lagrangian that determine the masses of the particles also fix the properties of such interactions. So we can conduct experiments to test quantitatively the presence of interaction terms of that type. The strength of the interaction and the amount of particle mass are uniquely connected. Third, different sets of Higgs fields, as occur in the Standard Model or in the various SSMs, imply different sets of Higgs bosons with various properties, so tests can distinguish these alternatives, too. All that we need to carry out the tests are appropriate particle colliders--ones that have sufficient energy to produce the different Higgs bosons, sufficient intensity to make enough of them and very good detectors to analyze what is produced.

A practical problem with performing such tests is that we do not yet understand the theories well enough to calculate what masses the Higgs bosons themselves should have, which makes searching for them more difficult because one must examine a range of masses. A combination of theoretical reasoning and data from experiments guides us about roughly what masses to expect. The Large Electron-Positron Collider (LEP) at CERN, the European laboratory for particle physics near Geneva, operated over a mass range that had a significant chance of including a Higgs boson. It did not find one--although there was tantalizing evidence for one just at the limits of the collider's energy and intensity--before it was shut down in 2000 to make room for constructing a newer facility, CERN's Large Hadron Collider (LHC). The Higgs must therefore be heavier than about 120 proton masses. Nevertheless, LEP did produce indirect evidence that a Higgs boson exists: experimenters at LEP made a number of precise measurements, which can be combined with similar measurements from the Tevatron and the collider at the Stanford Linear Accelerator Center. The entire set of data agrees well with theory only if certain interactions of particles with the lightest Higgs boson are included and only if the lightest Higgs boson is not heavier than about 200 proton masses. That provides researchers with an upper limit for the mass of the Higgs boson, which helps focus the search.

The LEP collider saw tantalizing evidence for the Higgs particle.

For the next few years, the only collider that could produce direct evidence for Higgs bosons will be the Tevatron. Its energy is sufficient to discover a Higgs boson in the range of masses implied by the indirect LEP evidence, if it can consistently achieve the beam intensity it was expected to have, which so far has not been possible. In 2007 the LHC, which is seven times more energetic and is designed to have far more intensity than the Tevatron, is scheduled to begin taking data. It will be a factory for Higgs bosons (meaning it will produce many of the particles a day). Assuming the LHC functions as planned, gathering the relevant data and learning how to interpret it should take one to two years. Carrying out the complete tests that show in detail that the interactions with Higgs fields are providing the mass will require a new electron-positron collider in addition to the LHC (which collides protons) and the Tevatron (which collides protons and antiprotons).

**Dark Matter**

What is discovered about Higgs bosons will not only test whether the Higgs mechanism is indeed providing mass, it will also point the way to how the Standard Model can be extended to solve problems such as the origin of dark matter. With regard to dark matter, a key particle of the SSM is the lightest super partner (LSP). Among the super partners of the known Standard Model particles predicted by the SSM, the LSP is the one with the lowest mass. Most super partners decay promptly to lower-mass super partners, a chain of decays that ends with the LSP, which is stable because it has no lighter particle that it can decay into. (When a super partners decays, at least one of the decay products should be another super partner; it should not decay entirely into Standard Model particles.) Superpartner particles would have been created early in the big bang but then promptly decayed into LSPs. The LSP
is the leading candidate particle for dark matter. The Higgs bosons may also directly affect the amount of dark matter in the universe. We know that the amount of LSPs today should be less than the amount shortly after the big bang, because some would have collided and annihilated into quarks and leptons and photons, and the annihilation rate may be dominated by LSPs interacting with Higgs bosons. As mentioned earlier, the two basic SSM Higgs fields give mass to the Standard Model particles and some mass to the super partners, such as the LSP. The super partners acquire more mass via additional interactions, which may be with still further Higgs fields or with fields similar to the Higgs. We have theoretical models of how these processes can happen, but until we have data on the super partners themselves we will not know how they work in detail. Such data are expected from the LHC or perhaps even from the Tevatron.

Neutrino masses may also arise from interactions with additional Higgs or Higgs-like fields, in a very interesting way. Neutrinos were originally assumed to be massless, but since 1979 theorists have predicted that they have small masses, and over the past decade several impressive experiments have confirmed the predictions [see "Solving the Solar Neutrino Problem," by Arthur B. McDonald, Joshua R. Klein and David L. Wark; Scientific American, April 2003]. The neutrino masses are less than a millionth the size of the next smallest mass, the electron mass. Because neutrinos are electrically neutral, the theoretical description of their masses is more subtle than for charged particles. Several processes contribute to the mass of each neutrino species, and for technical reasons the actual mass value emerges from solving an equation rather than just adding the terms. Thus, we have understood the three ways that mass arises: The main form of mass we are familiar with--that of protons and neutrons and therefore of atoms--comes from the motion of quarks bound into protons and neutrons. The proton mass would be about what it is even without the Higgs field. The masses of the quarks themselves, however, and also the mass of the electron, are entirely caused by the Higgs field. Those masses would vanish without the Higgs. Last, but certainly not least, most of the amount of super partner masses, and therefore the mass of the dark matter particle (if it is indeed the lightest super partner), comes from additional interactions beyond the basic Higgs one. Finally, we consider an issue known as the family problem. Over the past half a century physicists have shown that the world we see, from people to flowers to stars, is constructed from just six particles: three matter particles (up quarks, down quarks and electrons), two force quanta (photons and gluons), and Higgs bosons--a remarkable and surprisingly simple description. Yet there are four more quarks, two more particles similar to the electron, and three neutrinos. All are very short-lived or barely interact with the other six particles. They can be classified into three families: up, down, electron neutrino, electron; charm, strange, muon neutrino, muon; and top, bottom, tau neutrino, tau. The particles in each family have interactions identical to those of the particles in other families. They differ only in that those in the second family are heavier than those in the first, and those in the third family are heavier still. Because these masses arise from interactions with the Higgs field, the particles must have different interactions with the Higgs field. Hence, the family problem has two parts: Why are there three families when it seems only one is needed to describe the world we see? Why do the families differ in mass and have the masses they do? Perhaps it is not obvious why physicists are astonished that nature contains three almost identical families even if one would do. It is because we want to fully understand the laws of nature and the basic particles and forces. We expect that every aspect of the basic laws is a necessary one. The goal is to have a theory in which all the particles and their mass ratios emerge inevitably, without making ad hoc assumptions about the values of the masses and without adjusting parameters. If having three families is essential, then it is a clue whose significance is currently not understood.

**COSMIC STRING TO TIE UP:**

The standard model and the SSM can accommodate the observed family structure, but they cannot explain it. This is a strong statement. It is not that the SSM has not yet explained the family structure but that it cannot. For me, the most exciting aspect of string theory is not only that it may provide us
with a quantum theory of all the forces but also that it may tell us what the elementary particles are and why there are three families. String theory seems able to address the question of why the interactions with the Higgs field differ among the families. In string theory, repeated families can occur, and they are not identical. Their differences are described by properties that do not affect the strong, weak, electromagnetic or gravitational forces but that do affect the interactions with Higgs fields, which fits with our having three families with different masses. Although string theorists have not yet fully solved the problem of having three families, the theory seems to have the right structure to provide a solution. String theory allows many different family structures, and so far no one knows why nature picks the one we observe rather than some other [see "The String Theory Landscape," by Raphael Bousso and Joseph Polchinski; Scientific American, September 2004]. Data on the quark and lepton masses and on their super partner masses may provide major clues to teach us about string theory. One can now understand why it took so long historically to begin to understand mass. Without the Standard Model of particle physics and the development of quantum field theory to describe particles and their interactions, physicists could not even formulate the right questions. Whereas the origins and values of mass are not yet fully understood, it is likely that the framework needed to understand them is in place. Mass could not have been comprehended before theories such as the Standard Model and its supersymmetric extension and string theory existed. Whether they indeed provide the complete answer is not yet clear, but mass is now a routine research topic in particle physics.

SOME TIME in the next few years a great discovery will be unveiled, with appropriate fanfare. The headlines will read "ORIGIN OF MASS DISCOVERED". Many readers will be blown away; many will be cynical. Some will scratch their heads and wonder, what do these words actually mean? One doesn't normally think of mass as something with an origin. But a wise and happy few will be prepared. They will leaf through their treasured back issues of New Scientist, fish out the right one, and refresh their memories. Welcome back! What will have been discovered is a new kind of heavy, highly unstable particle, the so-called Higgs particle. And we might see it in just a few months, at one of two high-energy accelerators: the Large Electron Positron collider (LEP) at CERN near Geneva or the Tevatron at Batavia, Illinois. The Higgs is more than just another expensive, highly unstable particle: it embodies the mechanism that gives other fundamental particles mass. But isn't mass just a fact of life? Not necessarily. In fact, ours would be a much simpler world if particles didn't have mass. For one thing, mass disfigures the theory of the weak nuclear force. The weak force, as befits its name, is much weaker than the strong force which holds atomic nuclei together and the electromagnetic force that holds atoms together. But it does things that no other interaction can: it causes the slow decay of various otherwise stable particles, and it is the only interaction aware of neutrinos. So what's the problem? Well, the existence of mass means that particles feeling the weak force don't all spin in the same way (see "Messy mass"). It would be neater if they did. That is merely untidiness; but there is another, more disturbing problem with the particles that carry the weak force. All forces in nature work by the action of such carrier particles; photons carry the electromagnetic force, for example. And in 1954, Chen Ning Yang and Robert Mills hypothesized the existence of particles called vector mesons, generalized versions of the photon, which looked like good candidates to carry the weak nuclear force. Then in 1961 Sheldon Glashow used them in a theory that unified weak and electromagnetic forces. According to this theory, vector mesons are massless, like the photon. But unlike electromagnetism, the weak force is short-ranged, a sign that its carrier particles must have mass. To fix this, Glashow fudged the equations by just sticking in a mass, without understanding where it came from.

Cosmic molasses

It would be easier, then, to understand an imaginary world with only massless particles, forever whizzing around at the speed of light. But we know that in our world particles do have mass. So to get from that ideal world to ours, we need some kind of cosmic molasses that fills all space and slows down these massless speed demons. But if this molasses is everywhere, why can't we see it?
To understand, imagine you're living in a bar magnet. An ordinary magnet is really an extraordinary thing. For whereas the laws of physics do not have a preferred direction, the magnet does: its pole. Where does this direction come from? Each electron in any material acts as a small magnet, pointing in the direction of its spin axis. An isolated electron would be equally happy with its spin in any direction, indifference that we call rotational symmetry. But in some materials, such as iron, neighboring electrons prefer to point in the same direction. Like insecure teenagers, they don't care what they are doing, as long as they are all doing the same thing. So to make all the electrons happy or, in more dignified language, to obtain the configuration of minimum energy, all the spins have to pick a common direction—it doesn't matter which. That direction defines the magnetic pole.

The rotational symmetry of an isolated spin is gone, but not forgotten. For if we heat an iron magnet above 870 °C, the spins get enough energy to break free from their neighbors and point in random directions again - the material loses its magnetism. If the iron is later cooled, it will once again become magnetic. But the new pole will usually point in a different direction from the old. And rotational symmetry can reappear in another, subtler way. Give the spins just a little energy, and you can make the preferred spin direction (the local magnetic North) change slowly as a function of location. Configurations in which the preferred direction varies periodically are called spin waves. And just as quantum mechanics parcels up light waves into photons, it parcels these spin waves into particles known as magnons.

Particle swarm

Intelligent creatures living inside a magnet would be used to seeing magnons, but they would have trouble figuring out why magnons exist. Evolution would adapt their senses to ignore the unchanging aspects of their environment. So what we think of as the material of the magnet, they would commonly regard as empty space. And it would seem obvious that there was a preferred direction to space, because everything the creatures experienced would be colored by the pervasive magnetism of their world. Eventually, though, some visionary might imagine the true situation: an underlying set of laws with full rotation symmetry, a symmetry hidden by the spontaneous alignment of spins in the pervading medium. Our visionary would have deduced that the "vacuum" is really a structured medium, explained the existence of magnons, and so become a hero of physics.

This is just what happened on Earth. We have known since the 1930s that our vacuum is really a swarm of short-lived "virtual" particles, appearing and disappearing at random. But where is the organized structure in this melee? The visionaries who first saw it were Yoichiro Nambu and Jeffrey Goldstone. In the early 1960s they noticed a symmetry by which the laws of physics stay the same if certain particles are substituted for others. (It would take an article several times the length of this one to attach proper names and identifiable faces to these particles, and unless you are a very unusual person you would not stay awake to the end. Trust me.) But, just as in the magnet, at low temperature the symmetry is broken: from the symmetrical swarm of virtual particles, one kind condenses out in large numbers. So a preference is formed among the otherwise interchangeable types of particles. Instead of a preferred direction like the magnet, our space has a preferred particle composition.

And this is where the cosmic molasses oozes into our story. In 1966 Peter Higgs of the University of Edinburgh and his co-workers Robert Brout and François Englert of the Free University in Brussels added this idea to the theory of vector mesons. They discovered that when the symmetry breaks, producing a condensate of virtual Higgs particles, the vector mesons become massive. Better still, interactions with the condensate could generate the masses of all the other elementary particles, the quarks and leptons. Nambu and Goldstone had constructed a form of cosmic molasses using particles already known to exist. But this isn't quite enough because it exerts too little drag on the vector mesons, and none at all on the leptons. In 1967, however, Steven Weinberg (and later Abdus Salam) postulated an additional stickier form, and showed how it could give an improved, fudge-free
version of Glashow's weak interaction model. This stickier stuff is what physicists usually mean when they talk about the Higgs condensate.

How can we test this extraordinary conception? We could try to heat up the vacuum, by concentrating a lot of energy in a small space, and watch to see if its symmetry is restored as the condensate evaporates. All particles in this region would become massless. Unfortunately, that will only happen at temperatures approaching 1016 kelvin. Although such temperatures were universal in the early stages of the Big Bang, they are out of reach on Earth for the foreseeable future. The Relativistic Heavy Ion Collider at Brookhaven, New York, due to turn on this summer, will peak at only 1013 kelvin.

**Stir it up**

A much more modest project is feasible, however. Rather than restore symmetry completely, we can stir up the Higgs condensate a bit. This being a quantum world, we can only stir it up in discrete units. The minimal excitation - a ripple in the cosmic molasses - is the Higgs particle. How hard will it be to make this particle? Who gets to taste the joy of discovery depends on the value of the Higgs mass, as does the nature of particle physics. We can already narrow down the range. If the Higgs particle were lighter than 95 gigaelectronvolts (GeV), about 100 times the mass of a proton, LEP would already have seen it. If it were heavier than 600 GeV, virtual Higgs particles would affect many particle reactions in a way that experiments have already ruled out. And the promising theoretical idea of supersymmetry - an extension of the Standard Model that proposes a host of extra fundamental particles, partners of the familiar bunch - predicts masses well below 200 GeV for the Higgs particle; probably between 100 and 130 GeV. That is why so much excitement surrounds the upcoming explorations (see “The Higgs particle cookbook” below). Scientists at LEP will drive their machine to the limits of its energy and luminosity, pushing the mass window up to 105 GeV or so within two years. Meanwhile, scientists at the Tevatron hope to explore all the way up to 160 GeV. If they fail, then a final effort will be made at the Large Hadron Collider (LHC) being constructed in Geneva due to open around 2005. Its reach extends beyond 600 GeV. If that fails, we theoretical physicists will be exceedingly embarrassed, and I hesitate to predict what we'll do. The Standard Model requires just one Higgs particle. But theories with more symmetry imply several new particles -- Higgs galore. The theory of super symmetry predicts at least five Higgs-type particles. In the most popular version, the lightest member of the Higgs family has the properties we discussed above. There is no consensus on the masses of the others, although they should not be much heavier than 1000 GeV, and might be much lighter. The masses of these particles will tell us how the supersymmetric partners of ordinary particles hide themselves from us. At present it is a big mystery, and wild concepts are in the air, including their infection by otherwise inaccessible “dark” matter, or exotic condensates living only in extra dimensions of space. The LHC should shed light on this mystery. More ambitious models that unify the strong and electroweak forces predict a bizarre tribe of very much heavier Higgs particles. We probably won't be able to make them directly anytime soon, but we might sense the effect of their exchange as virtual particles. Some of them can make protons decay, at rates close to current experimental limits’ cosmic Remember the future headline, trumpeting the discovery of the origin of mass? Honesty compels me to call the headline writer to task: the statement is not entirely misleading, but it's literally false. Actually the lion's share of ordinary mass, in protons and neutrons, has nothing to do with Higgs particles. It comes instead from the energy of the gluon field that holds their constituent quarks together. Intrigued? Write to the editors, demanding a follow-up article.

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**Messy mass**

ONCE, we thought that the fundamental laws of physics made no distinction between left and right - for
any behaviour you can observe in the real world, its mirror image can also happen. So if you filmed the real world and its reflected image, someone watching the movies later wouldn't be able to tell which was which. This is called parity symmetry. Then in 1956 Tsao-Dai Lee at Columbia University, New York, and Chen Ning Yang at the Institute for Advanced Study, Princeton, suggested that the weak interaction breaks parity symmetry. They turned out to be right. For example, neutrons decay through the weak interaction into protons, electrons and electron antineutrinos. The electrons emitted in this decay are moving at nearly the speed of light, and they are also spinning. About 98 per cent of them are left-handed, meaning that if you pointed the thumb of your left hand in the direction of its motion your fingers would curl in the direction the electron spins. This bias violates parity symmetry, because it distinguishes left from right. So the weak interaction likes left-handed particles (electrons, muons, quarks and so on) and right-handed antiparticles. But, irritatingly, this seems to be no more than a rule of thumb. Weak processes involving right-handed particles or left-handed antiparticles are rare, but not absolutely nonexistent.

Political heavyweights

In 1993, researchers were challenged by William Waldegrave, then Britain's science minister, to come up with a concise description of the Higgs particle. For a complete list of the winners and their explanations see The Waldegrave Higgs Challenge. David Miller from University College London won a bottle of champagne for this “quasi-political explanation”.

Imagine a cocktail party of political party workers who are uniformly distributed across the floor, all talking to their nearest neighbors. The ex-prime minister enters and crosses the room. All of the workers in her neighborhood are strongly attracted to her and cluster round her. As she moves she attracts the people she comes close to, while the ones she has left return to their even spacing. Because of the knot of people always clustered around her she acquires a greater mass than normal, that is, she has more momentum for the same speed of movement across the room. Once moving she is harder to stop, and once stopped she is harder to get moving again because the clustering process has to be restarted. In three dimensions, and with the complications of relativity, this is the Higgs mechanism. In order to give particles mass, a background field is invented which becomes locally distorted whenever a particle moves through it. The distortion—the clustering of the field around the particle—generates the particle's mass.

The idea comes directly from the physics of solids. Instead of a field spread throughout all space a solid contains a lattice of positively charged crystal atoms. When an electron moves through the lattice the atoms are attracted to it, causing the electron's effective mass to be as much as 40 times bigger than the mass of a free electron. The postulated Higgs field in the vacuum is a sort of hypothetical lattice which fills our Universe. We need it because otherwise we cannot explain why the Z and W particles which carry the Weak Interactions are so heavy while the photon which carries electromagnetic forces is massless. Now consider a rumor passing through our room full of uniformly spread political workers. Those near the door hear of it first and cluster together to get the details, then they turn and move closer to their next neighbors who want to know about it too. A wave of clustering passes through the room. It may spread out to all the corners, or it may form a compact bunch which carries the news along a line of workers from the door to some dignitary at the other side of the room. Because the information is carried by clusters of people, and since it was clustering which gave extra mass to the ex-prime minister, then the rumor-carrying clusters also have mass. The Higgs particle is predicted to be just such a clustering in the Higgs field. We will find it much easier to believe that the field exists, and that the mechanism for giving other particles mass is true, if we actually see the Higgs particle itself. Again, there are analogies in the physics of solids. A crystal lattice can carry waves of clustering without needing an electron to move and attract the atoms. These waves can behave as if they are particles. They are called phonons. There could be a Higgs mechanism, and a Higgs field throughout our Universe, without there being a Higgs particle. The next generation of colliders will sort this out.
Physically, if you look at the low level of CMB anisotropy and the BAO power spectrum, you need some sort of mass that interacts gravitationally but is decoupled from the photons before recombination (i.e. - dark matter). Otherwise there would be no way to seed the galaxy formation that we see occurring at lower redshifts. That is to say, dark matter needs to be primordial, so that it can dodge Silk Damping and provide the gravitational "cores" around which galaxies form.

AFFECTIVE CONSCIOUSNESS:

Affect consciousness (Monsen, Monsen, Solbakken & Hansen, 2008) refers to the mutual relationship between activation of basic affective experiences and the individual's capacity to consciously perceive, tolerate, reflect upon and express these experiences (Monsen & Monsen, 1999; Solbakken, Hansen, Havik & Monsen, 2011). AC is traditionally operationalized as degrees of awareness, tolerance, nonverbal and conceptual expression for each of, in the newest revision of the model, eleven basic affects, and measured by the semi-structured Affect Consciousness Interview (ACI).

A vast array of ideas from a number of different approaches can be emphasized as inspirations for the development of the AC construct (Solbakken, Hansen & Monsen, 2011), most notably Silvan Tomkins' Affect theory, Script Theoretical formulations by the same author (Tomkins, 2008a, 2008b), Discrete Emotions Theory (Izard, 1976; 1993). The writings of Daniel Stern (1986) and the seminal studies by Emde (1983; 1996) on nonverbal affective communication with infants are also central, along with modern self psychological formulations as those advocated by Stolorow, Brandshaft & Atwood (1987), Stolorow & Atwood (1992) and Lichtenberg et al. (1992, 1996).

The affect consciousness construct posits that affect, along with pain and the cyclical drives constitute the primary motivating force in all human affairs (Solbakken, Hansen & Monsen, 2011). Of these motivating forces the affect system is the primary, and by far the most flexible (Tomkins, 2008a), it is hypothesized that affect is ever present in the organism, continually shaping and codetermining both the qualia of consciousness and the behavior initiated or terminated.

It is assumed that affect activation structures not only behavior, but also the coherence of experience, so that affect that is void of meaning to the individual constitute the core of meaningless and disorganized experience. Furthermore central importance is given to the function of affect expression as a primary channel of communication: The transactions of affect-displays among humans are regarded denser in information than any verbal communication. Importantly attunement of affective communication between child and caregiver is seen as paramount to the acquisition of coherent self experience and attainment of adequate affect integration and regulation in the course of development (Solbakken, Hansen & Monsen, 2011).

The affect system is viewed as an independent, evolutionary early, response apparatus developed specifically for adaptive purposes. This system is hypothesized to interact and transact with the other major adaptive systems of the human organism; drive, motor, perceptual, sensory, memory and cognitive, but is seen as the central core of direction for those other systems (Tomkins, 2008a). The adaptiveness of these interactions and their organization is the defining feature of affect consciousness.

PARTITION FUNCTION:

Partition function (quantum field theory) (We have used Wikipedia and author home pages and Stanford Encyclopedia for the preparation of the following introductory remarks).

In quantum field theory, we have a generating functional, Z[J] of correlation functions and this value,
called the partition function is usually expressed by something like the following functional integral:

$$Z[J] = \int \mathcal{D}\phi e^{i\{S[\phi] + \int d^4x J(x)\phi(x)\}}$$

Where $S$ is the action functional.

The partition function in quantum field theory is a special case of the mathematical partition function, and is related to the statistical partition function in statistical mechanics. The primary difference is that the countable collection of random variables seen in the definition of such simpler partition functions has been replaced by an uncountable set, thus necessitating the use of integrals over a field $\hat{\phi}$.

The prototypical use of the partition function is to obtain Feynman amplitudes by differentiating with respect to the auxiliary function (sometimes called the current) $J$. Thus, for example:

$$\{G(x_1, x_2)\} = -\left.\frac{\delta}{\delta J(x_1)} \frac{\delta}{\delta J(x_2)} \log Z[J]\right|_{J=0}$$

Is the Green's function, propagator or correlation function for the field $\hat{\phi}$ between points $x_1$ and $x_2$ in space?

**Reactionary Complex valued function:**

Unlike the partition function in statistical mechanics, that in quantum field theory contains an extra factor of $i$ in front of the action, making the integrand complex, not real. It is sometimes mistakenly implied that this has something to do with Wick rotations; this is not so. Rather, the $i$ has to do with the fact that the fields $\hat{\phi}$ are to be interpreted as quantum-mechanical probability amplitudes, taking on values in the complex projective space (complex Hilbert space, but the emphasis is placed on the word projective, because the probability amplitudes are still normalized to one). By contrast, more traditional partition functions involve random variables that are real-valued, and range over a simplex--a simplex, being the geometric way of saying that the total of probabilities sum to one. The factor of can be understood to arise as the Jacobian of the natural measure of volume in complex projective space. For the (highly unusual) situation where the complex-valued probability amplitude is to be replaced by some other field taking on values in some other mathematical, the $i$ would be replaced by the appropriate geometric factor (that is, the Jacobian) for that space.

Statistical mechanics was born at the beginning of the 20th century to provide a microscopic explanation of thermodynamics and then, for many years, it was used to understand the different aggregation states of matter. Quantum field theory instead was born in the context of sub nuclear physics as the basic method to describe the fundamental interactions between the ultimate constituents of matter. These two fields evolved independently for many years. Only at the end of the ‘60s was it realized that statistical mechanics was able to provide the basic tools for a non-perturbative and rigorous definition of quantum field theory. On the other hand, quantum-field-theory concepts like renormalization group and scale invariance found their application in statistical mechanics.
A three-arm polymer

Since then the cross-fertilization of the two fields has provided important methods and tools which have been applied not only in physics but also in several different contexts spanning from computer science, chemistry, biology, geology, and social sciences. Since many years our group is active both in quantum field theory and statistical mechanics, considering several problems which are relevant in elementary particle physics, condensed matter physics, biology, and mathematics, and which can be addressed by combining statistical-mechanics methods, quantum field theory, and the theory of stochastic processes.

In recent years much of our research concerns "random models", a vast class of systems which are the object of intense study in many disciplines — physics, biology, computer science, and mathematics to name a few. They have a complex phenomenology and many of their properties are still not understood. For instance, restricting to physical systems, random systems show a variety of new phenomena — slow relaxation and aging, memory and oblivion, and generalizations of the usual dissipation-fluctuation relations — which have been the object of intense experimental scrutiny. In this field we are extensively studying the critical behavior of several spin models with quenched disorder and phase separation in fluid systems, in particular colloid-polymer mixtures, in porous materials. Beside random systems, we are also investigating, both numerically and analytically, the critical behavior of less-common magnetic systems, some properties of high-Tc superconductors, and the finite-density behavior of solutions of polymers of different architecture. In the following model we concentrate on Quantum Field and Statistical mechanics, Markov fields, Hilbert spaces, Lie algebras and translations in space and time. Expansion of quantum field describing electrons or other fermions are also studied.

A simple neural network

Other subjects under intensive investigation by our group are spin glasses and neural nets. The methods at the basis of their study combine the physical intuition, accumulated through the use of the replica trick and numerical simulations, with the need for a rigorous mathematical treatment. The essential ingredients are given by powerful interpolation methods, and sum rules. These methods led in the past years to the proof of relevant results, in particular concerning the control of the infinite volume limit, and the mechanism of the spontaneous replica symmetry breaking. For the neural nets of Hopfield type many physicists have developed interpolation method which allows the characterization of the replica symmetric approximation, and the possibility of introducing functional order parameters for the description of the replica symmetry breaking.

The variational principle arising in the expression of the free energy in the infinite volume limit is of novel type, in that it involves a mini-max procedure, in contrast with the Sherrington-Kirkpatrick model for a spin glass. The theory of self-oscillating mechanical systems has been exploited for the study of speech formation, analysis and synthesis, and musical instrument functioning. It is possible to apply fully nonlinear schemes, by completely avoiding any kind of exploitation of the Fourier analysis. The role of the different peaks of the spectrum in the Fourier analysis is played by the intervention of successive Landau instability modes for the self-oscillating system. Finally, in recent times, scientists have developed the possibility of giving simple models for the immunological system, based on stochastic dynamical systems of statistical mechanics far from equilibrium. The models are simple enough to allow practical evaluations, in connection with the known phenomenology, but they
are very rich in the possibility of introducing all basic feature of the real system. In this connection mention can be made of the studies done of the quantum field theory formulation of the relativistic Majorana equations, introduced in 1932 in a famous paper on Nuovo Cimento, and the study of slowing down, scattering and absorption of neutrons, original methods of Fermi, Wick, Bothe, Heisenberg, with the purpose of a realistic assessment of the validity of the approximations introduced by them, in comparison with the modern methods of numerical simulations in the nuclear reactor theory.

(1) The Higgs Field is the fundamental quantum arena, quantitative form of means and ends that gives birth to all mass in the universe. In the finding of Higgs Boson, there exists an inferential thought and diversity and essential predications, predicational anteriority, character constitution of primordial exactitude and ontological resonance of the formation of the world itself. AND The particle Higgs Boson

Module Numbered One

NOTATION:

$G_{13}$ : CATEGORY ONE OF The Higgs Field is the fundamental quantum arena, quantitative form of means and ends that gives birth to all mass in the universe. In the finding of Higgs Boson, there exists an inferential thought and diversity and essential predications, predicational anteriority, character constitution of primordial exactitude and ontological resonance of the formation of the world itself

$G_{14}$ : CATEGORY TWO OF THE HIGGS FIELD is the fundamental quantum arena, quantitative form of means and ends that gives birth to all mass in the universe. In the finding of Higgs Boson, there exists an inferential thought and diversity and essential predications, predicational anteriority, character constitution of primordial exactitude and ontological resonance of the formation of the world itself

$G_{15}$ : Category Three of The Higgs Field is the fundamental quantum arena, quantitative form of means and ends that gives birth to all mass in the universe. In the finding of Higgs Boson, there exists an inferential thought and diversity and essential predications, predicational anteriority, character constitution of primordial exactitude and ontological resonance of the formation of the world itself

$T_{13}$ : Category One Of The Particle Higgs Boson (Again Classification Is For The Systems For Which The Theories Are Applicable Or For That Matter Systems Which Violates The Theories Mentioned)

$T_{14}$ : Category Two Of The Particle Higgs Boson (Again Classification Is For The Systems For Which The Theories Are Applicable Or For That Matter Systems Which Violates The Theories Mentioned)

$T_{15}$ : Category Three Of The Particle Higgs Boson (Again Classification Is For The Systems For Which The Theories Are Applicable Or For That Matter Systems Which Violates The Theories Mentioned)

(2) Higgs Quantum Field AND Mass of the matter

MODULE NUMBERED TWO:

In the most widely believed scenario, dark matter is composed of "weakly interacting massive particles" ("WIMP"). The adjective "weak" really means that the particles interact via the weak nuclear force. This pretty much guarantees that they interact with the Higgs boson, too: the WIMPs carry the hypercharge or the weak nuclear charge (that's what "WI" in "WIMP" guarantees). According to current theory, the most likely WIMP is the LSP, the lightest superpartner in a super symmetric theory. The LSP may be a gravitino in which case its interactions with the Higgs bosons are almost non-existent – gravity is the only relevant interaction; or they may be a "neutralino" (higgsino, photino, or zino – or, in a different basis, higgsino, neutral wino, or bino) and this interacts with the Higgs boson as strongly as the known superpartners of the
neutralino, namely the Z-boson or the Higgs boson itself. So if the dark matter is composed of neutralinos, then it interacts with the Higgs boson. However, the Higgs field isn’t the main reason why these particles are massive ("M" in "WIMP"). Instead, the main reason is a supersymmetry breaking mechanism, something independent from the Higgs mechanism which may also be called the electroweak symmetry breaking mechanism. It’s due to SUSY breaking, and not the electroweak symmetry breaking, that the LSP is massive. For example, SUSY breaking is needed to make photino (or bino/wino/neutralino that contains a piece of it) massive even though the photon, its known superpartner, is massless. If the dark matter is made of axions, its interactions with the Higgs field are virtually non-existent. If dark matter is composed of MACHOs or if it doesn’t exist at all, it makes no sense to describe its interactions at the level of particle physics because it is a composite object or because it doesn’t exist. For example, if dark matter were composed of black holes, they would interact like other black holes – pretty much only gravitationally.

\[ G_{16} : \text{Category One Of Lie Algebra Of The Groups} \]
\[ G_{17} : \text{Category Two Of Lie Algebra Of The Groups} \]
\[ G_{18} : \text{Category Three Of Lie Algebra Of The Groups} \]
\[ T_{16} : \text{Category One Of Translations In Space And Time} \]
\[ T_{17} : \text{Category Two Of Translations In Space And Time} \]
\[ T_{18} : \text{Category Three Of Translations In Space And Time} \]

**Markov Fields And Hilbert Spaces:**

**Module Numbered Three:**

\[ G_{20} : \text{Category One Of Markov Space( There Are Lot Of Markov Spaces And Some Spaces Could Be Approximated To Markov Spaces And Markov Spaces Could Be Made To Lose Their Characteristics And Parameter Under Some Transformations. These Points Are To Be Borne In Mind. We Here Speak Of The Characterized Systems For Which Markov Theory Is Applicable)} \]
\[ G_{21} : \text{Category Two Of Category One Of Markov Space( There Are Lot Of Markov Spaces And Some Spaces Could Be Approximated To Markov Spaces And Markov Spaces Could Be Made To Lose Their Characteristics And Parameter Under Some Transformations. These Points Are To Be Borne In Mind. We Here Speak Of The Characterized Systems For Which Markov Theory Is Applicable)} \]
\[ G_{22} : \text{Category Three Of Category One Of Markov Space( There Are Lot Of Markov Spaces And Some Spaces Could Be Approximated To Markov Spaces And Markov Spaces Could Be Made To Lose Their Characteristics And Parameter Under Some Transformations. These Points Are To Be Borne In Mind. We Here Speak Of The Characterized Systems For Which Markov Theory Is Applicable)} \]
\[ T_{20} : \text{Category One Of Hilbert Spaces And Quantum, Fields} \]
\[ T_{21} : \text{Category Two Of Hilbert Spaces And Quantum Fields} \]
\[ T_{22} : \text{Category Three Of Hilbert Spaces And Quantum Fields} \]

**Quantum Field Theory** (Again, Parametric zed Systems To Which Qft Could Be Applied Is Taken In To Consideration And Renormalization Theory (Based On Certain Variables Of The System Which Consequentially Classifiable On Parameters)

241
Module Numbered Four:

$G_{24}$: Category One Of Quantum Field Theory (Evaluative Parametricization Of Situational Orientations And Essential Cognitive Orientation And Choice Variables Of The System To Which QFT Is Applicable)

$G_{25}$: Category Two Of Quantum Field Theory

$G_{26}$: Category Three Of Quantum Field Theory

$T_{24}$: Category One Of Renormalization Theory

$T_{25}$: Category Two Of Renormalization Theory (Systemic Instrumental Characterisations And Action Orientations And Functional Imperatives Of Change Manifested Therein)

$T_{26}$: Category Three Of Quantum Field Theory

Quantum Connections Due To Due To Electron Positron Pairs And Linear Corrections To Maxwell's Equations:

Module Numbered Five:

$G_{28}$: Category One Of Quantum Corrections Due To Virtual Electron-Positron Pairs

$G_{29}$: Category Two Of Quantum Corrections Due To Virtual Electron-Positron Pairs

$G_{30}$: Category Three Of Quantum Corrections Due To Virtual Electron-Positron Pairs

$T_{28}$: Category One Of Linear Corrections In Maxwell's Equation Applicable To Various Systems With Defined Characteristics And The Concomitant Correction Factor. Classification Is Based On The Parametricization Of The Systems Even Though To Some Systems Linear Corrections To Maxwell's Equations Are The Same

$T_{29}$: Category Two Of Linear Corrections In Maxwell's Equation Applicable To Various Systems With Defined Characteristics And The Concomitant Correction Factor. Classification Is Based On The Parametricization Of The Systems Even Though To Some Systems Linear Corrections To Maxwell's Equations Are The Same

$T_{30}$: Category Three Of Linear Corrections In Maxwell's Equation Applicable To Various Systems With Defined Characteristics And The Concomitant Correction Factor. Classification Is Based On The Parametricization Of The Systems Even Though To Some Systems Linear Corrections To Maxwell's Equations Are The Same

Asymmetric Freedom Decisive Regularities In The Case When Sobolev Inequalities Are On The Borderline Of Transition:

Module Numbered Six:

$G_{32}$: Category One Of Decisive Regularities In Respect Of Cases Wherein Sobolev Inequalities Are In The Borderline Phase Transition

$G_{33}$: Category Two Of Decisive Regularities With Respect To Cases Wherein Sobolev Inequalities
Are In Border Line Transition

\( G_{34} \): Category Three Of Decisive Regularities With Respect Top Systemic Cases In Regard To Which Sobolev Inequalities Are In Border Line Transition

\( T_{32} \): Category One Of Asymptotic Freedom

\( T_{33} \): Category Two Of Asymptotic Freedom

\( T_{34} \): Category Three Of Asymptotic Freedom

**Expansion Rate Of Quantum Field Describing Electrons And Other Fermions And Creation And Annihilation Of Operators according To Pauli’s Exclusion Principle (Again We Talk Of The Systemic Characteristics To Which Pauli’s Exclusion Principle Is Applied, And The Parametricization And Stratification Follows)**

**Module Numbered Seven**

\( G_{36} \): Category One Of Creation And Annihilation Operators Due To Pauli’s Exclusion Principle (Note Pauli’s Exclusion Principle Denies The Two States For The Electrons)

\( G_{37} \): Category Two Of Creation And Annihilation Operators Due To Pauli’s Exclusion Principle (Note Pauli’s Exclusion Principle Denies The Two States For The Electrons)

\( G_{38} \): Category Three Of Orthogonal Energy State Of Vacuum (Energy Excitation Of The Vacuum And Concomitant Generation Of Energy Differential-Time Lag Or Instantaneousness might Exists Whereby Accentuation And Attritions Model May Assume Zero Positions)

\( T_{36} \): Category One Of Expansion Rates Of Quantum Fields Describing Electrons and Other Fermions

\( T_{37} \): Category Two Of Expansion Rates Of Quantum Fields Describing Electrons and Other Fermions

\( T_{38} \): Category Three Of Expansion Rates Of Quantum Fields Describing Electrons and Other Fermions

**Texture Zero Of Neutrinos Preservation And Renormalization**

**Module Numbered Eight**

\( G_{40} \): Category one of Neutrinos With Zero Texture (There May Or Might Not Be Neutrinos Without Texture And There Are Lot Of Neutrinos As There Are Lot Of Leptions, Photons, Electrons In The Universe)

\( G_{41} \): Category two of Neutrinos With Zero Texture (There May Or Might Not Be Neutrinos Without Texture And There Are Lot Of Neutrinos As There Are Lot Of Leptions, Photons, Electrons In The Universe)

\( G_{42} \): Category three of Neutrinos With Zero Texture (There May Or Might Not Be Neutrinos Without Texture And There Are Lot Of Neutrinos As There Are Lot Of Leptions, Photons, Electrons In The Universe)
$T_{40}$: Category one of Renormalization

