QUANTUM GRAVITY - THE EL DORADO – NAY A NE PLUS ULTRA --THE FINAL FINALE

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ABSTRACT: Motivation for quantizing gravity comes from the remarkable success of the quantum theories of the other three fundamental interactions, and from experimental evidence suggesting that gravity can be made to show quantum effects. Although some quantum gravity theories such as string theory and other unified field theories (or 'theories of everything') attempt to unify gravity with the other fundamental forces, others such as loop quantum gravity make no such attempt; they simply quantize the gravitational field while keeping it separate from the other forces. Observed physical phenomena can be described well by quantum mechanics or general relativity, without needing both. This can be thought of as due to an extreme separation of mass scales at which they are important. Quantum effects are usually important only for the "very small", that is, for objects no larger than typical molecules. General relativistic effects, on the other hand, show up mainly for the "very large" bodies such as collapsed stars. (Planets’ gravitational fields, as of 2011, are well-described by linearised except for Mercury's perihelion precession; so strong-field effects—any effects of gravity beyond lowest nonvanishing order in \( \phi /c^2 \)—have not been observed even in the gravitational fields of planets and main sequence stars). There is a lack of experimental evidence relating to quantum gravity, and classical physics adequately describes the observed effects of gravity over a range of 50 orders of magnitude of mass, i.e., for masses of objects from about 10⁻²³ to 10³⁰ kg. We present a complete Model which probably explains the positivities and discrepancies and inadequacies of each model. Physics is certainly moving in to the subterranean realm and ceratoid dualism of consciousness and subject object duality (Freud vouchsafed only at the mother's breast shall the subject and object shall be one), like a maverick trying to transcend the boundaries of space time, standing on the threshold of infinity trying to ponder what lies beyond the veil which separates the scene from unseen?
INTRODUCTION:

The following figurative representation is explains in best possible words the model that is proposed. A consummate model encompassing all the theories is presented. The theories are there to be applied to various physical systems which have different parametric representationalities. Concept of “Theory” is explained in previous examples. And the bank’s example of conservativeness of individual debits and credits and the holistic conservativeness of assets and Liability is pronouncedly predominant in this case also. We shall not repeat in the following the same argument. One more factor that is to be remarked is that there are possibilities of concatenation of same theory with different theories. That the name appeared twice in the Model should not foreclose its option for its relationship with others.

CLASSICAL MECHANICS AND NEWTONIAN GRAVITY:

MODULE NUMBERED ONE

NOTATION:

$G_{13}$ : CATEGORY ONE OF CLASSICAL MECHANICS

$G_{14}$ : CATEGORY TWO OF CLASSICAL MECHANICS

$G_{15}$ : CATEGORY THREE OF CLASSICAL MECHANICS

$T_{13}$ : CATEGORY ONE OF NEWTONIAN GRAVITY
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\[ T_{16} : \text{CATEGORY TWO OF NEWTONIAN GRAVITY} \]

\[ T_{15} : \text{CATEGORY THREE OF NEWTONIAN GRAVITY (WE ARE TALKING OF SYSTEMS; LAW IS THERE BUT IS APPLICABLE TO VARIOUS SYSTEMS) INARIANT SU(3), THE PHYSICAL PARAMETER STATES} \]

**QUANTUM MECHANICS AND QUANTUM FIELD THEORY:**

**MODULE NUMBERED TWO:**

\[ G_{16} : \text{CATEGORY ONE OF QUANTUM MECHANICS} \]

\[ G_{17} : \text{CATEGORY TWO OF QUANTUM MECHANICS} \]

\[ G_{18} : \text{CATEGORY THREE OF QUANTUM MECHANICS} \]

\[ T_{16} : \text{CATEGORY ONE OF QUANTUM FIELD THEORY} \]

\[ T_{17} : \text{CATEGORY TWO OF QUANTUM FIELD THEORY} \]

\[ T_{18} : \text{CATEGORY THREE OF QUANTUM FIELD THEORY} \]

**ELECTROMAGNETISM AND STR (SPECIAL THEORY OF RELATIVITY):**

**MODULE NUMBERED THREE:**

\[ G_{20} : \text{CATEGORY ONE OF ELECTROMAGNETISM} \]

\[ G_{21} : \text{CATEGORY TWO OF ELECTROMAGNETIC THEORY} \]

\[ G_{22} : \text{CATEGORY THREE OF ELECTROMAGNETIC THEORY} \]

\[ T_{20} : \text{CATEGORY ONE OF STR} \]

\[ T_{21} : \text{CATEGORY TWO OF STR} \]

\[ T_{22} : \text{CATEGORY THREE OF STR} \]

**GTR (GENERAL THEORY OF RELATIVITY) AND QFT (QUANTUM FIELD THEORY) IN CURVED SPACE TIME (BASED ON CERTAIN VARIABLES OF THE SYSTEM WHICH CONSEQUENTIALLY CLASSIFIABLE ON PARAMETERS):**

**MODULE NUMBERED FOUR:**

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$G_{24}$: CATEGORY ONE OF GTR EVALUATIVE PARAMETRICIZATION OF SITUATIONAL ORIENTATIONS AND ESSENTIAL COGNITIVE ORIENTATION AND CHOICE VARIABLES OF THE SYSTEM TO WHICH QFT IS APPLICABLE)

$G_{25}$: CATEGORY TWO OF GTR

$G_{26}$: CATEGORY THREE OF GTR

$T_{24}$: CATEGORY ONE OF QFT IN CURVED SPACE TIME

$T_{25}$: CATEGORY TWO OF QFT (SYSTEMIC INSTRUMENTAL CHARACTERISATIONS AND ACTION ORIENTATIONS AND FUNCTIONAL IMPERATIVES OF CHANGE MANIFESTED THEREIN)

