The Analytical Solution and Numerical Simulation for Ytterbium-Doped Silica Glass Fiber Laser Output Power

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Abstract
In this paper as the first rate equations in end pump fiber laser have been solved analytically with negligible the scattering loss and the output power versus input power has been derived. The result were applied for a single and double clad Yb³⁺-doped silica glass fiber laser, for lasing transition it acts as a quasi-four-level system the effect of the, type concentration core radius fiber length output reflectivity, pump power and figure of inner cladding on the output lasing power have been studied.

Keywords: Rare-Earth; Fiber Laser; Rate Equation; Quasi four levels.

1. Introduction
Fiber lasers have many advantages such as high conversion efficiency immunity from thermal lasing effect, simplicity of optical construction and excellent beam quality. Ytterbium doping is attractive for high power fiber laser because of its high efficiency and strong pump absorption. Yb³⁺-doped silica glass fiber exhibit very broad absorption and emission band, from (800) nm to (1064) nm for absorption and (970) nm to (1200) nm for emission [1,2]. The simplicity of the level structure provides freedom unwanted processes such as excited state absorption, multi-phonon nonradioactive decay, and concentration quenching [3,4]. However, pumping doped silica fiber with high concentrations can result in excess loss at the pump and lasing wavelengths owing to photo darkening, which can significantly reduce the overall conversion efficiency and degrade the long-term performance [5]. For high power applications, Single Clad Fiber (SCF) is not suitable because of the very low injection efficiency of large stripe laser diodes. Therefore, Double Clad Fiber (DCF) was design with an attractive medium to gain high power high brightness and broad wavelength tuning. Development of high power laser diode source with advance in design and fabrication of (DCF) have successfully demonstrated kilowatt level fiber lasers and amplifiers pump by laser diode [6].

In this paper, as the first, we analytically solved the rate equations in single end-pumped Yb³⁺-doped silica fiber laser so; we improved the analytical rate equation solutions and found the output laser power versus input parameters. The results were applied of Yb³⁺-doped silica (SCF) and (DCF) for lasing transitions it acts as a quasi-four-level system.

2. Theory Models
2.1 Energy Level Structure
Fig (1) shows the energy level of (Yb³⁺) ion in silica fiber. (Yb³⁺) processes a simple atomic structure with only two principal manifolds i.e. ground state (²F⁷/₂) and excited state (²F⁵/₂) separated by ~10000 cm⁻¹, which makes it an ideal rare-earth element for lasing [7]. Therefore sublevels of upper (²F⁵/₂) are labeled as (e, f, g) and the four sublevels of lower (²F⁷/₂) manifold are labeled as (a, b, c, d). Weak multi-phonon decay is practically the only nonradioactive channel that exists. The excited state has a lifetime of ~ (1X 10⁻⁸ sec) and acts as metastable level. The absence of higher energy levels near the upper manifold reduces the occurrence of multi-photon relaxation and excited state absorption (EAS) [7].
2.2 Cavity design
Our linear cavity is composed of Yb$^{3+}$-doped silica glass of length (L) and with reflectivity’s of $R_1$ and $R_2$ at the lasing wavelength Fig (2) shows schematic illustration of laser oscillator [8].

![Cavity Schematic](image)

Fig. (2): Schematic of End Pumped Linear Cavity Fiber laser.

2.3 Rate Equations
The relative population of (Yb$^{3+}$)ions in upper and lower energy levels are governed by the local rate equation [9, 10].

\[
\frac{dN_1}{dt} = -(R_{12} + W_{12})N_1 + (R_{21} + W_{21} + A_{21})N_2
\]

(1)

\[
\frac{dN_2}{dt} = (R_{12} + W_{12})N_1 - (R_{21} + W_{21} + R_{22})N_2
\]

(2)

Where (N) is the total number of ions per unit volume, (N$_i$) is the number of ions per unit volume in level (i), $R$ is the pump transition, (W) is the lasing transition and (A$_{21}$) the spontaneous emission transition rate coefficient. The pump and seed transition rate are governed by absorption and emission cross section for ion in the host medium and can be written as [8, 10].

\[
R_{12} = \frac{\sigma_{AP}P}{\hbar v_{pAP}}
\]