$T_{41}$: Category Two of Renormalization

$T_{42}$: Category Three of Renormalization

\[(a_{13}^{(1)}, a_{14}^{(1)}, a_{15}^{(1)}, b_{14}^{(1)}, b_{15}^{(1)}, b_{16}^{(1)}, a_{17}^{(1)}, a_{18}^{(1)}, a_{19}^{(2)}, b_{16}^{(2)}, b_{17}^{(2)}, b_{18}^{(2)}, a_{20}^{(3)}, a_{21}^{(3)}, a_{22}^{(3)}, b_{22}^{(3)}, b_{23}^{(3)}, b_{25}^{(3)}, a_{24}^{(4)}, a_{25}^{(4)}, a_{26}^{(4)}, b_{26}^{(4)}, b_{27}^{(4)}, b_{28}^{(4)}, b_{29}^{(5)}, b_{30}^{(5)}, a_{28}^{(5)}, a_{29}^{(5)}, a_{30}^{(5)}, a_{32}^{(6)}, a_{33}^{(6)}, a_{34}^{(6)}, b_{32}^{(6)}, b_{33}^{(6)}, b_{34}^{(6)}\]

are Accentuation coefficients

\[(a_{13}^{(1)}, a_{14}^{(1)}, a_{15}^{(1)}, b_{14}^{(1)}, b_{15}^{(1)}, b_{16}^{(1)}, b_{17}^{(1)}, b_{18}^{(1)}, a_{16}^{(2)}, a_{17}^{(2)}, a_{18}^{(2)}, b_{18}^{(2)}, b_{19}^{(2)}, b_{20}^{(2)}, a_{20}^{(3)}, a_{21}^{(3)}, a_{22}^{(3)}, b_{22}^{(3)}, b_{23}^{(3)}, b_{24}^{(3)}, a_{24}^{(4)}, a_{25}^{(4)}, b_{25}^{(4)}, b_{26}^{(4)}, b_{27}^{(4)}, b_{28}^{(4)}, b_{29}^{(5)}, b_{30}^{(5)}, a_{28}^{(5)}, a_{29}^{(5)}, a_{30}^{(5)}, a_{32}^{(6)}, a_{33}^{(6)}, a_{34}^{(6)}, b_{32}^{(6)}, b_{33}^{(6)}, b_{34}^{(6)}\]

are Dissipation coefficients

Quantum Field Theory And Statistical Mechanics (We Are Talking Of Systems To Which Theory Is Applicable And Holds. Bank Example Of Assets Being Equivalent To Liabilities And That Each Individual Debits And Credits Being Conservative In Addition To The Conservativeness Of Holistic System Is To Be Mentioned), Markov Field And Hilbert Space:

Module Numbered One

The differential system of this model is now (Module Numbered one)

\[
\begin{align*}
\frac{dg_{13}}{dt} &= (a_{13}^{(1)})G_{14} - [(a_{13}^{(1)} + (a_{14}^{(1)}(T_{14}, t))]G_{13} \\
\frac{dg_{14}}{dt} &= (a_{14}^{(1)})G_{13} - [(a_{13}^{(1)} + (a_{14}^{(1)}(T_{14}, t))]G_{14} \\
\frac{dg_{15}}{dt} &= (a_{15}^{(1)})G_{14} - [(a_{15}^{(1)} + (a_{14}^{(1)}(T_{14}, t))]G_{15} \\
\frac{dt_{13}}{dt} &= (b_{13}^{(1)})T_{14} - [(b_{13}^{(1)} - (b_{14}^{(1)}(G, t))]T_{13} \\
\frac{dt_{14}}{dt} &= (b_{14}^{(1)})T_{14} - [(b_{14}^{(1)} - (b_{14}^{(1)}(G, t))]T_{14} \\
\frac{dt_{15}}{dt} &= (b_{15}^{(1)})T_{14} - [(b_{15}^{(1)} - (b_{14}^{(1)}(G, t))]T_{15} \\
+(a_{13}^{(1)}(T_{14}, t) &= \text{First augmentation factor} \\
-(b_{14}^{(1)}(G, t) &= \text{First detritions factor}
\end{align*}
\]

LIE ALGEBRA OF THE GROUPS AND TRANSLATIONS IN SPACE TIME

Module Numbered Two

The differential system of this model is now (Module numbered two)

\[
\begin{align*}
\frac{dg_{16}}{dt} &= (a_{16}^{(2)})G_{17} - [(a_{16}^{(2)} + (a_{17}^{(2)}(T_{17}, t))]G_{16} \\
\frac{dg_{17}}{dt} &= (a_{17}^{(2)})G_{16} - [(a_{17}^{(2)} + (a_{17}^{(2)}(T_{17}, t))]G_{17}
\end{align*}
\]
\[
\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[ (a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t) \right] G_{18} \]

\[
\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[ (b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}, t) \right] T_{16} \]

\[
\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[ (b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}, t) \right] T_{17} \]

\[
\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[ (b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}, t) \right] T_{18} \]

\[
+ (a''_{19})^{(2)}(T_{17}, t) = \text{First augmentation factor} \]

\[
- (b''_{19})^{(2)}(G_{19}, t) = \text{First detritions factor} \]

**MARKOV FIELD AND HILBERT SPACE:**

**MODULE NUMBERED THREE**

The differential system of this model is now (Module numbered three)

\[
\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[ (a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t) \right] G_{20} \]

\[
\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[ (a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t) \right] G_{21} \]

\[
\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[ (a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t) \right] G_{22} \]

\[
\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[ (b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) \right] T_{20} \]

\[
\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[ (b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t) \right] T_{21} \]

\[
\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[ (b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t) \right] T_{22} \]

\[
+ (a''_{23})^{(3)}(T_{21}, t) = \text{First augmentation factor} \]

\[
- (b''_{23})^{(3)}(G_{23}, t) = \text{First detritions factor} \]

**QUANTUM FIELD THEORY (AGAIN, PARAMETRICIZED SYSTEMS TO WHICH QFT COULD BE APPLIED IS TAKEN INTO CONSIDERATION AND RENORMALIZATION THEORY (BASED ON CERTAIN VARIABLES OF THE SYSTEM WHICH CONSEQUENTIALLY CLASSIFIABLE ON PARAMETERS))**

**MODULE NUMBERED FOUR**

The differential system of this model is now (Module numbered Four)

\[
\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[ (a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t) \right] G_{24} \]

\[
\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[ (a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t) \right] G_{25} \]

\[
\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[ (a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t) \right] G_{26} \]

\[
\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[ (b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}, t) \right] T_{24} \]

\[
\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[ (b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}, t) \right] T_{25} \]
\[
\frac{dT_{26}}{dt} = (b_{26})^{(4)} T_{25} - \left[ (b_{26}')^{(4)} - (b_{26}'')^{(4)} \right] G_{27}, t \]

\[
+ (a''_{26})^{(4)} (T_{25}, t) = \text{First augmentation factor} \\
- (b''_{26})^{(4)} (G_{27}, t) = \text{First detritions factor}
\]

**QUANTUM CORRECTIONS DUE TO ELECTRON POSITRON PAIRS AND LINEAR CORRECTIONS TO MAXWELL’S EQUATIONS:**

**MODULE NUMBERED FIVE:**

The differential system of this model is now (Module number five)

\[
\frac{dG_{28}}{dt} = (a_{28})^{(5)} G_{29} - \left[ (a_{28}')^{(5)} + (a_{28}'')^{(5)} (T_{29}, t) \right] G_{28}
\]

\[
\frac{dG_{29}}{dt} = (a_{29})^{(5)} G_{28} - \left[ (a_{29}')^{(5)} + (a_{29}'')^{(5)} (T_{29}, t) \right] G_{29}
\]

\[
\frac{dG_{30}}{dt} = (a_{30})^{(5)} G_{29} - \left[ (a_{30}')^{(5)} + (a_{30}'')^{(5)} (T_{29}, t) \right] G_{30}
\]

\[
\frac{dT_{28}}{dt} = (b_{28})^{(5)} T_{29} - \left[ (b_{28}')^{(5)} - (b_{28}'')^{(5)} \right] G_{31}, t \]

\[
\frac{dT_{29}}{dt} = (b_{29})^{(5)} T_{28} - \left[ (b_{29}')^{(5)} - (b_{29}'')^{(5)} \right] G_{31}, t \]

\[
\frac{dT_{30}}{dt} = (b_{30})^{(5)} T_{29} - \left[ (b_{30}')^{(5)} - (b_{30}'')^{(5)} \right] G_{31}, t \]

\[
+ (a''_{28})^{(5)} (T_{29}, t) = \text{First augmentation factor} \\
- (b''_{28})^{(5)} (G_{31}, t) = \text{First detritions factor}
\]

**ASYMMETRIC FREEDOM DECISIVE REGULARITIES IN THE CASE WHEN SOBOLEV INEQUALITIES ARE ON THE BORDER LINE OF TRANSITION:**

**MODULE NUMBERED SIX**

The differential system of this model is now (Module numbered Six)

\[
\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} - \left[ (a_{32}')^{(6)} + (a_{32}'')^{(6)} (T_{33}, t) \right] G_{32}
\]

\[
\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \left[ (a_{33}')^{(6)} + (a_{33}'')^{(6)} (T_{33}, t) \right] G_{33}
\]

\[
\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \left[ (a_{34}')^{(6)} + (a_{34}'')^{(6)} (T_{33}, t) \right] G_{34}
\]

\[
\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \left[ (b_{32}')^{(6)} - (b_{32}'')^{(6)} \right] G_{35}, t \]

\[
\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \left[ (b_{33}')^{(6)} - (b_{33}'')^{(6)} \right] G_{35}, t \]

\[
\frac{dT_{34}}{dt} = (b_{34})^{(6)} T_{33} - \left[ (b_{34}')^{(6)} - (b_{34}'')^{(6)} \right] G_{35}, t \]

\[
+ (a''_{32})^{(6)} (T_{33}, t) = \text{First augmentation factor} \\
- (b''_{32})^{(6)} (G_{35}, t) = \text{First detritions factor}
\]

**EXPANSION RATE OF QUANTUM FIELD DESCRIBING ELECTRONS AND OTHER FERMIONS AND**
CREATION AND ANNHIILATION OF OPERATORS ACCORDING TO PAULI’S EXCLUSION PRINCIPLE
(Again we talk of the systemic characteristics to which Pauli’s exclusion principle is applied, and the parametricization and stratification follows)

MODULE NUMBERED SEVEN:

The differential system of this model is now (SEVENTH MODULE)

\[
\frac{dG_{36}}{dt} = (a_{36})^{(7)} G_{36} - \left[ (a_{36}')^{(7)} + (a_{36}'')^{(7)} (T_{37}, t) \right] G_{36} \tag{1127}
\]

\[
\frac{dG_{37}}{dt} = (a_{37})^{(7)} G_{37} - \left[ (a_{37}')^{(7)} + (a_{37}'')^{(7)} (T_{37}, t) \right] G_{37} \tag{1128}
\]

\[
\frac{dG_{38}}{dt} = (a_{38})^{(7)} G_{38} - \left[ (a_{38}')^{(7)} + (a_{38}'')^{(7)} (T_{37}, t) \right] G_{38} \tag{1129}
\]

\[
\frac{dT_{36}}{dt} = (b_{36})^{(7)} T_{36} - \left[ (b_{36}')^{(7)} - (b_{36}'')^{(7)} (G_{39}, t) \right] T_{36} \tag{1130}
\]

\[
\frac{dT_{37}}{dt} = (b_{37})^{(7)} T_{37} - \left[ (b_{37}')^{(7)} - (b_{37}'')^{(7)} (G_{39}, t) \right] T_{37} \tag{1131}
\]

\[
\frac{dT_{38}}{dt} = (b_{38})^{(7)} T_{38} - \left[ (b_{38}')^{(7)} - (b_{38}'')^{(7)} (G_{39}, t) \right] T_{38} \tag{1132}
\]

\[+(a_{36}'')^{(7)} (T_{37}, t) = \text{First augmentation factor} \tag{1133}\]

TEXTURE ZERO OF NEUTRINOS PRESERVATION AND RENORMALIZATION

MODULE NUMBERED EIGHT

GOVERNING EQUATIONS:

The differential system of this model is now

\[
\frac{dG_{40}}{dt} = (a_{40})^{(8)} G_{41} - \left[ (a_{40}')^{(8)} + (a_{40}'')^{(8)} (T_{41}, t) \right] G_{40} \tag{1134}
\]

\[
\frac{dG_{41}}{dt} = (a_{41})^{(8)} G_{40} - \left[ (a_{41}')^{(8)} + (a_{41}'')^{(8)} (T_{41}, t) \right] G_{41} \tag{1135}
\]

\[
\frac{dG_{42}}{dt} = (a_{42})^{(8)} G_{41} - \left[ (a_{42}')^{(8)} + (a_{42}'')^{(8)} (T_{41}, t) \right] G_{42} \tag{1136}
\]

\[
\frac{dT_{40}}{dt} = (b_{40})^{(8)} T_{41} - \left[ (b_{40}')^{(8)} - (b_{40}'')^{(8)} (G_{43}, t) \right] T_{40} \tag{1137}
\]

\[
\frac{dT_{41}}{dt} = (b_{41})^{(8)} T_{40} - \left[ (b_{41}')^{(8)} - (b_{41}'')^{(8)} (G_{43}, t) \right] T_{41} \tag{1138}
\]

\[
\frac{dT_{42}}{dt} = (b_{42})^{(8)} T_{41} - \left[ (b_{42}')^{(8)} - (b_{42}'')^{(8)} (G_{43}, t) \right] T_{42} \tag{1139}
\]

\[-(b_{40}'')^{(8)} (G_{39}, t) = \text{First detriments factor} \tag{1140}\]

\[
\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[ (a_{13}')^{(1)} (T_{14}, t) + (a_{13}'')^{(1)} (T_{17}, t) + (a_{16}'')^{(2,2)} (T_{37}, t) + (a_{20}'')^{(3,3)} (T_{37}, t) \right] G_{13} \tag{1140}
\]

\[-(a_{13}'')^{(1)} (T_{37}, t) + (a_{40}'')^{(8)} (T_{41}, t) \tag{1133}\]

247
\[
\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix}
(a_{14})^{(1)}(T_{14}, t) + (a_{14}^\prime)^{(1)}(T_{14}, t) + (a_{17}^\prime)^{(2.2)}(T_{17}, t) + (a_{21}^\prime)^{(3.3)}(T_{21}, t) \\
+ (a_{25})^{(4.4,4,4)}(T_{25}, t) + (a_{29}^\prime)^{(5.5,5,5)}(T_{29}, t) + (a_{33}^\prime)^{(6,6,6,6)}(T_{33}, t) \\
+ (a_{37})^{(7,7)}(T_{37}, t) + (a_{41})^{(8)}(T_{41}, t)
\end{bmatrix} G_{14}
\]

\[
\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix}
(a_{15})^{(1)}(T_{15}, t) + (a_{15}^\prime)^{(1)}(T_{15}, t) + (a_{18}^\prime)^{(2.2)}(T_{18}, t) + (a_{22}^\prime)^{(3.3)}(T_{22}, t) \\
+ (a_{26}^\prime)^{(4.4,4,4)}(T_{26}, t) + (a_{30}^\prime)^{(5.5,5,5)}(T_{30}, t) + (a_{34})^{(6,6,6,6)}(T_{34}, t) \\
+ (a_{38})^{(7,7)}(T_{38}, t) + (a_{42}^\prime)^{(8)}(T_{42}, t)
\end{bmatrix} G_{15}
\]

Where \( (a_{ij})^{(n)}(T_{ij}, t) \) are first augmentation coefficients for category 1, 2 and 3

\( + (a_{16})^{(2.2)}(T_{16}, t), + (a_{17})^{(2.2)}(T_{17}, t), + (a_{18})^{(2.2)}(T_{18}, t) \) are second augmentation coefficient for category 1, 2 and 3

\( + (a_{20})^{(3.3)}(T_{20}, t), + (a_{21})^{(3.3)}(T_{21}, t), + (a_{22})^{(3.3)}(T_{22}, t) \) are third augmentation coefficient for category 1, 2 and 3

\( + (a_{26}^\prime)^{(4.4,4,4)}(T_{26}, t), + (a_{30}^\prime)^{(4.4,4,4)}(T_{30}, t), + (a_{34}^\prime)^{(4.4,4,4)}(T_{34}, t) \) are fourth augmentation coefficient for category 1, 2 and 3

\( + (a_{29}^\prime)^{(5.5,5,5)}(T_{29}, t), + (a_{33}^\prime)^{(5.5,5,5)}(T_{33}, t), + (a_{37}^\prime)^{(5.5,5,5)}(T_{37}, t) \) are fifth augmentation coefficient for category 1, 2 and 3

\( + (a_{36}^\prime)^{(6,6,6,6)}(T_{36}, t), + (a_{39}^\prime)^{(6,6,6,6)}(T_{39}, t), + (a_{42}^\prime)^{(6,6,6,6)}(T_{42}, t) \) are sixth augmentation coefficient for category 1, 2 and 3

\( + (a_{38})^{(7,7)}(T_{38}, t) + (a_{41})^{(7,7)}(T_{41}, t) + (a_{44})^{(7,7)}(T_{44}, t) \) are seventh augmentation coefficients

\( + (a_{42}^\prime)^{(8)}(T_{42}, t) + (a_{44}^\prime)^{(8)}(T_{44}, t) + (a_{46}^\prime)^{(8)}(T_{46}, t) \) are eighth augmentation coefficients

\[
\frac{dF_{13}}{dt} = (b_{13})^{(1)}G_{14} - \begin{bmatrix}
(b_{13})^{(1)}(G, t) - (b_{14}^\prime)^{(4.4,4,4)}(G, t) - (b_{25})^{(7,7)}(G, t) - (b_{28})^{(3.3,3,3,3)}(G, t) \\
- (b_{30})^{(4.4,4,4,4)}(G, t) - (b_{33})^{(5.5,5,5,5)}(G, t) - (b_{36})^{(6,6,6,6,6)}(G, t) \\
- (b_{39})^{(7,7,7,7,7)}(G, t) - (b_{42})^{(8,8,8,8,8)}(G, t)
\end{bmatrix} T_{13}
\]

\[
\frac{dF_{14}}{dt} = (b_{14})^{(1)}G_{13} - \begin{bmatrix}
(b_{14})^{(1)}(G, t) - (b_{14}^\prime)^{(1)}(G, t) - (b_{17}^\prime)^{(2.2,2.2)}(G, t) - (b_{21}^\prime)^{(3.3,3,3)}(G, t) \\
- (b_{25}^\prime)^{(4.4,4,4,4)}(G, t) - (b_{29}^\prime)^{(5.5,5,5,5)}(G, t) - (b_{33}^\prime)^{(6,6,6,6,6)}(G, t) \\
- (b_{37}^\prime)^{(7,7,7,7,7,7)}(G, t) - (b_{41})^{(8,8,8,8,8,8)}(G, t)
\end{bmatrix} T_{14}
\]

\[
\frac{dF_{15}}{dt} = (b_{15})^{(1)}G_{14} - \begin{bmatrix}
(b_{15})^{(1)}(G, t) - (b_{15}^\prime)^{(1)}(G, t) - (b_{18}^\prime)^{(2.2,2.2)}(G, t) - (b_{22}^\prime)^{(3.3,3,3,3)}(G, t) \\
- (b_{26}^\prime)^{(4.4,4,4,4,4)}(G, t) - (b_{29}^\prime)^{(5.5,5,5,5,5)}(G, t) - (b_{33}^\prime)^{(6,6,6,6,6,6,6)}(G, t) \\
- (b_{38})^{(7,7,7,7,7,7,7,7,7)}(G, t) - (b_{42})^{(8,8,8,8,8,8,8,8,8)}(G, t)
\end{bmatrix} T_{15}
\]

Where \( -(b_{ij})^{(1)}(G, t), -(b_{ij}^\prime)^{(1)}(G, t), -(b_{ij}^\prime)^{(1)}(G, t) \) are first detritions coefficients for category 1, 2 and 3
are second detrition coefficients for category 1, 2 and 3

are third detrition coefficients for category 1, 2 and 3

are fourth detrition coefficients for category 1, 2 and 3

are fifth detrition coefficients for category 1, 2 and 3

are sixth detrition coefficients for category 1, 2 and 3

are seventh detrition coefficients

are eighth detrition coefficients

SECOND MODULE CONCATENATION:

Where are first detrition coefficients for category 1, 2 and 3

are second detrition coefficients for category 1, 2 and 3

are third detrition coefficients for category 1, 2 and 3

are fourth detrition coefficients for category 1, 2 and 3

are fifth detrition coefficients for category 1, 2 and 3

are sixth detrition coefficients for category 1, 2 and 3

SECOND MODULE CONCATENATION:

Where

are first detrition coefficients for category 1, 2 and 3

are second detrition coefficients for category 1, 2 and 3

are third detrition coefficients for category 1, 2 and 3

are fourth detrition coefficients for category 1, 2 and 3

are fifth detrition coefficients for category 1, 2 and 3

are sixth detrition coefficients for category 1, 2 and 3

SECOND MODULE CONCATENATION:
\[
\frac{dG_{18}}{dt} = (a_{18}^{(2)}) G_{17} - \begin{pmatrix}
(a_{10}^{(3)}) + (a_{10}^{(2)}) (T_{17}, t) & + (a_{15}^{(1,1)})(T_{14}, t) & + (a_{22}^{(3,3,3)})(T_{22}, t) \\
+ (a_{26}^{(4,4,4,4,4)})(T_{26}, t) & + (a_{30}^{(5,5,5,5,5)})(T_{29}, t) & + (a_{34}^{(6,6,6,6,6)})(T_{33}, t) \\
+ (a_{42}^{(8,8)})(T_{42}, t) & + (a_{42}^{(8,8)})(T_{42}, t) & + (a_{42}^{(8,8)})(T_{42}, t)
\end{pmatrix} G_{18}
\]

Where
\[
+ (a_{10}^{(3)})(T_{17}, t), + (a_{10}^{(2)})(T_{17}, t), + (a_{10}^{(2)})(T_{17}, t)
\]
are first augmentation coefficients for category 1, 2 and 3

\[
+ (a_{15}^{(1,1)})(T_{14}, t), + (a_{15}^{(1,1)})(T_{14}, t), + (a_{15}^{(1,1)})(T_{14}, t)
\]
are second augmentation coefficient for category 1, 2 and 3

\[
+ (a_{22}^{(3,3,3)})(T_{22}, t), + (a_{22}^{(3,3,3)})(T_{22}, t), + (a_{22}^{(3,3,3)})(T_{22}, t)
\]
are third augmentation coefficient for category 1, 2 and 3

\[
+ (a_{26}^{(4,4,4,4,4)})(T_{26}, t), + (a_{26}^{(4,4,4,4,4)})(T_{26}, t), + (a_{26}^{(4,4,4,4,4)})(T_{26}, t)
\]
are fourth augmentation coefficient for category 1, 2 and 3

\[
+ (a_{30}^{(5,5,5,5,5)})(T_{29}, t), + (a_{30}^{(5,5,5,5,5)})(T_{29}, t), + (a_{30}^{(5,5,5,5,5)})(T_{29}, t)
\]
are fifth augmentation coefficient for category 1, 2 and 3

\[
+ (a_{34}^{(6,6,6,6,6)})(T_{33}, t), + (a_{34}^{(6,6,6,6,6)})(T_{33}, t), + (a_{34}^{(6,6,6,6,6)})(T_{33}, t)
\]
are sixth augmentation coefficient for category 1, 2 and 3

\[
+ (a_{38}^{(7,7,7)})(T_{37}, t), + (a_{38}^{(7,7,7)})(T_{37}, t), + (a_{38}^{(7,7,7)})(T_{37}, t)
\]
are Seventh Detrition Coefficients

\[
+ (a_{42}^{(9)})(T_{42}, t), + (a_{42}^{(9)})(T_{42}, t), + (a_{42}^{(9)})(T_{42}, t)
\]
are Eight Augmentation Coefficients

\[
\frac{dT_{16}}{dt} = (b_{16}^{(2)}) T_{17} - \begin{pmatrix}
(b_{10}^{(2)}) & - (b_{10}^{(2)})(G_{19}, t) & - (b_{10}^{(1,1)})(G_{19}, t) & - (b_{20}^{(3,3,3)})(G_{23}, t) \\
- (b_{24}^{(4,4,4,4)})(G_{27}, t) & - (b_{24}^{(5,5,5,5,5)})(G_{31}, t) & - (b_{24}^{(6,6,6,6,6)})(G_{35}, t) \\
- (b_{42}^{(8,8)})(G_{42}, t) & - (b_{42}^{(8,8)})(G_{42}, t) & - (b_{42}^{(8,8)})(G_{42}, t)
\end{pmatrix} T_{16}
\]

\[
\frac{dT_{17}}{dt} = (b_{17}^{(2)}) T_{16} - \begin{pmatrix}
(b_{10}^{(2)}) & - (b_{10}^{(2)})(G_{19}, t) & - (b_{10}^{(1,1)})(G_{19}, t) & - (b_{20}^{(3,3,3)})(G_{23}, t) \\
- (b_{26}^{(4,4,4,4,4)})(G_{27}, t) & - (b_{26}^{(5,5,5,5,5,5)})(G_{31}, t) & - (b_{26}^{(6,6,6,6,6,6)})(G_{35}, t) \\
- (b_{42}^{(8,8)})(G_{42}, t) & - (b_{42}^{(8,8)})(G_{42}, t) & - (b_{42}^{(8,8)})(G_{42}, t)
\end{pmatrix} T_{17}
\]

\[
\frac{dT_{18}}{dt} = (b_{18}^{(2)}) T_{17} - \begin{pmatrix}
(b_{10}^{(2)}) & - (b_{10}^{(2)})(G_{19}, t) & - (b_{10}^{(1,1)})(G_{19}, t) & - (b_{20}^{(3,3,3)})(G_{23}, t) \\
- (b_{26}^{(4,4,4,4,4,4)})(G_{27}, t) & - (b_{26}^{(5,5,5,5,5,5,5)})(G_{31}, t) & - (b_{26}^{(6,6,6,6,6,6,6)})(G_{35}, t) \\
- (b_{42}^{(8,8)})(G_{42}, t) & - (b_{42}^{(8,8)})(G_{42}, t) & - (b_{42}^{(8,8)})(G_{42}, t)
\end{pmatrix} T_{18}
\]

Third Module Concatenation:

\[
\frac{dG_{20}}{dt} = (a_{20}^{(3)}) G_{21} - \begin{pmatrix}
(a_{20}^{(3)}) & + (a_{20}^{(3)})(T_{21}, t) & + (a_{20}^{(2,2,2)})(T_{17}, t) & + (a_{20}^{(1,1,1)})(T_{14}, t) \\
+ (a_{24}^{(4,4,4,4,4)})(T_{25}, t) & + (a_{26}^{(5,5,5,5,5,5)})(T_{29}, t) & + (a_{32}^{(6,6,6,6,6,6)})(T_{33}, t) \\
+ (a_{42}^{(8,8)})(T_{42}, t) & + (a_{42}^{(8,8)})(T_{42}, t)
\end{pmatrix} G_{20}
\]

250
\[
\frac{dg_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix}
(a_{21})^{(3)} + (a_{21})^{(3)}(T_{21}, t) + (a_{17})^{(2,2,2)}(T_{17}, t) + (a_{13})^{(1,1,1)}(T_{14}, t) \\
+ (a_{25})^{(4,4,4,4,4)}(T_{25}, t) + (a_{25})^{(5,5,5,5,5)}(T_{29}, t) + (a_{25})^{(6,6,6,6,6)}(T_{23}, t) \\
+ (a_{27}^{(7,7,7)}(T_{37}, t) + (a_{41}^{(8,8,8)}(T_{41}, t)
\end{bmatrix} G_{21}
\]

\[
\frac{dg_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix}
(a_{22})^{(3)} + (a_{22})^{(3)}(T_{21}, t) + (a_{19})^{(2,2,2)}(T_{17}, t) + (a_{15})^{(1,1,1)}(T_{14}, t) \\
+ (a_{25})^{(4,4,4,4,4)}(T_{25}, t) + (a_{25})^{(5,5,5,5,5)}(T_{29}, t) + (a_{25})^{(6,6,6,6,6)}(T_{23}, t) \\
+ (a_{29}^{(7,7,7)}(T_{37}, t) + (a_{43}^{(8,8,8)}(T_{41}, t)
\end{bmatrix} G_{22}
\]

are first augmentation coefficients for category 1, 2 and 3

\[
+ (a_{16}^{(2,2,2)}(T_{17}, t), (a_{17}^{(2,2,2)}(T_{17}, t), + (a_{18}^{(2,2,2)}(T_{17}, t)
\]

are second augmentation coefficients for category 1, 2 and 3

\[
+ (a_{13}^{(1,1,1)}(T_{14}, t), (a_{14}^{(1,1,1)}(T_{14}, t), + (a_{15}^{(1,1,1)}(T_{14}, t)
\]

are third augmentation coefficients for category 1, 2 and 3

\[
+ (a_{24}^{(4,4,4,4,4)}(T_{25}, t), (a_{26}^{(4,4,4,4,4)}(T_{25}, t), (a_{26}^{(4,4,4,4,4)}(T_{25}, t)
\]

are fourth augmentation coefficients for category 1, 2 and 3

\[
+ (a_{26}^{(5,5,5,5,5)}(T_{29}, t), (a_{26}^{(5,5,5,5,5)}(T_{29}, t), (a_{26}^{(5,5,5,5,5)}(T_{29}, t)
\]

are fifth augmentation coefficients for category 1, 2 and 3

\[
+ (a_{32}^{(6,6,6,6,6)}(T_{33}, t), (a_{32}^{(6,6,6,6,6)}(T_{33}, t), (a_{32}^{(6,6,6,6,6)}(T_{33}, t)
\]

are sixth augmentation coefficients for category 1, 2 and 3

\[
+ (a_{36}^{(7,7,7)}(T_{37}, t), (a_{38}^{(7,7,7)}(T_{37}, t), (a_{36}^{(7,7,7)}(T_{37}, t)
\]

are seventh augmentation coefficient

\[
+ (a_{40}^{(8)}(T_{35}, t), (a_{40}^{(8)}(T_{41}, t), (a_{40}^{(8)}(T_{41}, t)
\]

Are Eighth Augmentation Coefficient
\[
(b_{22})^{(3)}T_{21} = \begin{bmatrix}
(b_{22}^n)^{(3)}(G_{23}, t) & -(b_{22}^n)^{(2,2,2)}(G_{19}, t) & -(b_{22}^n)^{(1,1,1)}(G, t) \\
-(b_{21}^n)^{(1,1,1)}(G_{23}, t) & -(b_{22}^n)^{(4,4,4,4,4)}(G_{27}, t) & -(b_{21}^n)^{(5,5,5,5,5,5)}(G_{31}, t) \\
-(b_{19}^n)^{(7,7,7)}(G_{39}, t) & -(b_{19}^n)^{(8,8,8)}(G_{43}, t) & 0 \\
\end{bmatrix}
\]

\[
-(b_{20}^n)^{(3)}(G_{23}, t), -(b_{21}^n)^{(3)}(G_{23}, t), -(b_{22}^n)^{(3)}(G_{23}, t)
\]

are first detriments coefficients for category 1, 2 and 3

\[
-(b_{19}^n)^{(2,2,2)}(G_{19}, t), -(b_{19}^n)^{(2,2,2)}(G_{19}, t), -(b_{19}^n)^{(2,2,2)}(G_{19}, t)
\]

are second detriments coefficients for category 1, 2 and 3

\[
-(b_{17}^n)^{(1,1,1)}(G, t), -(b_{17}^n)^{(1,1,1)}(G, t), -(b_{17}^n)^{(1,1,1)}(G, t)
\]

are third detriments coefficients for category 1, 2 and 3

\[
-(b_{17}^n)^{(4,4,4,4,4)}(G_{27}, t), -(b_{17}^n)^{(4,4,4,4,4)}(G_{27}, t), -(b_{17}^n)^{(4,4,4,4,4)}(G_{27}, t)
\]

are fourth detriments coefficients for category 1, 2 and 3

\[
-(b_{17}^n)^{(5,5,5,5,5)}(G_{31}, t), -(b_{17}^n)^{(5,5,5,5,5)}(G_{31}, t), -(b_{17}^n)^{(5,5,5,5,5)}(G_{31}, t)
\]

are fifth detriments coefficients for category 1, 2 and 3

\[
-(b_{17}^n)^{(6,6,6,6,6,6)}(G_{35}, t), -(b_{17}^n)^{(6,6,6,6,6,6)}(G_{35}, t), -(b_{17}^n)^{(6,6,6,6,6,6)}(G_{35}, t)
\]

are sixth detriments coefficients for category 1, 2 and 3

\[
\begin{aligned}
&\text{Where } [a_{24}^n]^{(6)}(T_{25}, t), [a_{25}^n]^{(4)}(T_{25}, t), [a_{26}^n]^{(4)}(T_{25}, t) \text{ are first augmentation coefficients for category 1, 2 and 3} \\
&\text{+ } [a_{28}^n]^{(5,5)}(T_{29}, t), [a_{28}^n]^{(5,5)}(T_{29}, t), [a_{28}^n]^{(5,5)}(T_{29}, t) \text{ are second augmentation coefficients for category 1, 2 and 3} \\
&\text{+ } [a_{26}^n]^{(6,6)}(T_{33}, t), [a_{26}^n]^{(6,6)}(T_{33}, t), [a_{26}^n]^{(6,6)}(T_{33}, t) \text{ are third augmentation coefficients for category 1, 2 and 3} \\
&\text{+ } [a_{16}^n]^{(1,1,1,1)}(T_{14}, t), [a_{16}^n]^{(1,1,1,1)}(T_{14}, t), [a_{16}^n]^{(1,1,1,1)}(T_{14}, t) \text{ are fourth augmentation coefficients for category 1, 2 and 3} \\
&\text{+ } [a_{16}^n]^{(2,2,2,2,2)}(T_{15}, t), [a_{16}^n]^{(2,2,2,2,2)}(T_{15}, t), [a_{16}^n]^{(2,2,2,2,2)}(T_{15}, t) \text{ are fifth augmentation coefficients for category 1, 2 and 3} \\
&\text{+ } [a_{16}^n]^{(3,3,3,3)}(T_{21}, t), [a_{16}^n]^{(3,3,3,3)}(T_{21}, t), [a_{16}^n]^{(3,3,3,3)}(T_{21}, t) \text{ are sixth augmentation coefficients for category 1, 2 and 3} \\
&\text{+ } [a_{16}^n]^{(7,7,7,7,7)}(T_{37}, t), [a_{16}^n]^{(7,7,7,7,7)}(T_{37}, t), [a_{16}^n]^{(7,7,7,7,7)}(T_{37}, t) \text{ are seventh augmentation coefficients for category 1, 2 and 3} \\
\end{aligned}
\]

**FOURTH MODULE CONCATENATION:**

\[
\begin{bmatrix}
(a_{24}^n)^{(4)}(G_{25}) & + (a_{28}^n)^{(5,5)}(T_{29}, t) & + (a_{32}^n)^{(6,6)}(T_{33}, t) \\
+ (a_{14}^n)^{(1,1,1,1)}(T_{14}, t) & + (a_{16}^n)^{(2,2,2,2,2)}(T_{15}, t) & + (a_{16}^n)^{(3,3,3,3)}(T_{21}, t) \\
+ (a_{16}^n)^{(7,7,7,7,7)}(T_{37}, t) & + (a_{16}^n)^{(8,8,8,8)}(T_{41}, t) & 0 \\
\end{bmatrix}
\]

\[
\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \]

\[
\begin{bmatrix}
(a_{23}^n)^{(4)} & + (a_{29}^n)^{(5,5)}(T_{29}, t) & + (a_{33}^n)^{(6,6)}(T_{33}, t) \\
+ (a_{13}^n)^{(1,1,1,1)}(T_{14}, t) & + (a_{15}^n)^{(2,2,2,2)}(T_{15}, t) & + (a_{15}^n)^{(3,3,3,3)}(T_{21}, t) \\
+ (a_{15}^n)^{(7,7,7,7,7)}(T_{37}, t) & + (a_{15}^n)^{(8,8,8,8)}(T_{41}, t) & 0 \\
\end{bmatrix}
\]

\[
\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \]

\[
\begin{bmatrix}
(a_{23}^n)^{(4)} & + (a_{29}^n)^{(5,5)}(T_{29}, t) & + (a_{33}^n)^{(6,6)}(T_{33}, t) \\
+ (a_{13}^n)^{(1,1,1,1)}(T_{14}, t) & + (a_{15}^n)^{(2,2,2,2)}(T_{15}, t) & + (a_{15}^n)^{(3,3,3,3)}(T_{21}, t) \\
+ (a_{15}^n)^{(7,7,7,7,7)}(T_{37}, t) & + (a_{41}^n)^{(8,8,8,8)}(T_{41}, t) & 0 \\
\end{bmatrix}
\]
\[
\frac{dG_{26}}{dt} = (a_{26}^{(4)})G_{25} - \left[ (a_{25}^{(4)})^{(4)}(T_{25}, t) + (a_{30}^{(5,5)})^{(5,5)}(T_{29}, t) + (a_{34}^{(6,6)})^{(6,6)}(T_{33}, t) \\
+ (a_{12}^{(11,1,1,1)})^{(11,1,1,1)}(T_{14}, t) + (a_{17}^{(2,2,2,2)})^{(2,2,2,2)}(T_{17}, t) + (a_{22}^{(3,3,3,3)})^{(3,3,3,3)}(T_{21}, t) \\
+ (a_{26}^{(7,7,7,7)})^{(7,7,7,7)}(T_{26}, t) + (a_{42}^{(8,8,8,8)})^{(8,8,8,8)}(T_{42}, t) \right] G_{26}
\]

Where \( (a_{25}^{(4)})^{(4)}(T_{25}, t), (a_{25}^{(5,5)})^{(5,5)}(T_{29}, t), (a_{25}^{(6,6)})^{(6,6)}(T_{33}, t) \) are first augmentation coefficients for category 1, 2 and 3

\( + (a_{12}^{(11,1,1,1)})^{(11,1,1,1)}(T_{14}, t) \) are second augmentation coefficient for category 1, 2 and 3

\( + (a_{17}^{(2,2,2,2)})^{(2,2,2,2)}(T_{17}, t) \) are third augmentation coefficient for category 1, 2 and 3

\( + (a_{22}^{(3,3,3,3)})^{(3,3,3,3)}(T_{21}, t) \) are fourth augmentation coefficients for category 1, 2 and 3

\( + (a_{26}^{(7,7,7,7)})^{(7,7,7,7)}(T_{26}, t) \) are fifth augmentation coefficients 1, 2 and 3

\( + (a_{42}^{(8,8,8,8)})^{(8,8,8,8)}(T_{42}, t) \) are sixth augmentation coefficients for category 1, 2 and 3

\( + (a_{46}^{(7,7,7,7,7)})(T_{46}, t) \) and \( + (a_{46}^{(7,7,7,7,7)})(T_{46}, t) \) are seventh augmentation coefficients

\( + (a_{48}^{(8,8,8,8,8)}(T_{48}, t) \) and \( + (a_{48}^{(8,8,8,8,8)}(T_{48}, t) \) are eighth augmentation coefficients

\[
\frac{dT_{24}}{dt} = (b_{24}^{(4)})T_{25} - \left[ (b_{24}^{(4)})^{(4)}(G_{27}, t) - (b_{28}^{(5,5)})^{(5,5)}(G_{31}, t) - (b_{32}^{(6,6)})^{(6,6)}(G_{35}, t) \\
- (b_{15}^{(1,1,1,1)})^{(1,1,1,1)}(G_{17}, t) - (b_{17}^{(2,2,2,2)})^{(2,2,2,2)}(G_{17}, t) - (b_{21}^{(3,3,3,3)})^{(3,3,3,3)}(G_{21}, t) \\
- (b_{26}^{(7,7,7,7,7)})(G_{26}, t) - (b_{40}^{(8,8,8,8,8)})(G_{43}, t) \right] T_{24}
\]

Where \( -(b_{24}^{(4)})^{(4)}(G_{27}, t), -(b_{28}^{(5,5)})^{(5,5)}(G_{31}, t), -(b_{32}^{(6,6)})^{(6,6)}(G_{35}, t) \) are first detrition coefficients for category 1, 2 and 3

\( -(b_{15}^{(1,1,1,1)})^{(1,1,1,1)}(G_{17}, t), -(b_{17}^{(2,2,2,2)})^{(2,2,2,2)}(G_{17}, t), -(b_{21}^{(3,3,3,3)})^{(3,3,3,3)}(G_{21}, t) \) are second detrition coefficients for category 1, 2 and 3

\( -(b_{26}^{(7,7,7,7,7)})(G_{26}, t), -(b_{40}^{(8,8,8,8,8)})(G_{43}, t) \) are third detrition coefficients for category 1, 2 and 3

\( -(b_{30}^{(1,1,1,1,1)})^{(1,1,1,1,1)}(G_{14}, t), -(b_{34}^{(3,3,3,3,3)})(G_{19}, t), -(b_{38}^{(8,8,8,8,8)})(G_{39}, t) \) are fourth detrition coefficients for category 1, 2 and 3

\( -(b_{10}^{(1,1,1,1,1,1)})(G_{10}, t), -(b_{17}^{(2,2,2,2,2)})(G_{17}, t), -(b_{22}^{(3,3,3,3,3)})(G_{21}, t) \) are fifth detrition coefficients for category 1, 2 and 3

\( -(b_{25}^{(3,3,3,3,3)})(G_{25}, t), -(b_{30}^{(5,5,5,5,5)})(G_{30}, t), -(b_{35}^{(7,7,7,7,7)})(G_{35}, t) \) are sixth detrition coefficients for category 1, 2 and 3

\( -(b_{34}^{(7,7,7,7,7)})(G_{34}, t), -(b_{40}^{(8,8,8,8,8,8)})(G_{40}, t), -(b_{45}^{(8,8,8,8,8,8,8)})(G_{45}, t) \) are seventh detrition coefficients

\( -(b_{40}^{(8,8,8,8,8,8)})(G_{40}, t), -(b_{45}^{(8,8,8,8,8,8,8)})(G_{45}, t), -(b_{50}^{(8,8,8,8,8,8,8,8)})(G_{50}, t) \) are eighth detrition coefficients.
**Fifth Module Concatenation:**

\[
\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[ (a_{28}^{(5)} + (a_{24})^{(6)}(T_{29}, t) + (a_{26}^{(4,4)}(T_{29}, t)) + (a_{22}^{(6,6)}(T_{33}, t)) + 
\right.
\left. + (a_{15}^{(1,1,1,1)}(T_{14}, t)) + (a_{16}^{(2,2,2,2)}(T_{17}, t)) + (a_{20}^{(3,3,3,3)}(T_{21}, t))
\right] \]

\[
\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{29} - \left[ (a_{29}^{(5)} + (a_{25})^{(6,6)}(T_{29}, t) + (a_{26}^{(4,4)}(T_{29}, t)) + (a_{23}^{(6,6)}(T_{33}, t)) + 
\right.\]
\left. + (a_{14}^{(1,1,1,1)}(T_{14}, t)) + (a_{17}^{(2,2,2,2)}(T_{17}, t)) + (a_{21}^{(3,3,3,3)}(T_{21}, t)) + (a_{30}^{(5,5)}(T_{29}, t)) \right] \]

\[
\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[ (a_{30}^{(5)} + (a_{26}^{(6,6)}(T_{29}, t)) + (a_{27}^{(4,4)}(T_{29}, t)) + (a_{23}^{(6,6)}(T_{33}, t)) + 
\right.\]
\left. + (a_{15}^{(1,1,1,1)}(T_{14}, t)) + (a_{18}^{(2,2,2,2)}(T_{17}, t)) + (a_{20}^{(3,3,3,3)}(T_{21}, t)) + (a_{30}^{(5,5)}(T_{29}, t)) \right] \]

Where \((a_{28}^{(5)}(T_{29}, t))\), \((a_{29}^{(5)}(T_{29}, t))\), and \((a_{30}^{(5)}(T_{29}, t))\) are first augmentation coefficients for category 1, 2, and 3.