$T_{26}$: CATEGORY THREE OF QUANTUM FIELD THEORY

GTR (GENERAL THEORY OF RELATIVITY (THERE ARE MANY OBSERVES AND GTR IS APPLICABLE TO BILLION SYSTEMS NOTWITHSTANDING THE GENERALISATIONAL NATURE OF THE THEORY) AND QUANTUM GRAVITY

MODULE NUMBERED FIVE:

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CATEGORY ONE OF GTR

CATEGORY TWO OF QUANTUM GRAVITY

CATEGORY THREE OF QUANTUM GRAVITY

THE FINAL THEORY MUST POSSESS THE SAME CHARACTERSTICS OF ITS CONSTITUENTS-IT CANNOT SIT IN IVORY TOWER WITHOUT APPLICABILITY TO VARIOUS SYSTEMS)

$T_{28}$: CATEGORY ONE OF QUANTUM GRAVITY

$T_{29}$: CATEGORY TWO OF QUANTUM GRAVITY

$T_{30}$: CATEGORY THREE OF QUANTUM GRAVITY

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QFT IN CURVED SPACE TIME AND QUANTUM GRAVITY:

MODULE NUMBERED SIX:

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CATEGORY ONE OF QFT IN CURVED SPACE AND TIME

CATEGORY TWO OF QFT IN SPACE AND TIME
\[ G_{34} : \text{CATEGORY THREE OF QFT IN CURVED SPACE AND TIME} \]

\[ T_{32} : \text{CATEGORY ONE OF QUANTUM GRAVITY} \]

\[ T_{33} : \text{CATEGORY TWO OF QUANTUM GRAVITY} \]

\[ T_{34} : \text{CATEGORY THREE OF QUANTUM GRAVITY} \]

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\[ G_{36} : \text{CATEGORY ONE OF GTR} \]

\[ G_{37} : \text{CATEGORY TWO OF GTR} \]

\[ G_{38} : \text{CATEGORY THREE OF GTR} \]

\[ T_{36} : \text{CATEGORY ONE OF QFT IN CURVED SPACE TIME} \]

\[ T_{37} : \text{CATEGORY TWO OF QFT IN CURVED SPACE TIME} \]

\[ T_{38} : \text{CATEGORY THREE OF QFT IN CURVED SPACE AND TIME} \]

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\[ (a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}, (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}, (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}, (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}, (a_{24})^{(4)}, (a_{25})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)} \]

are Accentuation coefficients

\[ (a_{13}')^{(1)}, (a_{14}')^{(1)}, (a_{15}')^{(1)}, (b_{13}')^{(1)}, (b_{14}')^{(1)}, (b_{15}')^{(1)}, (a_{16}')^{(2)}, (a_{17}')^{(2)}, (a_{18}')^{(2)}, (b_{16}')^{(2)}, (b_{17}')^{(2)}, (b_{18}')^{(2)}, (a_{20}')^{(3)}, (a_{21}')^{(3)}, (a_{22}')^{(3)}, (b_{20}')^{(3)}, (b_{21}')^{(3)}, (b_{22}')^{(3)}, (a_{24}')^{(4)}, (a_{25}')^{(4)}, (b_{24}')^{(4)}, (b_{25}')^{(4)}, (b_{26}')^{(4)}, (b_{28}')^{(5)}, (b_{29}')^{(5)}, (b_{30}')^{(5)}, (a_{28}')^{(5)}, (a_{29}')^{(5)}, (a_{30}')^{(5)}, (a_{32}')^{(6)}, (a_{33}')^{(6)}, (a_{34}')^{(6)}, (b_{32}')^{(6)}, (b_{33}')^{(6)}, (b_{34}')^{(6)} \]

are Dissipation coefficients

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**CLASSICAL MECHANICS AND NEWTONIAN GRAVITY:**

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\[ \frac{dG_{13}}{dt} = (a_{13})^{(1)}g_{14} - \left[ (a_{13}')^{(1)} + (a_{13}'')^{(1)}(T_{14}, t) \right] G_{13} \]
$$\frac{dg_{14}}{dt} = (a_{14}')^{(1)}G_{13} - [(a_{14}')^{(1)} + (a_{14}'')^{(1)}(T_{14}, t)]G_{14}$$

$$\frac{dg_{15}}{dt} = (a_{15}')^{(1)}G_{14} - [(a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14}, t)]G_{15}$$

$$\frac{dt_{13}}{dt} = (b_{13}')^{(1)}T_{14} - [(b_{13}')^{(1)} - (b_{13}'')^{(1)}(G, t)]T_{13}$$

$$\frac{dt_{14}}{dt} = (b_{14}')^{(1)}T_{13} - [(b_{14}')^{(1)} - (b_{14}'')^{(1)}(G, t)]T_{14}$$

$$\frac{dt_{15}}{dt} = (b_{15}')^{(1)}T_{14} - [(b_{15}')^{(1)} - (b_{15}'')^{(1)}(G, t)]T_{15}$$

$$+(a_{13}'')^{(1)}(T_{14}, t) = \text{First augmentation factor}$$

$$-(b_{13}'')^{(1)}(G, t) = \text{First detritions factor}$$

**QUANTUM MECHANICS AND QUANTUM FIELD THEORY:**

**MODULE NUMBERED TWO:**

The differential system of this model is now (Module numbered two)

$$\frac{dg_{16}}{dt} = (a_{16}')^{(2)}G_{17} - [(a_{16}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)]G_{16}$$

$$\frac{dg_{17}}{dt} = (a_{17}')^{(2)}G_{16} - [(a_{17}')^{(2)} + (a_{17}'')^{(2)}(T_{17}, t)]G_{17}$$

$$\frac{dg_{18}}{dt} = (a_{18}')^{(2)}G_{17} - [(a_{18}')^{(2)} + (a_{18}'')^{(2)}(T_{17}, t)]G_{18}$$