(3)

\[
R_{21} = \frac{\sigma_{LP}P}{\hbar v_{LAP}}
\]

(4)

\[
W_{12} = \frac{\sigma_{AL}P}{\hbar v_{LAP}}
\]

(5)

\[
W_{21} = \frac{\sigma_{EL}P}{\hbar v_{LAP}}
\]

(6)

The pump cross-sections ($\sigma_P = \sigma_{AP} + \sigma_{LP}$), $v_p$ is the pump frequency ($v_p = c/\lambda_p$) and lasing cross-sections ($\sigma_L = \sigma_{AL} + \sigma_{EL}$), $v_L$ is the lasing frequency ($v_L = c/\lambda_L$), the spontaneous emission rate write as following:-

\[
A_{21} = \frac{1}{\tau}
\]

(7)

Where (\tau) is the lifetime of Yb$^{3+}$ion in excited state, ($P_p$), ($P_L$) are the pump and lasing power respectively [8].
The power filling factor for pump ($\Gamma_p$) is the ratio of core area and cladding area $\Gamma_p = \frac{A_{core}}{A_{clad}}$, where $(A_{eff} = \pi a^2)$ in Multi-Mode Fiber and $(A_{eff} = \pi w_p^2)$ in Single Mode Fiber ($w_p$) is the mode field radius, for fundamental mode defined by [11]

$$w_p = a[0.761 + \frac{1.237}{V_t^2} + \frac{1.429}{V_t^3}]$$

Where ($V$) is normalized frequency at pump wavelength ($V = 2\pi aN_d/\lambda_p$), $N_d$ is the numerical aperture and ($A_{clad} = \pi b^2$) where (a) and (b) is the radius of core and cladding respectively [12]

And the power filling factor for lasing $[\Gamma_L = 1+\exp(-2(r/W_L)^2)]$, Where ($r$) shows the radius of doped area, ($W_L$) is the mode field radius, for a fundamental mode at ($r=a$) is defined by [12]

$$w_L = a[0.616 + \frac{1.660}{V_t^2} + \frac{0.987}{V_t^3}]$$

Where $U$ is the normalized frequency at lasing wavelength ($U = \frac{2\pi N_d}{\lambda_L}$).

By applying the energy conservation law $N_1 + N_2 = N$ and under steady state condition

$$\frac{dN_1}{dt} = 0, (i = 1, 2)$$

The eqs (1 and 2) became as

$$N_1 = \frac{\frac{\alpha_p \sigma_p F_p \sigma_{al} F_{N_2} L + \frac{\partial \Gamma_L}{\partial v}}{\Gamma_{P} \frac{\sigma_{al} F_{N_2} L}{\sigma_{ap} F_{N_1} L} + \frac{\partial \Gamma_L}{\partial v}}}{\frac{\Gamma_{P} \frac{\sigma_{al} F_{N_2} L}{\sigma_{ap} F_{N_1} L} + \frac{\partial \Gamma_L}{\partial v}}}{N_2}$$

(10)

$$N_2 = \frac{\frac{\alpha_p \sigma_p F_p \sigma_{al} F_{N_2} L + \frac{\partial \Gamma_L}{\partial v}}{\Gamma_{P} \frac{\sigma_{al} F_{N_2} L}{\sigma_{ap} F_{N_1} L} + \frac{\partial \Gamma_L}{\partial v}}}{\frac{\Gamma_{P} \frac{\sigma_{al} F_{N_2} L}{\sigma_{ap} F_{N_1} L} + \frac{\partial \Gamma_L}{\partial v}}}{N_2}$$

(11)

2.4 Propagation Equation

A linear cavity for Yb$^{3+}$ doped fiber laser can be modeled from the 2-level system equations described by applying the boundary condition for resonator cavity and by setting the net round trip gain to unity. The boundary condition for forward $P_L^+(0)$ and back-word $P_L^-(L)$ propagation laser power respectively see fig. (2) Are [8].

$$P_L^+(0) = R_1 P_L^-(0)$$

(12)

$$P_L^-(L) = R_2 P_L^+(L)$$

(13)

The difference of pump and laser power and small signal gain coefficient along the fiber length ($N_1 = N - N_2$) and negligible the scattering loss are given by propagation equation [8, 13].