And \((a_{24}^{(6,6)}(T_{29}, t))\), \((a_{26}^{(4,4)}(T_{29}, t))\), and \((a_{22}^{(6,6)}(T_{33}, t))\) are second augmentation coefficient for category 1, 2, and 3.

\((a_{15}^{(1,1,1,1)}(T_{14}, t))\), \((a_{16}^{(1,1,1,1)}(T_{14}, t))\), and \((a_{20}^{(3,3,3,3)}(T_{21}, t))\) are third augmentation coefficients for category 1, 2, and 3.

\((a_{14}^{(1,1,1,1)}(T_{14}, t))\), \((a_{17}^{(2,2,2,2)}(T_{17}, t))\), and \((a_{21}^{(3,3,3,3)}(T_{21}, t))\) are fourth augmentation coefficients for category 1, 2, and 3.

\((a_{26}^{(6,6)}(T_{29}, t))\), \((a_{27}^{(4,4)}(T_{29}, t))\), and \((a_{23}^{(6,6)}(T_{33}, t))\) are fifth augmentation coefficients for category 1, 2, and 3.

\((a_{15}^{(1,1,1,1)}(T_{14}, t))\), \((a_{16}^{(1,1,1,1)}(T_{14}, t))\), and \((a_{20}^{(3,3,3,3)}(T_{21}, t))\) are sixth augmentation coefficients for category 1, 2, and 3.

\((a_{28}^{(5,5)}(T_{29}, t))\), \((a_{29}^{(5,5)}(T_{29}, t))\), and \((a_{30}^{(5,5)}(T_{29}, t))\) are Eighth Augmentation Coefficients.

\[
\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[ -(b_{28}^{(5)}(G_{31}, t)) - (b_{24}^{(6,6)}(G_{23}, t)) - (b_{22}^{(6,6)}(G_{33}, t)) - 
\right.\]
\left. - (b_{13}^{(1,1,1,1)}(G_{14}, t)) - (b_{16}^{(2,2,2,2)}(G_{17}, t)) - (b_{20}^{(3,3,3,3)}(G_{21}, t)) - (b_{30}^{(5,5)}(T_{33}, t)) \right] \]

\[
\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{29} - \left[ -(b_{29}^{(5)}(G_{31}, t)) - (b_{25}^{(6,6)}(G_{23}, t)) - (b_{23}^{(6,6)}(G_{33}, t)) - 
\right.\]
\left. - (b_{14}^{(1,1,1,1)}(G_{14}, t)) - (b_{17}^{(2,2,2,2)}(G_{17}, t)) - (b_{21}^{(3,3,3,3)}(G_{21}, t)) - (b_{30}^{(5,5)}(T_{33}, t)) \right] \]

\[
\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[ -(b_{30}^{(5)}(G_{31}, t)) - (b_{27}^{(6,6)}(G_{23}, t)) - (b_{25}^{(6,6)}(G_{33}, t)) - 
\right.\]
\left. - (b_{15}^{(1,1,1,1)}(G_{14}, t)) - (b_{18}^{(2,2,2,2)}(G_{17}, t)) - (b_{22}^{(3,3,3,3)}(G_{21}, t)) - (b_{28}^{(5,5)}(T_{33}, t)) \right] \]
SIXTH MODULE CONCATENATION

\[
\frac{dG_{32}}{dt} = (a_{32})^6 G_{33} - \left[ \begin{array}{c}
(a_{32})^6 + (a_{32})^6 (T_{33}, t) + (a_{29}^T) (5,5,5) (T_{29}, t) + (a_{29}^T) (4,4,4) (T_{25}, t) \\
+ (a_{29}^T) (1,1,1,1,1) (T_{14}, t) + (a_{16}^T) (2,2,2,2,2) (T_{17}, t) + (a_{20}^T) (3,3,3,3,3) (T_{21}, t) \\
+ (a_{40}^T) (7,7,7,7,7) (T_{37}, t) + (a_{40}^T) (8,8,8,8,8) (T_{41}, t)
\end{array} \right] G_{32}
\]

\[
\frac{dG_{33}}{dt} = (a_{33})^6 G_{32} - \left[ \begin{array}{c}
(a_{33})^6 + (a_{33})^6 (T_{33}, t) + (a_{29}^T) (5,5,5) (T_{29}, t) + (a_{29}^T) (4,4,4) (T_{25}, t) \\
+ (a_{29}^T) (1,1,1,1,1) (T_{14}, t) + (a_{16}^T) (2,2,2,2,2) (T_{17}, t) + (a_{20}^T) (3,3,3,3,3) (T_{21}, t) \\
+ (a_{41}^T) (7,7,7,7,7) (T_{37}, t) + (a_{41}^T) (8,8,8,8,8) (T_{41}, t)
\end{array} \right] G_{33}
\]

\[
\frac{dG_{34}}{dt} = (a_{34})^6 G_{33} - \left[ \begin{array}{c}
(a_{34})^6 + (a_{34})^6 (T_{33}, t) + (a_{29}^T) (5,5,5) (T_{29}, t) + (a_{29}^T) (4,4,4) (T_{25}, t) \\
+ (a_{29}^T) (1,1,1,1,1) (T_{14}, t) + (a_{16}^T) (2,2,2,2,2) (T_{17}, t) + (a_{20}^T) (3,3,3,3,3) (T_{21}, t) \\
+ (a_{42}^T) (7,7,7,7,7) (T_{37}, t) + (a_{42}^T) (8,8,8,8,8) (T_{41}, t)
\end{array} \right] G_{34}
\]

\[\begin{align*}
+ (a_{32}^T) (5,5,5) (T_{33}, t) + & (a_{32}^T) (5,5,5) (T_{29}, t) + (a_{20}^T) (5,5,5) (T_{29}, t) \\
+ (a_{32}^T) (4,4,4) (T_{25}, t) + & (a_{32}^T) (4,4,4) (T_{25}, t) + (a_{20}^T) (4,4,4) (T_{25}, t)
\end{align*}\]
are first augmentation coefficients for category 1, 2 and 3

\[\begin{align*}
+ (a_{33}^T) (1,1,1,1,1) (T_{14}, t) + & (a_{33}^T) (1,1,1,1,1) (T_{14}, t) + (a_{16}^T) (1,1,1,1,1) (T_{14}, t) \\
+ (a_{33}^T) (2,2,2,2,2) (T_{17}, t) + & (a_{33}^T) (2,2,2,2,2) (T_{17}, t) + (a_{20}^T) (2,2,2,2,2) (T_{17}, t)
\end{align*}\]
are second augmentation coefficients for category 1, 2 and 3

\[\begin{align*}
+ (a_{34}^T) (1,1,1,1,1) (T_{14}, t) + & (a_{34}^T) (1,1,1,1,1) (T_{14}, t) + (a_{16}^T) (1,1,1,1,1) (T_{14}, t) \\
+ (a_{34}^T) (2,2,2,2,2) (T_{17}, t) + & (a_{34}^T) (2,2,2,2,2) (T_{17}, t) + (a_{20}^T) (2,2,2,2,2) (T_{17}, t)
\end{align*}\]
are third augmentation coefficients for category 1, 2 and 3

\[\begin{align*}
+ (a_{32}^T) (5,5,5,5) (T_{14}, t) + & (a_{33}^T) (5,5,5,5) (T_{14}, t) + (a_{34}^T) (5,5,5,5) (T_{14}, t) \\
+ (a_{20}^T) (5,5,5,5) (T_{14}, t) + & (a_{20}^T) (5,5,5,5) (T_{14}, t) + (a_{20}^T) (5,5,5,5) (T_{14}, t)
\end{align*}\]
- are fourth augmentation coefficients

\[\begin{align*}
+ (a_{32}^T) (5,5,5,5,5) (T_{17}, t) + & (a_{33}^T) (5,5,5,5,5) (T_{17}, t) + (a_{34}^T) (5,5,5,5,5) (T_{17}, t) \\
+ (a_{20}^T) (5,5,5,5,5) (T_{17}, t) + & (a_{20}^T) (5,5,5,5,5) (T_{17}, t) + (a_{20}^T) (5,5,5,5,5) (T_{17}, t)
\end{align*}\]
- fifth augmentation coefficients

\[\begin{align*}
+ (a_{32}^T) (3,3,3,3,3) (T_{21}, t) + & (a_{33}^T) (3,3,3,3,3) (T_{21}, t) + (a_{34}^T) (3,3,3,3,3) (T_{21}, t) \\
+ (a_{20}^T) (3,3,3,3,3) (T_{21}, t) + & (a_{20}^T) (3,3,3,3,3) (T_{21}, t) + (a_{20}^T) (3,3,3,3,3) (T_{21}, t)
\end{align*}\]
sixth augmentation coefficients

\[\begin{align*}
+ (a_{32}^T) (7,7,7,7,7) (T_{37}, t) + & (a_{33}^T) (7,7,7,7,7) (T_{37}, t) + (a_{34}^T) (7,7,7,7,7) (T_{37}, t) \\
+ (a_{40}^T) (8,8,8,8,8) (T_{37}, t) + & (a_{40}^T) (8,8,8,8,8) (T_{37}, t) + (a_{40}^T) (8,8,8,8,8) (T_{37}, t)
\end{align*}\]
- are Seventh Augmentation Coefficients

\[\begin{align*}
+ (a_{32}^T) (8,8,8,8,8) (T_{41}, t) + & (a_{33}^T) (8,8,8,8,8) (T_{41}, t) + (a_{34}^T) (8,8,8,8,8) (T_{41}, t)
\end{align*}\]
- are Eight Augmentation Coefficients

\[
\frac{dT_{32}}{dt} = (b_{32})^6 T_{33} - \left[ \begin{array}{c}
(b_{32})^6 (b_{35}^T) (G_{35}, t) - (b_{29}^T) (5,5,5) (G_{31}, t) - (b_{29}^T) (4,4,4) (G_{27}, t) \\
- (b_{29}^T) (1,1,1,1,1) (G_{31}, t) - (b_{16}^T) (2,2,2,2,2) (G_{19}, t) - (b_{20}^T) (3,3,3,3,3) (G_{23}, t) \\
- (b_{40}^T) (7,7,7,7,7) (G_{39}, t) - (b_{40}^T) (8,8,8,8,8) (G_{43}, t)
\end{array} \right] T_{32}
\]
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\[
\frac{dG_{40}}{dt} = \left[ (a_{40}^{(8)}(T_{21},t) + (a_{40}^{(8)}(T_{23},t) + (a_{15}^{(8)}(T_{14},t) + (a_{16}^{(8)}(T_{17},t) + \right. \\
\left. + (a_{36}^{(8)}(T_{37},t) \right) G_{40}
\]

\[
\frac{dG_{41}}{dt} = \left[ (a_{41}^{(8)}(T_{21},t) + (a_{41}^{(8)}(T_{23},t) + (a_{16}^{(8)}(T_{14},t) + (a_{17}^{(8)}(T_{17},t) + \right. \\
\left. + (a_{37}^{(8)}(T_{37},t) \right) G_{41}
\]

\[
\frac{dG_{42}}{dt} = \left[ (a_{42}^{(8)}(T_{21},t) + (a_{42}^{(8)}(T_{23},t) + (a_{17}^{(8)}(T_{14},t) + (a_{18}^{(8)}(T_{17},t) + \right. \\
\left. + (a_{38}^{(8)}(T_{37},t) \right) G_{42}
\]

\[
\frac{dT_{41}}{dt} = \left[ (b_{40}^{(8)}(T_{41},t) + (b_{40}^{(8)}(T_{43},t) + (b_{13}^{(8)}(T_{44},t) + (b_{14}^{(8)}(T_{45},t) + \right. \\
\left. + (b_{15}^{(8)}(T_{37},t) \right) T_{41}
\]

\[
\frac{dG_{40}}{dt} = \left[ (a_{40}^{(8)}(T_{21},t) + (a_{40}^{(8)}(T_{23},t) + (a_{12}^{(8)}(T_{14},t) + (a_{15}^{(8)}(T_{17},t) + \right. \\
\left. + (a_{36}^{(8)}(T_{37},t) \right) G_{40}
\]

\[
\frac{dG_{41}}{dt} = \left[ (a_{41}^{(8)}(T_{21},t) + (a_{41}^{(8)}(T_{23},t) + (a_{16}^{(8)}(T_{14},t) + (a_{17}^{(8)}(T_{17},t) + \right. \\
\left. + (a_{37}^{(8)}(T_{37},t) \right) G_{41}
\]

\[
\frac{dG_{42}}{dt} = \left[ (a_{42}^{(8)}(T_{21},t) + (a_{42}^{(8)}(T_{23},t) + (a_{17}^{(8)}(T_{14},t) + (a_{18}^{(8)}(T_{17},t) + \right. \\
\left. + (a_{38}^{(8)}(T_{37},t) \right) G_{42}
\]

\[
\frac{dT_{41}}{dt} = \left[ (b_{40}^{(8)}(T_{41},t) + (b_{40}^{(8)}(T_{43},t) + (b_{13}^{(8)}(T_{44},t) + (b_{14}^{(8)}(T_{45},t) + \right. \\
\left. + (b_{15}^{(8)}(T_{37},t) \right) T_{41}
\]
First augmentation factor

$$\frac{dT_{41}}{dt} = (b_{42})^{(8)}T_{41} - \begin{bmatrix} (b_{41})^{(8)} - (b_{42})^{(8)}(G_{43}, t) - (b_{43})^{(8)}(G_{44}, t) - (b_{47})^{(8)}(G_{15}, t) - (b_{41})^{(8)}(G_{23}, t) - \\ (b_{42})^{(8)}(G_{27}, t) - (b_{49})^{(8)}(G_{33}, t) - (b_{4})^{(8)}(G_{35}, t) - (b_{4})^{(8)}(G_{39}, t) \end{bmatrix} T_{42}$$

$$+(a_{40}^{(8)}(T_{41}, t) = \text{First augmentation factor}$$

$$-(b_{40}^{(8)}(G_{43}, t) = \text{First detritions factor}$$

Where we suppose

(A) \((a_j)^{(1)}, (a_i)^{(1)}, (a_i^{(1)}), (b_i)^{(1)}, (b_i^{(1)}) > 0, i, j = 13, 14, 15\)

(B) \(\text{The functions } (a_i^{(1)}), (b_i^{(1)}) \text{ are positive continuous increasing and bounded.}\)

Definition of \((p_i)^{(1)}, (r_i)^{(1)}:\)

\((a_i^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}\)

\((b_i^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i^{(1)}) \leq (\hat{B}_{13})^{(1)}\)

\(\lim_{T_{14} \to \infty}(a_i^{(1)}(T_{14}, t) = (p_i)^{(1)}\)

\(\lim_{G \to \infty}(b_i^{(1)}(G, t) = (r_i)^{(1)}\)

Definition of \((\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}:\)

Where \((\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}\) are positive constants and \(i = 13, 14, 15\)

They satisfy Lipschitz condition:

\(|(a_i^{(1)}(T_{14}, t) - (a_i^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}^t|e^{-(\hat{R}_{13})^{(1)}t}\)

\(|(b_i^{(1)}(G, t) - (b_i^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{R}_{13})^{(1)}t}\)

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i^{(1)}(T_{14}, t)\) and \((a_i^{(1)}(T_{14}, t). T_{14}^t\) and \((T_{14}, t)\) are points belonging to the interval \([\hat{k}_{13})^{(1)}, (\hat{R}_{13})^{(1)}\].

It is to be noted that \((a_i^{(1)}(T_{14}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{A}_{13})^{(1)} = 1\) then the function \((a_i^{(1)}(T_{14}, t)\), the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

Definition of \((\hat{A}_{13})^{(1)}, (\hat{k}_{13})^{(1)}:\)

(C) \((\hat{A}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, \) are positive constants
\[
\frac{a_j^{(1)}}{(M_{13})^{(1)}} < 1 \quad \frac{b_j^{(1)}}{(M_{13})^{(1)}} < 1
\]

**Definition of** \((\hat{p}_{13})^{(1)}, (\hat{q}_{13})^{(1)}): \hspace{1cm} 1191

(D) There exists two constants \((\hat{p}_{13})^{(1)}\) and \((\hat{q}_{13})^{(1)}\) which together with \((M_{13})^{(1)}, (k_{13})^{(1)}, (A_{13})^{(1)}\) and \((B_{13})^{(1)}\) and the constants \((a_i)^{(1)}, (a_j)^{(1)}, (b_i)^{(1)}, (b_j)^{(1)}, (b_i')^{(1)}, (b_j')^{(1)}, (\hat{r}_i)^{(1)}\), \(i = 13, 14, 15,\)

satisfy the inequalities

\[
\frac{1}{(M_{13})^{(1)}} [(a_i)^{(1)} + (a_j)^{(1)} + (A_{13})^{(1)} + (\hat{p}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1
\]

\[
\frac{1}{(M_{13})^{(1)}} [(b_i)^{(1)} + (b_j)^{(1)} + (B_{13})^{(1)} + (\hat{q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1
\]

\[+(a_{26})^{(2)}(T_{37}, t) = \text{First augmentation factor}\]

(1) \((a_{1})^{(2)}, (a_{j})^{(2)}, (a_{i})^{(2)}, (b_{i})^{(2)}, (b_{j})^{(2)}, (b_{i'})^{(2)} > 0, \quad i, j = 16, 17, 18\]

(E) (2) The functions \((a_{i})^{(2)}, (b_{i})^{(2)}\) are positive continuous increasing and bounded.

**Definition of** \((p_{1})^{(2)}, (r_{i})^{(2)}): \hspace{1cm} 1192

\[a_{i}^{(2)}(T_{17}, t) \leq (p_{1})^{(2)} \leq (\hat{A}_{16})^{(2)}\]

\[b_{i}^{(2)}(G_{19}, t) \leq (r_{i})^{(2)} \leq (\hat{B}_{16})^{(2)}\]

(F) (3) \(\lim_{t_{2} \to \infty} (a_{i})^{(2)}(T_{17}, t) = (p_{1})^{(2)}\)

\[\lim_{t_{2} \to \infty} (b_{i})^{(2)}(G_{19}, t) = (r_{i})^{(2)}\]

**Definition of** \((\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}): \hspace{1cm} 1193

\[\text{Where} \quad (\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_{1})^{(2)}, (r_{i})^{(2)}\]

They satisfy Lipschitz condition:

\[
|b_{i}^{(2)}(G_{19}, t) - (b_{i})^{(2)}(G_{19}, t)| < (k_{16})^{(2)} ||G_{19} - G_{19}'|| e^{-(\hat{B}_{16})^{(2)} t}\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_{i})^{(2)}(T_{17}, t)\) and \((b_{i})^{(2)}(G_{19}, t)\). And \((T_{17}, t)\) are points belonging to the interval \([\hat{k}_{16})^{(2)}, (M_{16})^{(2)}\].

It is to be noted that if \((M_{16})^{(2)} = 1\) then the function \((a_{i})^{(2)}(T_{17}, t)\) is continuously increasing. In the eventual case of the fact, that if \((M_{16})^{(2)} \neq 1\) then the second augmentation coefficient would be absolutely continuous.

**Definition of** \((\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}): \hspace{1cm} 1194

(G) (4) \((\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, \) are positive constants

259
There exists two constants \(( \hat{P}_{13} )^{(2)}, ( \hat{Q}_{13} )^{(2)} \):

Definition of \(( \hat{P}_{13} )^{(2)}, ( \hat{Q}_{13} )^{(2)} \):

There exists two constants \(( \hat{P}_{16} )^{(2)}\) and \(( \hat{Q}_{16} )^{(2)}\) which together with \(( \hat{\alpha}_{16} )^{(2)}, ( \hat{A}_{16} )^{(2)}\) and \(( \hat{B}_{16} )^{(2)}\) and the constants \((a_{ij})^{(2)}, (a_{ij}^{''})^{(2)}, (b_{ij})^{(2)}, (b_{ij}^{''})^{(2)}, (p_{ij})^{(2)}, (r_{ij})^{(2)}, i = 16,17,18,\)

satisfy the inequalities

\[
\frac{1}{(M_{16})^{(2)}} \left[ (a_{ij})^{(2)} + (a_{ij}^{''})^{(2)} + ( \hat{A}_{16} )^{(2)} + ( \hat{P}_{16} )^{(2)} ( \hat{\alpha}_{16} )^{(2)} \right] < 1
\]

\[
\frac{1}{(M_{16})^{(2)}} \left[ (b_{ij})^{(2)} + (b_{ij}^{''})^{(2)} + ( \hat{B}_{16} )^{(2)} + ( \hat{Q}_{16} )^{(2)} ( \hat{\alpha}_{16} )^{(2)} \right] < 1
\]

Where we suppose

\[
(H) \quad (a_{ij})^{(3)}, (a_{ij}^{''})^{(3)}, (b_{ij})^{(3)}, (b_{ij}^{''})^{(3)} > 0, \quad i, j = 20,21,22
\]

The functions \((a_{ij}^{''})^{(3)}, (b_{ij}^{''})^{(3)}\) are positive continuous increasing and bounded.

Definition of \((p_{ij})^{(3)}, (r_{ij})^{(3)}\):

\[
(a_{ij}^{''})^{(3)}(T_{21}, t) \leq (p_{ij})^{(3)} \leq ( \hat{A}_{20} )^{(3)}
\]

\[
(b_{ij}^{''})^{(3)}(G_{23}, t) \leq (r_{ij})^{(3)} \leq ( \hat{B}_{20} )^{(3)}
\]

\[
\lim_{T_{21} \to 0}(a_{ij}^{''})^{(3)}(T_{21}, t) = (p_{ij})^{(3)}
\]

\[
\lim_{G_{23} \to 0}(b_{ij}^{''})^{(3)}(G_{23}, t) = (r_{ij})^{(3)}
\]

Definition of \(( \hat{A}_{20} )^{(3)}, ( \hat{B}_{20} )^{(3)}\):

Where \([ ( \hat{A}_{20} )^{(3)}, ( \hat{B}_{20} )^{(3)}], (p_{ij})^{(3)}, (r_{ij})^{(3)}, i = 20,21,22\]

They satisfy Lipschitz condition:

\[
| (a_{ij}^{''})^{(3)}(T_{21}, t) - (a_{ij}^{''})^{(3)}(T_{21}, t) | \leq ( \hat{\alpha}_{20} )^{(3)} |T_{21} - T_{21}^{'}| e^{- ( \hat{\alpha}_{20} )^{(3)} t}
\]

\[
| (b_{ij}^{''})^{(3)}(G_{23}, t) - (b_{ij}^{''})^{(3)}(G_{23}, t) | < ( \hat{\alpha}_{20} )^{(3)} |G_{23} - G_{23}^{'}| e^{- ( \hat{\alpha}_{20} )^{(3)} t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_{ij}^{''})^{(3)}(T_{21}, t)\)

and \((a_{ij}^{''})^{(3)}(T_{21}, t) \cdot (T_{21}, t)\) And \((T_{21}, t)\) are points belonging to the interval \([ ( \hat{\alpha}_{20} )^{(3)}, ( \hat{\alpha}_{20} )^{(3)} ]\).

It is to be noted that \((a_{ij}^{''})^{(3)}(T_{21}, t)\) is uniformly continuous. In the eventuality of the fact, that if \(( \hat{\alpha}_{20} )^{(3)} = 1\) then the function \((a_{ij}^{''})^{(3)}(T_{21}, t)\), the THIRD augmentation coefficient, would be absolutely continuous.

Definition of \(( \hat{\alpha}_{20} )^{(3)}, ( \hat{\alpha}_{20} )^{(3)}\):

\[
( \hat{\alpha}_{20} )^{(3)}, ( \hat{\alpha}_{20} )^{(3)}, \text{ are positive constants}
\]

\[
\frac{(a_{ij})^{(3)}}{(M_{20})^{(3)}}, \frac{(b_{ij})^{(3)}}{(M_{20})^{(3)}} < 1
\]

There exists two constants \(( \hat{P}_{20} )^{(3)}\) and \(( \hat{Q}_{20} )^{(3)}\) which together with \(( \hat{\alpha}_{20} )^{(3)}, ( \hat{\alpha}_{20} )^{(3)}, ( \hat{\alpha}_{20} )^{(3)}\) and \(( \hat{\alpha}_{20} )^{(3)}\) and the constants \((a_{ij})^{(3)}, (a_{ij}^{''})^{(3)}, (b_{ij})^{(3)}, (b_{ij}^{''})^{(3)}, (p_{ij})^{(3)}, (r_{ij})^{(3)}, i = 20,21,22,\)
satisfy the inequalities

$$\frac{1}{(M_{20})^3} \left[ (a_i)^(3) + (a_j)^(3) + (\hat{A}_{20})^(3) + (\hat{B}_{20})^(3) (\hat{k}_{20})^(3) \right] < 1$$

$$\frac{1}{(M_{20})^3} \left[ (b_i)^(3) + (b_j)^(3) + (\hat{B}_{20})^(3) + (\hat{Q}_{20})^(3) (\hat{k}_{20})^(3) \right] < 1$$

Where we suppose

$$(a_i)^(4), (a_j)^(4), (\hat{a}_i')^(4), (b_i)^(4), (b_j')^(4), (b_j'')^(4) > 0, \quad i, j = 24, 25, 26$$

The functions $$(a_i')^(4), (b_i')^(4)$$ are positive continuous increasing and bounded.

**Definition of** $$(\hat{p})^(4), (\hat{r})^(4):$$

$$(a_i')^(4) (T_{25}, t) \leq (p_i)^(4) \leq (\hat{A}_{24})^(4)$$

$$(b_i')^(4) (G_{27}, t) \leq (r_i)^(4) \leq (B_{24})^(4)$$

$$\lim_{T_{2} \to \infty} (a_i'')^(4) (T_{25}, t) = (p_i)^(4)$$

$$\lim_{G_{27} \to \infty} (b_i'')^(4) (G_{27}, t) = (r_i)^(4)$$

**Definition of** $$(\hat{A}_{24})^(4), (\hat{B}_{24})^(4):$$

Where $$(\hat{A}_{24})^(4), (\hat{B}_{24})^(4), (p_i)^(4), (r_i)^(4)$$ are positive constants and $$(I) = 24, 25, 26$$

They satisfy Lipschitz condition:

$$| (a_i'')^(4) (T_{25}, t) - (a_i')^(4) (T_{25}, t) | \leq (\hat{k}_{24})^(4) |T_{25} - T_{25}'| e^{- (\hat{B}_{24})^(4) t}$$

$$| (b_i'')^(4) (G_{27}, t) - (b_i')^(4) (G_{27}, t) | < (\hat{k}_{24})^(4) |(G_{27}) - (G_{27}')| e^{- (\hat{B}_{24})^(4) t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $$(a_i')^(4) (T_{25}, t)$$ and $$(a_i'')^(4) (T_{25}, t) . (T_{25}', t)$$ And $$(T_{25}, t)$$ are points belonging to the interval $$[ (\hat{k}_{24})^(4), (\hat{M}_{24})^(4) ] .$$

It is to be noted that $$(a_i')^(4) (T_{25}, t)$$ is uniformly continuous. In the eventuality of the fact, that if $$(\hat{M}_{24})^(4) = 4$$ then the function $$(a_i'')^(4) (T_{25}, t)$$, the FOURTH augmentation coefficient WOULD be absolutely continuous.

**Definition of** $$(\hat{M}_{24})^(4), (\hat{k}_{24})^(4):$$

$$(\hat{M}_{24})^(4), (\hat{k}_{24})^(4),$$ are positive constants

$$\frac{(a_{i})^{(4)}}{(M_{24})^{(4)}} < 1$$

**Definition of** $$(\hat{P}_{24})^(4), (\hat{Q}_{24})^(4):$$

There exists two constants $$(\hat{P}_{24})^(4)$$ and $$(\hat{Q}_{24})^(4)$$ which together with $$(\hat{M}_{24})^(4), (\hat{k}_{24})^(4), (\hat{A}_{24})^(4)$$ and $$(\hat{B}_{24})^(4)$$ and the constants

$$(a_{i})^{(4)}, (a_{i})^{(4)}, (b_{i})^{(4)}, (b_{i})^{(4)}, (p_{i})^{(4)}, (r_{i})^{(4)}, (I) = 24, 25, 26,$$

satisfy the inequalities
Where we suppose

\((a_j)^{(5)}, (a_j')^{(5)}, (a_j'')^{(5)}; (b_j)^{(5)}, (b_j')^{(5)}, (b_j'')^{(5)} > 0, \quad i, j = 28, 29, 30\)

The functions \((a_j'')^{(5)}, (b_j'')^{(5)}\) are positive continuous increasing and bounded.

**Definition of** \((p_j)^{(5)}, (r_j)^{(5)}\):

\[ (a_j'')^{(5)}(T_{29}, t) \leq (p_j)^{(5)} \leq (\hat{A}_{28})^{(5)} \]

\[ (b_j'')^{(5)}(G_{31}, t) \leq (r_j)^{(5)} \leq (b_j')^{(5)} \leq (\hat{B}_{28})^{(5)} \]

\[ \lim_{t_2 \to \infty} (a_j'')^{(5)}(T_{29}, t) = (p_j)^{(5)} \]

\[ \lim_{G \to \infty} (b_j'')^{(5)}(G_{31}, t) = (r_j)^{(5)} \]

**Definition of** \((\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}\):

Where \([\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_j)^{(5)}, (r_j)^{(5)}\] are positive constants and \(t = 28, 29, 30\)

They satisfy Lipschitz condition:

\[ |(a_j'')^{(5)}(T_{29}', t) - (a_j'')^{(5)}(T_{29}, t)| \leq (\hat{K}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t} \]

\[ |(b_j'')^{(5)}((G_{31})', t) - (b_j'')^{(5)}(G_{31}, t)| \leq (\hat{K}_{28})^{(5)}|((G_{31})' - (G_{31}))(G_{31}) - (G_{31})'|e^{-(\hat{M}_{28})^{(5)}t} \]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_j'')^{(5)}(T_{29}', t)\) and \((a_j'')^{(5)}(T_{29}, t)\). \(T_{29}, t\) and \((T_{29}, t)\) are points belonging to the interval \([\hat{K}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\].

It is to be noted that \((a_j'')^{(5)}(T_{29}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{28})^{(5)} = 5\) then the function \((a_j'')^{(5)}(T_{29}, t)\), the FIFTH augmentation coefficient attributable would be absolutely continuous.

**Definition of** \((\hat{M}_{28})^{(5)}, (\hat{K}_{28})^{(5)}\):

\((\hat{M}_{28})^{(5)}, (\hat{K}_{28})^{(5)}\), are positive constants

\[ (a_j)^{(5)}(b_j)^{(5)} \leq (\hat{M}_{28})^{(5)} < 1 \]

**Definition of** \((\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}\):

There exists two constants \((\hat{P}_{28})^{(5)}\) and \((\hat{Q}_{28})^{(5)}\) which together with \((\hat{M}_{28})^{(5)}, (\hat{K}_{28})^{(5)}\), \((\hat{A}_{28})^{(5)}\) and \((\hat{B}_{28})^{(5)}\) and the constants \((a_j)^{(5)}, (a_j')^{(5)}, (b_j)^{(5)}, (b_j')^{(5)}, (p_j)^{(5)}, (r_j)^{(5)}\), \(i = 28, 29, 30\), satisfy the inequalities

\[ \frac{1}{(\hat{M}_{28})^{(5)}} [(a_j)^{(5)} + (a_j')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{K}_{28})^{(5)}] < 1 \]

\[ \frac{1}{(\hat{M}_{28})^{(5)}} [(b_j)^{(5)} + (b_j')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{K}_{28})^{(5)}] < 1 \]

Where we suppose
(a_i^{(6)}, a_{i'}^{(6)}, a_{i''}^{(6)}, b_i^{(6)}, b_{i'}^{(6)}, b_{i''}^{(6)}) > 0, \ i, j = 32,33,34

The functions (a_{i''}^{(6)}, b_{i''}^{(6)}) are positive continuous increasing and bounded.

**Definition of** (p_i^{(6)}, (r_i)^{(6)}):

\begin{align*}
(a_i^{(6)}(T_{33}, t) & \leq (p_i)^{(6)} \leq (A_{32})^{(6)} \\
(b_i^{(6)})(G_{33}, t) & \leq (r_i)^{(6)} \leq (B_{32})^{(6)} \\
\lim_{T_{33} \to 0}(a_i^{(6)}(T_{33}, t) &= (p_i)^{(6)} \\
\lim_{G_{33} \to 0}(b_i^{(6)})(G_{33}, t) = & (r_i)^{(6)}
\end{align*}

**Definition of** (A_{32}^{(6)}, (B_{32})^{(6)}):

Where \( (A_{32})^{(6)}, (B_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)} \) are positive constants and \( \ell = 32,33,34 \)

They satisfy Lipschitz condition:

\begin{align*}
|(a_i^{(6)}(T_{33}', t) - (a_i^{(6)}(T_{33}, t)) & \leq (A_{32})^{(6)} |T_{33} - T_{33}'| e^{-\lambda_{32}^{(6)} t} \\
|(b_i^{(6)})(G_{33}', t) - (b_i^{(6)})(G_{33}, t)) & < (B_{32})^{(6)} |(G_{33} - G_{33}')| e^{-\lambda_{32}^{(6)} t}
\end{align*}

With the Lipschitz condition, we place a restriction on the behavior of functions \( (a_i^{(6)}(T_{33}, t) \) and \( (b_i^{(6)}(G_{33}, t) \). \( T_{33}', t) \) and \( (T_{33}, t) \) are points belonging to the interval \( ([A_{32}]^{(6)}, ([B_{32}]^{(6)}]. \)

It is to be noted that \( (A_{32})^{(6)} = 1 \) then the function \( (a_i^{(6)}(T_{33}, t) \), the SIXTH augmentation coefficient would be absolutely continuous.

**Definition of** (\(\tilde{A}_{32}^{(6)}, (\tilde{B}_{32})^{(6)}\) :

\begin{align*}
(A_{32})^{(6)}, (\tilde{B}_{32})^{(6)} \text{ are positive constants} \\
\frac{(a_i^{(6)})(b_i^{(6)})}{(\tilde{A}_{32})^{(6)}(\tilde{B}_{32})^{(6)}} < 1
\end{align*}

**Definition of** (\(\tilde{P}_{32}^{(6)}, (\tilde{Q}_{32})^{(6)}\) :

There exists two constants \( (\tilde{P}_{32})^{(6)} \) and \( (\tilde{Q}_{32})^{(6)} \) which together with \( (\tilde{A}_{32})^{(6)}, (\tilde{B}_{32})^{(6)}, (\tilde{A}_{32})^{(6)}(\tilde{B}_{32})^{(6)} \) and \( (\tilde{B}_{32})^{(6)}(\tilde{Q}_{32})^{(6)} \) and the constants \( (a_i)^{(6)}, (a_{i'}^{(6)}, b_i^{(6)}, b_{i'}^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34, \) satisfy the inequalities

\begin{align*}
\frac{1}{(\tilde{A}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (A_{32})^{(6)} + (\tilde{P}_{32})^{(6)}(\tilde{A}_{32})^{(6)}] < 1 \\
\frac{1}{(\tilde{B}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (B_{32})^{(6)} + (\tilde{Q}_{32})^{(6)}(\tilde{B}_{32})^{(6)}] < 1
\end{align*}

Where we suppose
The functions \((a_i^{(7)}, b_i^{(7)})\) are positive continuous increasing and bounded.