$$\frac{dt_{16}}{dt} = (b_{16}')^{(2)}T_{17} - [(b_{16}')^{(2)} - (b_{16}'')^{(2)}(G_{19}, t)]T_{16}$$

$$\frac{dt_{17}}{dt} = (b_{17}')^{(2)}T_{16} - [(b_{17}')^{(2)} - (b_{17}'')^{(2)}(G_{19}, t)]T_{17}$$

$$\frac{dt_{18}}{dt} = (b_{18}')^{(2)}T_{17} - [(b_{18}')^{(2)} - (b_{18}'')^{(2)}(G_{19}, t)]T_{18}$$

$$+(a_{16}'')^{(2)}(T_{17}, t) = \text{First augmentation factor}$$

$$-(b_{16}'')^{(2)}(G_{19}, t) = \text{First detritions factor}$$

**ELECTROMAGNETISM AND STR(SPECIAL THEORY OF RELATIVITY):**

**MODULE NUMBERED THREE:**

The differential system of this model is now (Module numbered three)

$$\frac{dg_{20}}{dt} = (a_{20}')^{(3)}G_{21} - [(a_{20}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)]G_{20}$$

$$\frac{dg_{21}}{dt} = (a_{21}')^{(3)}G_{20} - [(a_{21}')^{(3)} + (a_{21}'')^{(3)}(T_{21}, t)]G_{21}$$

$$\frac{dg_{22}}{dt} = (a_{22}')^{(3)}G_{21} - [(a_{22}')^{(3)} + (a_{22}'')^{(3)}(T_{21}, t)]G_{22}$$

$$\frac{dt_{20}}{dt} = (b_{20}')^{(3)}T_{21} - [(b_{20}')^{(3)} - (b_{20}'')^{(3)}(G_{23}, t)]T_{20}$$

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\[
\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b_{21}')^{(3)} - (b_{21}'')^{(3)}(G_{23}, t)\right]T_{21}
\]

\[
\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b_{22}')^{(3)} - (b_{22}'')^{(3)}(G_{23}, t)\right]T_{22}
\]

\[+(a_{20}')^{(3)}(T_{21}, t) = \text{First augmentation factor}
\]

\[-(b_{20}')^{(3)}(G_{23}, t) = \text{First detriments factor}
\]

GTR (GENERAL THEORY OF RELATIVITY) AND QFT (QUANTUM FIELD THEORY) IN CURVED SPACE TIME (BASED ON CERTAIN VARIABLES OF THE SYSTEM WHICH CONSEQUENTIALY CLASSIFIABLE ON PARAMETERS):

: MODULE NUMBERED FOUR

The differential system of this model is now (Module number four)

\[
\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a_{24}')^{(4)} + (a_{24}'')^{(4)}(T_{25}, t)\right]G_{24}
\]

\[
\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a_{25}')^{(4)} + (a_{25}'')^{(4)}(T_{25}, t)\right]G_{25}
\]

\[
\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a_{26}')^{(4)} + (a_{26}'')^{(4)}(T_{25}, t)\right]G_{26}
\]

\[
\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b_{24}')^{(4)} - (b_{24}'')^{(4)}(G_{27}, t)\right]T_{24}
\]

\[
\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b_{25}')^{(4)} - (b_{25}'')^{(4)}(G_{27}, t)\right]T_{25}
\]

\[
\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b_{26}')^{(4)} - (b_{26}'')^{(4)}(G_{27}, t)\right]T_{26}
\]

\[+(a_{24}')^{(4)}(T_{25}, t) = \text{First augmentation factor}
\]

\[-(b_{24}')^{(4)}(G_{27}, t) = \text{First detriments factor}
\]

GTR (GENERAL THEORY OF RELATIVITY) (THERE ARE MANY OBSERVES AND GTR IS APPLICABLE TO BILLION SYSTEMS NOTWITHSTANDING THE GENERALISATIONAL NATURE OF THE THEORY) AND QUANTUM GRAVITY

: MODULE NUMBERED FIVE

The differential system of this model is now (Module number five)

\[
\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a_{28}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right]G_{28}
\]

\[
\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a_{29}')^{(5)} + (a_{29}'')^{(5)}(T_{29}, t)\right]G_{29}
\]

\[
\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a_{30}')^{(5)} + (a_{30}'')^{(5)}(T_{29}, t)\right]G_{30}
\]

\[
\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b_{28}')^{(5)} - (b_{28}'')^{(5)}(G_{31}, t)\right]T_{28}
\]
\[
\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{29} - \left[(b_{29}')^{(5)} - (b_{29}'')^{(5)}(G_{31}, t)\right]T_{29} \\
\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b_{30}')^{(5)} - (b_{30}'')^{(5)}(G_{31}, t)\right]T_{30} \\
+(a_{28}'')^{(5)}(T_{29}, t) = \text{First augmentation factor} \\
-(b_{28}'')^{(5)}((G_{31}), t) = \text{First detritions factor} \\
\]

QFT IN CURVED SPACE TIME AND QUANTUM GRAVITY:

MODULE NUMBERED SIX:

The differential system of this model is now (Module numbered Six)

\[
\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{32} - \left[(a_{32}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right]G_{32} \\
\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a_{33}')^{(6)} + (a_{33}'')^{(6)}(T_{33}, t)\right]G_{33} \\
\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a_{34}')^{(6)} + (a_{34}'')^{(6)}(T_{33}, t)\right]G_{34} \\
\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b_{32}')^{(6)} - (b_{32}'')^{(6)}((G_{35}), t)\right]T_{32} \\
\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b_{33}')^{(6)} - (b_{33}'')^{(6)}((G_{35}), t)\right]T_{33} \\
\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b_{34}')^{(6)} - (b_{34}'')^{(6)}((G_{35}), t)\right]T_{34} \\
+(a_{32}'')^{(6)}(T_{33}, t) = \text{First augmentation factor} \\
\]