$$\frac{dP_L(z)}{dz} = \Gamma_L \left\{ \sigma_{al} + \sigma_{ap} \right\} N_2 - \sigma_{ap} N \frac{dP_L(z)}{dz}$$

(14)

$$g(z) = \Gamma_L \left\{ \sigma_{al} + \sigma_{ap} \right\} N_2 - \left( \gamma - 1 \right) \sigma_{al} N$$

(16)

The term ($\gamma$) is given by [13].

$$\gamma = \frac{1 + f_t/\Gamma_L}{\Gamma_L}$$

(17)

Where $f_t$ and $f_u$ are the thermal Boltzmann factor in lower and upper laser levels respectively, given by [14, 15].

$$f_t = \frac{\exp[-E_t/kT]}{\sum \exp[-E_i/kT]}$$

(18)

$$f_u = \frac{\exp[-E_u/kT]}{\sum \exp[-E_i/kT]}$$

(19)

When ($\gamma = 1$) the laser system is a true four-level, ($\gamma = 2$) is a true three-level, ($\gamma < 1.5$) a quasi four-level, and ($\gamma > 1.5$) a quasi-three-level since ($KT=207cm^{-1}$) where (T) is the temperature of host material in ($K^o$) is Boltzmann constant [13].

2.5 Pump Power

The gain by pumping is given by integrated the eq. (13) along the fiber length (L)

$$G_p = \ln \frac{P_L^+(L)}{P_L^+(0)} = \Gamma_p \left\{ \sigma_{ap} + \sigma_{ap} \right\} \int_0^L N_2(z) dz - \Gamma_p \sigma_{ap} N L$$

(20)

And the gain by laser is obtained by integration small signal gain coefficient $g(z)$

In equation (15) a long fiber length (L)

$$G_L = \int_0^L g(z) dz = \int_{P_L^+(0)}^{P_L^+(L)} \frac{dP_L(z)}{dz}$$

(21)

The stationary condition for the linear cavity fiber laser is given by

$$R_1 R_2 \exp(2G_L) = 1$$

$$G_L = \ln \frac{1}{\sqrt{R_1 R_2}}$$

(22)
The difference relationship of pump power over the fiber length can be written as:

\[ P_p(L) = P_p(0) \exp \left[ \frac{\gamma L}{\gamma_t L_{\text{NL}}} \ln \frac{1}{\sqrt{R_1 R_2}} - \frac{\sigma_p \sigma_{\text{el}} - \Sigma \sigma_{\text{el}} \gamma_{\text{el}}}{\sigma_L} (\sigma_p \sigma_{\text{el}} + \sigma_{\text{el}} \gamma_{\text{el}}) \Gamma_p \right] \]  

(23)

2.6 Lasing Power \( P_L(L) \)

We combine equations (11 and 12) to obtain as:

\[ \frac{dP_L}{dz} + \frac{1}{v_p} \frac{dP_L}{dt} + \frac{h v_p \Delta \text{eff}}{t} N_2 = 0 \]  

(24)

The equation may be integrated a long fiber length (L) to obtain as:

\[ P_L(L) - P_L(0) + \frac{v_p}{v_p} [P_p(L) - P_p(0)] + \frac{h v_p \Delta \text{eff}}{t} \int_0^L N_2(z) \, dz = 0 \]  

(25)

Which is used \( P_p(L) = P_p(0) \exp G_p \) and

\[ \int_0^L N_2(z) \, dz = \frac{1}{\Gamma_{\text{eff}}} \left( G_p + \Gamma_p \sigma_{\text{ap}} \right) \]  

(26)

Then, equation (24) can be rewritten as:

\[ P_L(L) - P_L(0) = \frac{v_p}{v_p} P_p(0) \left[ 1 - \exp G_p \right] - (\Gamma_p \sigma_{\text{ap}} \right) \]  

(28)

2.7 Out Lasing Power \( P_{\text{out}} \)

The output power of the lasing in Fig. (2) can be obtained as:

\[ P_{\text{out}} = (1 - R_2) P_{\text{sat}}^+ (L) \]  

(29)

When \( G_L = \frac{P_L(L)}{P_L(0)} = \frac{1}{\sqrt{R_1 R_2}} \) then

\[ P_{\text{sat}}^+ (0) = P_{\text{sat}}^+ (L) \sqrt{R_1 R_2} \]  