**Definition of** \((p_i^{(7)}, r_i^{(7)})\):

\[
(a_i^{(7)}(T_{37}, t) \leq (p_i^{(7)} \leq (\tilde{A}_{36})^{(7)} \\
(b_i^{(7)}(G, t) \leq (r_i^{(7)} \leq (\tilde{B}_{36})^{(7)})
\]

\[
\lim_{t \to \infty} (a_i^{(7)}(T_{37}, t) = (p_i)^{(7)} \\
\lim_{G \to \infty}(b_i^{(7)}((G_{39})), t) = (r_i)^{(7)}
\]

**Definition of** \((\tilde{A}_{36})^{(7)}, (\tilde{B}_{36})^{(7)}\):

Where \((\tilde{A}_{36})^{(7)}, (\tilde{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}\) are positive constants and \(i = 36, 37, 38\)

They satisfy Lipschitz condition:

\[
|(a_i^{(7)}(T_{37}, t)) - (a_i^{(7)}(T_{37}, t))| \leq (\tilde{A}_{36})^{(7)}(T_{37} - T_{37}')e^{-(\tilde{A}_{36})^{(7)}t} \\
|(b_i^{(7)}((G_{39})), t) - (b_i^{(7)}((G_{39})), T_{37}))| < (\tilde{B}_{36})^{(7)}|((G_{39}) - (G_{39}))|e^{-(\tilde{B}_{36})^{(7)}t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i^{(7)}(T_{37}, t))\) and \((a_i^{(7)}(T_{37}, t))\). \((T_{37}, t)\) is a point belonging to the interval \([\tilde{A}_{36})^{(7)}, (\tilde{B}_{36})^{(7)}\) .

It is to be noted that if \((\tilde{A}_{36})^{(7)} = 7\) then the function \((a_i^{(7)}(T_{37}, t))\) the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

**Definition of** \((\tilde{M}_{36})^{(7)}, (\tilde{k}_{36})^{(7)}\):

\[
(K) \quad (\tilde{M}_{36})^{(7)}, (\tilde{k}_{36})^{(7)}, \text{ are positive constants} \\
\left(\frac{a_i^{(7)}}{M_{36}^{(7)}}\right), \left(\frac{b_i^{(7)}}{M_{36}^{(7)}}\right) < 1
\]

**Definition of** \((\tilde{p}_{36})^{(7)}, (\tilde{q}_{36})^{(7)}\):

There exists two constants \((\tilde{p}_{36})^{(7)}\) and \((\tilde{q}_{36})^{(7)}\) which together with \((\tilde{M}_{36})^{(7)}, (\tilde{k}_{36})^{(7)}, (\tilde{A}_{36})^{(7)}\) and \((\tilde{B}_{36})^{(7)}\) and the constants \((a_i^{(7)}), (a_i^{(7)}), (b_i^{(7)}), (b_i^{(7)}), (p_i^{(7)}), (r_i^{(7)}), i = 36, 37, 38\), satisfy the inequalities

\[
\frac{1}{(\tilde{M}_{36})^{(7)}}[(a_i^{(7)} + (a_i^{(7)}) + (\tilde{A}_{36})^{(7)} + (\tilde{p}_{36})^{(7)}(\tilde{k}_{36})^{(7)}] < 1 \\
\frac{1}{(\tilde{M}_{36})^{(7)}}[(b_i^{(7)} + (b_i^{(7)}) + (\tilde{B}_{36})^{(7)} + (\tilde{q}_{36})^{(7)}(\tilde{k}_{36})^{(7)}] < 1
\]
Definition of $G_0(0), T_0(0):$

\[
G_0(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{\phi}28)^{(5)} t}, \quad G_0(0) = G_0^0 > 0
\]

\[
T_0(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{\phi}28)^{(5)} t}, \quad T_0(0) = T_0^0 > 0
\]

Where we suppose

\[(L) \quad (a_i^{(8)}, a_i''^{(8)}, b_i^{(8)}, b_i''^{(8)}) > 0, \quad i, j = 40, 41, 42\]

\[(M) \quad \text{The functions } (a_i''^{(8)}, b_i''^{(8)}) \text{ are positive continuous increasing and bounded.}
\]

Definition of $(p_i)^{(8)}, (r_i)^{(8)}$: \[a_i''^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \]

\[(b_i''^{(8)})(G_{43}, t) \leq (r_i)^{(8)} \leq (b_i^{(8)}) \leq (\hat{B}_{40})^{(8)} \]

\[(N) \quad \lim_{T_{41} \to 0}(a_i''^{(8)}(T_{41}, t) = (p_i)^{(8)} \]

\[\lim_{G_{43} \to 0}(b_i''^{(8)}(G_{43}, t) = (r_i)^{(8)} \]

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$: \[\text{Where } \{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}\} \text{ are positive constants and } i = 40, 41, 42\]

They satisfy Lipschitz condition:

\[
|a_i''^{(8)}(T_{41}', t) - a_i''^{(8)}(T_{41}, t)| \leq (\hat{K}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\hat{M}_{40})^{(8)} t}
\]

\[
|b_i''^{(8)}(G_{43}', t) - b_i''^{(8)}(G_{43}, t)| \leq (\hat{K}_{40})^{(8)}|G_{43}' - G_{43}|e^{-(\hat{M}_{40})^{(8)} t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i''^{(8)}(T_{41}, t)$ and $(a_i''^{(8)}(T_{41}, t$). $(T_{41}, t$ and $(T_{41}, t$ are points belonging to the interval $[\hat{K}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i''^{(8)}(T_{41}, t$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i''^{(8)}(T_{41}, t$, the EIGHT augmentation coefficient would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{K}_{40})^{(8)}$: \[(O) \quad (\hat{M}_{40})^{(8)}, (\hat{K}_{40})^{(8)}\), are positive constants

\[
\left(\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\right) < 1
\]

\[
\hat{G}_{15}(t) = G_{15}^0 + \int_0^t \left[a_{15}^{(1)}G_{14}(s_{(1)}) - \left(a_{15}'^{(1)}(1) + a_{15}''^{(1)}(1)T_{14}(s_{(1)}), s_{(1)}\right)G_{15}(s_{(1)})\right] ds_{(1)}
\]

By

\[
\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[a_{13}^{(1)}G_{14}(s_{(1)}) - \left(a_{13}'^{(1)} + a_{13}''^{(1)}(1)T_{14}(s_{(1)}), s_{(1)}\right)G_{13}(s_{(1)})\right] ds_{(1)}
\]
Consider operator conditions if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions where \( s_{(13)} \) is the integrand that is integrated over an interval \((0, t)\)

**Definition of** \( G_i(0), T_l(0) : \)

\[
G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{Q}_{36})^{(7)} t}, \quad G_i(0) = G_i^0 \geq 0
\]

\[
T_l(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{Q}_{36})^{(7)} t}, \quad T_l(0) = T_l^0 > 0
\]

Consider operator \( \mathcal{A}^{(7)} \) defined on the space of sextuples of continuous functions \( G_i, T_l : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[
G_i(0) = G_i^0, \quad T_l(0) = T_l^0, \quad G_i^0 \leq (\hat{P}_{36})^{(7)}, \quad T_l^0 \leq (\hat{Q}_{36})^{(7)},
\]

\[
0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{Q}_{36})^{(7)} t}
\]

\[
0 \leq T_l(t) - T_l^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{Q}_{36})^{(7)} t}
\]

By

\[
\tilde{G}_{36}(t) = G_i^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{36}) - \left( (a'_{36})^{(7)} + a''_{36}^{(7)} T_{37}(s_{36}), s_{(36)} \right) G_{36}(s_{36}) \right] ds_{(36)}
\]

\[
\tilde{G}_{37}(t) = G_i^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{36}) - \left( (a'_{37})^{(7)} + a''_{37}^{(7)} T_{36}(s_{36}), s_{(36)} \right) G_{37}(s_{36}) \right] ds_{(36)}
\]

\[
\tilde{G}_{38}(t) = G_i^0 + \int_0^t \left[ (a_{38})^{(7)} G_{36}(s_{36}) - \left( (a'_{38})^{(7)} + a''_{38}^{(7)} T_{36}(s_{36}), s_{(36)} \right) G_{38}(s_{36}) \right] ds_{(36)}
\]

\[
\tilde{T}_{36}(t) = T_l^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{36}) - \left( (b'_{36})^{(7)} + b''_{36}^{(7)} G(s_{36}), s_{(36)} \right) T_{36}(s_{36}) \right] ds_{(36)}
\]

\[
\tilde{T}_{37}(t) = T_l^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{36}) - \left( (b'_{37})^{(7)} + b''_{37}^{(7)} G(s_{36}), s_{(36)} \right) T_{37}(s_{36}) \right] ds_{(36)}
\]

\[
\tilde{T}_{38}(t) = T_l^0 + \int_0^t \left[ (b_{38})^{(7)} T_{36}(s_{36}) - \left( (b'_{38})^{(7)} + b''_{38}^{(7)} G(s_{36}), s_{(36)} \right) T_{38}(s_{36}) \right] ds_{(36)}
\]

Where \( s_{(36)} \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( \mathcal{A}^{(2)} \) defined on the space of sextuples of continuous functions \( G_i, T_l : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[
G_i(0) = G_i^0, \quad T_l(0) = T_l^0, \quad G_i^0 \leq (\hat{P}_{16})^{(2)}, \quad T_l^0 \leq (\hat{Q}_{16})^{(2)},
\]

\[
0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{Q}_{16})^{(2)} t}
\]
\[ 0 \leq T_i(t) - T_i^0 \leq (\tilde{Q}_{16})^{(2)}e^{(\tilde{A}_{16})^{(2)}t} \]

By
\[ \tilde{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16}^{(2)}G_{16}(s_{16}) - (a_{16}^{(2)} + a_{16}^{(2)}(T_{17}(s_{16}), s_{16}))G_{16}(s_{16}) \right] ds_{16} \]
\[ \tilde{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17}^{(2)}G_{16}(s_{16}) - (a_{17}^{(2)} + a_{17}^{(2)}(T_{17}(s_{16}), s_{17}))G_{17}(s_{16}) \right] ds_{16} \]
\[ \tilde{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18}^{(2)}G_{17}(s_{16}) - (a_{18}^{(2)} + a_{18}^{(2)}(T_{17}(s_{16}), s_{16}))G_{18}(s_{16}) \right] ds_{16} \]
\[ \tilde{T}_{16}(t) = T_{16}^0 + \int_0^t \left[ (b_{16}^{(2)}T_{17}(s_{16}) - (b_{16}^{(2)} - b_{16}^{(2)}(G(s_{16}), s_{16}))T_{16}(s_{16}) \right] ds_{16} \]
\[ \tilde{T}_{17}(t) = T_{17}^0 + \int_0^t \left[ (b_{17}^{(2)}T_{16}(s_{16}) - (b_{17}^{(2)} - b_{17}^{(2)}(G(s_{16}), s_{16}))T_{17}(s_{16}) \right] ds_{16} \]
\[ \tilde{T}_{18}(t) = T_{18}^0 + \int_0^t \left[ (b_{18}^{(2)}T_{17}(s_{16}) - (b_{18}^{(2)} - b_{18}^{(2)}(G(s_{16}), s_{16}))T_{18}(s_{16}) \right] ds_{16} \]

Where \( s_{16} \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( \mathcal{A}^{(3)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}^+ \rightarrow \mathbb{R}^+ \) which satisfy
\[ G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\tilde{P}_{20})^{(3)}, T_i^0 \leq (\tilde{Q}_{20})^{(3)}, \]
\[ 0 \leq G_i(t) - G_i^0 \leq (\tilde{P}_{20})^{(3)}e^{(\tilde{B}_{20})^{(3)}t}, \]
\[ 0 \leq T_i(t) - T_i^0 \leq (\tilde{Q}_{20})^{(3)}e^{(\tilde{B}_{20})^{(3)}t} \]

By
\[ \tilde{G}_{20}(t) = G_{20}^0 + \int_0^t \left[ (a_{20}^{(3)}G_{21}(s_{20}) - (a_{20}^{(3)} + a_{20}^{(3)}(T_{21}(s_{20}), s_{20}))G_{20}(s_{20}) \right] ds_{20} \]
\[ \tilde{G}_{21}(t) = G_{21}^0 + \int_0^t \left[ (a_{21}^{(3)}G_{20}(s_{20}) - (a_{21}^{(3)} + a_{21}^{(3)}(T_{21}(s_{20}), s_{20}))G_{21}(s_{20}) \right] ds_{20} \]
\[ \tilde{G}_{22}(t) = G_{22}^0 + \int_0^t \left[ (a_{22}^{(3)}G_{21}(s_{20}) - (a_{22}^{(3)} + a_{22}^{(3)}(T_{21}(s_{20}), s_{20}))G_{22}(s_{20}) \right] ds_{20} \]
\[ \tilde{T}_{20}(t) = T_{20}^0 + \int_0^t \left[ (b_{20}^{(3)}T_{21}(s_{20}) - (b_{20}^{(3)} - b_{20}^{(3)}(G(s_{20}), s_{20}))T_{20}(s_{20}) \right] ds_{20} \]
\[ \tilde{T}_{21}(t) = T_{21}^0 + \int_0^t \left[ (b_{21}^{(3)}T_{20}(s_{20}) - (b_{21}^{(3)} - b_{21}^{(3)}(G(s_{20}), s_{20}))T_{21}(s_{20}) \right] ds_{20} \]
\[ \tilde{T}_{22}(t) = T_{22}^0 + \int_0^t \left[ (b_{22}^{(3)}T_{21}(s_{20}) - (b_{22}^{(3)} - b_{22}^{(3)}(G(s_{20}), s_{20}))T_{22}(s_{20}) \right] ds_{20} \]

Where \( s_{20} \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( \mathcal{A}^{(4)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}^+ \rightarrow \mathbb{R}^+ \) which satisfy
\[ G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\tilde{P}_{24})^{(4)}, T_i^0 \leq (\tilde{Q}_{24})^{(4)}, \]
\[-G_i(t) - G^0_i \leq (\hat{P}_{24})^{(4)} e^{(\hat{R}_{24})^{(4)}} t \]
\[-T_i(t) - T^0_i \leq (\hat{Q}_{24})^{(4)} e^{(\hat{R}_{24})^{(4)}} t \]

**By**

\[ \check{G}_{24}(t) = G^0_{24} + \int_0^t [(a_{24})^{(4)} G_{24}(s_{(24)}) - \left((a_{24}^{(4)} + a_{24}^{(2)} T_{25}(s_{(24)}), s_{(24)})\right)G_{24}(s_{(24)})] ds_{(24)} \]
\[ \check{G}_{25}(t) = G^0_{25} + \int_0^t [(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a_{25}^{(4)} + a_{25}^{(2)} T_{25}(s_{(24)}), s_{(24)})\right)G_{24}(s_{(24)})] ds_{(24)} \]
\[ \check{G}_{26}(t) = G^0_{26} + \int_0^t [(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a_{26}^{(4)} + a_{26}^{(2)} T_{25}(s_{(24)}), s_{(24)})\right)G_{25}(s_{(24)})] ds_{(24)} \]
\[ \check{T}_{24}(t) = T^0_{24} + \int_0^t [(b_{24})^{(4)} T_{24}(s_{(24)}) - \left((b_{24}^{(4)} - b_{24}^{(2)} G(s_{(24)}), s_{(24)})\right)T_{24}(s_{(24)})] ds_{(24)} \]
\[ \check{T}_{25}(t) = T^0_{25} + \int_0^t [(b_{25})^{(4)} T_{25}(s_{(24)}) - \left((b_{25}^{(4)} - b_{25}^{(2)} G(s_{(24)}), s_{(24)})\right)T_{25}(s_{(24)})] ds_{(24)} \]
\[ \check{T}_{26}(t) = T^0_{26} + \int_0^t [(b_{26})^{(4)} T_{26}(s_{(24)}) - \left((b_{26}^{(4)} - b_{26}^{(2)} G(s_{(24)}), s_{(24)})\right)T_{26}(s_{(24)})] ds_{(24)} \]

Where \( s_{(24)} \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( A^{(5)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G^0_i, T_i(0) = T^0_i, G^0_i \leq (\hat{P}_{28})^{(5)}, T^0_i \leq (\hat{Q}_{28})^{(5)}, \]
\[ 0 \leq G_i(t) - G^0_i \leq (\hat{P}_{28})^{(5)} e^{(\hat{R}_{28})^{(5)}} t \]
\[ 0 \leq T_i(t) - T^0_i \leq (\hat{Q}_{28})^{(5)} e^{(\hat{R}_{28})^{(5)}} t \]

**By**

\[ \check{G}_{28}(t) = G^0_{28} + \int_0^t [(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a_{28}^{(5)} + a_{28}^{(3)} T_{29}(s_{(28)}), s_{(28)})\right)G_{29}(s_{(28)})] ds_{(28)} \]
\[ \check{G}_{29}(t) = G^0_{29} + \int_0^t [(a_{29})^{(5)} G_{29}(s_{(28)}) - \left((a_{29}^{(5)} + a_{29}^{(3)} T_{29}(s_{(28)}), s_{(28)})\right)G_{29}(s_{(28)})] ds_{(28)} \]
\[ \check{G}_{30}(t) = G^0_{30} + \int_0^t [(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a_{30}^{(5)} + a_{30}^{(3)} T_{29}(s_{(28)}), s_{(28)})\right)G_{30}(s_{(28)})] ds_{(28)} \]
\[ \check{T}_{28}(t) = T^0_{28} + \int_0^t [(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b_{28}^{(5)} - b_{28}^{(3)} G(s_{(28)}), s_{(28)})\right)T_{29}(s_{(28)})] ds_{(28)} \]
\[ \check{T}_{29}(t) = T^0_{29} + \int_0^t [(b_{29})^{(5)} T_{29}(s_{(28)}) - \left((b_{29}^{(5)} - b_{29}^{(3)} G(s_{(28)}), s_{(28)})\right)T_{29}(s_{(28)})] ds_{(28)} \]
\[ \check{T}_{30}(t) = T^0_{30} + \int_0^t [(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b_{30}^{(5)} - b_{30}^{(3)} G(s_{(28)}), s_{(28)})\right)T_{30}(s_{(28)})] ds_{(28)} \]

Where \( s_{(28)} \) is the integrand that is integrated over an interval \((0, t)\)
Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

\[ G_i(0) = G^0_i, \ T_i(0) = T^0_i, \ G^0_i \leq (\hat{P}_{32})^{(6)}, \ T^0_i \leq (\hat{Q}_{32})^{(6)} , \]
\[ 0 \leq G_i(t) - G^0_i \leq (\hat{P}_{32})^{(6)} e^{(\mathcal{A}_{32})^{(6)}t} \]
\[ 0 \leq T_i(t) - T^0_i \leq (\hat{Q}_{32})^{(6)} e^{(\mathcal{A}_{32})^{(6)}t} \]

By

\[ G_{32}(t) = G^0_{32} + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{32}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} \right) T_{33}(s_{32}), s_{32}) \right] G_{32}(s_{32}) \right] ds_{32} \]
\[ \tilde{G}_{33}(t) = G^0_{33} + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{32}) - \left( (a'_{33})^{(6)} + a''_{33})^{(6)} \right) T_{33}(s_{32}), s_{32}) \right] G_{33}(s_{32}) \right] ds_{32} \]
\[ \tilde{G}_{34}(t) = G^0_{34} + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{32}) - \left( (a'_{34})^{(6)} + a''_{34})^{(6)} \right) T_{33}(s_{32}), s_{32}) \right] G_{34}(s_{32}) \right] ds_{32} \]
\[ T_{32}(t) = T^0_{32} + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{32}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} \right) G(s_{32}), s_{32}) \right] T_{32}(s_{32}) \right] ds_{32} \]
\[ \tilde{T}_{33}(t) = T^0_{33} + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{32}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} \right) G(s_{32}), s_{32}) \right] T_{33}(s_{32}) \right] ds_{32} \]
\[ \tilde{T}_{34}(t) = T^0_{34} + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{32}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} \right) G(s_{32}), s_{32}) \right] T_{34}(s_{32}) \right] ds_{32} \]

Where $s_{32}$ is the integrand that is integrated over an interval $(0, t)$

If the conditions are fulfilled, there exists a solution satisfying the conditions

**Definition of** $G_i(0), T_i(0)$:

\[ G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\mathcal{A}_{36})^{(7)}t}, \quad G_i(0) = G^0_i > 0 \]
\[ T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\mathcal{A}_{36})^{(7)}t}, \quad T_i(0) = T^0_i > 0 \]

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

\[ G_i(0) = G^0_i, \ T_i(0) = T^0_i, \ G^0_i \leq (\hat{P}_{36})^{(7)}, \ T^0_i \leq (\hat{Q}_{36})^{(7)} , \]
\[ 0 \leq G_i(t) - G^0_i \leq (\hat{P}_{36})^{(7)} e^{(\mathcal{A}_{36})^{(7)}t} \]
\[ 0 \leq T_i(t) - T^0_i \leq (\hat{Q}_{36})^{(7)} e^{(\mathcal{A}_{36})^{(7)}t} \]

269
By

\[ \tilde{G}_{36}(t) = G_{36}^{0} + \int_{0}^{t} \left[ \left( a_{36}^{(7)}(t)^{7} G_{37}(s_{36}) \right) - \left( \left( a_{36}^{(7)} + a_{236}^{(7)}(T_{32}(s_{36}), s_{36}) \right) G_{36}(s_{36}) \right) \right] ds_{36} \]

\[ \tilde{G}_{37}(t) = G_{37}^{0} + \int_{0}^{t} \left[ \left( a_{37}^{(7)}(t)^{7} G_{36}(s_{36}) \right) - \left( \left( a_{37}^{(7)} + a_{237}^{(7)}(T_{32}(s_{36}), s_{36}) \right) G_{37}(s_{36}) \right) \right] ds_{36} \]

\[ \tilde{G}_{38}(t) = G_{38}^{0} + \int_{0}^{t} \left[ \left( a_{38}^{(7)}(t)^{7} G_{37}(s_{36}) \right) - \left( \left( a_{38}^{(7)} + a_{238}^{(7)}(T_{32}(s_{36}), s_{36}) \right) G_{38}(s_{36}) \right) \right] ds_{36} \]

\[ \tilde{T}_{36}(t) = T_{36}^{0} + \int_{0}^{t} \left[ \left( b_{36}^{(7)}(t)^{7} T_{36}(s_{36}) \right) - \left( \left( b_{36}^{(7)} + b_{236}^{(7)}(G(s_{36}), s_{36}) \right) T_{36}(s_{36}) \right) \right] ds_{36} \]

\[ \tilde{T}_{37}(t) = T_{37}^{0} + \int_{0}^{t} \left[ \left( b_{37}^{(7)}(t)^{7} T_{36}(s_{36}) \right) - \left( \left( b_{37}^{(7)} + b_{237}^{(7)}(G(s_{36}), s_{36}) \right) T_{37}(s_{36}) \right) \right] ds_{36} \]

\[ \tilde{T}_{38}(t) = T_{38}^{0} + \int_{0}^{t} \left[ \left( b_{38}^{(7)}(t)^{7} T_{37}(s_{36}) \right) - \left( \left( b_{38}^{(7)} + b_{238}^{(7)}(G(s_{36}), s_{36}) \right) T_{38}(s_{36}) \right) \right] ds_{36} \]

Where \( s_{36} \) is the integrand that is integrated over an interval \((0, t)\)

If the conditions above are fulfilled, there exists a solution satisfying the conditions

Definition of \( G_{i}(0), T_{i}(0) \):

\[ G_{i}(t) \leq \left( \tilde{P}_{40}^{(8)} e^{(R_{40}) t} \right)^{0} , \quad G_{i}(0) = G_{i}^{0} > 0 \]

\[ T_{i}(t) \leq \left( \tilde{Q}_{40}^{(8)} e^{(R_{40}) t} \right)^{0} , \quad T_{i}(0) = T_{i}^{0} > 0 \]

Proof:

Consider operator \( A^{(8)} \) defined on the space of sextuples of continuous functions \( G_{i}, T_{i} : \mathbb{R}_{+} \rightarrow \mathbb{R}_{+} \) which satisfy

\[ G_{i}(0) = G_{i}^{0}, \quad T_{i}(0) = T_{i}^{0}, \quad G_{i}^{0} \leq \left( \tilde{P}_{40}^{(8)} \right)^{0}, \quad T_{i}^{0} \leq \left( \tilde{Q}_{40}^{(8)} \right)^{0}, \]

\[ 0 \leq G_{i}(t) - G_{i}^{0} \leq \left( \tilde{P}_{40}^{(8)} e^{(R_{40}) t} \right)^{0} \]

\[ 0 \leq T_{i}(t) - T_{i}^{0} \leq \left( \tilde{Q}_{40}^{(8)} e^{(R_{40}) t} \right)^{0} \]

By

\[ \tilde{G}_{40}(t) = G_{40}^{0} + \int_{0}^{t} \left[ \left( a_{40}^{(8)}(s_{40}) - \left( \left( a_{40}^{(8)} + a_{240}^{(8)}(T_{41}(s_{40}), s_{40}) \right) G_{40}(s_{40}) \right) \right) ds_{40} \]

\[ \tilde{G}_{41}(t) = G_{41}^{0} + \int_{0}^{t} \left[ \left( a_{41}^{(8)}(s_{40}) - \left( \left( a_{41}^{(8)} + a_{241}^{(8)}(T_{41}(s_{40}), s_{40}) \right) G_{41}(s_{40}) \right) \right) ds_{40} \]
\[
\mathcal{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42}^{(8)} G_{41}(s_{(40)}) - \left((a_{42}^{(8)} + a_{42}^{(8)}(T_{41}(s_{(40)}),s_{(40)}) G_{42}(s_{(40)})) \right) dS_{(40)} \right.
\]

\[
\hat{\mathcal{T}}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40}^{(8)} T_{41}(s_{(40)}) - \left((b_{40}^{(8)} - b_{40}^{(8)}(G(s_{(40)}),s_{(40)}) \right) T_{40}(s_{(40)}) \right) dS_{(40)} \right.
\]

\[
\hat{\mathcal{T}}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41}^{(8)} T_{40}(s_{(40)}) - \left((b_{41}^{(8)} - b_{41}^{(8)}(G(s_{(40)}),s_{(40)}) \right) T_{41}(s_{(40)}) \right) dS_{(40)} \right.
\]

\[
\hat{\mathcal{T}}_{42}(t) = T_{42}^0 + \int_0^t \left[ (b_{42}^{(8)} T_{41}(s_{(40)}) - \left((b_{42}^{(8)} - b_{42}^{(8)}(G(s_{(40)}),s_{(40)}) \right) T_{42}(s_{(40)}) \right) dS_{(40)} \right.
\]

Where \(s_{(40)}\) is the integrand that is integrated over an interval \((0,t)\).

The operator \(\mathcal{A}^{(8)}\) maps the space of functions satisfying global equations into itself.

Indeed it is obvious that

\[
G_{40}(t) \leq G_{40}^0 + \int_0^t \left[ (a_{40}^{(8)} G_{41}^0 + (\hat{P}_{40}^{(8)} e^{(\mathcal{A}_{40})^{(8)}} s_{(40)}) \right) dS_{(40)} =

(1 + (a_{40}^{(8)} t) G_{41}^0 + \frac{a_{40}^{(8)} (\hat{P}_{40}^{(8)})}{(M_{40})^{(8)}} e^{(\mathcal{A}_{40})^{(8)} t} - 1)
\]

From which it follows that

\[
(G_{40}(t) - G_{40}^0) e^{-(\mathcal{A}_{40})^{(8)} t} \leq \frac{(a_{40}^{(8)} (\hat{P}_{40}^{(8)})}{(M_{40})^{(8)}} \left[(\hat{P}_{40}^{(8)}) + G_{41}^0 \right) e^{-\frac{(\hat{P}_{40}^{(8)}) g_{41}^0}{a_{41}^{(8)}}} + (\hat{P}_{40}^{(8)}) \right]
\]

\((G_{40}^0)\) is as defined in the statement of theorem 1.

In order that the operator \(\mathcal{A}^{(1)}\) transforms the space of sextuples of functions \(G_i, T_i\) satisfying GLOBAL EQUATIONS into itself.

The operator \(\mathcal{A}^{(1)}\) is a contraction with respect to the metric

\[
d \left((G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)})\right) =
\]

\[
\sup_{t \in \mathbb{R}_+} \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-(\mathcal{A}_{13})^{(1)} t} \max_{t \in \mathbb{R}_+} \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-(\mathcal{A}_{13})^{(1)} t}
\]
Indeed if we denote

**Definition of \( \mathcal{G}, \mathcal{T} : (\mathcal{G}, \mathcal{T}) = \mathcal{A}^{(1)}(\mathcal{G}, \mathcal{T}) \)**

It results

\[
|G_{13}^{(1)} - G_{13}^{(2)}| \leq \int_{0}^{t} (a_{13}^{(1)}) |G_{14}^{(1)} - G_{14}^{(2)}| e^{-\mathcal{K}_{13}(1)_{s_{(13)}}} e^{(\mathcal{M}_{13})^{(1)}_{s_{(13)}}} ds_{(13)} + \\
\int_{0}^{t} [(a_{13}^{(1)}) G_{13}^{(1)} - G_{13}^{(2)}] e^{-\mathcal{K}_{13}(1)_{s_{(13)}}} e^{(\mathcal{M}_{13})^{(1)}_{s_{(13)}}} + \\
(a_{13}^{(1)''} (T_{14}^{(1)}, s_{(13)}) G_{13}^{(1)} - G_{13}^{(2)})[ e^{-\mathcal{K}_{13}(1)_{s_{(13)}}} e^{(\mathcal{M}_{13})^{(1)}_{s_{(13)}}} + \\
G_{13}^{(2)} (a_{13}^{(1)''} (T_{14}^{(1)}, s_{(13)}) - a_{13}^{(1)''} (T_{14}^{(2)}, s_{(13)})) e^{-\mathcal{K}_{13}(1)_{s_{(13)}}} e^{(\mathcal{M}_{13})^{(1)}_{s_{(13)}}}] ds_{(13)}
\]

Where \( s_{(13)} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[
|G_{13}^{(1)} - G_{13}^{(2)}| e^{-\mathcal{K}_{13}(1)_{t}} \leq \frac{1}{(\mathcal{M}_{13})^{(1)}} [(a_{13}^{(1)}) + (a_{13}^{(1)})^{(1)} + (\mathcal{A}_{13})^{(1)} + (\mathcal{P}_{13})^{(1)} (\mathcal{K}_{13})^{(1)}] d \left( G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)} \right)
\]

And analogous inequalities for \( G_{i}, T_{i} \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{13}^{(1)''})\) and \((b_{13}^{(1)''})\) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\mathcal{P}_{13})^{(1)} e^{(\mathcal{M}_{13})^{(1)}_{t}}\) and \((\mathcal{Q}_{13})^{(1)} e^{(\mathcal{M}_{13})^{(1)}_{t}}\) respectively of \( \mathbb{R}_{+} \).

If instead of proving the existence of the solution on \( \mathbb{R}_{+} \), we have to prove it only on a compact then it suffices to consider that \((a_{i}^{(1)''})\) and \((b_{i}^{(1)''})\), \(i = 13, 14, 15\) depend only on \( T_{14} \) and respectively on \( G(\text{and not on } t) \) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_{i}(t) = 0 \) and \( T_{i}(t) = 0 \)

From 19 to 24 it results

\[
G_{i}(t) \geq G_{i}^{0} e^{-\int_{0}^{t} [(a_{i}^{(1)} - (a_{i}^{(1)''}) (T_{14}(s_{(13)}) s_{(13)})] ds_{(13)}} \geq 0
\]

\[
T_{i}(t) \geq T_{i}^{0} e^{(-b_{i}^{(1)}) t} > 0 \text{ for } t > 0
\]

**Definition of \( (\mathcal{M}_{13})^{(1)} \) and \( (\mathcal{M}_{13})^{(1)} \):**

**Remark 3:** if \( G_{13} \) is bounded, the same property have also \( G_{14} \) and \( G_{15} \). instead if

\[
G_{13} \leq (\mathcal{M}_{13})^{(1)} \text{ it follows } \frac{d G_{14}}{dt} \leq (\mathcal{M}_{13})^{(1)}_{1} - (a_{14})^{(1)} G_{14} \text{ and by integrating }
\]

\[
G_{14} \leq (\mathcal{M}_{13})^{(1)}_{2} = G_{14}^{0} + 2(a_{14})^{(1)} (\mathcal{M}_{13})^{(1)}_{1} (a_{14})^{(1)}
\]
In the same way, one can obtain
\[ G_{15} \leq ((M_{13})^{(1)})_3 = G_0^2 + 2(a_{15})^{(1)}((M_{13})^{(1)})_2/(a_{15})^{(1)} \]

If \( G_{14} \) or \( G_{15} \) is bounded, the same property follows for \( G_{13}, G_{15} \) and \( G_{13}, G_{14} \) respectively.

Remark 4: If \( G_{13} \) is bounded, from below, the same property holds for \( G_{14} \) and \( G_{15} \).

The proof is analogous with the preceding one. An analogous property is true if \( G_{14} \) is bounded from below.

Remark 5: If \( T_{13} \) is bounded from below and \( \lim_{t \to \infty} ((b_{1''})^{(1)} (G(t), t)) = (b_{1''})^{(1)} \) then \( T_{14} \to \infty \).

**Definition of** \((m)^{(1)}\) **and** \(\varepsilon_1\):

Indeed let \( t_1 \) be so that for \( t > t_1 \)

\[ (b_{1''})^{(1)} (G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)} \]

Then
\[ \frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14} \]

which leads to
\[ T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_0^2 e^{-\varepsilon_1 t} \]

If we take \( t \) such that \( e^{-\varepsilon_1 t} = \frac{1}{2} \) it results

\[ T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{2} \right), \quad t \log \frac{2}{\varepsilon_1} \]

By taking now \( \varepsilon_1 \) sufficiently small one sees that \( T_{14} \) is unbounded.

The same property holds for \( T_{15} \) if \( \lim_{t \to \infty} ((b_{1''})^{(1)} (G(t), t)) = (b_{1''})^{(1)} \)

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take \( (a_{13})^{(2)} \), \( (b_{13})^{(2)} \) \( (M_{16})^{(2)} \) \( (a_{16})^{(2)} \) and \( (Q_{16})^{(2)} \) large to have

\[ \frac{(a_{13})^{(2)}}{(M_{16})^{(2)}} \left( (P_{16})^{(2)} + ((P_{16})^{(2)} + G_0^0) e^{-(P_{16})^{(2)} (G_0^0)} \right) \leq (P_{16})^{(2)} \]

\[ \frac{(b_{13})^{(2)}}{(M_{16})^{(2)}} \left( (Q_{16})^{(2)} + T_0^0 e^{-\left(\frac{(Q_{16})^{(2)} + T_0^0}{T_0^0}\right)} \right) \leq (Q_{16})^{(2)} \]

In order that the operator \( \mathcal{A}^{(2)} \) transforms the space of sextuples of functions \( G_i, T_i \) satisfying

The operator \( \mathcal{A}^{(2)} \) is a contraction with respect to the metric

\[ d \left( ((G_{19})^{(1)}(T_{19})^{(1)}), ((G_{19})^{(2)}(T_{19})^{(2)}) \right) = \]
\[ \sup_{i} \max_{t \in \mathbb{R}_{+}} \left| G_{1}^{(1)}(t) - G_{i}^{(2)}(t) \right| e^{-\left( M_{16}(2) t \right)} \max_{t \in \mathbb{R}_{+}} \left| T_{i}^{(1)}(t) - T_{i}^{(2)}(t) \right| e^{-\left( M_{16}(2) t \right)} \]

Indeed if we denote

Definition of \( \tilde{G}_{19}, \tilde{T}_{19} : \quad \tilde{G}_{19} \tilde{T}_{19} = \mathcal{A}(G_{19}, T_{19}) \)

It results

\[ | \tilde{G}_{16}^{(1)} - \tilde{G}_{16}^{(2)} | \leq \int_{0}^{t} (a_{16}^{(2)}) (b_{16}^{(2)}) e^{-\left( M_{16}(2) t \right)} \]

\[ \int_{0}^{t} \left( (a_{16}^{(2)}) (b_{16}^{(2)}) e^{-\left( M_{16}(2) t \right)} \right) \]}

Where \( s_{16} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[ | (G_{19}^{(1)} - G_{19}^{(2)}) | e^{-\left( M_{16}(2) t \right)} \]

\[ \leq \frac{1}{(M_{16}(2))} (a_{16}^{(2)}) + (a_{16}^{(2)}) + (M_{16}(2)) \]

\[ + (P_{16}(2), k_{16}(2)) d((G_{19}^{(1)}), (T_{19}^{(1)}), (G_{19}^{(2)}), (T_{19}^{(2)})) \]

And analogous inequalities for \( G_{i} \) and \( T_{i} \) Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed \( (a_{16}^{(2)}) \) and \( (b_{16}^{(2)}) \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( (P_{16}(2)) e^{-\left( M_{16}(2) t \right)} \) and \( (Q_{16}(2)) e^{-\left( M_{16}(2) t \right)} \) respectively of \( \mathbb{R}_{+} \).

If instead of proving the existence of the solution on \( \mathbb{R}_{+} \), we have to prove it only on a compact then it suffices to consider that \( (a_{i}^{(2)}) \) and \( (b_{i}^{(2)}) \), \( i = 16, 17, 18 \) depend only on \( T_{17} \) and respectively on \( G_{19} \) and not on \( t \) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any \( t \) where \( G_{i}(t) = 0 \) and \( T_{i}(t) = 0 \)

From 19 to 24 it results

\[ G_{i}^{(t)} \geq G_{i}^{(t)} e^{-\left( M_{16}(2) t \right) + \left( T_{17}(S_{16}(2)) \right)} + d \geq 0 \]

\[ T_{i}^{(t)} \geq T_{i}^{(t)} e^{-\left( M_{16}(2) t \right)} \geq 0 \quad \text{for } t > 0 \]
Definition of \((\mathcal{M}_{16})^{(2)}_1, (\mathcal{M}_{16})^{(2)}_2\) and \((\mathcal{M}_{16})^{(2)}_3\):

Remark 3: if \(G_{16}\) is bounded, the same property have also \(G_{17}\) and \(G_{18}\). Indeed if \(G_{16} < (\mathcal{M}_{16})^{(2)}\) it follows \(\frac{dG_{17}}{dt} \leq (\mathcal{M}_{16})^{(2)}_1 - (a_{17})^{(2)}G_{17}\) and by integrating
\[
G_{17} \leq (\mathcal{M}_{16})^{(2)}_2 = G_{17}^0 + 2(a_{17})^{(2)}(\mathcal{M}_{16})^{(2)}_1/(a_{17})^{(2)}
\]
In the same way, one can obtain
\[
G_{18} \leq (\mathcal{M}_{16})^{(2)}_3 = G_{18}^0 + 2(a_{18})^{(2)}(\mathcal{M}_{16})^{(2)}_2/(a_{18})^{(2)}
\]
If \(G_{17}\) or \(G_{18}\) is bounded, the same property follows for \(G_{16}, G_{18}\) and \(G_{16}, G_{17}\) respectively.

Remark 4: If \(G_{16}\) is bounded, from below, the same property holds for \(G_{17}\) and \(G_{18}\).

The proof is analogous with the preceding one. An analogous property is true if \(G_{17}\) is bounded from below.

Remark 5: If \(T_{16}\) is bounded from below and \(\lim_{t \to \infty} (b_{17}^{(2)}(G_{19})(t), t) = (b_{17}^{(2)}(2)\) then \(T_{17} \to \infty\).