GTR AND QFT IN CURVED SPACE TIME

MODULE NUMBERED SEVEN:

The differential system of this model is now (SEVENTH MODULE)

\[
\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a_{36}')^{(7)} + (a_{36}'')^{(7)}(T_{37}, t)\right]G_{36} \\
\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a_{37}')^{(7)} + (a_{37}'')^{(7)}(T_{37}, t)\right]G_{37} \\
\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a_{38}')^{(7)} + (a_{38}'')^{(7)}(T_{37}, t)\right]G_{38} \\
\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b_{36}')^{(7)} - (b_{36}'')^{(7)}((G_{39}), t)\right]T_{36} \\
\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b_{37}')^{(7)} - (b_{37}'')^{(7)}((G_{39}), t)\right]T_{37} \\
\]

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\[
\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[ (b_{38}^{1})^{(7)} - (b_{38}^{2})^{(7)}((G_{39}), t) \right] T_{38}
\]

\[ (a_{38})^{(7)}(T_{37}, t) = \text{First augmentation factor} \]

\[ -(b_{38})^{(7)}((G_{39}), t) = \text{First detritions factor} \]

**FIRST MODULE CONCATENATION:**

\[
\frac{dG_{14}}{dt} = (a_{13})^{(1)}G_{14} - \left[ (a_{14})^{(1)}G_{14} + (a_{15})^{(1)}T_{14}, t] + (a_{16})^{(2,2)}[(T_{17}, t)] + (a_{17})^{(3,3)}[(T_{21}, t)] + (a_{18})^{(4,4,4,4)}[(T_{25}, t)] + (a_{19})^{(5,5,5,5)}[(T_{29}, t)] + (a_{20})^{(6,6,6,6)}[(T_{33}, t)] \right] G_{14}
\]

\[
\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{14} - \left[ (a_{14})^{(1)}G_{14} + (a_{15})^{(1)}T_{14}, t] + (a_{16})^{(2,2)}[(T_{17}, t)] + (a_{17})^{(3,3)}[(T_{21}, t)] + (a_{18})^{(4,4,4,4)}[(T_{25}, t)] + (a_{19})^{(5,5,5,5)}[(T_{29}, t)] + (a_{20})^{(6,6,6,6)}[(T_{33}, t)] \right] G_{14}
\]

\[
\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[ (a_{15})^{(1)}G_{14} + (a_{16})^{(1)}T_{14}, t] + (a_{17})^{(2,2)}[(T_{17}, t)] + (a_{18})^{(3,3)}[(T_{21}, t)] + (a_{19})^{(4,4,4,4)}[(T_{25}, t)] + (a_{20})^{(5,5,5,5)}[(T_{29}, t)] + (a_{21})^{(6,6,6,6)}[(T_{33}, t)] \right] G_{15}
\]

Where \( [a_{13}]^{(1)}(T_{14}, t) \), \( [a_{14}]^{(1)}(T_{14}, t) \), \( [a_{15}]^{(1)}(T_{14}, t) \) are first augmentation coefficients for category 1, 2 and 3

\( + (a_{16})^{(2,2)}(T_{17}, t) \), \( + (a_{17})^{(3,3)}(T_{21}, t) \) are second augmentation coefficient for category 1, 2 and 3

\( + (a_{18})^{(4,4,4,4)}(T_{25}, t) \), \( + (a_{19})^{(5,5,5,5)}(T_{29}, t) \), \( + (a_{20})^{(6,6,6,6)}(T_{33}, t) \) are third augmentation coefficient for category 1, 2 and 3

\( + (a_{21})^{(4,4,4,4)}(T_{25}, t) \), \( + (a_{22})^{(5,5,5,5)}(T_{29}, t) \), \( + (a_{23})^{(6,6,6,6)}(T_{33}, t) \) are fourth augmentation coefficient for category 1, 2 and 3

\( + (a_{24})^{(5,5,5,5)}(T_{29}, t) \), \( + (a_{25})^{(6,6,6,6)}(T_{33}, t) \) are fifth augmentation coefficient for category 1, 2 and 3

\( + (a_{26})^{(6,6,6,6)}(T_{33}, t) \), \( + (a_{27})^{(7,7)}(T_{37}, t) \), \( + (a_{28})^{(7,7)}(T_{37}, t) \), \( + (a_{29})^{(7,7)}(T_{37}, t) \), \( + (a_{30})^{(7,7)}(T_{37}, t) \) ARE SEVENTH AUGMENTATION COEFFICIENTS

\[
\frac{dT_{14}}{dt} = (b_{13})^{(1)}T_{14} - \left[ (b_{13})^{(1)}(G_{14}, t) - (b_{14})^{(7)}(G_{39}, t) - (b_{15})^{(7)}(G_{39}, t) - (b_{16})^{(3,3)}(G_{39}, t) - (b_{17})^{(4,4,4,4)}(G_{39}, t) - (b_{18})^{(5,5,5,5)}(G_{39}, t) - (b_{19})^{(6,6,6,6)}(G_{39}, t) \right] \]
\[
\frac{d\mathbf{T}_{14}}{dt} = (b_{14})^{(1)} T_{14} - \begin{bmatrix}
(b'_{14})^{(1)}(G,t) & -(b''_{15})^{(1)}(G,t) & -(b''_{21})^{(2,2,3)}(G_{19}, t) & -(b''_{23})^{(3,3)}(G_{23}, t) \\
-(b'_{20})^{(4,4,4)}(G_{27}, t) & -(b''_{26})^{(5,5,5,5)}(G_{31}, t) & -(b''_{34})^{(6,6,6,6)}(G_{35}, t) & -(b''_{30})^{(7,7)}(G_{39}, t)
\end{bmatrix}
\]

\[
\frac{d\mathbf{T}_{15}}{dt} = (b_{15})^{(1)} T_{14} - \begin{bmatrix}
(b'_{15})^{(1)}(G,t) & -(b''_{14})^{(1)}(G,t) & -(b''_{19})^{(2,2,3)}(G_{19}, t) & -(b''_{21})^{(3,3)}(G_{23}, t) \\
-(b'_{20})^{(4,4,4)}(G_{27}, t) & -(b''_{26})^{(5,5,5,5)}(G_{31}, t) & -(b''_{34})^{(6,6,6,6)}(G_{35}, t) & -(b''_{30})^{(7,7)}(G_{39}, t)
\end{bmatrix}
\]