(30)

\[ P_L(L) - P_L(0) = P_{\text{sat}}^+ (L) \left[ 1 + R_2 - \sqrt{R_1 R_2} - \frac{\sqrt{R_1}}{R_2} \right] \]  

(31)

Which are used the equations. (11 and 12 and 29) to obtain as:

\[ P_{\text{sat}}^+ = \frac{\lambda_p}{\lambda_{\text{L}} + 1 + R_2 - \sqrt{R_1 R_2} - \frac{\sqrt{R_1}}{R_2}} \left( 1 - \exp G_p \right) \left( P_p(0) - P_{\text{sat}}^+ \right) \]  

(32)

When \( P_{\text{out}} = 0 \), the laser threshold power obtained as

\[ P_{\text{th}} = \frac{P_{\text{sat}}^+ \sigma_{\text{ap}} \Gamma_p}{(1 - \exp G_p)} \]  

(33)

And we combine equations (31) from \( P_{\text{out}} = \eta(P_{\text{abs}} - P_{\text{th}}) \) efficiency (\( \eta \)) is obtained as

\[ P_{\text{abs}} = P_p(0) \]  

(35)

3. Result and discussion

Firstly, in this research, we calculated (\( \gamma \)) by the equation (16) when the pumping of the Yb\(^{3+}\) doped silica fiber laser with a wavelength (\( \lambda_p = 920 \text{nm} \)), where through the parameter value (\( \gamma \)) we can determine the type of the pumping plan for the laser emission (\( \lambda_{\text{L}} \)). The table (3-1) shows parameter values (\( \gamma \)) in each laser wavelength emitted at the above (\( \lambda_p \)).

Table (3-1): Lasing wavelength \( (\lambda_{\text{L}}) \) and transition factor at pumping wavelength \( (\lambda_p = 920 \text{nm}) \).

<table>
<thead>
<tr>
<th>Transition</th>
<th>Lasing Wavelength ( \lambda_{\text{L}} \times 10^{-9} ) (m)</th>
<th>( \gamma )-Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>e → a</td>
<td>975</td>
<td>1.9694</td>
</tr>
<tr>
<td>e → b</td>
<td>1035</td>
<td>1.0534</td>
</tr>
<tr>
<td>e → c</td>
<td>1090</td>
<td>1.0058</td>
</tr>
<tr>
<td>e → d</td>
<td>1140</td>
<td>1.0007</td>
</tr>
</tbody>
</table>

We relied in the search on \( \lambda_{\text{L}} = 109 \text{nm} \) as a typical example of a quasi-four level pumping plan so table (3-2) shows all transactions with both \( \lambda_p \) and \( \lambda_{\text{L}} \) [8, 12].
Table (3-2): Default values for used parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_p$</td>
<td>$920 \times 10^{-9}$</td>
<td>m</td>
</tr>
<tr>
<td>$\alpha_{\text{eff}}$</td>
<td>$6\times10^{-25}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$\alpha_{\text{clad}}$</td>
<td>$0.25 \times 10^{-25}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$\alpha_{\text{clad}}$</td>
<td>$1090\times10^{-9}$</td>
<td>m</td>
</tr>
<tr>
<td>$\alpha_{\text{clad}}$</td>
<td>$0.014 \times 10^{-25}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$\alpha_{\text{clad}}$</td>
<td>$2 \times 10^{-25}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$T_o$</td>
<td>$1 \times 10^{-3}$</td>
<td>sec</td>
</tr>
</tbody>
</table>

In the event that the optic fiber from the Single-Clad Fiber (SCF) type, the laser design that was used in this research is (Lycom) specifically, since all of this design transactions that were used in the numerical simulation through Matlab program (8.1) to calculate (Pout) shown in table (3-3).where (*) in this table means that the parameter is calculated in the present study.