Definition of \((m)^{(2)}\) and \(\varepsilon_2\):

Indeed let \(t_2\) be so that for \(t > t_2\)
\[(b_{17})^{(2)} - (b_{17}^{(2)}(G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}\]

Then \(\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}\) which leads to
\[
T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{16}^0e^{-\varepsilon_2 t} \text{ if we take } t \text{ such that } e^{-\varepsilon_2 t} = \frac{1}{2} \text{ it results}
\]
\[
T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), t \log \frac{2}{\varepsilon_2} \text{ By taking now } \varepsilon_2 \text{ sufficiently small one sees that } T_{17} \text{ is unbounded.}
\]

The same property holds for \(T_{18}\) if \(\lim_{t \to \infty} (b_{18}^{(2)}(2)(G_{19})(t), t) = (b_{18}^{(2)}(2)\)

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take \(\frac{(a_{20})^{(3)}}{(M_{20})^{(3)}} , \frac{(b_{20})^{(3)}}{(M_{20})^{(3)}} < 1\) and to choose

\((\dot{P}_{20})^{(3)}\) and \((\dot{Q}_{20})^{(3)}\) large to have
\[
\frac{(a_{20})^{(3)}}{(M_{20})^{(3)}} \left[ (\dot{P}_{20})^{(3)} + (\dot{P}_{20})^{(3)} + G_{20}^0 \right] \leq (\dot{P}_{20})^{(3)}
\]
\[
\frac{(b_i)^{(3)}}{(M_20)^{(3)}} \left[ \left( \bar{Q}_{20}^{(3)} + T_{i}^{0} \right) e^{\left( \frac{(\bar{Q}_{20}^{(3)} + T_{i}^{0})}{T_{i}^{0}} \right)} + \left( \bar{Q}_{20}^{(3)} \right) \right] \leq \left( \bar{Q}_{20}^{(3)} \right) 
\]

In order that the operator \( \mathcal{A}^{(3)} \) transforms the space of sextuples of functions \( G_i, T_i \) into itself.

The operator \( \mathcal{A}^{(3)} \) is a contraction with respect to the metric

\[
d \left( (G_{23}^{(1)}, T_{23}^{(1)}), (G_{23}^{(2)}, T_{23}^{(2)}) \right) = 
\]

\[
\sup_{t \in \mathbb{R}^4} \max_{i} \left| G_{i}^{(1)}(t) - G_{i}^{(2)}(t) \right| e^{-\left( M_{20}^{(3)} \right)} \max_{t \in \mathbb{R}^4} \left| T_{i}^{(1)}(t) - T_{i}^{(2)}(t) \right| e^{-\left( M_{20}^{(3)} \right)} \}
\]

Indeed if we denote

Definition of \( \bar{G}_{23}^{(1)}, \bar{T}_{23}^{(1)} : \left( (G_{23}^{(1)}, T_{23}^{(1)}), (G_{23}^{(2)}, T_{23}^{(2)}) \right) = \mathcal{A}^{(3)}( (G_{23}^{(1)}, T_{23}^{(1)})) \)

It results

\[
\left| \bar{G}_{20}^{(1)} - \bar{G}_{i}^{(2)} \right| \leq \int_{0}^{t} (a_{20}^{(3)}) (G_{20}^{(1)} - G_{20}^{(2)}) e^{-\left( M_{20}^{(3)} \right)} e^{-\left( M_{20}^{(3)} \right)} ds_{(20)} + 
\]

\[
\int_{0}^{t} \left( (a_{20}^{(3)}) (G_{20}^{(1)} - G_{20}^{(2)}) e^{-\left( M_{20}^{(3)} \right)} e^{-\left( M_{20}^{(3)} \right)} ds_{(20)} + 
\]

Where \( s_{(20)} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[
\left| G^{(1)} - G^{(2)} \right| e^{-\left( M_{20}^{(3)} \right)} \leq \frac{1}{(M_{20}^{(3)})} \left( (a_{20}^{(3)}) + (a_{20}^{(3)}) + (\lambda_{20}^{(3)}) + (P_{20}^{(3)}) (P_{20}^{(3)}) ds_{(20)} \right) \left( (G_{23}^{(1)}, T_{23}^{(1)}); (G_{23}^{(2)}, T_{23}^{(2)}) \right) 
\]

And analogous inequalities for \( G_{i} \) and \( T_{i} \). Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed \( (a_{20}^{(3)}) \) and \( (b^{(3)}_{20}) \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( (P_{20}^{(3)}) e^{-\left( M_{20}^{(3)} \right)} \) and \( (Q_{20}^{(3)}) e^{-\left( M_{20}^{(3)} \right)} \) respectively of \( \mathbb{R}_{+} \).

If instead of proving the existence of the solution on \( \mathbb{R}_{+} \), we have to prove it only on a compact then it suffices to consider that \( (a_{i}^{(3)}) \) and \( (b_{i}^{(3)}) \), \( i = 20, 21, 22 \) depend only on \( T_{21} \) and respectively on \( G_{23} \) (and not on \( t \)) and hypothesis.
can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where \( G_1(t) = 0 \) and \( T_1(t) = 0 \)

From 19 to 24 it results

\[
G_1(t) \geq G_1^0 e^{\left[-\int_0^t (a_1'(s) - a_1''(s)(r_1(x(s), x_0(s)))) dx_0(s)\right]} \geq 0
\]

\[
T_1(t) \geq T_1^0 e^{-(b_1'(s))} > 0 \text{ for } t > 0
\]

Definition of \( (\mathcal{M}_{20})_1 ), (\mathcal{M}_{20})_2 \) and \( (\mathcal{M}_{20})_3 ) : \]

Remark 3: if \( G_{20} \) is bounded, the same property have also \( G_{21} \) and \( G_{22} \). Indeed if \( \]

\[
G_{20} < (\mathcal{M}_{20})_1 \text{ it follows } \frac{dG_{21}}{dt} \leq (\mathcal{M}_{20})_1 - (a_{21})^{(3)} G_{21} \text{ and by integrating}
\]

\[
G_{21} \leq (\mathcal{M}_{20})_2 = G_{21}^0 + 2(a_{21})^{(3)} (\mathcal{M}_{20})_1 / (a_{21})^{(3)}
\]

In the same way, one can obtain

\[
G_{22} \leq (\mathcal{M}_{20})_3 = G_{22}^0 + 2(a_{22})^{(3)} (\mathcal{M}_{20})_2 / (a_{22})^{(3)}
\]

If \( G_{21} \) or \( G_{22} \) is bounded, the same property follows for \( G_{20}, G_{22} \) and \( G_{20}, G_{21} \) respectively.

Remark 4: If \( G_{20} \) is bounded, from below, the same property holds for \( G_{21} \) and \( G_{22} \).

The proof is analogous with the preceding one. An analogous property is true if \( G_{21} \) is bounded from below.

Remark 5: If \( T_{20} \) is bounded from below and \( \lim_{t \to \infty} ((b_{21}''(3)) (G_{22}(t), t)) = (b_{21}''(3)) \) then \( T_{21} \)

\[
= \infty.
\]

Definition of \( (m)(3) \) and \( \varepsilon_3 ) : \]

Indeed let \( t_3 \) be so that for \( t > t_3 \)

\[
(b_{21})^{(3)} - (b_{21}'')^{(3)} ((G_{23})(t), t) < \varepsilon_3, T_{20}(t) > (m)(3)
\]

Then \( \frac{dT_{21}}{dt} \geq (a_{21})^{(3)} (m)(3) - \varepsilon_3 T_{21} \) which leads to

\[
T_{21} \geq \left( \frac{(a_{21})^{(3)} (m)(3)}{\varepsilon_3} \right) (1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_3 t} = \frac{1}{2} \text{ it results}
\]

\[
T_{21} \geq \left( \frac{(a_{21})^{(3)} (m)(3)}{2} \right), \text{ by taking now } \varepsilon_3 \text{ sufficiently small one sees that } T_{21} \text{ is}
\]

unbounded.

The same property holds for \( T_{22} \) if \( \lim_{t \to \infty} ((b_{22}''(3)) (G_{23})(t), t) = (b_{22}''(3)) \)

We now state a more precise theorem about the behaviors at infinity of the solutions
It is now sufficient to take \( \frac{(a_i)^{(4)}}{(M_{24})^{(4)}} \), \( \frac{(b_i)^{(4)}}{(M_{24})^{(4)}} \) < 1 and to choose
\[ (P_{24})^{(4)} \) and \( (Q_{24})^{(4)} \) large to have
\[
\left( \frac{(a_i)^{(4)}}{(M_{24})^{(4)}} \right) \left( (P_{24})^{(4)} + \left( (P_{24})^{(4)} + G_i^0 \right) e \left( \frac{(P_{24})^{(4)} + G_i^0}{g_j} \right) \right) \leq (P_{24})^{(4)}
\]
\[
\left( \frac{(b_i)^{(4)}}{(M_{24})^{(4)}} \right) \left( (Q_{24})^{(4)} + T_i^0 \right) e \left( \frac{(Q_{24})^{(4)} + T_i^0}{g_j} \right) \leq (Q_{24})^{(4)}
\]

In order that the operator \( \mathcal{A}^{(4)} \) transforms the space of sextuples of functions \( G_i, T_i \)
satisfying \( IN \) to itself

The operator \( \mathcal{A}^{(4)} \) is a contraction with respect to the metric
\[
d \left( (G_{27})^{(1)}, (T_{27})^{(1)} \right), (G_{27})^{(2)}, (T_{27})^{(2)} \right) =
\]
\[
\sup_{t \in [0, \infty]} \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-((\theta_{24})^{(4)} s(24))} \max_{i \in [0, \infty]} \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-((\theta_{24})^{(4)} s(24))}
\]

Indeed if we denote

Definition of \( (G_{27}), (T_{27}) \) : \( (\overline{G_{27}}, \overline{T_{27}}) = \mathcal{A}^{(4)}((G_{27}), (T_{27})) \)

It results
\[
\left( G_{24}^{(1)} - G_{24}^{(2)} \right) \leq \int_0^t \left( a_{24}^{(4)} \right) G_{25}^{(1)} - G_{25}^{(2)} \right| e^{-((\theta_{24})^{(4)} s(24))} e^{(\theta_{24})^{(4)} s(24)} ds(24) + 
\]
\[
\int_0^t \left( a_{24}^{(4)} \right) G_{24}^{(1)} - G_{24}^{(2)} \right| e^{-((\theta_{24})^{(4)} s(24))} e^{(\theta_{24})^{(4)} s(24)} + 
\]
\[
(a_{24}^{(4)}(T_{25}^{(1)} \cdot s(24))) \left( G_{24}^{(1)} - G_{24}^{(2)} \right) e^{(\theta_{24})^{(4)} s(24)} e^{(\theta_{24})^{(4)} s(24)} + 
\]
\[
(a_{24}^{(4)}(T_{25}^{(2)} \cdot s(24))) \left( G_{24}^{(2)} - (a_{24}^{(4)}(T_{25}^{(1)} \cdot s(24))) e^{(\theta_{24})^{(4)} s(24)} e^{(\theta_{24})^{(4)} s(24)} ds(24)
\]

Where \( s(24) \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows
\[
\left( (G_{27})^{(1)} - (G_{27})^{(2)} \right) e^{-((\theta_{24})^{(4)} t}_{p q r s (24)} \left( (a_{24})^{(4)} + (a_{24}')^{(4)} + (\theta_{24})^{(4)} + (P_{24})^{(4)} \right) \] \( \cdot (G_{27})^{(4)} \) \( (T_{27})^{(1)} \), \( (G_{27})^{(2)}, (T_{27})^{(2)} \)

And analogous inequalities for \( G_i, T_i \). Taking into account the hypothesis the result follows
Remark 1: The fact that we supposed \((a_{24}^{(4)})\) and \((b_{24}^{(4)})\) depending also on \(t\) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((P_{24}^{(4)})e^{(R_{24}^{(4)})t}\) and \((Q_{24}^{(4)})e^{(R_{24}^{(4)})t}\) respectively of \(\mathbb{R}_+\).

If instead of proving the existence of the solution on \(\mathbb{R}_+\), we have to prove it only on a compact then it suffices to consider that \((a_i^{(4)})\) and \((b_i^{(4)})\), \(i = 24, 25, 26\) depend only on \(T_{25}\) and respectively on \((G_{27})(\text{and not on } t)\) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any \(t\) where \(G_i(t) = 0\) and \(T_i(t) = 0\)

From GLOBAL EQUATIONS it results

\[
G_i(t) \geq G_i^0 \left| - \int_a^t (a_i^{(4)} - (a_i^{(4)}) (T_{25}^{(2)}(x_{24}(t)))) ds_{24} \right| \geq 0
\]

\[
T_i(t) \geq T_i^0 e^{-(b_i^{(4)} t)} > 0 \text{ for } t > 0
\]

Definition of \((\overline{M}_{24})^{(4)}_1\), \((\overline{M}_{24})^{(4)}_2\) and \((\overline{M}_{24})^{(4)}_3\) :

Remark 3: if \(G_{24}\) is bounded, the same property have also \(G_{25}\) and \(G_{26}\), indeed if \(G_{24} < (\overline{M}_{24})^{(4)}\) it follows \(\frac{dG_{25}}{dt} \leq (\overline{M}_{24})^{(4)}_1 - (a_{25}^{(4)}) G_{25}\) and by integrating

\[
G_{25} \leq (\overline{M}_{24})^{(4)}_2 G_{25} + 2(a_{25}^{(4)}) (\overline{M}_{24})^{(4)}_1 / (a_{25}^{(4)})
\]

In the same way, one can obtain

\[
G_{26} \leq (\overline{M}_{24})^{(4)}_3 G_{26} + 2(a_{26}^{(4)}) (\overline{M}_{24})^{(4)}_2 / (a_{26}^{(4)})
\]

If \(G_{25} or G_{26}\) is bounded, the same property follows for \(G_{24}, G_{26} and G_{25}, G_{26}\) respectively.

Remark 4: If \(G_{24}\) is bounded, from below, the same property holds for \(G_{25}\) and \(G_{26}\).

The proof is analogous with the preceding one. An analogous property is true if \(G_{25}\) is bounded from below.

Remark 5: If \(T_{24}\) is bounded from below and \(\lim_{t \to \infty} ((b_i^{(4)} ((G_{27})(t), t)) = (b_{25}^{(4)})\) then \(T_{25} \to \infty\).

Definition of \((m)^{(4)}\) and \(\varepsilon_4\):

Indeed let \(t_4\) be so that for \(t > t_4\)

\[
(b_{25})^{(4)} - (b_i^{(4)} ((G_{27})(t), t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}
\]
Then \( \frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25} \) which leads to

\[
T_{25} \geq \left( \frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4} \right) (1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}
\]

If we take \( t \) such that \( e^{-\varepsilon_4 t} = \frac{1}{2} \) it results

\[
T_{25} \geq \left( \frac{(a_{25})^{(4)}(m)^{(4)}}{2} \right),
\]

By taking now \( \varepsilon_4 \) sufficiently small one sees that \( T_{25} \) is unbounded. The same property holds for \( T_{26} \) if \( \lim_{t \to \infty} (b_{26}^{(4)})((G_{27})(t), t) = (b_{26}^{(4)}) \)

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for \( G_{29}, G_{30}, T_{29}, T_{29}, T_{30} \)

It is now sufficient to take \( \frac{(a_i)^{(5)}}{(M_{28})^{(5)}} \), \( \frac{(b_i)^{(5)}}{(M_{28})^{(5)}} \) < 1 and to choose

\( (\hat{P}_{28})^{(5)} \) and \( (\hat{Q}_{28})^{(5)} \) large to have

\[
\frac{(a_i)^{(5)}}{(M_{28})^{(5)}} \left( P_{28}^{(5)} + (\hat{P}_{28})^{(5)} + G_i^0 \right) e^{-\left(\frac{\hat{P}_{28}^{(5)} + G_i^0}{G_i^0}\right)} \leq (\hat{P}_{28})^{(5)}
\]

\[
\frac{(b_i)^{(5)}}{(M_{28})^{(5)}} \left( (\hat{Q}_{28})^{(5)} + T_0^0 \right) e^{-\left(\frac{\hat{Q}_{28}^{(5)} + T_0^0}{T_0^0}\right)} + (\hat{Q}_{28})^{(5)} \leq (\hat{Q}_{28})^{(5)}
\]

In order that the operator \( \mathcal{A}^{(5)} \) transforms the space of sextuples of functions \( G_i, T_i \) into itself

The operator \( \mathcal{A}^{(5)} \) is a contraction with respect to the metric

\[
d \left( \left(G_{31}^{(1)}, (T_{31})^{(1)} \right), \left(G_{31}^{(2)}, (T_{31})^{(2)} \right) \right) = \sup_{i \in \mathbb{R}} \max_{t \in \mathbb{R}} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-((M_{28})^{(5)} t)}, \max_{t \in \mathbb{R}} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-((M_{28})^{(5)} t)}
\]

Indeed if we denote

Definition of \( (G_{31}, (T_{31}) : (G_{31}, (T_{31}) \big| = \mathcal{A}^{(5)}((G_{31}), (T_{31})) \)

It results

\[
|\tilde{G}_{28}^{(1)} - \tilde{G}_{28}^{(2)}| \leq \int_0^t (a_{28})^{(5)} |G_{29}^{(1)} - G_{29}^{(2)}| e^{-((M_{28})^{(5)} t(28))} e^{((M_{28})^{(5)} t(28))} ds_{(28)} +
\]

\[
\int_0^t ((a_{28})^{(5)} |G_{28}^{(1)} - G_{28}^{(2)}| e^{-((M_{28})^{(5)} t(28))} e^{((M_{28})^{(5)} t(28))} ds_{(28)} +
\]

280
\[(a_{28}^{(2)}(s_{28}^{(2)})) (T_{29}^{(1)}, s_{28}^{(2)}) \big| G_{28}^{(1)} - G_{28}^{(2)} \big| e^{-(\bar{\mathcal{M}}_{28})^{(5)} s_{28}^{(2)} e^{(\bar{\mathcal{M}}_{28})^{(5)} s_{28}^{(2)}} + G_{28}^{(2)}(a_{28}^{(2)}(s_{28}^{(2)})) (T_{29}^{(1)}, s_{28}^{(2)}) - (a_{28}^{(2)}(s_{28}^{(2)})) (T_{29}^{(2)}, s_{28}^{(2)}) \big| e^{-(\bar{\mathcal{M}}_{28})^{(5)} s_{28}^{(2)} e^{(\bar{\mathcal{M}}_{28})^{(5)} s_{28}^{(2)}}} d s_{28}^{(2)}
\]

Where \(s_{28}^{(2)}\) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[\left| (G_{31}^{(1)} - (G_{31}^{(2)}) e^{-((\bar{\mathcal{M}}_{28})^{(5)t})} \right| \leq \frac{1}{(\bar{\mathcal{M}}_{28})^{(5)}} \left| (a_{28}^{(5)} + (a_{28}^{(5)} + (\bar{\mathcal{M}}_{28})^{(5)}) + (P_{28}(5) + \bar{\mathcal{M}}_{28})^{(5)}) d \left( ((G_{31}^{(1)}), (T_{31}^{(1)}), (G_{31}^{(2)}), (T_{31}^{(2)})) \right) \right| \]

And analogous inequalities for \(G_i\) and \(T_i\). Taking into account the hypothesis (35,35,36) the result follows

Remark 1: The fact that we supposed \((a_{28}^{(5)}(s)\) and \((b_{28}^{(5)}(s)\) depending also on \(t\) can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((P_{28}(5) e^{(\bar{\mathcal{M}}_{28})^{(5)t}}\) and \((Q_{28}(5) e^{(\bar{\mathcal{M}}_{28})^{(5)t}}\) respectively of \(R_+\).

If instead of proving the existence of the solution on \(R_+\), we have to prove it only on a compact then it suffices to consider that \((a_i^{(5)}(s)\) and \((b_i^{(5)}(s)\),

\(i = 28, 29, 30\) depend only on \(T_{29}\) and respectively on \(G_{31}\)

(and not on \(t\)) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any \(t\) where \(G_i(t) = 0\) and \(T_i(t) = 0\)

From GLOBAL EQUATIONS it results

\[G_i(t) \geq G_i^0 e^{-\int_0^t ((a_i^{(5)}) - (a_i^{(5)}(T_{29}(s_{28})) s_{28}^{(2)})) ds_{28}^{(2)}} \geq 0\]

\[T_i(t) \geq T_i^0 e^{-((b_i^{(5)}))} > 0 \text{ for } t > 0\]

Definition of \((\bar{\mathcal{M}}_{28})^{(5)}\), \((\bar{\mathcal{M}}_{28})^{(5)}\) and \((\bar{\mathcal{M}}_{28})^{(5)}\):

Remark 3: if \(G_{28}\) is bounded, the same property have also \(G_{29}\) and \(G_{30}\). indeed if

\[G_{28} \leq (\bar{\mathcal{M}}_{28})^{(5)} \text{ it follows } \frac{dG_{28}}{dt} \leq (\bar{\mathcal{M}}_{28})^{(5)} - (a_{28}^{(5)} G_{28}\) and by integrating

\[G_{29} \leq (\bar{\mathcal{M}}_{28})^{(5)} = G_{29}^0 + 2(a_{29}^{(5)}((\bar{\mathcal{M}}_{29})^{(5)}))_1 / (a_{29}^{(1)}(s)\)

In the same way, one can obtain

\[G_{30} \leq (\bar{\mathcal{M}}_{28})^{(5)} = G_{30}^0 + 2(a_{30}^{(5)}((\bar{\mathcal{M}}_{29})^{(5)}))_1 / (a_{30}^{(1)}(s)\)

If \(G_{29}\) or \(G_{30}\) is bounded, the same property follows for \(G_{28}\), \(G_{30}\) and \(G_{28}\), \(G_{29}\) respectively.
Remark 4: If $G_{28}$ is bounded, from below, the same property holds for $G_{29}$ and $G_{30}$. The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below.

Remark 5: If $T_{28}$ is bounded from below and \( \lim_{t \to \infty} \left( (b_{29})^{(5)} \left( (G_{31}) (t), t \right) \right) = (b_{29})^{(5)} \) then $T_{29}$

\[ \text{Definition of } (m)^{(5)} \text{ and } \varepsilon_5 : \]

Indeed let $t_5$ be so that for $t > t_5$

\[ (b_{29})^{(5)} - (b_{29}')^{(5)}((G_{31})(t), t) < \varepsilon_5, \]

Then \( \frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29} \) which leads to

\[ T_{29} \geq \frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5} \left( 1 - e^{-\varepsilon_5 t} \right) + T_{29}^0 e^{-\varepsilon_5 t} \]

If we take $t$ such that \( e^{-\varepsilon_5 t} = \frac{1}{2} \) it results

\[ T_{29} \geq \frac{(a_{29})^{(5)}(m)^{(5)}}{2 \varepsilon_5}, \]

\( t = \log \frac{2}{\varepsilon_5} \) By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded. The same property holds for $T_{30}$ if \( \lim_{t \to \infty} (b_{30}')^{(5)}((G_{31})(t), t) = (b_{30})^{(5)} \)

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take \( \frac{(a_i)^{(6)}}{(M_{32})^{(6)}} \), \( \frac{(b_i)^{(6)}}{(M_{32})^{(6)}} < 1 \) and to choose \( (\bar{P}_{32})^{(6)} \) and \( (\bar{Q}_{32})^{(6)} \) large to have

\[ \frac{(a_i)^{(6)}}{(M_{32})^{(6)}} \left[ (\bar{P}_{32})^{(6)} + ((\bar{P}_{32})^{(6)} + G_j^0) e^{-\left( \frac{(\bar{P}_{32})^{(6)} + G_j^0}{G_j^0} \right) \frac{t_j}{2}} \right] \leq (\bar{P}_{32})^{(6)} \]

\[ \frac{(b_i)^{(6)}}{(M_{32})^{(6)}} \left[ ((\bar{Q}_{32})^{(6)} + T_j^0) e^{-\left( \frac{(\bar{Q}_{32})^{(6)} + T_j^0}{T_j} \right) \frac{t_j}{2}} + (\bar{Q}_{32})^{(6)} \right] \leq (\bar{Q}_{32})^{(6)} \]

In order that the operator $A^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ into itself

The operator $A^{(6)}$ is a contraction with respect to the metric

\[ d \left( ((G_{33})^{(1)}, (T_{33})^{(1)}), ((G_{33})^{(2)}, (T_{33})^{(2)}) \right) = \]

282
\[ \sup_{t \in \mathbb{R}_+} \max_{i \in \mathbb{R}} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\mathcal{R}_{32})^{(6)}t} \max_{i \in \mathbb{R}} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\mathcal{R}_{32})^{(6)}t} \]

Indeed if we denote

**Definition of \((G_{35}, T_{35})\):** \((G_{35}, T_{35}) = \mathcal{A}^{(6)}((G_{35}, T_{35}))\)

It results

\[ |\tilde{G}_i^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{32}^{(2)})^{(6)} |G_{33}^{(1)} - G_{33}^{(2)}| e^{-(\mathcal{R}_{32})^{(6)}x_{32}} e^{(\mathcal{R}_{32})^{(6)}s_{32}} ds_{32} + \]

\[ \int_0^t (a_{32}^{(2)})^{(6)} |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\mathcal{R}_{32})^{(6)}x_{32}} e^{(\mathcal{R}_{32})^{(6)}s_{32}} + \]

\[ (a_{32}^{(2)})^{(6)} (T_{33}^{(1)}, s_{32}) |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\mathcal{R}_{32})^{(6)}x_{32}} e^{(\mathcal{R}_{32})^{(6)}s_{32}} + \]

\[ G_{32}^{(2)} (a_{32}^{(2)})^{(6)} (T_{33}^{(2)}, s_{32}) - (a_{32}^{(2)})^{(6)} (T_{33}^{(2)}, s_{32}) | e^{-(\mathcal{R}_{32})^{(6)}x_{32}} e^{(\mathcal{R}_{32})^{(6)}s_{32}} | ds_{32} \]

Where \(s_{32}\) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

(1) \((a_i^{(1)}), (a_i^{''(1)}), (b_i^{(1)}), (b_i^{''(1)}) > 0, \quad i, j = 13, 14, 15\)

(2) The functions \((a_i^{''})^{(1)}), (b_i^{''})^{(1)}\) are positive continuous increasing and bounded.

**Definition of \((p_i^{(1)}), (r_i^{(1)})\):**

\((a_i^{''(1)})(T_{14}, t) \leq (p_i^{(1)}) \leq (A_{13})^{(1)}\)

\((b_i^{''(1)})(G, t) \leq (r_i^{(1)}) \leq (B_{13})^{(1)}\)

(3) \(\lim_{T_{14} \to \infty} (a_i^{''(1)})(T_{14}, t) = (p_i^{(1)})\)

\(\lim_{G \to \infty} (b_i^{''(1)})(G, t) = (r_i^{(1)})\)

**Definition of \((A_{13})^{(1)}, (B_{13})^{(1)}, (p_i^{(1)}), (r_i^{(1)})\):**

Where \((A_{13})^{(1)}, (B_{13})^{(1)}, (p_i^{(1)}), (r_i^{(1)})\) are positive constants and \(i = 13, 14, 15\)

They satisfy Lipschitz condition:

\[ |(a_i^{''(1)})(T_{14}, t) - (a_i^{''(1)})(T_{14}, t)| \leq (\tilde{k}_{13}^{(1)}) |T_{14} - T_{14}'| e^{-(\mathcal{R}_{13})^{(1)}t} \]

\[ |(b_i^{''(1)})(G, t) - (b_i^{''(1)})(G, T)| \leq (\tilde{k}_{13}^{(1)}) |G - G'| e^{-(\mathcal{R}_{13})^{(1)}t} \]
With the Lipschitz condition, we place a restriction on the behavior of functions $(a'_i)'(T_{14}, t)$ and $(a''_{ij})'(T_{14}, t)$. $(T_{14}, t)$ and $(T_{14}, t)$ are points belonging to the interval $[\hat{k}_{13}(1), \hat{M}_{13}(1)]$.

It is to be noted that $(a''_{ij})'(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13}(1) = 1$ then the function $(a''_{ij})'(T_{14}, t)$, the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

**Definition of $(\hat{M}_{13}(1), (\hat{k}_{13}(1))$:**

$(\hat{M}_{13}(1), (\hat{k}_{13}(1))$, are positive constants

\[
\frac{(a_i)(1)}{(M_{13})^{(1)}}, \frac{(b_j)(1)}{(M_{13})^{(1)}} < 1
\]

**Definition of $(\hat{P}_{13}(1), (\hat{Q}_{13}(1))$:**

There exists two constants $(\hat{P}_{13}(1) and (\hat{Q}_{13}(1)$ which together with $(\hat{M}_{13}(1), (\hat{k}_{13}(1), (\hat{A}_{13}(1), and (\hat{B}_{13}(1)$ and the constants $(a_i)(1), (a'_i)(1), (b_j)(1), (b'_{ij})(1), (p_j)(1), (r_j)(1), i = 13, 14, 15,$ satisfy the inequalities

\[
\frac{1}{(\hat{M}_{13})^{(1)}} \left( (a_i)(1) + (a'_i)(1) + (\hat{A}_{13}(1) + (\hat{P}_{13}(1) (\hat{k}_{13}(1) < 1
\]

\[
\frac{1}{(\hat{M}_{13})^{(1)}} \left( (b_j)(1) + (b'_{ij})(1) + (\hat{B}_{13}(1) + (\hat{Q}_{13}(1) (\hat{k}_{13}(1) < 1
\]

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$

It is now sufficient to take $(\frac{(a_i)(1)}{(M_{36})^{(1)}}, \frac{(b_j)(1)}{(M_{36})^{(1)}} < 1 and to choose

$(\hat{P}_{36}(1)$ and $(\hat{Q}_{36}(1)$ large to have

\[
\frac{(a_i)(1)}{(M_{36})^{(1)}} \left[ (\hat{P}_{36}(1) + (\hat{P}_{36}(1) + (a'_i)(1) e^{-\left(\frac{(\hat{P}_{36}(1)+T_{36}}{7_j}\right)} \right] \leq (\hat{P}_{36}(1)
\]

\[
\frac{(b_j)(1)}{(M_{36})^{(1)}} \left[ (\hat{Q}_{36}(1) + T_{36}) e^{-\left(\frac{(\hat{Q}_{36}(1)+T_{36}}{7_j}\right)} \right] \leq (\hat{Q}_{36}(1)
\]

In order that the operator $\mathcal{A}(7)$ transforms the space of sextuples of functions $G_i, T_i$ satisfying into itself

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

It is now sufficient to take $(\frac{(a_i)(1)}{(M_{40})^{(1)}}, \frac{(b_j)(1)}{(M_{40})^{(1)}} < 1 and to choose

$(\hat{P}_{40}(1)$ and $(\hat{Q}_{40}(1)$ large to have
In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying GLOBAL EQUATIONS into itself

The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d \left( \left( (G_{43})^{(1)}, (T_{43})^{(1)} \right), \left( (G_{43})^{(2)}, (T_{43})^{(2)} \right) \right) =$$

$$\sup_{t \in [0, T]} \max_{t_1 \in [0, t]} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-\left(\theta_{40}(8) + \theta_{40}(8)\right) t} + \max_{t_1 \in [0, t]} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-\left(\theta_{40}(8) + \theta_{40}(8)\right) t}$$

Indeed if we denote

**Definition of** $\mathcal{A}^{(8)}((G_{43}), (T_{43})) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results

$$|\tilde{G}_{40}^{(1)} - \tilde{G}_{40}^{(2)}| \leq \int_0^t (a_{40}^{(8)}) e^{-\theta_{40}(8) x_{40} \theta_{40}(8) x_{40}} dS_{40} +$$

$$\int_0^t \left( a_{40}^{(8)} \right) e^{-\theta_{40}(8) x_{40} \theta_{40}(8) x_{40}} dS_{40} +$$

Where $S_{40}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses IT follows

$$\left| (G_{43})^{(1)} - (G_{43})^{(2)} \right| e^{-\theta_{40}(8) t} \leq$$

$$\frac{1}{(M_{40})^{(8)}} \left( (a_{40}^{(8)}) + (a_{40}^{(8)}) + (\bar{A}_{40})^{(8)} + (\bar{P}_{40})^{(8)} (\bar{K}_{40})^{(8)} \right) d \left( \left( (G_{43})^{(1)}, (T_{43})^{(1)} \right), \left( (G_{43})^{(2)}, (T_{43})^{(2)} \right) \right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis (38,35,36) the result follows

**Remark 1:** The fact that we supposed $(a_{40}^{(8)})$ and $(b_{40}^{(8)})$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(P_{40})^{(8)} e^{-\theta_{40}(8) t}$ and $(Q_{40})^{(8)} e^{-\theta_{40}(8) t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_{40}^{(8)})$ and $(b_{40}^{(8)})$, $i = 40, 41, 42$ depend only on $T_{41}$ and respectively on $(G_{43})(and not on t)$ and hypothesis can replaced by a usual Lipschitz condition.
**Remark 2:** There does not exist any \( t \) where \( G_i(t) = 0 \) and \( T_i(t) = 0 \)

From global equations it results

\[
G_i(t) \geq G_i^0 e^{-\int_0^T (a_i''(\theta) - a_i''(\theta)(T_3(t, x_{3(2)}))(x_{3(2)})) dx_{3(2)}} \geq 0
\]

\[
T_i(t) \geq T_i^0 e^{-b_i''(\theta)(1)} > 0 \quad \text{for} \quad t > 0
\]

**Definition of** \((\mathcal{M}_{40})^{(\theta)}_1, (\mathcal{M}_{40})^{(\theta)}_2 \) and \((\mathcal{M}_{40})^{(\theta)}_3 \):

**Remark 3:** if \( G_{40} \) is bounded, the same property have also \( G_{41} \) and \( G_{42} \). indeed if

\[
G_{40} < (\mathcal{M}_{40})^{(\theta)}
\]

it follows \( \frac{dG_{41}}{dt} \leq (\mathcal{M}_{40})^{(\theta)}_1 - (a_{41})^{(\theta)}G_{41} \) and by integrating

\[
G_{41} \leq (\mathcal{M}_{40})^{(\theta)}_2 = G_{41}^0 + 2(a_{41})^{(\theta)}((\mathcal{M}_{40})^{(\theta)}_1)/(a_{41})^{(\theta)}
\]

In the same way, one can obtain

\[
G_{42} \leq (\mathcal{M}_{40})^{(\theta)}_3 = G_{42}^0 + 2(a_{42})^{(\theta)}((\mathcal{M}_{40})^{(\theta)}_2)/(a_{42})^{(\theta)}
\]

If \( G_{41} \) or \( G_{42} \) is bounded, the same property follows for \( G_{40}, G_{42} \) and \( G_{40}, G_{41} \) respectively.

**Remark 1:** If \( G_{40} \) is bounded, from below, the same property holds for \( G_{41} \) and \( G_{42} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{41} \) is bounded from below.

**Remark 5:** If \( T_{40} \) is bounded from below and \( \lim_{t \to \infty} ((b_1''(\theta))(G_{43})(t), t)) = (b_1''(\theta)) \) then \( T_{41} \to \infty \).

**Definition of** \((m)^{(\theta)}\) and \( \epsilon_{\theta} \):

Indeed let \( t_0 \) be so that for \( t > t_0 \)

\[
(b_{41})^{(\theta)} - (b_{01})^{(\theta)}((G_{43})(t), t) < \epsilon_{\theta}, T_{40}(t) > (m)^{(\theta)}
\]

Then \( \frac{dT_{41}}{dt} \geq (a_{41})^{(\theta)}(m)^{(\theta)} - \epsilon_{\theta}T_{41} \) which leads to

\[
T_{41} \geq \left(\frac{(a_{41})^{(\theta)}(m)^{(\theta)}}{\epsilon_{\theta}}\right)(1 - e^{-\epsilon_{\theta}t}) + T_{41}^0 e^{-\epsilon_{\theta}t}
\]

If we take \( t \) such that \( e^{-\epsilon_{\theta}t} = \frac{1}{2} \) it results

\[
T_{41} \geq \left(\frac{(a_{41})^{(\theta)}(m)^{(\theta)}}{2}\right), \quad t = \log \frac{1}{\epsilon_{\theta}}
\]

By taking now \( \epsilon_{\theta} \) sufficiently small one sees that \( T_{41} \) is unbounded. The same property holds for \( T_{42} \) if \( \lim_{t \to \infty} ((b_{42})''(\theta))(G_{43})(t), t) = (b_1''(\theta)) \)

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 82

**Remark 2:** There does not exist any \( t \) where \( G_i(t) = 0 \) and \( T_i(t) = 0 \)

From CONCATENATED GLOBAL EQUATIONS it results

\[
G_i(t) \geq G_i^0 e^{-\int_0^T (a_i''(\theta) - a_i''(\theta)(T_3(t, x_{3(2)}))(x_{3(2)})) dx_{3(2)}} \geq 0
\]
\[ T_i(t) \geq T_i^0 e^{-\left(b_i^0\right)^{(7)} t} > 0 \quad \text{for} \ t > 0 \]

**Definition of** \((\mathcal{M}_{36})^{(7)}\) , \((\mathcal{M}_{36})^{(7)}_1\), \((\mathcal{M}_{36})^{(7)}_2\), and \((\mathcal{M}_{36})^{(7)}_3\) : \[ 1403 \]

**Remark 3:** if \(G_{36}\) is bounded, the same property have also \(G_{37}\) and \(G_{38}\). indeed if \(G_{36} < (\mathcal{M}_{36})^{(7)}\) it follows \(G_{37} \leq (\mathcal{M}_{36})^{(7)}_1 = (a_{37}^{(7)})^2 G_{37}\) and by integrating

\[ G_{37} \leq (\mathcal{M}_{36})^{(7)}_2 = G_{37}^0 + 2(a_{37}^{(7)})((\mathcal{M}_{36})^{(7)}_1)/\left(a_{57}^{(7)}\right) \]

In the same way, one can obtain

\[ G_{38} \leq (\mathcal{M}_{36})^{(7)}_3 = G_{38}^0 + 2(a_{38}^{(7)})((\mathcal{M}_{36})^{(7)}_2)/\left(a_{58}^{(7)}\right) \]

If \(G_{37}\) or \(G_{38}\) is bounded, the same property follows for \(G_{36}\), \(G_{38}\) and \(G_{36}, G_{37}\) respectively.

**Remark 7:** If \(G_{36}\) is bounded, from below, the same property holds for \(G_{37}\) and \(G_{38}\). The proof is analogous with the preceding one. An analogous property is true if \(G_{37}\) is bounded from below.

**Remark 5:** If \(T_{36}\) is bounded from below and \(\lim_{t \to \infty} (b_{36}^{(7)}((G_{39}(t), t)) = (b_{37}^{(7)}(t)\) then \(T_{37} \to \infty\).