Where 
- \((b''_{14})^{(1)}(G,t)\), \((b''_{15})^{(1)}(G,t)\), \((b''_{19})^{(1)}(G,t)\) are first detrition coefficients for category 1, 2 and 3
- \((b''_{14})^{(2,2)}(G_{19}, t)\), \((b''_{15})^{(2,2)}(G_{19}, t)\), \((b''_{19})^{(2,2)}(G_{19}, t)\) are second detrition coefficients for category 1, 2 and 3
- \((b''_{20})^{(3,3)}(G_{23}, t)\), \((b''_{21})^{(3,3)}(G_{23}, t)\), \((b''_{23})^{(3,3)}(G_{23}, t)\) are third detriments coefficients for category 1, 2 and 3
- \((b''_{26})^{(4,4,4,4)}(G_{31}, t)\), \((b''_{26})^{(4,4,4,4)}(G_{31}, t)\), \((b''_{26})^{(4,4,4,4)}(G_{31}, t)\) are fourth detriments coefficients for category 1, 2 and 3
- \((b''_{30})^{(5,5,5,5)}(G_{35}, t)\), \((b''_{30})^{(5,5,5,5)}(G_{35}, t)\), \((b''_{30})^{(5,5,5,5)}(G_{35}, t)\) are fifth detriments coefficients for category 1, 2 and 3
- \((b''_{30})^{(6,6,6,6)}(G_{39}, t)\), \((b''_{30})^{(6,6,6,6)}(G_{39}, t)\), \((b''_{30})^{(6,6,6,6)}(G_{39}, t)\) are sixth detriments coefficients for category 1, 2 and 3
- \((b''_{30})^{(7,7)}(G_{39}, t)\), \((b''_{30})^{(7,7)}(G_{39}, t)\), \((b''_{30})^{(7,7)}(G_{39}, t)\) are SEVENTH DETRITION COEFFICIENTS

\[
\frac{d\mathbf{G}_{16}}{dt} = (a_{16})^{(2)} G_{17} - \begin{bmatrix}
(a'_{16})^{(2)}(T_{13}, t) & +(a''_{16})^{(2)}(T_{13}, t) & +(a''_{16})^{(1,1)}(T_{14}, t) & +(a''_{20})^{(3,3,3)}(T_{21}, t) \\
+(a'_{24})^{(4,4,4,4)}(T_{29}, t) & +(a''_{24})^{(5,5,5,5)}(T_{29}, t) & +(a''_{20})^{(6,6,6,6)}(T_{33}, t) & +(a''_{30})^{(7,7,7)}(T_{37}, t)
\end{bmatrix}
\]

SECOND MODULE CONCATENATION
\[ \frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[ \begin{array}{c} (a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t) + (a_{14})^{(1,1)}(T_{14}, t) + (a_{21})^{(3,3,3)}(T_{21}, t) \\ - (a'_{23})^{(4,4,4,4)}(T_{23}, t) + (a_{29})^{(5,5,5,5)}(T_{29}, t) + (a_{33})^{(6,6,6,6,6)}(T_{33}, t) \\ + (a_{37})^{(7,7,7)}(T_{37}, t) \end{array} \right] \]

\[ \frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[ \begin{array}{c} (a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{18}, t) + (a_{15})^{(1,1)}(T_{14}, t) + (a_{22})^{(3,3,3)}(T_{22}, t) \\ + (a'_{26})^{(4,4,4,4)}(T_{26}, t) + (a_{30})^{(5,5,5,5)}(T_{30}, t) + (a_{34})^{(6,6,6,6,6)}(T_{34}, t) \\ + (a_{38})^{(7,7,7)}(T_{38}, t) \end{array} \right] \]

Where \( + (a_{17})^{(2)}(T_{17}, t) \), \( + (a_{18})^{(2)}(T_{18}, t) \) and \( + (a_{19})^{(2)}(T_{19}, t) \) are first augmentation coefficients for category 1, 2 and 3.

\( + (a_{21})^{(1,1)}(T_{14}, t) \), \( + (a_{22})^{(1,1)}(T_{14}, t) \) and \( + (a_{29})^{(1,1)}(T_{14}, t) \) are second augmentation coefficient for category 1, 2 and 3.

\( + (a_{23})^{(3,3,3)}(T_{21}, t) \), \( + (a_{24})^{(3,3,3)}(T_{21}, t) \) and \( + (a_{30})^{(3,3,3)}(T_{21}, t) \) are third augmentation coefficient for category 1, 2 and 3.

\( + (a_{26})^{(4,4,4,4)}(T_{26}, t) \), \( + (a_{27})^{(4,4,4,4)}(T_{26}, t) \) and \( + (a_{33})^{(4,4,4,4)}(T_{26}, t) \) are fourth augmentation coefficient for category 1, 2 and 3.

\( + (a_{29})^{(5,5,5,5,5)}(T_{29}, t) \), \( + (a_{34})^{(5,5,5,5,5)}(T_{29}, t) \) and \( + (a_{35})^{(5,5,5,5,5)}(T_{29}, t) \) are fifth augmentation coefficient for category 1, 2 and 3.