Table (3-3): Default values for used parameters (Lycom design).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$2.5 \times 10^{-6}$</td>
<td>m</td>
</tr>
<tr>
<td>$b$</td>
<td>$62.5 \times 10^{-6}$</td>
<td>m</td>
</tr>
<tr>
<td>$N_A$</td>
<td>0.15</td>
<td>-</td>
</tr>
<tr>
<td>$R_1$</td>
<td>0.998</td>
<td>-</td>
</tr>
<tr>
<td>$R_2$</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>$L$</td>
<td>$(4 – 100) \times 10^{-2}$</td>
<td>m</td>
</tr>
<tr>
<td>$P_F(o)$</td>
<td>$(2 – 150)$</td>
<td>w</td>
</tr>
<tr>
<td>$V(*)$</td>
<td>2.561</td>
<td>-</td>
</tr>
<tr>
<td>$M(*)$</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>$A_{\text{eff}}(*)$</td>
<td>$1.964 \times 10^{-11}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A_{\text{clad}}(*)$</td>
<td>$1.227 \times 10^{-8}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$\Gamma_p(*)$</td>
<td>0.0016</td>
<td>-</td>
</tr>
<tr>
<td>$\Gamma(*)$</td>
<td>0.78</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure (3) shows how (Pout) calculated by equation (31) increases linearly with the increase of pumping power $P_F(o)$, and note that both of the laser output (Pout) and efficiency ($\eta$) would be higher in case the concentration Is of (CorActive) type of this laser design, as for Figure (4), it explains how the (Pout) linearly decreases with the increases in the radius of the core of the optical fiber (a), while Figures (5), (6) and (7) show how (Pout) increases linearly with the increase in the length of the optical fiber (L), and the reflectivity of the output laser mirror (R$_2$) and the pumping power $P_F(o)$ respectively, and we also note in the latter form that all of (Pout) and (\eta) would be higher if the SCF is of the (Single Mode) type.

While, In the case that the optic fiber is of the Double – Clad Fiber (DCF) type and for a laser design of the type of Large Mode Area (LMA), where all transactions with this design shown in table (3-4) we note in Figure (8) how (Pout) increases linearly with the increase of $P_F(o)$, whether the form of the inner cladding is a rectangle or square or circle. We also find that both of (Pout) and (\eta) would be higher for the rectangular shape because ($A_{\text{clad}}$) of the rectangle is less than they are in the other shape.

Table (3-4): large mode area design.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>$4\times10^{23}$</td>
<td>ion/m$^3$</td>
</tr>
<tr>
<td>$N_A$</td>
<td>0.05</td>
<td>-</td>
</tr>
<tr>
<td>$T$</td>
<td>$1\times10^{-3}$</td>
<td>sec</td>
</tr>
<tr>
<td>$R_1$</td>
<td>0.988</td>
<td>-</td>
</tr>
<tr>
<td>$a$</td>
<td>$10\times10^{-6}$</td>
<td>m</td>
</tr>
<tr>
<td>$b$</td>
<td>$200\times10^{-6}$</td>
<td>m</td>
</tr>
<tr>
<td>$d$</td>
<td>$400\times10^{-6}$</td>
<td>m</td>
</tr>
</tbody>
</table>
Fig. (3) The laser output Power Vs pump power for SCF.

Fig. (4) The laser output Power Vs Core radius for SCF.

Fig. (5) The laser output Power Vs Fiber Length for SCF.

Fig. (6) The laser output Power Vs Output Reflectivity for SCF.

Fig. (7) The laser output Power Vs Pump Power for SCF.

Fig. (8) The laser output Power Vs Pump power for (DCF).
4. Conclusions

In this research the equivalent of the laser output and the calculation of this power was derived through numerical simulation (Yb3+dope silica fiber laser) at (\(\lambda_P=920\) nm) and (\(\lambda_L=1090\) nm) and for two types of the optical fiber, in the case that the optic fiber is (SCF), and the design of the laser is (Lycom ) type. We found that the concentration of the (CorActive) type is the best for the purpose of obtaining laser efficiency and output power, and (Pout) decreases with the increases in the radius of the core of the optical fiber while increases with each of the length of optical fiber, reflectively of the output laser mirror and pumping power, and that both the efficiency and the output laser power would be better in the case the (SCF) is of the (Single Mode) type. While, In case the optic fiber is (DCF) and the design of the laser is of the (LMA) type, we found that (Pout) increases with the increase in the pumping power, whether the form of the inner cladding was a rectangle or square or circle, and that both of the efficiency and the laser output would be better in the rectangular shape.

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