**Definition of** \((m)^{(7)}\) and \(e_{7}\):

Indeed let \(t_7\) be so that for \(t > t_7\)

\[ (b_{37}^{(7)} - (b_{37}^{(7)})((G_{39}(t), t)) < e_{7}, T_{36} (t) > (m)^{(7)} \]

Then \(\frac{dT_{37}}{dt} \geq (a_{37}^{(7)}(m)^{(7)}) - e_{7} T_{37}\) which leads to

\[ T_{37} \geq \left(\frac{(a_{37}^{(7)}(m)^{(7)})}{e_{7}}\right) (1 - e^{-e_{7} t}) + T_{37}^0 e^{-e_{7} t} \]

If we take \(t\) such that \(e^{-e_{7} t} = \frac{1}{2}\) it results

\[ T_{37} \geq \left(\frac{(a_{37}^{(7)}(m)^{(7)})}{2}\right) , \quad t = \log \frac{2}{e_{7}} \]

By taking now \(e_{7}\) sufficiently small one sees that \(T_{37}\) is unbounded. The same property holds for \(T_{38}\) if \(\lim_{t \to \infty} (b_{38}^{(7)}((G_{39}(t), t) = (b_{38}^{(7)}(t)\)

We now state a more precise theorem about the behaviors at infinity of the solutions

\[ -(a_1^{(7)} - (a_1^{(7)}(2) - (a_1^{(7)}(2)(T_{17}, t) + (a_1^{(7)}(2)(T_{17}, t) \leq -\left(a_1\right)^{(2)} \]

\[ -(\tau_2^{(7)} \leq -(b_1^{(7)} + b_1^{(7)}(2) - (b_1^{(7)}(2)((G_{19}(t), t) - (b_1^{(7)}(2)((G_{19}(t), t) \leq -\left(\tau_2\right)^{(2)} \]

**Definition of** \((v_1)^{(2)}(v_2)^{(2)}(u_1)^{(2)}(u_2)^{(2)}(\)

By \((v_1)^{(2)} > 0, (v_2)^{(2)} < 0\) and respectively \((u_1)^{(2)} > 0, (u_2)^{(2)} < 0\) the roots

\[ (v_1)^{(2)}(v_2)^{(2)}(u_1)^{(2)}(u_2)^{(2)}(\]

and \((b_1)^{(2)}(u_1)^{(2)}(b_1)^{(2)} - (b_1)^{(2)} = 0\)

**Definition of** \((\bar{v}_1)^{(2)}(\bar{v}_2)^{(2)}(\bar{u}_1)^{(2)}(\bar{u}_2)^{(2)}(\)

By \((\bar{v}_1)^{(2)} > 0, (\bar{v}_2)^{(2)} < 0\) and respectively \((\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0\) the

287
roots of the equations \((a_{12})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0\)

and \((b_{17})^{(2)}(u^{(2)})^2 + (\pi_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0\)

**Definition of** \((m_1)^{(2)} , (m_2)^{(2)} , (\mu_1)^{(2)} , (\mu_2)^{(2)} : -\)

\((oo)\) If we define \((m_1)^{(2)} , (m_2)^{(2)} , (\mu_1)^{(2)} , (\mu_2)^{(2)}\) by

\((m_2)^{(2)} = (v_0)^{(2)}\), \((m_1)^{(2)} = (v_1)^{(2)}\), **if** \((v_0)^{(2)} < (v_1)^{(2)}\)

\((m_2)^{(2)} = (v_1)^{(2)}\), \((m_1)^{(2)} = (v_0)^{(2)}\), **if** \((v_1)^{(2)} < (v_0)^{(2)} < (v_1)^{(2)}\),

and \(v_0)^{(2)} = \frac{a_{16}}{a_{17}}\)

\((m_1)^{(2)} = (v_1)^{(2)} , (m_2)^{(2)} = (v_0)^{(2)}\), **if** \((v_1)^{(2)} < (v_0)^{(2)}\)

and analogously

\((\mu_2)^{(2)} = (u_0)^{(2)} , (\mu_1)^{(2)} = (u_1)^{(2)}\), **if** \((u_0)^{(2)} < (u_1)^{(2)}\)

\((\mu_2)^{(2)} = (u_1)^{(2)} , (\mu_2)^{(2)} = (\bar{u}_1)^{(2)}\), **if** \((u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}\),

and \(u_0)^{(2)} = \frac{T_{16}^{(0)}}{T_{17}^{(0)}}\)

\((\mu_2)^{(2)} = (u_1)^{(2)} , (\mu_2)^{(2)} = (u_0)^{(2)}\), **if** \((\bar{u}_1)^{(2)} < (u_0)^{(2)}\)

Then the solution satisfies the inequalities

\(G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}^0 e^{(S_1)^{(2)}t} \leq G_{16}^0 e^{(S_1)^{(2)}t}\)

\((p_j)^{(2)}\) is defined

\(\frac{1}{(m_2)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}^0 (t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}\)

\(\left(\frac{(a_{16})^{(2)} G_{16}^0}{(m_2)^{(2)}(S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)} - (a_{16})^{(2)}}\right) \left[e^{(S_1)^{(2)}t} - e^{-(S_2)^{(2)}t}\right] + G_{16}^0 e^{-(S_2)^{(2)}t} \leq G_{18}^0 (t) \leq \)

\(\left(\frac{(a_{16})^{(2)} G_{16}^0}{(m_2)^{(2)}(S_1)^{(2)} - (a_{16})^{(2)}}\right) \left[e^{(S_1)^{(2)}t} - e^{-(a_{16})^{(2)}t}\right] + G_{16}^0 e^{-(a_{16})^{(2)}t}\)

\(T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}^0 (t) \leq T_{16}^0 e^{((R_1)^{(2)} + (R_1)^{(2)})t}\)

\(\frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}^0 (t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (R_1)^{(2)})t}\)

\(\left(\frac{(b_{16})^{(2)} T_{16}^0}{(\mu_2)^{(2)}(R_1)^{(2)} - (b_{16})^{(2)}}\right) \left[e^{(R_1)^{(2)}t} - e^{-(b_{16})^{(2)}t}\right] + T_{18}^0 e^{-(b_{16})^{(2)}t} \leq T_{18}^0 (t) \leq \)

\(\left(\frac{(a_{16})^{(2)} T_{16}^0}{(\mu_2)^{(2)}(R_1)^{(2)} + (R_1)^{(2)} + (R_2)^{(2)}}\right) \left[e^{(R_1)^{(2)}t} - e^{-(R_2)^{(2)}t}\right] + T_{16}^0 e^{-(R_2)^{(2)}t}\)

**Definition of** \((S_1)^{(2)} , (S_2)^{(2)} , (R_1)^{(2)} , (R_2)^{(2)} : -\)

Where \((S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a_{16})^{(2)}\)

\((S_2)^{(2)} = (a_{16})^{(2)} - (p_{16})^{(2)}\)
\[(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}\]
\[(R_2)^{(2)} = (b'_{16})^{(2)} - (r_{18})^{(2)}\]

**BEHAVIOR OF THE SOLUTIONS**

If we denote and define

**Definition of** \((\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}\):

\((pp)\) \((\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}\) four constants satisfying

\[-(\sigma_2)^{(3)} \leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}\]
\[-(\tau_2)^{(3)} \leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G, t) - (b''_{21})^{(3)}(G_{23}, t) \leq -(\tau_1)^{(3)}\]

**Definition of** \((v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}\):

\((qq)\) By \((v_1)^{(3)} > 0, (v_2)^{(3)} < 0\) and respectively \((u_1)^{(3)} > 0, (u_2)^{(3)} < 0\) the roots of the equations \((a_{21})^{(3)}(v^{(3)})^2 + (a_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0\)

and \((b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0\)

By \((\bar{v}_1)^{(3)} > 0, (\bar{v}_2)^{(3)} < 0\) and respectively \((\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0\) the roots of the equations \((a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0\)

and \((b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0\)

**Definition of** \((m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}\):

\((rr)\) If we define \((m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}\) by

\[(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}\] if \((v_0)^{(3)} < (v_1)^{(3)}\)

\[(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}\] if \((v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}\)

and

\[(v_0)^{(3)} = \frac{a_{20}}{a_{21}}\]

\[(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}\] if \((\bar{v}_1)^{(3)} < (v_0)^{(3)}\)

and analogously

\[(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}\] if \((u_0)^{(3)} < (u_1)^{(3)}\)

\[(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}\] if \((u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}\), and

\[(u_0)^{(3)} = \frac{a_{20}}{a_{21}}\]

Then the solution satisfies the inequalities

\[G_{20}e^{(S_1)^{(3)}-(P_{20})^{(3)}t} \leq G_{20}(t) \leq G_{20}e^{(S_1)^{(3)}t}\]

\((p_1)^{(3)}\) is defined
\[
\frac{1}{(m_1)^3} G_0 e^{(S_1)(-p_2)(t)} \leq G_{21}(t) \leq \frac{1}{(m_2)^3} G_0 e^{(S_1)(t)} \\
\left(\frac{(a_{22})(G_0)^2}{(m_1)^3((S_1)^3(-p_2)(-S_2)(S_2)(t))} + (G_22)e^{-(S_2)(t)}\right) \leq G_{22}(t) \leq \left(\frac{(a_{22})(G_0)^2}{(m_2)^3((S_1)^3(-a_{22})(S_2)(t))} + (G_22)e^{-(a_{22})(t)}\right) \\
T_0 e^{(R_1)(t)} \leq T_{20}(t) \leq \left(\frac{(b_{22})(G_0)^2}{(m_1)^3((R_1)(t)(R_2)(t))} + (T_0)e^{-(R_2)(t)}\right) \\
\left(\frac{(a_{22})(G_0)^2}{(m_1)^3((R_1)(t)(R_2)(t))} + (T_0)e^{-(R_2)(t)}\right) \\
\left(\frac{(a_{22})(G_0)^2}{(m_1)^3((R_1)(t)(R_2)(t))} + (T_0)e^{-(R_2)(t)}\right)
\]

**Definition of** \((S_1)(t), (S_2)(t), (R_1)(t), (R_2)(t)\):

Where \((S_1)(t) = (a_{20})(m_2)(-a_{20})\)
\[(S_2)(t) = (a_{22})(-p_{22})\]
\[(R_1)(t) = (b_{20})(\mu_2)(-b_{20})\]
\[(R_2)(t) = (b_{22})(-r_{22})\]

If we denote and define

**Definition of** \((\sigma_1)(4), (\sigma_2)(4), (\tau_1)(4), (\tau_2)(4)\):

\((\sigma_1)(4), (\sigma_2)(4), (\tau_1)(4), (\tau_2)(4)\) four constants satisfying
\[-(\sigma_2)(4) \leq -(a_{24})(4) + (a_{25})(4) - (a_{22})(4)(T_{25}, t) + (a_{25})(4)(T_{25}, t) \leq -(\sigma_1)(4)\]
\[-(\tau_2)(4) \leq -(b_{24})(4) + (b_{25})(4) - (b_{22})(4)((G_{27}, t)) - (b_{25})(4)((G_{27}, t)) \leq -(\tau_1)(4)\]

**Definition of** \((\nu_1)(4), (\nu_2)(4), (u_1)(4), (u_2)(4), (v)(4), (u)(4)\):

By \((\nu_1)(4) > 0, (\nu_2)(4) < 0\) and respectively \((u_1)(4) > 0, (u_2)(4) < 0\) the roots of the equations
\[(a_{25})(4)(v)(4)^2 + (\sigma_2)(4)v(4) - (a_{24})(4) = 0\]
and \((b_{25})(4)(u)(4)^2 + (\tau_2)(4)u(4) - (b_{24})(4) = 0\)

**Definition of** \((\bar{\nu}_1)(4), (\bar{\nu}_2)(4), (\bar{u}_1)(4), (\bar{u}_2)(4)\):

By \((\bar{\nu}_1)(4) > 0, (\bar{\nu}_2)(4) < 0\) and respectively \((\bar{u}_1)(4) > 0, (\bar{u}_2)(4) < 0\) the roots of the equations
\[(a_{25})(4)(\nu)(4)^2 + (\sigma_2)(4)v(4) - (a_{24})(4) = 0\]
and \((b_{25})(4)(u)(4)^2 + (\tau_2)(4)u(4) - (b_{24})(4) = 0\)

**Definition of** \((m_1)(4), (m_2)(4), (\mu_1)(4), (\mu_2)(4), (\nu_0)(4)\):

(ss) If we define \((m_1)(4), (m_2)(4), (\mu_1)(4), (\mu_2)(4), (\nu_0)(4)\) by
\[(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}; \text{ if } (v_0)^{(4)} < (v_1)^{(4)}\]
\[(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}; \text{ if } (v_2)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}\]
and \[\left(\frac{v_0}{v_2}\right)^{(4)} = \frac{v_0}{v_2}\]
\[(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}; \text{ if } (v_4)^{(4)} < (v_0)^{(4)}\]

and analogously
\[(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}; \text{ if } (u_0)^{(4)} < (u_1)^{(4)}\]
\[(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}; \text{ if } (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}\]
and \[\left(\frac{u_0}{u_2}\right)^{(4)} = \frac{u_0}{u_2}\]
\[(\mu_2)^{(4)} = (u_4)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}; \text{ if } (u_4)^{(4)} < (u_0)^{(4)}\]

where \((u_1)^{(4)}\) and \((u_0)^{(4)}\) are defined respectively.

Then the solution satisfies the inequalities
\[G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)}))t} \leq G_{24}^t \leq G_{24}^0 e^{((S_1)^{(4)}))t}\]

where \((p_{1j})^{(4)}\) is defined

\[\frac{1}{(m_2)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)}))t} \leq G_{24}^t \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{((S_1)^{(4)}))t}\]

\[\left(\frac{a_{20}}{m_1}\right)^{(4)} \left[ e^{((S_1)^{(4)} - (p_{24})^{(4)}))t} - e^{((S_1)^{(4)}))t} \right] + \left(\frac{a_{20}}{m_1}\right)^{(4)} e^{(-((S_1)^{(4)}))t} \leq G_{26}^t \leq G_{26}^0 e^{((S_1)^{(4)}))t}\]

\[T_{24}^0 e^{((R_1)^{(4)}))t} \leq T_{24}^t \leq T_{24}^0 e^{((R_1)^{(4)} + (R_{24})^{(4)}))t}\]

\[\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{((R_1)^{(4)}))t} \leq T_{24}^t \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (R_{24})^{(4)}))t}\]

\[\left(\frac{a_{26}}{\mu_1}\right)^{(4)} \left[ e^{((R_1)^{(4)}))t} - e^{(-((B_{26})^{(4)}))t} \right] + T_{26}^0 e^{(-((B_{26})^{(4)}))t} \leq T_{26}^t \leq T_{26}^0 e^{((R_1)^{(4)}))t}\]

\[\left(\frac{a_{26}}{\mu_1}\right)^{(4)} \left[ e^{((R_1)^{(4)} + (R_{24})^{(4)}))t} - e^{((R_1)^{(4)}))t} \right] + T_{26}^0 e^{(-((R_2)^{(4)}))t}\]

**Definition of \((S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}\):**

Where \((S_1)^{(4)} = (a_{24})^{(4)} (m_2)^{(4)} - (a_{24})^{(4)}\)
\[(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}\]
\[(R_1)^{(4)} = (b_{24})^{(4)} (\mu_2)^{(4)} - (b_{24})^{(4)}\]

291
\[(R_2)^{(4)} = (b_{26}')(4) - (r_{26})^{(4)}\]

**Behavior of the solutions**

If we denote and define

**Definition of** \( (\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}: \)

\( (\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)} \) four constants satisfying

\[-(\sigma_2)^{(5)} \leq -(a_{20}')(5) + (a_{20}(5)) + (a_{29}')(5)(T_{29}, t) \leq -(\sigma_1)^{(5)}\]

\[-(\tau_2)^{(5)} \leq -(b_{20}')(5) + (b_{20}(5)) - (b_{29}')(5)(G_{31}, t) \leq -(\tau_1)^{(5)}\]

**Definition of** \( (v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, \mu^{(5)}: \)

(tt) By \( (v_1)^{(5)} > 0, (v_2)^{(5)} < 0 \) and respectively \( (u_1)^{(5)} > 0, (u_2)^{(5)} < 0 \) the roots of the equations \( (a_{20}(5))(\nu^{(5)})^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0 \)

and \( (b_{20}(5))(\mu^{(5)})^2 + (\tau_1)^{(5)}\mu^{(5)} - (b_{28})^{(5)} = 0 \)

**Definition of** \( (\bar{v}_1)^{(5)}, (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}: \)

By \( (\bar{v}_1)^{(5)} > 0, (\bar{v}_2)^{(5)} < 0 \) and respectively \( (\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0 \) the roots of the equations \( (a_{20}(5))(\bar{\nu}^{(5)})^2 + (\sigma_1)^{(5)}\bar{\nu}^{(5)} - (a_{28})^{(5)} = 0 \)

and \( (b_{20}(5))(\bar{\mu}^{(5)})^2 + (\tau_1)^{(5)}\bar{\mu}^{(5)} - (b_{28})^{(5)} = 0 \)

**Definition of** \( (m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}: \)

(uu) If we define \( (m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)} \) by

\( (m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_2)^{(5)}, if \ (v_0)^{(5)} < (v_1)^{(5)} \)

\( (m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}, if \ (v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)} \)

and \( (v_0)^{(5)} = \frac{v_2}{v_2}, (v_1)^{(5)} = \frac{v_0}{v_2} \)

\( (m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (v_0)^{(5)}, if \ (\bar{v}_1)^{(5)} < (v_0)^{(5)} \)

and analogously

\( (\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, if \ (u_0)^{(5)} < (u_1)^{(5)} \)

\( (\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, if \ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)} \)

and \( (u_0)^{(5)} = \frac{u_2}{u_2}, (u_1)^{(5)} = \frac{u_0}{u_2} \)

\( (\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, if \ (\bar{u}_1)^{(5)} < (u_0)^{(5)} \) where \( (u_1)^{(5)}, (\bar{u}_1)^{(5)} \)
are defined respectively

Then the solution satisfies the inequalities

\[ G_{29}^{\theta}e^{((S_1)^{(5)}-(P_{29})^{(5)})t} \leq G_{29}^{\theta}e^{((S_1)^{(5)})t} \leq \frac{1}{(m_2)^{(5)}}G_{29}^{\theta}e^{((S_1)^{(5)}-(P_{29})^{(5)})t} \]

where \((p_i)^{(5)}\) is defined

\[ \frac{1}{(m_2)^{(5)}}\frac{1}{(m_2)^{(5)}}G_{29}^{\theta}e^{((S_1)^{(5)}-(P_{29})^{(5)})t} \leq G_{29}^{\theta}e^{((S_1)^{(5)}-(P_{29})^{(5)})t} \]

\[ T_{29}^{\theta}e^{((R_1)^{(5)})(t)} \leq T_{29}^{\theta}e^{((R_1)^{(5)})(t)} \leq \frac{1}{(m_2)^{(5)}}T_{29}^{\theta}e^{((R_1)^{(5)})(t)} \]

\[ T_{29}^{\theta}e^{((R_1)^{(5)})(t)} - T_{29}^{\theta}e^{((R_1)^{(5)})(t)} \leq \frac{1}{(m_2)^{(5)}}T_{29}^{\theta}e^{((R_1)^{(5)})(t)} \]

**Definition of \((S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}\):**

Where \((S_2)^{(5)} = (a_{29})^{(5)}(m_2)^{(5)} - (a_{29}^\prime)^{(5)}\)

\[(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}\]

\[(R_1)^{(5)} = (b_{29})^{(5)}(m_2)^{(5)} - (b_{29}^\prime)^{(5)}\]

\[(R_2)^{(5)} = (b_{30})^{(5)} - (r_{30})^{(5)}\]

**Behavior of the solutions**

If we denote and define

**Definition of \((\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}\):**

\[(\sigma_1)^{(6)} \cdot (\sigma_2)^{(6)} \cdot (\tau_1)^{(6)} \cdot (\tau_2)^{(6)} \text{ four constants satisfying}\]

\[ -\sigma_2^{(6)} \leq -(a_{32}^{(6)}) + (a_{33}^{(6)}) - (a_{32}^\prime)^{(6)}(T_{33}, t) + (a_{33}^\prime)^{(6)}(T_{33}, t) \leq -\sigma_1^{(6)}\]

\[ -\tau_2^{(6)} \leq -(b_{32}^{(6)}) + (b_{33}^{(6)}) - (b_{32}^\prime)^{(6)}(G_{33}, t) - (b_{33}^\prime)^{(6)}(G_{33}, t) \leq -\tau_1^{(6)}\]

**Definition of \((v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, \mu^{(6)}\):**

\[(v_1)^{(6)} > 0, (v_2)^{(6)} < 0 \text{ and respectively } (u_1)^{(6)} > 0, (u_2)^{(6)} < 0 \text{ the roots of } \text{ the equations}\]

\[ (a_{33})^{(6)}(\nu^{(6)})^{2} + (\sigma_1)^{(6)}(\nu^{(6)}) - (a_{32})^{(6)} = 0\]

\[ (b_{33})^{(6)}(\mu^{(6)})^{2} + (\tau_1)^{(6)}(\mu^{(6)}) - (b_{32})^{(6)} = 0 \text{ and}\]
Definition of \( \bar{\psi}_1^{(6)} \), \( \bar{\psi}_2^{(6)} \), \( \bar{u}_1^{(6)} \), \( \bar{u}_2^{(6)} \):

By \( (\bar{\psi}_1^{(6)}) > 0 \), \( (\bar{\psi}_2^{(6)}) < 0 \) and respectively \( (\bar{u}_1^{(6)}) > 0 \), \( (\bar{u}_2^{(6)}) < 0 \) the roots of the equations \( (a_{31})^{(6)}(\nu)^2 + (\sigma_1)^{(6)}\nu - (a_{32})^{(6)} = 0 \) and \( (b_{33})^{(6)}(u)^2 + (\tau_2)^{(6)}u - (b_{32})^{(6)} = 0 \)

Definition of \( (m_1)^{(6)} \), \( (m_2)^{(6)} \), \( (\mu_1)^{(6)} \), \( (\mu_2)^{(6)} \), \( (\nu_0)^{(6)} \):

If we define \( (m_1)^{(6)} \), \( (m_2)^{(6)} \), \( (\mu_1)^{(6)} \), \( (\mu_2)^{(6)} \) by

\( (m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}, \text{ if } (v_0)^{(6)} < (v_1)^{(6)} \)

\( (m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \text{ if } (v_1)^{(6)} < (v_0)^{(6)} \)

and

\( (v_0)^{(6)} = \frac{v_0^{(6)}}{v_0^{(6)}} \)

\( (m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \text{ if } (v_1)^{(6)} < (v_0)^{(6)} \)

and analogously

\( (\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \text{ if } (u_0)^{(6)} < (u_1)^{(6)} \)

\( (\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (u_1)^{(6)} < (u_0)^{(6)} \)

and

\( (u_0)^{(6)} = \frac{u_0^{(6)}}{u_0^{(6)}} \)

\( (\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (u_1)^{(6)} < (u_0)^{(6)} \)

where \( (u_0)^{(6)}, (\bar{u}_1)^{(6)} \) are defined respectively

Then the solution satisfies the inequalities

\[ G_{32}^0 e^{((s_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}^0(t) \leq G_{32}^0 e^{((s_1)^{(6)})t} \]

where \( (p_l)^{(6)} \) is defined

\[ \frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((s_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}^0(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{((s_1)^{(6)})t} \]

\[ \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((s_1)^{(6)} - (p_{32})^{(6)})(s_1^{(6)} - (p_{32})^{(6)})} \left[ e^{((s_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(s_1)^{(6)}t} \right] + G_{34}^0 e^{-(s_2)^{(6)}t} \leq G_{34}^0(t) \leq \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((s_1)^{(6)} - (p_{32})^{(6)})} \left[ e^{((s_1)^{(6)})t} - e^{-(s_1)^{(6)}t} \right] + G_{34}^0 e^{-(a_{34})^{(6)}t} \]

\[ T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}^0 e^{((R_1)^{(6)} + (p_{32})^{(6)})t} \]

294
\[
\frac{1}{\mu_1(\mu_2)}T_{32}e^{(\mu_1(\mu_2))t} \leq T_{34}(t) = \frac{1}{\mu_2(\mu_1)}T_{34}e^{(\mu_1(\mu_2))t} \\
\frac{(b_{34})^{(\mu_2)(\mu_1)} - b_{34}^{(\mu_2)(\mu_1)}}{(b_{34})^{(\mu_2)(\mu_1)}} e^{((b_{34})^{(\mu_2)(\mu_1)} + (b_{34})^{(\mu_2)(\mu_1)})t} + T_{34}^0 e^{((b_{34})^{(\mu_2)(\mu_1)} + (b_{34})^{(\mu_2)(\mu_1)})t} \leq T_{34}(t) \leq \frac{(a_{34})^{(\mu_2)(\mu_1)} + a_{34}^{(\mu_2)(\mu_1)}}{(a_{34})^{(\mu_2)(\mu_1)}} e^{((a_{34})^{(\mu_2)(\mu_1)} + (a_{34})^{(\mu_2)(\mu_1)})t} + T_{34}^0 e^{((a_{34})^{(\mu_2)(\mu_1)} + (a_{34})^{(\mu_2)(\mu_1)})t}
\]

**Definition of** \((S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}\):-

Where \((S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a_{32})^{(6)}\)
\((S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}\)
\((R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b_{32})^{(6)}\)
\((R_2)^{(6)} = (b_{34})^{(6)} - (r_{34})^{(6)}\)

If we denote and define

**Definition of** \((\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\sigma_2)^{(7)}, (\sigma_2)^{(7)}\):

\((\sigma_2)^{(7)} \leq -(a_{36})^{(7)} + (a_{37})^{(7)} - (a_{36})^{(7)}(T_{37}, t) + (a_{37})^{(7)}(T_{37}, t) \leq -(\sigma_1)^{(7)}\)
\((\tau_2)^{(7)} \leq -(b_{36})^{(7)} + (b_{37})^{(7)} - (b_{36})^{(7)}(t) + (b_{37})^{(7)}(t) \leq -(\tau_1)^{(7)}\)

**Definition of** \((v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, (v_1)^{(7)}, (u_2)^{(7)}\):

\((v_2)^{(7)} > 0, (v_2)^{(7)} < 0 \) and respectively \((u_1)^{(7)} > 0, (u_2)^{(7)} < 0 \) the roots of the equations
\((a_{37})^{(7)}(v_1)^{(7)} + (\sigma_1)^{(7)}v_1 - (a_{36})^{(7)} = 0\)
\((b_{37})^{(7)}(u_1)^{(7)} + (\tau_2)^{(7)}u_1 - (b_{36})^{(7)} = 0\)

**Definition of** \((\bar{v}_1)^{(7)}, (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}\):

\((\bar{v}_2)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0 \) and respectively \((\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0 \) the roots of the equations
\((a_{37})^{(7)}(v_1)^{(7)} + (\sigma_2)^{(7)}v_2 - (a_{36})^{(7)} = 0\)
\((b_{37})^{(7)}(u_1)^{(7)} + (\tau_2)^{(7)}u_2 - (b_{36})^{(7)} = 0\)

**Definition of** \((m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}\):

If we define \((m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}\) by
\((m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}, \) if \((v_0)^{(7)} < (v_1)^{(7)}\)
\((m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}, \) if \((v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}\)
and
\((v_0)^{(7)} = \frac{e_{36}^0}{e_{37}^0}\)
\((m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}, \) if \((v_1)^{(7)} < (v_0)^{(7)}\)
and analogously
\( (\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \text{if } (u_0)^{(7)} < (u_1)^{(7)} \)

\( (\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \text{if } (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}, \)

and \( (u_0)^{(7)} = \frac{T_{0\beta}^6}{T_{0\gamma}^7} \)

\( (\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \text{if } (\bar{u}_1)^{(7)} < (u_0)^{(7}) \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)} \)

are defined respectively

Then the solution satisfies the inequalities

\[
G_{36e}^0 ((S_1)^{(7)} - (p_{38})^{(7)}) t \leq G_{36}^0 (t) \leq G_{36e}^0 ((S_1)^{(7)} + (p_{38})^{(7)}) t
\]

where \( (p_1)^{(7)} \) is defined

\[
\frac{1}{(m_2)^{(7)}} G_{36e}^0 ((S_1)^{(7)} - (p_{38})^{(7)}) t \leq G_{37}^0 (t) \leq \frac{1}{(m_2)^{(7)}} G_{36e}^0 ((S_1)^{(7)} + (p_{38})^{(7)}) t
\]

\[
\left( \frac{(a_{38})^{(7)} e_{36}}{(m_2)^{(7)}} \right) e_{36}^0 ((S_1)^{(7)} - (p_{38})^{(7)} - (S_2)^{(7)}) t \left( e^{((S_1)^{(7)} - (p_{38})^{(7)}) t} e^{-(S_2)^{(7)} t} \right) + G_{38e}^0 e^{-(S_2)^{(7)} t} \leq G_{39}^0 (t) \leq \frac{(a_{38})^{(7)} e_{36}}{(m_2)^{(7)}} \left( e^{((S_1)^{(7)} - (p_{38})^{(7)}) t} e^{-(S_2)^{(7)} t} \right) + G_{38e}^0 e^{-(S_2)^{(7)} t}
\]

\[
T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}^0 (t) \leq T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)}) t}
\]

\[
\frac{1}{(m_2)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}^0 (t) \leq \frac{1}{(m_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)}) t}
\]

\[
\left( \frac{(b_{38})^{(7)} e_{36}}{(m_1)^{(7)}} \right) e_{36}^0 ((R_1)^{(7)} - (b_{38})^{(7)} + (S_2)^{(7)}) t \left( e^{((R_1)^{(7)} - (b_{38})^{(7)} + (S_2)^{(7)}) t} e^{-(S_2)^{(7)} t} \right) + T_{38}^0 e^{-(S_2)^{(7)} t} \leq T_{38}^0 (t) \leq \frac{(b_{38})^{(7)} e_{36}}{(m_1)^{(7)}} \left( e^{((R_1)^{(7)} - (b_{38})^{(7)} + (S_2)^{(7)}) t} e^{-(S_2)^{(7)} t} \right) + T_{38}^0 e^{-(S_2)^{(7)} t}
\]

**Definition of** \((S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}\):

Where \((S_1)^{(7)} = (a_{36})^{(7)} (m_2)^{(7)} - (a_{36}'^{(7)})\)

\( (S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)} \)

\( (R_1)^{(7)} = (b_{38})^{(7)} (\mu_2)^{(7)} - (b_{38}'^{(7)}) \)

\( (R_2)^{(7)} = (b_{38}')^{(7)} - (r_{38})^{(7)} \)

**Behavior of the solutions**

**Theorem 2:** If we denote and define

**Definition of** \((\sigma_3)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}\):

\((bbb) (\sigma_3)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}\) four constants satisfying

\[-(\sigma_2)^{(8)} \leq - (a_{40})^{(8)} + (a_{41})^{(8)} - (a_{40}')^{(8)} (T_{41}, t) + (a_{41}')^{(8)} (T_{41}, t) \leq -(\sigma_3)^{(8)} \]

\[-(\tau_2)^{(8)} \leq - (b_{40})^{(8)} + (b_{41})^{(8)} - (b_{40}')^{(8)} ((G_{43}), t) - (b_{41}')^{(8)} ((G_{43}), t) \leq -(\tau_1)^{(8)} \]
Definition of \((v_1)^{(0)}, (v_2)^{(0)}, (u_1)^{(0)}, (u_2)^{(0)}, \nu^{(0)}, u^{(0)}\):

(c) By \((v_1)^{(0)} > 0, (v_2)^{(0)} < 0\) and respectively \((u_1)^{(0)} > 0, (u_2)^{(0)} < 0\) the roots of the equations

\[ (a_{41})^{(0)}v^{(0)} - (a_{40})^{(0)} = 0 \]

and \((b_{41})^{(0)}u^{(0)} - (b_{40})^{(0)} = 0\)

Definition of \((\bar{v}_1)^{(0)}, (\bar{v}_2)^{(0)}, (\bar{u}_1)^{(0)}, (\bar{u}_2)^{(0)}\):

By \((\bar{v}_1)^{(0)} > 0, (\bar{v}_2)^{(0)} < 0\) and respectively \((\bar{u}_1)^{(0)} > 0, (\bar{u}_2)^{(0)} < 0\) the roots of the equations

\[ (a_{41})^{(0)}v^{(0)} - (a_{40})^{(0)} = 0 \]

and \((b_{41})^{(0)}u^{(0)} - (b_{40})^{(0)} = 0\)

Definition of \((m_1)^{(0)}, (m_2)^{(0)}, (\mu_1)^{(0)}, (\mu_2)^{(0)}, (v_0)^{(0)}\):

(d) If we define \((m_1)^{(0)}, (m_2)^{(0)}, (\mu_1)^{(0)}, (\mu_2)^{(0)}\) by

\[ (m_2)^{(0)} = (v_0)^{(0)}, (m_1)^{(0)} = (v_1)^{(0)}, \text{ if } (v_0)^{(0)} < (v_1)^{(0)} \]

\[ (m_2)^{(0)} = (v_1)^{(0)}, (m_1)^{(0)} = (v_0)^{(0)}, \text{ if } (v_1)^{(0)} < (v_0)^{(0)} \]

and analogously

\[ (\mu_2)^{(0)} = (u_0)^{(0)}, (\mu_1)^{(0)} = (u_1)^{(0)}, \text{ if } (u_0)^{(0)} < (u_1)^{(0)} \]

\[ (\mu_2)^{(0)} = (u_1)^{(0)}, (\mu_1)^{(0)} = (u_0)^{(0)}, \text{ if } (u_1)^{(0)} < (u_0)^{(0)} \]

are defined respectively

Then the solution of GLOBAL EQUATIONS satisfies the inequalities

\[ G_{40}e^{((s_1)^{(0)}-(p_{40})^{(0)})t} \leq G_{40}e^{((s_1)^{(0)})t} \leq G_{40}e^{((s_1)^{(0)}+1/t)} \]

where \((p_1)^{(0)}\) is defined

\[ \frac{1}{(m_1)^{(0)}} G_{40}e^{((s_1)^{(0)}-(p_{40})^{(0)})t} \leq G_{41}e^{((s_1)^{(0)})t} \leq \frac{1}{(m_1)^{(0)}} G_{40}e^{((s_1)^{(0)}+1/t)} \]

\[ \frac{(a_{42})^{(0)}}{(m_1)^{(0)}} \left[ e^{((s_1)^{(0)}-(p_{40})^{(0)})t} - e^{((s_2)^{(0)})t} \right] + G_{42}e^{((s_2)^{(0)}+1/t)} \leq G_{43}e^{((s_2)^{(0)}+1/t)} \]

\[ T_{40}e^{((\tau_1)^{(0)}+1/t)} \leq T_{40}e^{((\tau_1)^{(0)}+1/t)} \]
\[
\frac{1}{(\mu_4)(8)} T_{40} e^{(R_1)(8)t} \leq T_{40}(t) \leq \frac{1}{(\mu_2)(8)} T_{40} e^{((R_1)(8)+(R_4)(8))t}
\]

\[
(\frac{(b_{42})(8)^2}{(\mu_1)(8)((R_1)(8)-(R_4)(8))})(e^{(R_1)(8)t} - e^{-(b_{42})(8)t}) + T_{42} e^{-(b_{42})(8)t} \leq T_{42}(t) \leq \frac{(a_{42})(8)\mu_2}{(\mu_2)(8)((R_1)(8)+(R_4)(8)+(R_2)(8))}(e^{((R_1)(8)+(R_4)(8))t} - e^{-(R_2)(8)t}) + T_{42} e^{-(R_2)(8)t}
\]

**Definition of** \((S_1)(8), (S_2)(8), (R_1)(8), (R_2)(8);\)

Where \((S_1)(8) = (a_{40})(8)(m_{2})(8) - (a_{40})(8)\)

\((S_2)(8) = (a_{42})(8) - (p_{42})(8)\)

\((R_1)(8) = (b_{40})(8)(\mu_2)(8) - (b_{40})(8)\)

\((R_2)(8) = (b_{42})(8) - (r_{42})(8)\)

From GLOBAL EQUATIONS we obtain

\[
\frac{d\psi(8)}{dt} = (a_{40})(8) - ((a_{40})(8) - (a_{14})(8) + (a_{40})(8)(T_{41}, t)) - (a_{41})(8)(T_{41}, t)\psi(8) - (a_{41})(8)\psi(8)
\]

**Definition of** \(\psi(8);\)

\[
\psi(8) = \frac{a_{40}}{a_{41}}
\]

It follows

\[-(a_{41})(8)(\psi(8))^2 + (a_{41})(8)\psi(8) - (a_{40})(8) \leq \frac{d\psi(8)}{dt} \leq -(a_{41})(8)(\psi(8))^2 + (a_{41})(8)\psi(8) - (a_{40})(8)\]

From which one obtains

**Definition of** \((\bar{V}_0)(8), (V_0)(8);\)

(g) For \(0 < (V_0)(8) = \frac{C_0}{C_{11}} < (V_1)(8) < (\bar{V}_1)(8)\)

\[
\psi(8)(t) \geq \frac{\frac{(V_0)(8)+C(8)(V_1)(8)}{(V_0)(8)+C(8)(V_2)(8)} e^{-[(a_{41})(8)(V_1)(8)-(V_0)(8)]t}}{1+[(a_{41})(8)(V_1)(8)-(V_0)(8)]} \Rightarrow \frac{(C)(8) = (V_1)(8)-(V_0)(8)}{(V_0)(8)-(V_2)(8)}
\]

it follows \((V_0)(8) \leq \psi(8)(t) \leq (V_1)(8)\)

In the same manner, we get

\[
\psi(8)(t) \leq \frac{\frac{(V_1)(8)+C(8)(V_2)(8)}{(V_1)(8)+C(8)(V_2)(8)} e^{-[(a_{41})(8)(V_1)(8)-(V_2)(8)]t}}{1+[(a_{41})(8)(V_1)(8)-(V_2)(8)]} \Rightarrow \frac{(C)(8) = (V_1)(8)-(V_0)(8)}{(V_0)(8)-(V_2)(8)}
\]

From which we deduce \((V_0)(8) \leq \psi(8)(t) \leq (\bar{V}_0)(8)\)

(h) If \(0 < (V_1)(8) < (V_0)(8) = \frac{C_0}{C_{11}} < (\bar{V}_1)(8)\) we find like in the previous case,
From GLOBAL EQUATIONS we obtain consequence of the relation between \( \bar{a}_{24} \) defines \( \bar{\alpha}_{41} \).

Analogously if

\[
\frac{v(t)}{v_0(t)} = \frac{G_{41}}{G_{40}}
\]

Particular case:

If \( (a_{40})^{(b)} = (a_{41})^{(b)} \), then \( (s_1)^{(b)} = (s_2)^{(b)} \) and in this case \( (v_1)^{(b)} = (\bar{v}_1)^{(b)} \) if in addition \( (v_0)^{(b)} = (v_1)^{(b)} \) then \( v(t) = v_0(t) \) and as a consequence \( G_{40}(t) = (v_0(t)G_{41}(t) \) this also defines \( v_0(t) \) for the special case.

And so with the notation of the first part of condition (c), we have

**Definition of** \( v^{(b)}(t) \):

\[
(m_0)^{(b)} \leq v^{(b)}(t) \leq (m_1)^{(b)}, \quad v^{(b)}(t) = \frac{G_{40}(t)}{G_{41}(t)}
\]

In a completely analogous way, we obtain

**Definition of** \( u^{(b)}(t) \):

\[
(\mu_0)^{(b)} \leq u^{(b)}(t) \leq (\mu_1)^{(b)}, \quad u^{(b)}(t) = \frac{T_{40}(t)}{T_{41}(t)}
\]

Now, using this result and replacing it in CONCATENATED GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case**:

If \( (a_{40})^{(b)} = (a_{41})^{(b)} \), then \( (s_1)^{(b)} = (s_2)^{(b)} \) and in this case \( (v_1)^{(b)} = (\bar{v}_1)^{(b)} \) if in addition \( (v_0)^{(b)} = (v_1)^{(b)} \) then \( v^{(b)}(t) = v_0(t) \) and as a consequence \( G_{40}(t) = (v_0(t)G_{41}(t) \) this also defines \( v_0(t) \) for the special case.

Analogously if \( (b_{40})^{(b)} = (b_{41})^{(b)} \), then \( (r_1)^{(b)} = (r_2)^{(b)} \) and then \( (u_1)^{(b)} = (\bar{u}_1)^{(b)} \) if in addition \( (u_0)^{(b)} = (u_1)^{(b)} \) then \( T_{40}(t) = (u_0(t)T_{41}(t) \). This is an important consequence of the relation between \( (v_1)^{(b)} \) and \( (\bar{v}_1)^{(b)} \), and definition of \( (u_0)^{(b)} \).