\( + (a_{30})^{(6,6,6,6,6)}(T_{30}, t) \), \( + (a_{31})^{(6,6,6,6,6)}(T_{30}, t) \) and \( + (a_{36})^{(6,6,6,6,6)}(T_{30}, t) \) are sixth augmentation coefficient for category 1, 2 and 3.

\( + (a_{37})^{(7,7,7)}(T_{37}, t) \), \( + (a_{38})^{(7,7,7)}(T_{38}, t) \) and \( + (a_{39})^{(7,7,7)}(T_{39}, t) \) are seventh detrition coefficients.

\[ \frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{15} - \left[ \begin{array}{c} (b''_{16})^{(2)}(G_{19}, t) - (b'_{13})^{(1,1,1)}(G_{13}, t) - (b''_{20})^{(3,3,3)}(G_{23}, t) \\ - (b''_{24})^{(4,4,4,4)}(G_{27}, t) - (b''_{29})^{(5,5,5,5,5)}(G_{31}, t) - (b''_{32})^{(6,6,6,6,6,6)}(G_{35}, t) \\ + (b''_{36})^{(7,7,7)}(G_{39}, t) \end{array} \right] \]

\[ \frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[ \begin{array}{c} (b''_{17})^{(2)}(G_{19}, t) - (b'_{14})^{(1,1,1)}(G_{14}, t) - (b''_{21})^{(3,3,3)}(G_{23}, t) \\ - (b''_{25})^{(4,4,4,4)}(G_{27}, t) - (b''_{29})^{(5,5,5,5,5)}(G_{31}, t) - (b''_{33})^{(6,6,6,6,6,6)}(G_{35}, t) \\ - (b''_{37})^{(7,7,7)}(G_{39}, t) \end{array} \right] \]

\[ \frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[ \begin{array}{c} (b''_{18})^{(2)}(G_{19}, t) - (b''_{15})^{(1,1,1)}(G_{15}, t) - (b''_{22})^{(3,3,3)}(G_{23}, t) \\ - (b''_{26})^{(4,4,4,4)}(G_{27}, t) - (b''_{30})^{(5,5,5,5,5)}(G_{31}, t) - (b''_{34})^{(6,6,6,6,6,6)}(G_{35}, t) \\ - (b''_{38})^{(7,7,7)}(G_{39}, t) \end{array} \right] \]

Where \( - (b''_{10})^{(2)}(G_{10}) \), \( - (b''_{10})^{(2)}(G_{10}) \) and \( - (b''_{10})^{(2)}(G_{10}) \) are first detrition coefficients for category 1, 2 and 3.

\( - (b''_{12})^{(1,1)}(G_{12}) \), \( - (b''_{13})^{(1,1)}(G_{13}) \) and \( - (b''_{13})^{(1,1)}(G_{13}) \) are second detrition coefficients for category 1, 2 and 3.

\( - (b''_{20})^{(3,3,3)}(G_{23}) \), \( - (b''_{21})^{(3,3,3)}(G_{23}) \) and \( - (b''_{21})^{(3,3,3)}(G_{23}) \) are third detrition coefficients for category 1, 2 and 3.

\( - (b''_{24})^{(4,4,4,4)}(G_{27}) \), \( - (b''_{25})^{(4,4,4,4)}(G_{27}) \) and \( - (b''_{25})^{(4,4,4,4)}(G_{27}) \) are fourth detrition coefficients for category 1, 2 and 3.

\( - (b''_{29})^{(5,5,5,5,5)}(G_{31}) \), \( - (b''_{33})^{(5,5,5,5,5)}(G_{31}) \) and \( - (b''_{33})^{(5,5,5,5,5)}(G_{31}) \) are fifth detrition coefficients for category 1, 2 and 3.
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\[ (a_{20}^{(3)})G_{21} = \left[ (a_{20}^{(3)})^{(1)} + (a_{20}^{(3)})^{(2)}(T_{21}, t) + (a_{20}^{(3)})^{(3)}(T_{21}, t) \right] \]

\[ \frac{dG_{20}}{dt} = \left[ \begin{array}{c}
(a_{20}^{(3)})^{(1)} + (a_{20}^{(3)})^{(2)}(T_{21}, t) + (a_{20}^{(3)})^{(3)}(T_{21}, t) \\
+ (a_{20}^{(3)})^{(4,4,4,4,4)}(T_{25}, t) + (a_{20}^{(3)})^{(5,5,5,5,5)}(T_{29}, t) + (a_{20}^{(3)})^{(1,6,6,6,6,6)}(T_{33}, t)
\end{array} \right] G_{20} \]

\[ \frac{dG_{21}}{dt} = (a_{21}^{(3)})^{(1)} G_{20} - \left[ \begin{array}{c}
(a_{21}^{(3)})^{(1)} + (a_{21}^{(3)})^{(2)}(T_{21}, t) + (a_{21}^{(3)})^{(3)}(T_{21}, t) \\
+ (a_{21}^{(3)})^{(4,4,4,4,4)}(T_{25}, t) + (a_{21}^{(3)})^{(5,5,5,5,5)}(T_{29}, t) + (a_{21}^{(3)})^{(1,6,6,6,6,6)}(T_{33}, t)
\end{array} \right] G_{21} \]

\[ \frac{dG_{22}}{dt} = (a_{22}^{(3)})^{(1)} G_{21} - \left[ \begin{array}{c}
(a_{22}^{(3)})^{(1)} + (a_{22}^{(3)})^{(2)}(T_{21}, t) + (a_{22}^{(3)})^{(3)}(T_{21}, t) \\
+ (a_{22}^{(3)})^{(4,4,4,4,4)}(T_{25}, t) + (a_{22}^{(3)})^{(5,5,5,5,5)}(T_{29}, t) + (a_{22}^{(3)})^{(1,6,6,6,6,6)}(T_{33}, t)
\end{array} \right] G_{22} \]