From GLOBAL EQUATIONS we obtain

\[
\frac{dv^{(b)}}{dt} = (a_{24})^{(b)} - (a_{23})^{(b)}(a_{25})^{(b)}(T_{25}, t) - (a_{24})^{(b)}(a_{25})^{(b)}(T_{25}, t)\nu^{(b)} - (a_{25})^{(b)}\nu^{(b)}
\]

**Definition of** \( \nu^{(b)} \):

\[
\nu^{(b)} = \frac{G_{24}}{G_{25}}
\]

It follows

\[
- \left( (a_{25})^{(b)}\nu^{(b)} \right)^2 + (\nu^{(b)})^2 - (a_{24})^{(b)} \leq \frac{dv^{(b)}}{dt} \leq - \left( (a_{25})^{(b)}\nu^{(b)} \right)^2 + (\nu^{(b)})^2 - (a_{24})^{(b)}
\]

From which one obtains

\[
\frac{dv^{(b)}}{dt} \leq - \left( (a_{25})^{(b)}\nu^{(b)} \right)^2 + (\nu^{(b)})^2 - (a_{24})^{(b)}
\]

\[
\frac{dv^{(b)}}{dt} \leq - \left( (a_{25})^{(b)}\nu^{(b)} \right)^2 + (\nu^{(b)})^2 - (a_{24})^{(b)}
\]
Definition of $(\bar{v}_1)^{(4)}$, $(v_0)^{(4)}$:

(j) For $0 < \frac{\sigma_2}{\sigma_{25}} < (v_1)^{(4)} < (\bar{v}_1)^{(4)}$

$$v^{(4)}(t) \geq \frac{(v_2)^{(4)}(v_2)^{(4)}e^{-(a_{25}^{(4)})(v_1)^{(4)}-v_0)^{(4)}]}{4 + (C)^{(4)}e^{-(a_{25}^{(4)})(v_1)^{(4)}-v_0)^{(4)}]} , \quad (C)^{(4)} = \frac{(v_2)^{(4)}-v_0)^{(4)}}{(v_0)^{(4)}-v_2)^{(4)}}$$

it follows $(v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)}$

In the same manner, we get

$$v^{(4)}(t) \leq \frac{(v_2)^{(4)}(v_2)^{(4)}e^{-(a_{25}^{(4)})(v_1)^{(4)}-v_0)^{(4)}]}{4 + (C)^{(4)}e^{-(a_{25}^{(4)})(v_1)^{(4)}-v_0)^{(4)}]} , \quad (C)^{(4)} = \frac{(v_2)^{(4)}-v_0)^{(4)}}{(v_0)^{(4)}-v_2)^{(4)}}$$

From which we deduce $(v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)}$

(k) If $0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{\sigma_2}{\sigma_{25}} < (\bar{v}_1)^{(4)}$ we find like in the previous case,

$$(v_1)^{(4)} \leq \frac{(v_2)^{(4)}+\tau(C)^{(4)}(v_2)^{(4)}e^{-(a_{25}^{(4)})(v_1)^{(4)}-v_0)^{(4)}]}{1 + (C)^{(4)}e^{-(a_{25}^{(4)})(v_1)^{(4)}-v_0)^{(4)}]} \leq v^{(4)}(t) \leq \frac{(v_2)^{(4)}+\tau(C)^{(4)}(v_2)^{(4)}e^{-(a_{25}^{(4)})(v_1)^{(4)}-v_0)^{(4)}]}{1 + (C)^{(4)}e^{-(a_{25}^{(4)})(v_1)^{(4)}-v_0)^{(4)}]} \leq (\bar{v}_1)^{(4)}$$

(l) If $0 < (v_1)^{(4)} \leq (\bar{v}_1)^{(4)} \leq \frac{(v_0)^{(4)}}{G_{25}(t)}$, we obtain

$$(v_1)^{(4)} \leq v^{(4)}(t) \leq \frac{(v_2)^{(4)}+\tau(C)^{(4)}(v_2)^{(4)}e^{-(a_{25}^{(4)})(v_1)^{(4)}-v_0)^{(4)}]}{1 + (C)^{(4)}e^{-(a_{25}^{(4)})(v_1)^{(4)}-v_0)^{(4)}]} \leq (v_0)^{(4)}$$

And so with the notation of the first part of condition (c), we have

Definition of $v^{(4)}(t)$:

$$(m_1)^{(4)} \leq v^{(4)}(t) \leq (m_2)^{(4)}, \quad v^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:

$$(\mu_1)^{(4)} \leq u^{(4)}(t) \leq (\mu_2)^{(4)}, \quad u^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case:

If $(a_{24}^{(4)}) = (a_{25}^{(4)})$, then $(\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(v_1)^{(4)} = (\bar{v}_1)^{(4)}$ if in addition $(v_0)^{(4)} = (v_1)^{(4)}$ then $v^{(4)}(t) = (v_0)^{(4)}$ and as a consequence $G_{24}(t) = (v_0)^{(4)}G_{25}(t)$ this also
defines \((v_0)\) for the special case.

Analogously if \((b_{24}^{(4)}) = (b_{25}^{(4)})\), then \((r_1) = (r_2)\) and then
\[(u_1) = (\bar{u}_1)\]
if in addition \((u_0) = (u_1)\) then \(T_{24}(t) = (u_0)T_{25}(t)\) This is an important consequence of the relation between \((v_1)\) and \((\bar{v}_1)\), and definition of \((u_0)\).

From GLOBAL EQUATIONS we obtain

\[
\frac{dv^{(5)}}{dt} = (a_{28}^{(5)}) - \left( (a_{28}^{(5)}) + (a_{29}^{(5)})(T_{29}, t) - (a_{29}^{(5)})(T_{29}, t)\right) y^{(5)} - (a_{29}^{(5)})y^{(5)}
\]

**Definition of** \(v^{(5)}\):

\[
v^{(5)} = \frac{a_{28}}{a_{29}}
\]

It follows

\[-(a_{29}^{(5)})(v^{(5)})^2 + (\sigma_2^{(5)})v^{(5)} - (a_{28}^{(5)}) \leq \frac{dv^{(5)}}{dt} \leq -(a_{29}^{(5)})(v^{(5)})^2 + (\sigma_1^{(5)})v^{(5)} - (a_{28}^{(5)})\]

From which one obtains

**Definition of** \((\bar{v}_1)\), \((v_0)\):

(m) For \(0 < \frac{v_0^{(5)}}{a_{29}} < (v_1)\):

\[v^{(5)}(t) \geq \frac{(v_1^{(5)})(\bar{v}_1^{(5)})}{5 + (C^{(5)})e^{-a_{29}^{(5)}(v_1^{(5)} - v_0^{(5)})}} \]

\[
(C^{(5)}) = \frac{(v_1^{(5)} - v_0^{(5)})}{(v_0^{(5)} - \bar{v}_1^{(5)})}
\]

From which one deduces \((v_0) \leq v^{(5)}(t) \leq (v_1)\)

(n) If \(0 < (v_1) < (v_0) = \frac{a_{28}}{a_{29}} < (\bar{v}_1)\) we find like in the previous case,

\[
(v_1^{(5)}) \leq \frac{(v_1^{(5)})(\bar{v}_1^{(5)})}{1 + (C^{(5)})\bar{v}_1^{(5)} - (v_1^{(5)} - v_0^{(5)})} \leq v^{(5)}(t) \leq \frac{(v_1^{(5)})(\bar{v}_1^{(5)})}{1 + (C^{(5)})v_0^{(5)} - (v_1^{(5)} - v_0^{(5)})} \leq (\bar{v}_1^{(5)})
\]

(o) If \(0 < (v_1) \leq (\bar{v}_1) \leq \frac{v_0^{(5)}}{a_{29}}\) we obtain

\[
(v_1^{(5)}) \leq v^{(5)}(t) \leq \frac{(v_1^{(5)})(\bar{v}_1^{(5)})}{1 + (C^{(5)})v_0^{(5)} - (v_1^{(5)} - v_0^{(5)})} \leq (v_0^{(5)})
\]

301
And so with the notation of the first part of condition (c), we have

**Definition of** \( v^{(5)}(t) :\)

\[(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}, \quad v^{(5)}(t) = \frac{g_{29}(t)}{g_{20}(t)}\]

In a completely analogous way, we obtain

**Definition of** \( u^{(5)}(t) :\)

\[(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad u^{(5)}(t) = \frac{g_{28}(t)}{g_{29}(t)}\]

Now, using this result and replacing it in **GLOBAL EQUATIONS** we get easily the result stated in the theorem.

**Particular case:**

If \((a_{29}^{(5)} = (a_{29}^{(5)}), then \( (a_1)^{(5)} = (a_2)^{(5)} \) and in this case \((v_1)^{(5)} = (v_1)^{(5)} \) if in addition \((v_0)^{(5)} = (v_0)^{(5)} \) then \( v^{(5)}(t) = (v_0)^{(5)} \)) and as a consequence \( G_{29}(t) = (v_0)^{(5)}G_{20}(t) \)** this also defines \((v_0)^{(5)} \) for the special case.

Analogously if \((b_{29}^{(5)} = (b_{29}^{(5)}), then \( (\tau_1)^{(5)} = (\tau_2)^{(5)} \) and then

\((u_1)^{(5)} = (u_1)^{(5)} \) if in addition \((u_0)^{(5)} = (u_0)^{(5)} \) then \( T_{28}(t) = (u_0)^{(5)}T_{29}(t) \) This is an important consequence of the relation between \((v_1)^{(5)} \) and \( (v_1)^{(5)} \), and definition of \((u_0)^{(5)} \).

we obtain

\[\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - (a_{32})^{(6)} + (a_{32})^{(6)}(T_{33}, t) - (a_{32})^{(6)}(T_{33}, t)v^{(6)} - (a_{32})^{(6)}v^{(6)}\]

**Definition of** \( v^{(6)} :\)

\[v^{(6)} = \frac{a_{32}}{g_{23}}\]

It follows

\[- (a_{32})^{(6)}v^{(6)}(t)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{42})^{(6)} \leq \frac{dv^{(6)}}{dt} \leq - (a_{32})^{(6)}v^{(6)}(t)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{42})^{(6)}\]

From which one obtains

**Definition of** \((\bar{v}_2)^{(6)}, (v_0)^{(6)} :\)

(p) For \(0 < (v_0)^{(6)} = \frac{g_{22}}{g_{25}} < (v_1)^{(6)} < (\bar{v}_2)^{(6)}\)

\[v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (v_2)^{(6)}e^{-(a_{33})^{(6)}(v_1)^{(6)} - (v_0)^{(6)}}}{1 + (v_0)^{(6)}e^{-(a_{33})^{(6)}(v_1)^{(6)} - (v_0)^{(6)}}} \]

**Definition of** \((\bar{C})^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}\)

it follows \((v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}\)

In the same manner, we get

\[v^{(6)}(t) \leq \frac{(v_2)^{(6)} + (\bar{C})^{(6)}(v_2)^{(6)}e^{-(a_{33})^{(6)}(v_1)^{(6)} - (v_2)^{(6)}}}{1 + (\bar{C})^{(6})e^{-(a_{33})^{(6)}(v_1)^{(6)} - (v_2)^{(6)}}} \]

**Definition of** \((\bar{C})^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}\)
From which we deduce \( v_0^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)} \)

\((q)\) If \( 0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_0}{G_{23}} < (\bar{v}_1)^{(6)} \) we find like in the previous case,

\[ v_0^{(6)}(t) \leq \frac{(v_0)^{(6)} + (\bar{v}_1)^{(6)}(v_2)^{(6)} e^{-[a_{23}]^{(6)}(v_1)^{(6)} - (v_2)^{(6)} t]}{1 + (\bar{v}_1)^{(6)} e^{-[a_{23}]^{(6)}(v_1)^{(6)} - (v_2)^{(6)} t]}} \leq v^{(6)}(t) \leq \frac{(v_0)^{(6)} + (\bar{v}_1)^{(6)}(v_2)^{(6)} e^{-[a_{23}]^{(6)}(v_1)^{(6)} - (v_2)^{(6)} t]}{1 + (\bar{v}_1)^{(6)} e^{-[a_{23}]^{(6)}(v_1)^{(6)} - (v_2)^{(6)} t]}} \leq (\bar{v}_1)^{(6)}\]

\[(s)\) If \( 0 < (v_1)^{(6)} < (\bar{v}_2)^{(6)} \leq (v_0)^{(6)} = \frac{G_0}{G_{23}} \), we obtain

\[ v_0^{(6)}(t) \leq \frac{(v_0)^{(6)} + (\bar{v}_2)^{(6)}(v_2)^{(6)} e^{-[a_{23}]^{(6)}(v_1)^{(6)} - (v_2)^{(6)} t]}{1 + (\bar{v}_2)^{(6)} e^{-[a_{23}]^{(6)}(v_1)^{(6)} - (v_2)^{(6)} t]}} \leq v^{(6)}(t) \leq (v_0)^{(6)}\]

And so with the notation of the first part of condition \((c)\), we have

\textbf{Definition of} \( v^{(6)}(t) \) ::

\[ (m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)}, \quad v^{(6)}(t) = \frac{G_0}{G_{23}} \]

In a completely analogous way, we obtain

\textbf{Definition of} \( u^{(6)}(t) \) ::

\[ (\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad u^{(6)}(t) = \frac{T_{32}}{T_{33}}(t) \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

\textbf{Particular case}:

If \( (a'_{32})^{(6)} = (a'_{33})^{(6)}, \) then \( (\sigma_1)^{(6)} = (\sigma_2)^{(6)} \) and in this case \( (v_1)^{(6)} = (\bar{v}_1)^{(6)} \) if in addition \( (v_0)^{(6)} = (v_1)^{(6)} \) then \( v^{(6)}(t) = (v_0)^{(6)} \) and as a consequence \( G_{32}(t) = (v_0)^{(6)}G_{33}(t) \) this also defines \( v^{(6)}(t) \) for the special case.

Analogously if \( (b'_{32})^{(6)} = (b'_{33})^{(6)}, \) then \( (\tau_1)^{(6)} = (\tau_2)^{(6)} \) and then

\( (u_1)^{(6)} = (\bar{u})^{(6)} \) if in addition \( (u_0)^{(6)} = (u_2)^{(6)} \) then \( T_{32}(t) = (u_0)^{(6)}T_{33}(t) \) This is an important consequence of the relation between \( (v_1)^{(6)} \) and \( (\bar{v}_1)^{(6)} \) and definition of \( (u_0)^{(6)} \).

\textbf{Behavior of the solutions}:

If we denote and define

\textbf{Definition of} \( (\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)} \):

\[ -(\sigma_2)^{(7)} \leq -a'_{36}(T_3 - T_{37}, t) + (a'_{36})^{(7)} - (T_{37}, t)(T_{37}, t) \leq -(\sigma_1)^{(7)} \]

\[ -(\tau_2)^{(7)} \leq -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b'_{36})^{(7)}((G_{39}, t)) - (b'_{37})^{(7)}((G_{39}, t)) \]

303
Definition of \((v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}\):

(ff) By \((v_1)^{(7)} > 0\), \((v_2)^{(7)} < 0\) and respectively \((u_1)^{(7)} > 0\), \((u_2)^{(7)} < 0\) the roots of the equations 
\[\left(\frac{a_{50}}{a_{30}}\right)\left(\nu^{(7)}\right)^2 + \left(\frac{a_{30}}{a_{10}}\right)\nu^{(7)} - \left(\frac{a_{10}}{a_{0}}\right) = 0\]
and 
\[\left(\frac{b_{30}}{b_{10}}\right)\left(u^{(7)}\right)^2 + \left(\frac{b_{10}}{b_{0}}\right)u^{(7)} - \left(\frac{b_{0}}{b_{0}}\right) = 0\]

Definition of \((\bar{v}_1)^{(7)}, (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}\):

By \((\bar{v}_1)^{(7)} > 0\), \((\bar{v}_2)^{(7)} < 0\) and respectively \((\bar{u}_1)^{(7)} > 0\), \((\bar{u}_2)^{(7)} < 0\) the roots of the equations 
\[\left(\frac{a_{50}}{a_{30}}\right)\left(\nu^{(7)}\right)^2 + \left(\frac{a_{30}}{a_{10}}\right)\nu^{(7)} - \left(\frac{a_{10}}{a_{0}}\right) = 0\]
and 
\[\left(\frac{b_{30}}{b_{10}}\right)\left(u^{(7)}\right)^2 + \left(\frac{b_{10}}{b_{0}}\right)u^{(7)} - \left(\frac{b_{0}}{b_{0}}\right) = 0\]

Definition of \((m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}\):

(gg) If we define \((m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}\) by 
\[(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}, \text{ if } (v_0)^{(7)} < (v_1)^{(7)}\]
\[(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}, \text{ if } (v_0)^{(7)} < (v_1)^{(7)}\]
and 
\[(v_0)^{(7)} = G_{30}^{\nu_0} G_{30}^{v_1}\]

\((m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}, \text{ if } (\bar{v}_1)^{(7)} < (v_0)^{(7)}\)
and analogously

\((\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \text{ if } (u_0)^{(7)} < (u_1)^{(7)}\)

\((\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \text{ if } (u_0)^{(7)} < (u_1)^{(7)}\)
and 
\[(u_0)^{(7)} = G_{30}^{\nu_0} G_{30}^{v_1}\]

\((\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \text{ if } (\bar{u}_1)^{(7)} < (u_0)^{(7)}\) where \((u_1)^{(7)}, (\bar{u}_1)^{(7)}\)
are defined respectively

Then the solution of GLOBAL EQUATIONS satisfies the inequalities

\[G_{30}^{\theta_0}\left((s_1)^{(7)}-(p_{30})^{(7)}\right)t \leq G_{30}^{\theta_0}\left(s_1\right)^{(7)}t \leq G_{30}^{\theta_0}\left((s_1)^{(7)}-(p_{30})^{(7)}\right)t\]

where \((\theta_1)^{(7)}\) is defined

\[\frac{1}{(m_2)^{(7)}} G_{30}^{\theta_0}\left((s_1)^{(7)}-(p_{30})^{(7)}\right)t \leq G_{30}^{\theta_0}\left(s_1\right)^{(7)}t \leq \frac{1}{(m_2)^{(7)}} G_{30}^{\theta_0}\left((s_1)^{(7)}-(p_{30})^{(7)}\right)t \]

\[\left(\frac{a_{30}}{(m_1)^{(7)}(s_1)^{(7)}-(p_{30})^{(7)}}\right)\left(1 + e^{-\left(\frac{a_{30}}{a_{30}}\right)\left(s_1\right)^{(7)}}\right) \leq G_{30}^{\theta_0}\left(s_1\right)^{(7)}t \leq \left(\frac{a_{30}}{(m_2)^{(7)}(s_1)^{(7)}-(p_{30})^{(7)}}\right)\left(1 + e^{-\left(\frac{a_{30}}{a_{30}}\right)\left(s_1\right)^{(7)}}\right)\]

\[T_{30}^{\theta_0}\left(r_1\right)^{(7)}t \leq T_{30}^{\theta_0}\left((r_1)^{(7)}+(p_{30})^{(7)}\right)t\]
\[
\frac{1}{(\mu_1)^2} T_{36}^0 e^{(R_1)(7)t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^2} T_{36}^0 e^{(R_1)(7)+\text{up}(R_3)(7)t}
\]

\[
\frac{(b_{16b})^2}{(\mu_1)^2(T(R_4)(7)-b_{16b})^2} \left[ e^{(R_2)(7)t} - e^{-b_{16b}(7)t} \right] + T_{38}^0 e^{-(b_{16b}'(7))t} \leq T_{38}(t) \leq \frac{(a_{16b})^2}{(\mu_2)^2(T(R_4)(7)+\text{up}(R_3)(7)+R_2)(7))} \left[ e^{(R_2)(7)+\text{up}(R_3)(7)t} - e^{-R_2(7)t} \right] + T_{38}^0 e^{-R_2(7)t}
\]

**Definition** of \((S_1)(7), (S_2)(7), (R_1)(7), (R_2)(7)\):

Where \((S_1)(7) = (a_{36})(7)(m_{27})(7) - (a_{36})(7)\)

\((S_2)(7) = (a_{38})(7) - (p_{38})(7)\)

\((R_1)(7) = (b_{38})(7)(\mu_2)(7) - (b_{38})(7)\)

\((R_2)(7) = (b_{38})(7) - (r_{38})(7)\)

From **CONCATENATED GLOBAL EQUATIONS** we obtain

\[
\frac{dv^{(7)}}{dt} = (a_{36})(7) - \left( (a'_{36})(7) - (a'_{38})(7) + (a''_{38})(7)(T_{37}, t) \right) - (a''_{38})(7)(T_{37}, t)v^{(7)} - (a_{38})(7)v^{(7)}
\]

**Definition** of \(v^{(7)}\):

\[
v^{(7)} = \frac{a_{38}}{a_{37}}
\]

It follows

\[-(a_{37})(7)v^{(7)} + (a_{38})(7)v^{(7)} - (a_{36})(7) \leq \frac{dv^{(7)}}{dt} \leq -(a_{37})(7)v^{(7)} + (a_{38})(7)v^{(7)} - (a_{36})(7)\]

From which one obtains

**Definition** of \((\bar{v}_1)(7), (v_0)(7)\):

\((u)\) For \(0 < (v_0)(7) = \frac{a_{27}}{a_{37}} < (v_1)(7) < (\bar{v}_1)(7)\)

\[
v^{(7)}(t) \geq \frac{(v_1)(7) + (\hat{C})(7)(v_2)(7) e^{-[(a_{37})(7)(v_1)(7) - (v_0)(7)] t}}{1 + (\hat{C})(7) e^{-[(a_{37})(7)(v_1)(7) - (v_0)(7)] t}}
\]

where \((\hat{C})(7) = \frac{(v_1)(7) - (v_0)(7)}{(v_0)(7) - (v_2)(7)}\)

it follows \((v_0)(7) \leq v^{(7)}(t) \leq (v_1)(7)\)

In the same manner, we get

\[
v^{(7)}(t) \leq \frac{(v_2)(7) + (\hat{C})(7)(v_1)(7) e^{-[(a_{37})(7)(v_1)(7) - (v_2)(7)] t}}{1 + (\hat{C})(7) e^{-[(a_{37})(7)(v_1)(7) - (v_2)(7)] t}}
\]

where \((\hat{C})(7) = \frac{(v_2)(7) - (v_0)(7)}{(v_0)(7) - (v_2)(7)}\)

From which we deduce \((v_0)(7) \leq v^{(7)}(t) \leq (\bar{v}_1)(7)\)

\((u)\) If \(0 < (v_1)(7) < (v_0)(7) = \frac{a_{27}}{a_{37}} < (\bar{v}_1)(7)\) we find like in the previous case,
Theorem 3: If \((a_i''(0))\) and \((b_i''(0))\) are independent on \(t\), and the conditions

\[
(a_{44}'(8))(a_{45}'(8)) - (a_{44}(8))(a_{45}(8)) < 0
\]

\[
(a_{44}'(8))(a_{45}'(8)) - (a_{44}(8))(a_{45}(8)) + (a_{44}(8))(p_{44}(8)) + (a_{45}(8))(p_{45}(8)) + (p_{44}(8))(p_{45}(8)) > 0
\]

\[
(b_{41}'(8))(b_{41}(8)) - (b_{42}(8))(b_{42}(8)) > 0,
\]

\[
(b_{41}'(8))(b_{41}(8)) - (b_{42}(8))(b_{42}(8)) - (b_{40}(8))(r_{41}(8)) - (b_{41}(8))(r_{41}(8)) + (r_{43}(8))(r_{42}(8)) < 0
\]

with \((p_{40}(8)), (r_{41}(8))\) as defined are satisfied, then the system

\[
(a_{40}(8))G_{41} - [(a_{40}(8))(a_{41}(8))][G_{40}] = 0
\]

\[
(a_{41}(8))G_{40} - [(a_{41}(8))(a_{42}(8))][G_{41}] = 0
\]

\[
(a_{42}(8))G_{41} - [(a_{42}(8))(a_{43}(8))][G_{42}] = 0
\]

\[
(b_{40}(8))T_{41} - [(b_{40}(8))(b_{40}(8))][T_{40}] = 0
\]

\[
(b_{41}(8))T_{40} - [(b_{41}(8))(b_{41}(8))][T_{41}] = 0
\]

\[
(b_{42}(8))T_{41} - [(b_{42}(8))(b_{42}(8))][T_{42}] = 0
\]

has a unique positive solution, which is an equilibrium solution for the system.

Proof:

(a) Indeed the first two equations have a nontrivial solution \(G_{40}, G_{41}\) if

\[
F(T_{43}) = (a_{40}(8))(a_{41}(8)) - (a_{40}(8))(a_{41}(8)) + (a_{41}(8))(a_{42}(8)) + (a_{41}(8))(a_{42}(8))(T_{41}) + (a_{41}(8))(a_{42}(8))(T_{41}) + (a_{42}(8))(T_{41})(a_{41}(8))(T_{41}) = 0
\]

Definition and uniqueness of \(T_{41}^*\):

After hypothesis \(f'(0) < 0, f'(\infty) > 0\) and the functions \((a_i''(8))(T_{43})\) being increasing, it follows that there exists a unique \(T_{41}^*\) for which \(f(T_{41}^*) = 0\). With this value, we obtain from the three first equations

\[
G_{44} = \frac{(a_{41}(8))(a_{41}(8))}{[a_{41}(8)]^2 + (a_{42}(8))^2}[T_{41}]
\]

\[
G_{46} = \frac{(a_{42}(8))(a_{41}(8))}{[a_{42}(8)]^2 + (a_{42}(8))^2}[T_{41}]
\]

(q) By the same argument, the equations admit solutions \(G_{40}, G_{41}\) if

\[
\varphi(G_{43}) = (b_{40}(8))(b_{41}(8)) - (b_{40}(8))(b_{42}(8)) -
\]
\[ (b_{40}^{(0)})(b_{41}^{(0)})^{(0)}(G_{43}) + (b_{40}^{(8)})(b_{40}^{(0)})^{(0)}(G_{43}) + (b_{40}^{(8)})(G_{43})(b_{41}^{(0)})^{(0)}(G_{43}) = 0 \]

Where in \((G_{43})(G_{40}, \ G_{41}, \ G_{42})\), \(G_{40}, \ G_{42}\) must be replaced by their values. It is easy to see that \(\varphi\) is a decreasing function in \(G_{45}\) taking into account the hypothesis \(\varphi(0) > 0, \ \varphi(\infty) < 0\) it follows that there exists a unique \(G_{44}^*\) such that \(\varphi((G_{43})^*) = 0\)

Finally we obtain the unique solution

\[ G_{44}^* = \frac{(a_{40})^{(0)}G_{41}^*}{(a_{40})^{(0)} + (a_{40})^{(0)}(G_{43})} \quad G_{42}^* = \frac{(a_{41})^{(0)}G_{43}^*}{(a_{41})^{(0)} + (a_{41})^{(0)}(G_{43})} \]

\[ T_{44}^* = \frac{(b_{40})^{(0)}G_{43}^*}{(b_{40})^{(0)} - (b_{40})^{(0)}(G_{43})^*} \quad T_{42}^* = \frac{(b_{42})^{(0)}G_{43}^*}{(b_{42})^{(0)} - (b_{42})^{(0)}(G_{43})^*} \]

\[(b_{14})^{(1)}(T_{13}) - [(b_{14})^{(1)} - (b_{14})^{(1)}(G)]T_{14} = 0 \]

\[(b_{15})^{(1)}(T_{14}) - [(b_{15})^{(1)} - (b_{15})^{(1)}(G)]T_{15} = 0 \]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{16})^{(2)}G_{17} - [(a_{16})^{(2)} + (a_{16})^{(2)}(T_{17})]G_{16} = 0 \]

\[(a_{17})^{(2)}G_{16} - [(a_{17})^{(2)} + (a_{17})^{(2)}(T_{17})]G_{17} = 0 \]

\[(a_{18})^{(2)}G_{17} - [(a_{18})^{(2)} + (a_{18})^{(2)}(T_{17})]G_{18} = 0 \]

\[(b_{16})^{(2)}(T_{16}) - [(b_{16})^{(2)} - (b_{16})^{(2)}(G_{19})]T_{16} = 0 \]

\[(b_{17})^{(2)}(T_{16}) - [(b_{17})^{(2)} - (b_{17})^{(2)}(G_{19})]T_{17} = 0 \]

\[(b_{18})^{(2)}(T_{17}) - [(b_{18})^{(2)} - (b_{18})^{(2)}(G_{19})]T_{18} = 0 \]

\[(a_{24})^{(4)}G_{25} - [(a_{24})^{(4)} + (a_{24})^{(4)}(T_{25})]G_{24} = 0 \]

\[(a_{25})^{(4)}G_{24} - [(a_{25})^{(4)} + (a_{25})^{(4)}(T_{25})]G_{25} = 0 \]

\[(a_{26})^{(4)}G_{25} - [(a_{26})^{(4)} + (a_{26})^{(4)}(T_{25})]G_{26} = 0 \]

\[(b_{24})^{(4)}(T_{24}) - [(b_{24})^{(4)} - (b_{24})^{(4)}(G_{27})]T_{24} = 0 \]

\[(b_{25})^{(4)}(T_{24}) - [(b_{25})^{(4)} - (b_{25})^{(4)}(G_{27})]T_{25} = 0 \]

\[(b_{26})^{(4)}(T_{25}) - [(b_{26})^{(4)} - (b_{26})^{(4)}(G_{27})]T_{26} = 0 \]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{28})^{(5)}G_{29} - [(a_{28})^{(5)} + (a_{28})^{(5)}(T_{29})]G_{28} = 0 \]

\[(a_{29})^{(5)}G_{28} - [(a_{29})^{(5)} + (a_{29})^{(5)}(T_{29})]G_{29} = 0 \]

\[(a_{30})^{(5)}G_{29} - [(a_{30})^{(5)} + (a_{30})^{(5)}(T_{29})]G_{30} = 0 \]
has a unique positive solution, which is an equilibrium solution for the system

\[(a_{32})^{(6)}G_{32} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})]G_{32} = 0\]  
\[(a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})]G_{33} = 0\]  
\[(a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})]G_{34} = 0\]  
\[(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{33})]T_{32} = 0\]  
\[(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{33})]T_{33} = 0\]  
\[(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{33})]T_{34} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{36})^{(7)}G_{37} - [(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})]G_{36} = 0\]  
\[(a_{37})^{(7)}G_{36} - [(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})]G_{37} = 0\]  
\[(a_{38})^{(7)}G_{37} - [(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})]G_{38} = 0\]  
\[(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0\]  
\[(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})]T_{37} = 0\]  
\[(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})]T_{38} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{36})^{(9)}G_{39} - [(a'_{36})^{(9)} + (a''_{36})^{(9)}(T_{39})]G_{36} = 0\]  
\[(a_{37})^{(9)}G_{39} - [(a'_{37})^{(9)} + (a''_{37})^{(9)}(T_{39})]G_{37} = 0\]  
\[(a_{38})^{(9)}G_{39} - [(a'_{38})^{(9)} + (a''_{38})^{(9)}(T_{39})]G_{38} = 0\]  
\[(b_{36})^{(9)}T_{39} - [(b'_{36})^{(9)} - (b''_{36})^{(9)}(G_{39})]T_{36} = 0\]  
\[(b_{37})^{(9)}T_{36} - [(b'_{37})^{(9)} - (b''_{37})^{(9)}(G_{39})]T_{37} = 0\]  
\[(b_{38})^{(9)}T_{37} - [(b'_{38})^{(9)} - (b''_{38})^{(9)}(G_{39})]T_{38} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

(a) Indeed the first two equations have a nontrivial solution \(G_{36}, G_{37}\) if

\[F(T_{39}) = (a_{36})^{(9)}(a_{37})^{(9)} - (a_{36})^{(9)}(a_{37})^{(9)} + (a'_{36})^{(9)}(a''_{36})^{(9)}(T_{37}) + (a'_{37})^{(9)}(a''_{36})^{(9)}(T_{37}) + (a''_{36})^{(9)}(T_{37})(a''_{36})^{(9)}(T_{37}) = 0\]

**Definition and uniqueness of** \(T_{37}^*\) : 

After hypothesis \(f(0) < 0, f'(\infty) < 0\) and the functions \((a''_{36})^{(7)}(T_{37})\) are increasing, it follows that

there exists a unique \(T_{37}^*\) for which \(F(T_{39}) = 0\). With this value, we obtain from the three first equations

\[G_{36} = \frac{(a_{36})^{(9)}G_{39}}{[(a'_{36})^{(9)} + (a''_{36})^{(9)}(T_{37})]}, \quad G_{38} = \frac{(a_{38})^{(9)}G_{39}}{[(a'_{38})^{(9)} + (a''_{38})^{(9)}(T_{37})]}\]
(r) By the same argument, the equations (SOLUTIONAL) admit solutions \( G_{36}, G_{37} \) if

\[
\varphi(G_{39}) = (b_{36}'(r))^2(b_{37}'(r))^2 - (b_{36}(r))^2(b_{37}(r))^2 - \\
\left[ (b_{36}(r))^2(b_{37}(r))^2(G_{39}) + (b_{37}'(r))^2(b_{36}'(r))^2(G_{39}) \right] + (b_{36}'(r))^2(b_{37}'(r))^2(G_{39}) = 0
\]

Where in \( G_{36}, G_{37}, G_{38} \), \( G_{36} \) \( G_{36} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{37} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{37} \) such that \( \varphi(G^*) = 0 \)

Finally we obtain the unique solution OF THE SYSTEM

\[
G^*_{36} \text{ given by } \varphi(G_{39}(\cdot)) = 0, T^*_{37} \text{ given by } f(T^*_{37}) = 0 \text{ and }
G^*_{36} = \frac{(a_{36})^2 G_{21}}{(a_{36})^2 + (a_{36}')^2(T_{21})}, \quad G^*_{38} = \frac{(a_{38})^2 G_{22}}{(a_{38})^2 + (a_{38}')^2(T_{22})}
\]

\[
T^*_{36} = \frac{(b_{36}(r))^2 T_{21}(r)}{(b_{36}(r))^2 - (b_{36}'(r))^2((G_{39}(r))^2)}, \quad T^*_{38} = \frac{(b_{38}(r))^2 T_{22}(r)}{(b_{38}(r))^2 - (b_{38}'(r))^2((G_{39}(r))^2)}
\]

**Definition and uniqueness of \( T_{21} \):**

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \( (a_{11}'(r))^2(T_{21}) \) being increasing, it follows that there exists a unique \( T_{21}^* \) for which \( f(T_{21}) = 0 \). With this value, we obtain from the three first equations

\[
G_{20} = \frac{(a_{20})^3 G_{21}}{(a_{20})^3 + (a_{20}')^3(T_{21})}, \quad G_{22} = \frac{(a_{22})^3 G_{21}}{(a_{22})^3 + (a_{22}')^3(T_{22})}
\]

**Definition and uniqueness of \( T_{25} \):**

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \( (a_{11}'(r))^4(T_{25}) \) being increasing, it follows that there exists a unique \( T_{25}^* \) for which \( f(T_{25}) = 0 \). With this value, we obtain from the three first equations

\[
G_{24} = \frac{(a_{24})^4 G_{25}}{(a_{24})^4 + (a_{24}')^4(T_{25})}, \quad G_{26} = \frac{(a_{26})^4 G_{25}}{(a_{26})^4 + (a_{26}')^4(T_{25})}
\]

**Definition and uniqueness of \( T_{29} \):**

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \( (a_{11}'(r))^5(T_{29}) \) are increasing, it follows that there exists a unique \( T_{29}^* \) for which \( f(T_{29}) = 0 \). With this value, we obtain from the three first equations

\[
G_{28} = \frac{(a_{28})^5 G_{29}}{(a_{28})^5 + (a_{28}')^5(T_{29})}, \quad G_{30} = \frac{(a_{30})^5 G_{29}}{(a_{30})^5 + (a_{30}')^5(T_{29})}
\]

**Definition and uniqueness of \( T_{33} \):**

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \( (a_{11}'(r))^6(T_{33}) \) are increasing, it follows that there exists a unique \( T_{33}^* \) for which \( f(T_{33}) = 0 \). With this value, we obtain from the three first equations

\[
G_{32} = \frac{(a_{32})^6 G_{33}}{(a_{32})^6 + (a_{32}')^6(T_{33})}, \quad G_{34} = \frac{(a_{34})^6 G_{33}}{(a_{34})^6 + (a_{34}')^6(T_{33})}
\]
(s) By the same argument, the equations \( \text{GLOBAL} \) admit solutions \( G_{13}, G_{14} \) if
\[
\varphi(G) = (b_{13}'(1)(b_{14}'(1) - (b_{13})(b_{14})'(1) - \\
[(b_{13}'(1)(b_{14}'(1)(G) + (b_{14}'(1)(b_{13}'(1)(G))] + (b_{13}'(1)(G)(b_{14}'(1)(G) = 0
\]
Where in \( G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{14} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{14} \) such that \( \varphi(G_{14}) = 0 \)

(t) By the same argument, the equations 92, 93 admit solutions \( G_{16}, G_{17} \) if
\[
\varphi(G_{19}) = (b_{16}'(2)(b_{17}'(2) - (b_{16})(b_{17})'(2) - \\
[(b_{16}'(2)(b_{17}'(2)(G_{19}) + (b_{17}'(2)(b_{16}'(2)(G_{19}))] + (b_{16}'(2)(G_{19})(b_{17}'(2)(G_{19}) = 0
\]
Where in \( G(G_{16}, G_{17}, G_{18}), G_{16}, G_{18} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{17} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{14} \) such that \( \varphi(G_{14}) = 0 \)

(u) By the same argument, the concatenated equations admit solutions \( G_{20}, G_{21} \) if
\[
\varphi(G_{23}) = (b_{20}'(3)(b_{21}'(3) - (b_{20})(b_{21})'(3) - \\
[(b_{20}'(3)(b_{21}'(3)(G_{23}) + (b_{21}'(3)(b_{20}'(3)(G_{23}))] + (b_{20}'(3)(G_{23})(b_{21}'(3)(G_{23}) = 0
\]
Where in \( G(G_{20}, G_{21}, G_{22}), G_{20}, G_{22} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{17} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{21} \) such that \( \varphi(G_{21}) = 0 \)

(v) By the same argument, the equations of modules admit solutions \( G_{24}, G_{25} \) if
\[
\varphi(G_{27}) = (b_{24}'(4)(b_{25}'(4) - (b_{24})(b_{25})'(4) - \\
[(b_{24}'(4)(b_{25}'(4)(G_{27}) + (b_{25}'(4)(b_{24}'(4)(G_{27}))] + (b_{24}'(4)(G_{27})(b_{25}'(4)(G_{27}) = 0
\]
Where in \( G(G_{24}, G_{25}, G_{26}), G_{24}, G_{26} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{25} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{25} \) such that \( \varphi(G_{25}) = 0 \)

(w) By the same argument, the equations (modules) admit solutions \( G_{28}, G_{29} \) if
\[
\varphi(G_{31}) = (b_{28}'(5)(b_{29}'(5) - (b_{28})(b_{29})'(5) - \\
[(b_{28}'(5)(b_{29}'(5)(G_{31}) + (b_{29}'(5)(b_{28}'(5)(G_{31}))] + (b_{28}'(5)(G_{31})(b_{29}'(5)(G_{31}) = 0
\]
Where in \( G(G_{28}, G_{29}, G_{30}), G_{28}, G_{30} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{35} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{31} \) such that \( \varphi(G_{31}) = 0 \)

(x) By the same argument, the equations (modules) admit solutions \( G_{32}, G_{33} \) if
\[
\varphi(G_{32}) = (b_{28}'(5)(b_{29}'(5) - (b_{28})(b_{29})'(5) - \\
[(b_{28}'(5)(b_{29}'(5)(G_{32}) + (b_{29}'(5)(b_{28}'(5)(G_{32}))] + (b_{28}'(5)(G_{32})(b_{29}'(5)(G_{32}) = 0
\]
Where in \( G(G_{28}, G_{29}, G_{30}), G_{28}, G_{30} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{35} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{31} \) such that \( \varphi(G_{31}) = 0 \)
\[ \varphi(G_{35}) = (b_{32})^6(b_{33})^6 - (b_{32})(b_{33})^6 - \\
\left[(b_{32})^6(b_{33})^6(G_{35}) + (b_{32})^6(b_{33})^6(G_{35})\right] + (b_{32})^6(b_{33})^6(G_{35}) = 0 \]

Where in \((G_{35}, G_{32}, G_{33}, G_{34}, G_{32}G_{34})\) must be replaced by their values. It is easy to see that \(\varphi\) is a decreasing function in \(G_{33}\) taking into account the hypothesis \(\varphi(0) > 0, \varphi(\infty) < 0\) it follows that there exists a unique \(G_{33}\) such that \(\varphi(G^*) = 0\).