\[ \frac{dG_{23}}{dt} = (a_{23}^{(3)})^{(1)} G_{22} - \left[ \begin{array}{c}
(a_{23}^{(3)})^{(1)} + (a_{23}^{(3)})^{(2)}(T_{21}, t) + (a_{23}^{(3)})^{(3)}(T_{21}, t) \\
+ (a_{23}^{(3)})^{(4,4,4,4,4)}(T_{25}, t) + (a_{23}^{(3)})^{(5,5,5,5,5)}(T_{29}, t) + (a_{23}^{(3)})^{(1,6,6,6,6,6)}(T_{33}, t)
\end{array} \right] G_{23} \]

\[ + (a_{20}^{(3)})^{(4)}(T_{21}, t) + (a_{20}^{(3)})^{(5)}(T_{21}, t) + (a_{21}^{(3)})^{(4)}(T_{21}, t) \]

are first augmentation coefficients for category 1, 2 and 3

\[ + (a_{20}^{(3)})^{(2,2,2)}(T_{15}, t) + (a_{20}^{(3)})^{(2,2,2)}(T_{15}, t) + (a_{21}^{(3)})^{(2,2,2)}(T_{15}, t) \]

are second augmentation coefficients for category 1, 2 and 3

\[ + (a_{20}^{(3)})^{(1,1,1)}(T_{14}, t) + (a_{20}^{(3)})^{(1,1,1)}(T_{14}, t) + (a_{21}^{(3)})^{(1,1,1)}(T_{14}, t) \]

are third augmentation coefficients for category 1, 2 and 3

\[ + (a_{20}^{(3)})^{(4,4,4,4,4)}(T_{25}, t) + (a_{20}^{(3)})^{(4,4,4,4,4)}(T_{25}, t) + (a_{21}^{(3)})^{(4,4,4,4,4)}(T_{25}, t) \]

are fourth augmentation coefficients for category 1, 2 and 3

\[ + (a_{20}^{(3)})^{(5,5,5,5,5)}(T_{26}, t) + (a_{20}^{(3)})^{(5,5,5,5,5)}(T_{26}, t) + (a_{21}^{(3)})^{(5,5,5,5,5)}(T_{26}, t) \]

are fifth augmentation coefficients for category 1, 2 and 3

\[ + (a_{20}^{(3)})^{(6,6,6,6,6)}(T_{29}, t) + (a_{20}^{(3)})^{(6,6,6,6,6)}(T_{29}, t) + (a_{21}^{(3)})^{(6,6,6,6,6)}(T_{29}, t) \]

are sixth augmentation coefficients for category 1, 2 and 3

\[ + (a_{36}^{(7,7,7)})(T_{37}, t) + (a_{37}^{(7,7,7)})(T_{37}, t) + (a_{38}^{(7,7,7)})(T_{37}, t) \]

are seventh augmentation coefficient

\[ \frac{dG_{20}}{dt} = \left[ \begin{array}{c}
(b_{20}^{(3)})^{(1)} - (b_{20}^{(3)})^{(2)}(T_{23}, t) - (b_{20}^{(3)})^{(3)}(T_{23}, t) \\
- (b_{20}^{(3)})^{(4,4,4,4,4)}(T_{29}, t) - (b_{20}^{(3)})^{(5,5,5,5,5)}(T_{31}, t) - (b_{20}^{(3)})^{(1,6,6,6,6,6)}(T_{35}, t)
\end{array} \right] T_{20} \]

\[ \frac{dG_{21}}{dt} = \left[ \begin{array}{c}
(b_{21}^{(3)})^{(1)} - (b_{21}^{(3)})^{(2)}(T_{23}, t) - (b_{21}^{(3)})^{(3)}(T_{23}, t) \\
- (b_{21}^{(3)})^{(4,4,4,4,4)}(T_{29}, t) - (b_{21}^{(3)})^{(5,5,5,5,5)}(T_{31}, t) - (b_{21}^{(3)})^{(1,6,6,6,6,6)}(T_{35}, t)
\end{array} \right] T_{21} \]
\[(b_{21})^{(3)}T_{20} = \begin{bmatrix} 
(b_{21})^{(3)}(G_{23}, t) - (b_{22})^{(2,2.2)}(G_{19}, t) + (b_{22})^{(1,1,1)}(G, t) 
(b_{22})^{(4,4,4,4,4)}(G_{27}, t) - (b_{23})^{(5,5,5,5,5)}(G_{31}, t) - (b_{23})^{(6,6,6,6,6,6)}(G_{35}, t) 
(b_{23})^{(7,7,7)}(G_{39}, t) 
\end{bmatrix} T_{21} \]

de \[
\frac{dT_{22}}{dt} = \begin{bmatrix} 
(b_{22})^{(3)}(G_{23}, t) - (b_{22})^{(3)}(G_{23}, t) + (b_{22})^{(2,2.2)}(G_{19}, t) + (b_{22})^{(1,1,1)}(G, t) 
(b_{22})^{(4,4,4,4,4)}(G_{27}, t) - (b_{22})^{(4,4,4,4,4)}(G_{27}, t) - (b_{22})^{(6,6,6,6,6,6)}(G_{35}, t) 
(b_{22})^{(7,7,7)}(G_{39}, t) 
\end{bmatrix} T_{22} \]