Finally we obtain the unique solution:

\[ G^*_{14} \text{ given by } \varphi(G^*) = 0, T^*_{14} \text{ given by } f(T^*_{14}) = 0 \text{ and } \]
\[ G^*_{13} = \frac{(a_{13})^4(G^*_{14})}{(a_{13})^4 + (a_{13})^4(T^*_{14})}, \quad G^*_{15} = \frac{(a_{15})^4(G^*_{14})}{(a_{15})^4 + (a_{15})^4(T^*_{14})} \]
\[ T^*_{13} = \frac{(b_{13})^4(T^*_{14})}{(b_{13})^4 - (b_{13})^4(G^*)}, \quad T^*_{15} = \frac{(b_{15})^4(T^*_{14})}{(b_{15})^4 - (b_{15})^4(G^*)} \]

Obviously, these values represent an equilibrium solution.

Finally we obtain the unique solution:

\[ G^*_{17} \text{ given by } \varphi(G^*_{19}) = 0, T^*_{17} \text{ given by } f(T^*_{17}) = 0 \text{ and } \]
\[ G^*_{16} = \frac{(a_{16})^2(G^*_{17})}{(a_{16})^2 + (a_{16})^2(T^*_{17})}, \quad G^*_{18} = \frac{(a_{18})^2(G^*_{17})}{(a_{18})^2 + (a_{18})^2(T^*_{17})} \]
\[ T^*_{16} = \frac{(b_{16})^2(T^*_{17})}{(b_{16})^2 - (b_{16})^2(G^*)}, \quad T^*_{18} = \frac{(b_{18})^2(T^*_{17})}{(b_{18})^2 - (b_{18})^2(G^*)} \]

Obviously, these values represent an equilibrium solution.

Finally we obtain the unique solution:

\[ G^*_{21} \text{ given by } \varphi(G^*_{23}) = 0, T^*_{21} \text{ given by } f(T^*_{21}) = 0 \text{ and } \]
\[ G^*_{20} = \frac{(a_{20})^3(G^*_{21})}{(a_{20})^3 + (a_{20})^3(T^*_{21})}, \quad G^*_{22} = \frac{(a_{22})^3(G^*_{21})}{(a_{22})^3 + (a_{22})^3(T^*_{21})} \]
\[ T^*_{20} = \frac{(b_{20})^3(T^*_{21})}{(b_{20})^3 - (b_{20})^3(G^*)}, \quad T^*_{22} = \frac{(b_{22})^3(T^*_{21})}{(b_{22})^3 - (b_{22})^3(G^*)} \]

Obviously, these values represent an equilibrium solution.

Finally we obtain the unique solution:

\[ G^*_{25} \text{ given by } \varphi(G^*_{27}) = 0, T^*_{25} \text{ given by } f(T^*_{25}) = 0 \text{ and } \]
\[ G^*_{24} = \frac{(a_{24})^4(G^*_{25})}{(a_{24})^4 + (a_{24})^4(T^*_{25})}, \quad G^*_{26} = \frac{(a_{26})^4(G^*_{25})}{(a_{26})^4 + (a_{26})^4(T^*_{25})} \]
\[ T^*_{24} = \frac{(b_{24})^4(T^*_{25})}{(b_{24})^4 - (b_{24})^4(G^*)}, \quad T^*_{26} = \frac{(b_{26})^4(T^*_{25})}{(b_{26})^4 - (b_{26})^4(G^*)} \]

Obviously, these values represent an equilibrium solution.

Finally we obtain the unique solution.
Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions \((a^i_j)^{(1)}\) and \((b^i_j)^{(2)}\) Belong to \(C^1(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of \(G_i, T_i :\)**

\[ G_i = G_i^* + \xi_i, \quad T_i = T_i^* + \eta_i \]

\[ \frac{\partial(a^i_j)^{(1)}}{\partial t} (T_{14}) = (q_{14})^{(1)} , \quad \frac{\partial(b^i_j)^{(2)}}{\partial G_j} (G^*) = \xi_j \]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[ \frac{dG_{13}}{dt} = -((a_{13}^{(1)}) + (p_{13}^{(1)})G_{13} + (a_{13}^{(1)})G_{14} - (q_{13}^{(1)})G_{13}^* T_{14} \]

\[ \frac{dG_{14}}{dt} = -((a_{14}^{(1)}) + (p_{14}^{(1)})G_{14} + (a_{14}^{(1)})G_{13} - (q_{14}^{(1)})G_{14}^* T_{14} \]

\[ \frac{dG_{15}}{dt} = -((a_{15}^{(1)}) + (p_{15}^{(1)})G_{15} + (a_{15}^{(1)})G_{14} - (q_{15}^{(1)})G_{15}^* T_{14} \]

\[ \frac{dT_{13}}{dt} = -((b_{13}^{(1)}) - (r_{13}^{(1)})T_{13} + (b_{13}^{(1)})T_{14} + \sum_{j=13}^{15}(s_{13}(j))T_{14} \xi_j \]

\[ \frac{dT_{14}}{dt} = -((b_{14}^{(1)}) - (r_{14}^{(1)})T_{14} + (b_{14}^{(1)})T_{13} + \sum_{j=13}^{15}(s_{14}(j))T_{13} \xi_j \]

\[ \frac{dT_{15}}{dt} = -((b_{15}^{(1)}) - (r_{15}^{(1)})T_{15} + (b_{15}^{(1)})T_{14} + \sum_{j=13}^{15}(s_{15}(j))T_{14} \xi_j \]

If the conditions of the previous theorem are satisfied and if the functions \((a^i_j)^{(2)}\) and \((b^i_j)^{(2)}\) Belong to \(C^2(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable

**Definition of \(G_{i}, T_{i} :\)**
\[ G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i \]

\[ \frac{\delta(a'_{ij})^{(3)}}{\delta x_i} (T_1^{*2}) = (q_{17})^{(2)} , \quad \frac{\delta(b''_{ij})^{(3)}}{\delta y_j} (G) = s_{ij} \]

Taking into account equations (global) and neglecting the terms of power 2, we obtain

\[ \frac{dG_{16}}{dt} = -((a'_{16})^{(2)} + (p_{16})^{(2)}) G_{16} + (a_{16})^{(2)} G_{17} - (q_{16})^{(2)} G_{16} T_{17} \]

\[ \frac{dG_{17}}{dt} = -((a'_{17})^{(2)} + (p_{17})^{(2)}) G_{17} + (a_{17})^{(2)} G_{16} - (q_{17})^{(2)} G_{17} T_{17} \]

\[ \frac{dG_{18}}{dt} = -((a'_{18})^{(2)} + (p_{18})^{(2)}) G_{18} + (a_{18})^{(2)} G_{17} - (q_{18})^{(2)} G_{18} T_{17} \]

\[ \frac{dT_{16}}{dt} = -((b'_{16})^{(2)} - (r_{16})^{(2)}) T_{16} + (b_{16})^{(2)} T_{17} + \sum_{j=16}^{16}(s_{(16)(j)} T_{16} G_{j}) \]

\[ \frac{dT_{17}}{dt} = -((b'_{17})^{(2)} - (r_{17})^{(2)}) T_{17} + (b_{17})^{(2)} T_{16} + \sum_{j=16}^{18}(s_{(17)(j)} T_{17} G_{j}) \]

\[ \frac{dT_{18}}{dt} = -((b'_{18})^{(2)} - (r_{18})^{(2)}) T_{18} + (b_{18})^{(2)} T_{17} + \sum_{j=16}^{18}(s_{(18)(j)} T_{18} G_{j}) \]

If the conditions of the previous theorem are satisfied and if the functions \((a''_{ij})^{(3)}\) and \((b''_{ij})^{(3)}\) Belong to \(C^{(3)}(\mathbb{R}_{+})\), then the above equilibrium point is asymptotically stabl.

\textbf{Denote}

\textbf{Definition of} \(G_i, T_i \Rightarrow \)

\[ G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i \]

\[ \frac{\delta(a''_{ij})^{(3)}}{\delta x_i} (T_1^{*2}) = (q_{21})^{(3)} , \quad \frac{\delta(b''_{ij})^{(3)}}{\delta y_j} (G) = s_{ij} \]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[ \frac{dG_{20}}{dt} = -((a'_{20})^{(3)} + (p_{20})^{(3)}) G_{20} + (a_{20})^{(3)} G_{21} - (q_{20})^{(3)} G_{20} T_{21} \]

\[ \frac{dG_{21}}{dt} = -((a'_{21})^{(3)} + (p_{21})^{(3)}) G_{21} + (a_{21})^{(3)} G_{20} - (q_{21})^{(3)} G_{21} T_{21} \]

\[ \frac{dG_{22}}{dt} = -((a'_{22})^{(3)} + (p_{22})^{(3)}) G_{22} + (a_{22})^{(3)} G_{21} - (q_{22})^{(3)} G_{22} T_{21} \]

\[ \frac{dT_{20}}{dt} = -((b'_{20})^{(3)} - (r_{20})^{(3)}) T_{20} + (b_{20})^{(3)} T_{21} + \sum_{j=20}^{22}(s_{(20)(j)} T_{20} G_{j}) \]

\[ \frac{dT_{21}}{dt} = -((b'_{21})^{(3)} - (r_{21})^{(3)}) T_{21} + (b_{21})^{(3)} T_{20} + \sum_{j=20}^{22}(s_{(21)(j)} T_{21} G_{j}) \]

\[ \frac{dT_{22}}{dt} = -((b'_{22})^{(3)} - (r_{22})^{(3)}) T_{22} + (b_{22})^{(3)} T_{21} + \sum_{j=20}^{22}(s_{(22)(j)} T_{22} G_{j}) \]

If the conditions of the previous theorem are satisfied and if the functions \((a''_{ij})^{(4)}\) and \((b''_{ij})^{(4)}\) Belong to \(C^{(4)}(\mathbb{R}_{+})\), then the above equilibrium point is asymptotically stabl.

\textbf{Denote}

\textbf{Definition of} \(G_i, T_i \Rightarrow \)

\[ G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i \]

\[ \frac{\delta(a''_{ij})^{(4)}}{\delta x_i} (T_1^{*2}) = (q_{21})^{(4)} , \quad \frac{\delta(b''_{ij})^{(4)}}{\delta y_j} (G) = s_{ij} \]
Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_24}{dt} = -((a_{24})^{(4)}(p_{24})^{(4)})G_{24} + (a_{24})^{(4)}G_{25} - (q_{24})^{(4)}G_{24}^-T_{25} \tag{1609}
\]

\[
\frac{dG_{25}}{dt} = -((a_{25})^{(4)}(p_{25})^{(4)})G_{25} + (a_{25})^{(4)}G_{24} - (q_{25})^{(4)}G_{25}^-T_{25} \tag{1610}
\]

\[
\frac{dG_{26}}{dt} = -((a_{26})^{(4)}(p_{26})^{(4)})G_{26} + (a_{26})^{(4)}G_{25} - (q_{26})^{(4)}G_{26}^-T_{25} \tag{1611}
\]

\[
\frac{dT_{24}}{dt} = -((b_{24})^{(4)}(r_{24})^{(4)})T_{24} + (b_{24})^{(4)}T_{25} + \sum_{j=24}^{26}(s_{24}(j)T_{24}G_j) \tag{1612}
\]

\[
\frac{dT_{25}}{dt} = -((b_{25})^{(4)}(r_{25})^{(4)})T_{25} + (b_{25})^{(4)}T_{24} + \sum_{j=25}^{26}(s_{25}(j)T_{25}G_j) \tag{1613}
\]

\[
\frac{dT_{26}}{dt} = -((b_{26})^{(4)}(r_{26})^{(4)})T_{26} + (b_{26})^{(4)}T_{25} + \sum_{j=26}^{26}(s_{26}(j)T_{26}G_j) \tag{1614}
\]

If the conditions of the previous theorem are satisfied and if the functions \((a_i^{''})^{(5)}\) and \((b_i^{''})^{(5)}\) Belong to \(C^{(5)}(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

Denote

**Definition of** \(G_{i}, T_{i} :\)

\[
G_i = G_i^\ast + \hat{G}_i \quad , T_i = T_i^\ast + \hat{T}_i
\]

\[
\frac{\partial (a_i^{''})^{(5)}}{\partial T_{29}}(T_{29}) = (q_{29})^{(5)} \quad , \quad \frac{\partial (b_i^{''})^{(5)}}{\partial G_j}((G_s^{\ast})^\ast) = S_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{28}}{dt} = -((a_{28})^{(5)}(p_{28})^{(5)})G_{28} + (a_{28})^{(5)}G_{29} - (q_{28})^{(5)}G_{28}^-T_{29} \tag{1615}
\]

\[
\frac{dG_{29}}{dt} = -((a_{29})^{(5)}(p_{29})^{(5)})G_{29} + (a_{29})^{(5)}G_{28} - (q_{29})^{(5)}G_{29}^-T_{29} \tag{1616}
\]

\[
\frac{dG_{30}}{dt} = -((a_{30})^{(5)}(p_{30})^{(5)})G_{30} + (a_{30})^{(5)}G_{29} - (q_{30})^{(5)}G_{30}^-T_{29} \tag{1617}
\]

\[
\frac{dT_{28}}{dt} = -((b_{28})^{(5)}(r_{28})^{(5)})T_{28} + (b_{28})^{(5)}T_{29} + \sum_{j=28}^{30}(s_{28}(j)T_{28}G_j) \tag{1618}
\]

\[
\frac{dT_{29}}{dt} = -((b_{29})^{(5)}(r_{29})^{(5)})T_{29} + (b_{29})^{(5)}T_{28} + \sum_{j=29}^{30}(s_{29}(j)T_{29}G_j) \tag{1619}
\]

\[
\frac{dT_{30}}{dt} = -((b_{30})^{(5)}(r_{30})^{(5)})T_{30} + (b_{30})^{(5)}T_{29} + \sum_{j=28}^{30}(s_{30}(j)T_{30}G_j) \tag{1620}
\]

If the conditions of the previous theorem are satisfied and if the functions \((a_i^{''})^{(6)}\) and \((b_i^{''})^{(6)}\) Belong to \(C^{(6)}(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

Denote

**Definition of** \(G_{i}, T_{i} :\)
\[ G_i = G_i' + G_i \quad , \quad T_i = T_i' + T_i \]

\[ \frac{\delta(a_{ij}^{(6)})}{\delta T_{33}} (T_{33}^{*}) = (q_{33})^{(6)} , \quad \frac{\delta(b_{ij}^{(6)})}{\delta T_{33}} (G_{33}^{*}) = s_{ij} \]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[ \frac{dG_{32}}{dt} = -((a_{32}^{(6)}) + (p_{32}^{(6)})G_{32} + (a_{32}^{(6)})G_{33} - (q_{32}^{(6)})G_{32} T_{33} \]

\[ \frac{dG_{33}}{dt} = -((a_{33}^{(6)}) + (p_{33}^{(6)})G_{33} + (a_{33}^{(6)})G_{32} - (q_{33}^{(6)})G_{33} T_{33} \]

\[ \frac{dG_{34}}{dt} = -((a_{34}^{(6)}) + (p_{34}^{(6)})G_{34} + (a_{34}^{(6)})G_{33} - (q_{33}^{(6)})G_{34} T_{33} \]

\[ \frac{dT_{32}}{dt} = -((b_{32}^{(6)}) - (r_{32}^{(6)})T_{32} + (b_{32}^{(6)})T_{33} + \sum_{j=32}^{34} (s_{32}(j)T_{32} G_{j}) \]

\[ \frac{dT_{33}}{dt} = -((b_{33}^{(6)}) - (r_{33}^{(6)})T_{33} + (b_{33}^{(6)})T_{32} + \sum_{j=32}^{34} (s_{33}(j)T_{32} G_{j}) \]

\[ \frac{dT_{34}}{dt} = -((b_{34}^{(6)}) - (r_{34}^{(6)})T_{34} + (b_{34}^{(6)})T_{33} + \sum_{j=32}^{34} (s_{34}(j)T_{33} G_{j}) \]

Obviously, these values represent an equilibrium solution of 79,20,36,22,23.

If the conditions of the previous theorem are satisfied and if the functions \( a_{ij}^{(6)}(7) \) and \( b_{ij}^{(6)}(7) \) Belong to \( C^{(7)}(\mathbb{R}_+) \) then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** \( G, T, i : \)

\[ G_i = G_i' + G_i \quad , \quad T_i = T_i' + T_i \]

\[ \frac{\delta(a_{ij}^{(7)})}{\delta T_{37}} (T_{37}^{*}) = (q_{37})^{(7)} , \quad \frac{\delta(b_{ij}^{(7)})}{\delta T_{37}} (G_{37}^{*}) = s_{ij} \]

Then taking into account equations (SOLUTIONAL) and neglecting the terms of power 2, we obtain

\[ \frac{dG_{36}}{dt} = -((a_{36}^{(7)}) + (p_{36}^{(7)})G_{36} + (a_{36}^{(7)})G_{37} - (q_{36}^{(7)})G_{36} T_{37} \]

\[ \frac{dG_{37}}{dt} = -((a_{37}^{(7)}) + (p_{37}^{(7)})G_{37} + (a_{37}^{(7)})G_{36} - (q_{37}^{(7)})G_{37} T_{37} \]

\[ \frac{dG_{38}}{dt} = -((a_{38}^{(7)}) + (p_{38}^{(7)})G_{38} + (a_{38}^{(7)})G_{37} - (q_{38}^{(7)})G_{38} T_{37} \]

\[ \frac{dT_{36}}{dt} = -((b_{36}^{(7)}) - (r_{36}^{(7)})T_{36} + (b_{36}^{(7)})T_{37} + \sum_{j=36}^{38} (s_{36}(j)T_{36} G_{j}) \]

\[ \frac{dT_{37}}{dt} = -((b_{37}^{(7)}) - (r_{37}^{(7)})T_{37} + (b_{37}^{(7)})T_{36} + \sum_{j=36}^{38} (s_{37}(j)T_{37} G_{j}) \]

\[ \frac{dT_{38}}{dt} = -((b_{38}^{(7)}) - (r_{38}^{(7)})T_{38} + (b_{38}^{(7)})T_{37} + \sum_{j=36}^{38} (s_{38}(j)T_{38} G_{j}) \]

If the conditions of the previous theorem are satisfied and if the functions \( a_{ij}^{(8)}(8) \) and \( b_{ij}^{(8)}(8) \) Belong to \( C^{(8)}(\mathbb{R}_+) \) then the above equilibrium point is asymptotically stable.
Denote

**Definition of** \( G_{\nu} T_{i} : \)

\[
G_{i} = G_{i}^{*} + G_{i}, \quad T_{i} = T_{i}^{*} + T_{i}
\]

Then taking into account equations CONCATENATED EQUATIONS and neglecting the terms of power 2, we obtain

\[
\frac{d G_{40}}{d t} = -((a'_{40})^{(8)} + (p_{40})^{(8)} + (a_{40})^{(8)} G_{41} + (q_{40})^{(8)} G_{40} T_{41})
\]

\[
\frac{d G_{41}}{d t} = -((a'_{41})^{(8)} + (p_{41})^{(8)} + (a_{41})^{(8)} G_{41} + (q_{41})^{(8)} G_{41} T_{41})
\]

\[
\frac{d G_{42}}{d t} = -((a'_{42})^{(8)} + (p_{42})^{(8)} + (a_{42})^{(8)} G_{41} + (q_{42})^{(8)} G_{41} T_{41})
\]

\[
\frac{d T_{40}}{d t} = -((b'_{40})^{(8)} + (r_{40})^{(8)} T_{40} + (b_{40})^{(8)} T_{41} + \sum_{j=40}^{42} s_{(40)(j)} T_{40} G_{j})
\]

\[
\frac{d T_{41}}{d t} = -((b'_{41})^{(8)} + (r_{41})^{(8)} T_{41} + (b_{41})^{(8)} T_{41} + \sum_{j=40}^{42} s_{(41)(j)} T_{41} G_{j})
\]

\[
\frac{d T_{42}}{d t} = -((b'_{42})^{(8)} + (r_{42})^{(8)} T_{42} + (b_{42})^{(8)} T_{42} + \sum_{j=40}^{42} s_{(42)(j)} T_{42} G_{j})
\]

The characteristic equation of this system is

\[
(\lambda)^{(1)} + (b_{13}^{(1)}) - (r_{13}^{(1)}) + (( \lambda)^{(1)} + (a_{13}^{(1)}) + (p_{13}^{(1)}))
\]

\[
( ( \lambda)^{(1)} + (a_{13}^{(1)}) + (p_{13}^{(1)}) + (q_{13}^{(1)}) G_{14} + (a_{14}^{(1)}) (q_{13}^{(1)}) G_{13}^{*} )
\]

\[
( ( \lambda)^{(1)} + (b_{13}^{(1)} + (r_{13}^{(1)}) s_{(13)(14)} T_{14} + (b_{14}^{(1)} + (r_{13}^{(1)}) s_{(13)(14)} T_{14}^{*} )
\]

\[
+ ( ( \lambda)^{(1)} + (a_{14}^{(1)}) + (p_{14}^{(1)}) (q_{13}^{(1)}) G_{13} + (a_{13}^{(1)}) (q_{14}^{(1)}) G_{14}^{*} )
\]

\[
( ( \lambda)^{(1)} + (b_{13}^{(1)} + (r_{13}^{(1)}) s_{(13)(14)} T_{14} + (b_{14}^{(1)} + (r_{13}^{(1)}) s_{(13)(13)} T_{13}^{*} )
\]

\[
( ( \lambda)^{(1)} + (a_{13}^{(1)}) + (a_{14}^{(1)}) + (p_{13}^{(1)}) + (p_{14}^{(1)}))(\lambda)^{(1)}
\]

\[
( ( \lambda)^{(1)} + (b_{13}^{(1)} + (b_{14}^{(1)} + (r_{13}^{(1)}) + (r_{14}^{(1)}))(\lambda)^{(1)}
\]

\[
+ (( ( \lambda)^{(1)} + (a_{13}^{(1)} + (a_{14}^{(1)}) + (p_{13}^{(1)}) + (p_{14}^{(1)}))(\lambda)^{(1)} + (q_{13}^{(1)}) G_{15}^{*}
\]

\[
+ ( ( \lambda)^{(1)} + (a_{13}^{(1)} + (p_{13}^{(1)})(a_{13}^{(1)}) (q_{14}^{(1)}) G_{14} + (a_{14}^{(1)})(a_{15}^{(1)})(q_{13}^{(1)}) G_{13}^{*} )
\]

\[
( ( \lambda)^{(1)} + (b_{13}^{(1)} + (r_{13}^{(1)}) s_{(13)(15)} T_{14} + (b_{14}^{(1)} + (r_{13}^{(1)}) s_{(13)(13)} T_{13}^{*} ) = 0
\]

+ \( (\lambda)^{(2)} + (b_{16}^{(2)} + (r_{18}^{(2)}))((\lambda)^{(2)} + (a_{18}^{(2)} + (p_{18}^{(2)}))
\]

\[
( (( ( \lambda)^{(2)} + (a_{16}^{(2)} + (p_{16}^{(2)}))(q_{17}^{(2)} G_{17} + (a_{17}^{(2)})(q_{16}^{(2)} G_{16}^{*})
\]

316
\[
\left( (\lambda)^{(2)} + (b_{16}^{(2)}) - (r_{16})^{(2)} \right) S_{(17),(17)}T_{17} + (b_{17}^{(2)})S_{(16),(16)}T_{16} \\
+ \left( (\lambda)^{(2)} + (a_{16}^{(2)}) + (p_{17})^{(2)} \right) G_{16} + (a_{16}^{(2)})(q_{17})^{(2)}G_{17} \\
\left( (\lambda)^{(2)} + (b_{16}^{(2)} - (r_{16})^{(2)} \right) S_{(17),(17)}T_{17} + (b_{17}^{(2)})S_{(16),(16)}T_{16} \\
\left( (\lambda)^{(2)} + (a_{16}^{(2)} + (a_{17}^{(2)} + (p_{16}^{(2)} + (p_{17})^{(2)}) \lambda^{(2)}) \\
\left( (\lambda)^{(2)} + (b_{16}^{(2)} + (b_{17}^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}) \lambda^{(2)}) \\
+ (\lambda)^{(2)} + (a_{16}^{(2)} + (a_{17}^{(2)} + (p_{16}^{(2)} + (p_{17})^{(2)}) \lambda^{(2)}) (q_{16}^{(2)}G_{18} \\
+(\lambda)^{(2)} + (a_{16}^{(2)} + (p_{16}^{(2)})) \left( (a_{18}^{(2)})(q_{17})^{(2)}G_{17} + (a_{17}^{(2)})(a_{18}^{(2)})(q_{16}^{(2)}G_{16} \\
\left( (\lambda)^{(2)} + (b_{16}^{(2)}) - (r_{16})^{(2)} \right) S_{(17),(18)}T_{17} + (b_{17}^{(2)})S_{(16),(16)}T_{16} \right) = 0 \\
+ \\
(\lambda)^{(3)} + (b_{22}^{(3)}) - (r_{22})^{(3)} \left( ((\lambda)^{(3)} + (a_{22}^{(3)} + (p_{22})^{(3)}) \\
\left( ((\lambda)^{(3)} + (a_{20}^{(3)} + (p_{20})^{(3)})(q_{21})^{(3)}G_{21} + (a_{21})^{(3)}(q_{20})^{(3)}G_{20} \right) \\
\left( ((\lambda)^{(3)} + (b_{20}^{(3)} - (r_{20})^{(3)} \right) S_{(21),(21)}T_{21} + (b_{21})^{(3)}S_{(20),(20)}T_{20} \\
+ \left( ((\lambda)^{(3)} + (a_{21}^{(3)} + (p_{21})^{(3)} \right) (q_{20})^{(3)}G_{20} + (a_{20}^{(3)}(q_{21})^{(3)}G_{21} \\
\left( ((\lambda)^{(3)} + (b_{20}^{(3)} - (r_{20})^{(3)} \right) S_{(21),(20)}T_{21} + (b_{21})^{(3)}S_{(20),(20)}T_{20} \\
\left( (\lambda)^{(3)} + (a_{21}^{(3)} + (p_{21})^{(3)} \right) (a_{21}^{(3)} + (p_{21})^{(3)} \lambda^{(3)}) \\
\left( (\lambda)^{(3)} + (b_{20}^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)}) \lambda^{(3)} \right) \\
+ \left( ((\lambda)^{(3)} + (a_{20}^{(3)} + (a_{21}^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}) \lambda^{(3)} \right) (q_{22})^{(3)}G_{22} \\
+((\lambda)^{(3)} + (a_{20}^{(3)} + (p_{20})^{(3)})(a_{21}^{(3)} + (p_{21}^{(3)} \lambda^{(3)}) (q_{21})^{(3)}G_{21} + (a_{21}^{(3)}(a_{21}^{(3)}(q_{20})^{(3)}G_{20} \\
\left( ((\lambda)^{(3)} + (b_{20}^{(3)} - (r_{20})^{(3)} \right) S_{(21),(22)}T_{21} + (b_{21})^{(3)}S_{(20),(22)}T_{20} \right) = 0 \\
+ \\
(\lambda)^{(4)} + (b_{26}^{(4)}) - (r_{26})^{(4)} \left( ((\lambda)^{(4)} + (a_{26}^{(4)} + (p_{26})^{(4)}) \\
\left( ((\lambda)^{(4)} + (a_{24}^{(4)} + (p_{24})^{(4)})(q_{25})^{(4)}G_{25} + (a_{25})^{(4)}(q_{24})^{(4)}G_{24} \right) \\
\left( (\lambda)^{(4)} + (b_{24}^{(4)} - (r_{24})^{(4)} \right) S_{(25),(25)}T_{25} + (b_{25})^{(4)}S_{(24),(25)}T_{25} \\
+ \left( ((\lambda)^{(4)} + (a_{25}^{(4)} + (p_{25})^{(4)})(q_{24})^{(4)}G_{24} + (a_{24}^{(4)}(q_{25})^{(4)}G_{25} \right) \\
317
\[
\begin{align*}
&\left(\left((\lambda)^{(4)} + (b_{24}^{(4)} - (r_{24})^{(4)})s_{(25),(24)}T_{25}^{*} + (b_{25})^{(4)}s_{(24),(24)}T_{24}^{*}\right)\right)^{2} + \left((a_{24}^{(4)} + (a_{25}^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)})\right)(\lambda)^{(4)}
\end{align*}
\]

\[
\begin{align*}
&\left(\left((\lambda)^{(4)} + (b_{24}^{(4)} + (b_{25}^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)})\right)(\lambda)^{(4)}
\end{align*}
\]

\[
\begin{align*}
+ \left(\left((\lambda)^{(4)} + (a_{24}^{(4)} + (a_{25}^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)})\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}g_{26}
\end{align*}
\]

\[
\begin{align*}
+ \left((\lambda)^{(4)} + (a_{24}^{(4)} + (p_{24})^{(4)}\right)\left((a_{26}^{(4)}q_{26})^{(4)}g_{26}^{*} + (a_{26}^{(4)}a_{26}^{(4)}q_{24})^{(4)}g_{24}^{*}\right)
\end{align*}
\]

\[
\begin{align*}
\left(\left((\lambda)^{(4)} + (b_{24}^{(4)} - (r_{24})^{(4)})s_{(25),(24)}T_{25}^{*} + (b_{25})^{(4)}s_{(24),(24)}T_{24}^{*}\right)\right) = 0
\end{align*}
\]

\[
\begin{align*}
+ \left((\lambda)^{(5)} + (b_{30})^{(5)} - (r_{30})^{(5)}\right)\left((\lambda)^{(5)} + (a_{30})^{(5)} + (p_{30})^{(5)}\right)
\end{align*}
\]

\[
\begin{align*}
\left(\left((\lambda)^{(5)} + (a_{28}^{(5)} + (p_{28})^{(5)})(q_{29})^{(5)}g_{29}^{*} + (a_{28}^{(5)}q_{28})^{(5)}g_{28}^{*}\right)\right)
\end{align*}
\]

\[
\begin{align*}
\left(\left((\lambda)^{(5)} + (b_{28}^{(5)} - (r_{28})^{(5)}s_{(29),(28)}T_{29}^{*} + (b_{29})^{(5)}s_{(28),(28)}T_{29}^{*}\right)\right)
\end{align*}
\]

\[
\begin{align*}
\left(\left((\lambda)^{(5)} + (a_{29}^{(5)} + (p_{29})^{(5)})(q_{28})^{(5)}g_{28}^{*} + (a_{29}^{(5)}q_{29})^{(5)}g_{29}^{*}\right)\right)
\end{align*}
\]

\[
\begin{align*}
\left(\left((\lambda)^{(5)} + (b_{29}^{(5)} - (r_{29})^{(5)}s_{(29),(29)}T_{29}^{*} + (b_{29})^{(5)}s_{(28),(28)}T_{29}^{*}\right)\right)
\end{align*}
\]

\[
\begin{align*}
\left(\left((\lambda)^{(5)} + (a_{28}^{(5)} + (a_{29}^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}\right)(\lambda)^{(5)}\right)
\end{align*}
\]

\[
\begin{align*}
\left(\left((\lambda)^{(5)} + (b_{28}^{(5)} + (b_{29}^{(5)} - (r_{28})^{(5)} + (r_{29})^{(5)}\right)(\lambda)^{(5)}\right)
\end{align*}
\]

\[
\begin{align*}
+ \left((\lambda)^{(5)} + (a_{28}^{(5)} + (a_{29}^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}g_{30}
\end{align*}
\]

\[
\begin{align*}
+ \left((\lambda)^{(5)} + (a_{28}^{(5)} + (p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}g_{29}^{*} + (a_{28}^{(5)}q_{28})^{(5)}g_{28}^{*}\right)
\end{align*}
\]

\[
\begin{align*}
\left(\left((\lambda)^{(5)} + (b_{28}^{(5)} - (r_{28})^{(5)}s_{(29),(30)}T_{29}^{*} + (b_{29})^{(5)}s_{(28),(30)}T_{29}^{*}\right)\right) = 0
\end{align*}
\]

\[
\begin{align*}
+ \left((\lambda)^{(6)} + (b_{34}^{(6)} - (r_{34})^{(6)}\right)\left((\lambda)^{(6)} + (a_{34})^{(6)} + (p_{34})^{(6)}\right)
\end{align*}
\]

\[
\begin{align*}
\left(\left((\lambda)^{(6)} + (a_{32})^{(6)} + (p_{32})^{(6)}(q_{33})^{(6)}g_{33}^{*} + (a_{33})^{(6)}(q_{32})^{(6)}g_{32}^{*}\right)\right)
\end{align*}
\]

\[
\begin{align*}
\left(\left((\lambda)^{(6)} + (b_{32}^{(6)} - (r_{32})^{(6)}s_{(33),(33)}T_{33}^{*} + (b_{33})^{(6)}s_{(32),(33)}T_{33}^{*}\right)\right)
\end{align*}
\]

\[
\begin{align*}
+ \left((\lambda)^{(6)} + (a_{32})^{(6)} + (p_{32})^{(6)}(q_{33})^{(6)}g_{33}^{*} + (a_{32})^{(6)}(q_{32})^{(6)}g_{32}^{*}\right)
\end{align*}
\]

\[
\begin{align*}
\left(\left((\lambda)^{(6)} + (b_{32}^{(6)} - (r_{32})^{(6)}s_{(33),(32)}T_{33}^{*} + (b_{33})^{(6)}s_{(32),(32)}T_{33}^{*}\right)\right)
\end{align*}
\]

\[
\begin{align*}
\left(\left((\lambda)^{(6)} + (a_{32})^{(6)} + (a_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)}\right)(\lambda)^{(6)}\right)
\end{align*}
\]
\[
\begin{align*}
&\left[\left((\lambda)^{(6)}\right)^2 + \left((b_{32}^6)^{(6)} + (b_{33}^6)^{(6)} - (r_{32})^{(6)} + (r_{33})^{(6)})(\lambda)\right)^{(6)}\right] \\
&+ \left[\left((\lambda)^{(6)}\right)^2 + \left((a_{32}^6)^{(6)} + (a_{33}^6)^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)})(\lambda)\right)\right][(q_{34})^{(6)}G_{34}^{(6)}] \\
&+ \left((\lambda)^{(6)} + (a_{33}^6) + (p_{32})^{(6)}\right)((a_{34})^{(6)}(q_{33})^{(6)}G_{34}^{(6)} + (a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G_{32}^{(6)}) \\
&- \left((\lambda)^{(6)} + (b_{32}^6)^{(6)} - (r_{32})^{(6)}\right)S_{(33),(34)}T_{33}^{*} + (b_{33})^{(6)}S_{(32),(34)}T_{32}^{*}) = 0
\end{align*}
\]

+ 

\[
\begin{align*}
&\left((\lambda)^{(7)} + (b_{38}^7)^{(7)} - (r_{38})^{(7)}\right)[(\lambda)^{(7)} + (a_{38}^7) + (p_{38})^{(7)}] \\
&\left[\left((\lambda)^{(7)} + (a_{36}^7)^{(7)} + (p_{36})^{(7)}(q_{37})^{(7)}G_{37}^{(7)} + (a_{37})^{(7)}(q_{36})^{(7)}G_{36}^{(7)}\right)\right] \\
&\left((\lambda)^{(7)} + (b_{36}^7)^{(7)} - (r_{36})^{(7)}\right)S_{(37),(37)}T_{37}^{*} + (b_{37})^{(7)}S_{(36),(37)}T_{37}^{*} \\
&+ \left((\lambda)^{(7)} + (a_{37}^7) + (p_{37})^{(7)}(q_{36})^{(7)}G_{36}^{(7)} + (a_{36})^{(7)}(q_{37})^{(7)}G_{37}^{(7)}\right) \\
&\left((\lambda)^{(7)} + (b_{36}^7)^{(7)} - (r_{36})^{(7)}\right)S_{(37),(36)}T_{37}^{*} + (b_{37})^{(7)}S_{(36),(36)}T_{36}^{*} \\
&\left((\lambda)^{(7)} + (a_{37}^7) + (p_{37})^{(7)}(q_{36})^{(7)}G_{36}^{(7)} + (a_{36})^{(7)}(q_{37})^{(7)}G_{37}^{(7)}\right) \\
&\left((\lambda)^{(7)} + (b_{36}^7)^{(7)} - (r_{36})^{(7)}\right)(\lambda)^{(7)} \\
&\left((\lambda)^{(7)} + (b_{36}^7)^{(7)} + (b_{37}^7)^{(7)} - (r_{36})^{(7)} + (r_{37})^{(7)}\right)(\lambda)^{(7)} \\
&+ \left((\lambda)^{(7)} + (a_{36}^7) + (a_{37}^7) + (p_{36})^{(7)} + (p_{37})^{(7)}\right)(\lambda)^{(7)} \left(q_{36}^{(7)}G_{38}\right) \\
&+ \left((\lambda)^{(7)} + (a_{36}^7) + (p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G_{37}^{(7)} + (a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G_{36}^{(7)}\right) \\
&\left((\lambda)^{(7)} + (b_{38}^7)^{(7)} - (r_{38})^{(7)}\right)S_{(37),(38)}T_{37}^{*} + (b_{37})^{(7)}S_{(36),(38)}T_{38}^{*} = 0
\end{align*}
\]

Acknowledgments:

The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Journal Reviews, Nature ‘s Letters, Article Abstracts, Research papers, Abstracts Of Research Papers, Stanford Encyclopedia, Web Pages, Ask a Physicist Column, deliberations with Professors, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, we regret with great deal of compunction, contrition, regret , trepidation and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive.

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22. For an English source, see Haar, T. The Old Quantum Theory.


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NJ: Analytic Press.


55. Cockcroft-Walton experiment

56. Conversions used: 1956 International (Steam) Table (IT) values where one calorie = 4.1868 J and one BTU = 1055.05585262 J. Weapons designers’ conversion value of one gram TNT = 1000 calories used.

57. Assuming the dam is generating at its peak capacity of 6,809 MW.

58. Assuming a 90/10 alloy of Pt/Ir by weight, a \( C_p \) of 25.9 for Pt and 25.1 for Ir, a Pt-dominated average \( C_p \) of 25.8, 5.134 moles of metal, and 132 J.K\(^{-1}\) for the prototype. A variation of ±1.5 picograms is of course, much smaller than the actual uncertainty in the mass of the international prototype, which are ±2 micrograms.

59. Article on Earth rotation energy. Divided by \( c^2 \).

60. Earth's gravitational self-energy is \( 4.6 \times 10^{-10} \) that of Earth's total mass, or 2.7 trillion metric tons. Citation: The Apache Point Observatory Lunar Laser-Ranging Operation (APOLLO), T. W. Murphy, Jr. et al. University of Washington, Dept. of Physics.

61. There is usually more than one possible way to define a field energy, because any field can be made to couple to gravity in many different ways. By general scaling arguments, the correct answer at everyday distances, which are long compared to the quantum gravity scale, should be minimal coupling, which means that no powers of the curvature tensor appear. Any non-minimal couplings, along with other higher order terms, are presumably only determined by a theory of quantum gravity, and within string theory, they only start to contribute to experiments at the string scale.


70. Eriksen, Erik; Vøyenli, Kjell (1976), "The classical and relativistic concepts of mass", Foundations of Physics (Springer) 6: 115–124, Bibcode 1976FoPh...6..115E, DOI:10.1007/BF00708670