\begin{align*}
- (b_{23})^{(1,2,2)}(G_{23}, t) - (b_{23})^{(3)}(G_{23}, t) - (b_{23})^{(1,1,1)}(G_{19}, t) - (b_{23})^{(2,2,2)}(G_{19}, t) - (b_{23})^{(1,1,1)}(G, t) \\
- (b_{23})^{(4,4,4,4,4)}(G_{27}, t) - (b_{23})^{(5,5,5,5,5)}(G_{31}, t) - (b_{23})^{(6,6,6,6,6,6,6)}(G_{35}, t) - (b_{23})^{(7,7,7)}(G_{39}, t) \\
- (b_{23})^{(1,2,2)}(G_{23}, t) - (b_{23})^{(3)}(G_{23}, t) - (b_{23})^{(1,1,1)}(G_{19}, t) - (b_{23})^{(2,2,2)}(G_{19}, t) - (b_{23})^{(1,1,1)}(G, t) \\
- (b_{23})^{(4,4,4,4,4)}(G_{27}, t) - (b_{23})^{(5,5,5,5,5)}(G_{31}, t) - (b_{23})^{(6,6,6,6,6,6,6)}(G_{35}, t) - (b_{23})^{(7,7,7)}(G_{39}, t) \\
- (b_{23})^{(1,2,2)}(G_{23}, t) - (b_{23})^{(3)}(G_{23}, t) - (b_{23})^{(1,1,1)}(G_{19}, t) - (b_{23})^{(2,2,2)}(G_{19}, t) - (b_{23})^{(1,1,1)}(G, t) \\
- (b_{23})^{(4,4,4,4,4)}(G_{27}, t) - (b_{23})^{(5,5,5,5,5)}(G_{31}, t) - (b_{23})^{(6,6,6,6,6,6,6)}(G_{35}, t) - (b_{23})^{(7,7,7)}(G_{39}, t) \\
\end{align*}

\[\text{are first detritions coefficients for category 1, 2 and 3}\]

\[\text{are second detritions coefficients for category 1, 2 and 3}\]

\[\text{are third detritions coefficients for category 1, 2 and 3}\]

\[\text{are fourth detritions coefficients for category 1, 2 and 3}\]

\[\text{are fifth detritions coefficients for category 1, 2 and 3}\]

\[\text{are sixth detritions coefficients for category 1, 2 and 3}\]

\[\text{are seventh detritions coefficients}\]

\[\text{===============================================================================}\]

\[\text{FOURTH MODULE CONCATENATION:}\]

\[\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} 
(a_{24})^{(4)}(T_{25}, t) + (a_{25})^{(4)}(T_{25}, t) + (a_{25})^{(6,6)}(T_{25}, t) 
+ (a_{25})^{(1,1,1,1)}(T_{14}, t) + (a_{25})^{(2,2,2,2)}(T_{17}, t) + (a_{25})^{(3,3,3,3)}(T_{21}, t) 
+ (a_{25})^{(7,7,7,7)}(T_{37}, t) 
\end{bmatrix} G_{24} \]

\[\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} 
(a_{25})^{(4)}(T_{25}, t) + (a_{25})^{(4)}(T_{25}, t) + (a_{25})^{(6,6)}(T_{25}, t) 
+ (a_{25})^{(1,1,1,1)}(T_{14}, t) + (a_{25})^{(2,2,2,2)}(T_{17}, t) + (a_{25})^{(3,3,3,3)}(T_{21}, t) 
+ (a_{25})^{(7,7,7,7)}(T_{37}, t) 
\end{bmatrix} G_{25} \]

\[\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} 
(a_{26})^{(4)}(T_{25}, t) + (a_{26})^{(4)}(T_{25}, t) + (a_{26})^{(6,6)}(T_{25}, t) 
+ (a_{26})^{(1,1,1,1)}(T_{14}, t) + (a_{26})^{(2,2,2,2)}(T_{17}, t) + (a_{26})^{(3,3,3,3)}(T_{21}, t) 
+ (a_{26})^{(7,7,7,7)}(T_{37}, t) 
\end{bmatrix} G_{26} \]

\[\text{Where } (a_{24})^{(4)}(T_{25}, t), (a_{25})^{(4)}(T_{25}, t), (a_{26})^{(4)}(T_{25}, t) \text{ are first augmentation coefficients for category 1 and 2}\]
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are fourth augmentation coefficients for category 1, 2, and 3

are fifth augmentation coefficients for category 1, 2, and 3

are sixth augmentation coefficients for category 1, 2, and 3

are SEVENTH augmentation coefficients

ARE SEVENTH augmentation coefficients

FIFTH MODULE CONCATENATION:
\[
\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{28} - \left[ (a_{28}^{(5)} + (a_{26}^{(5)}) (T_{29}, t) + (a_{24}^{(4,4)}) (T_{25}, t) + (a_{22}^{(6,6)}) (T_{33}, t) + (a_{13}^{(1,1,1,1,1)}) (T_{14}, t) + (a_{12}^{(2,2,2,2,2)}) (T_{17}, t) + (a_{11}^{(3,3,3,3,3)}) (T_{21}, t) \right] G_{28}
\]

\[
\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{29} - \left[ (a_{29}^{(5)} + (a_{27}^{(5)}) (T_{29}, t) + (a_{25}^{(4,4)}) (T_{25}, t) + (a_{23}^{(6,6)}) (T_{33}, t) + (a_{14}^{(1,1,1,1,1)}) (T_{14}, t) + (a_{13}^{(2,2,2,2,2)}) (T_{17}, t) + (a_{12}^{(3,3,3,3,3)}) (T_{21}, t) \right] G_{29}
\]

\[
\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{30} - \left[ (a_{30}^{(5)} + (a_{28}^{(5)}) (T_{29}, t) + (a_{26}^{(4,4)}) (T_{25}, t) + (a_{24}^{(6,6)}) (T_{33}, t) + (a_{15}^{(1,1,1,1,1)}) (T_{14}, t) + (a_{14}^{(2,2,2,2,2)}) (T_{17}, t) + (a_{13}^{(3,3,3,3,3)}) (T_{21}, t) \right] G_{30}
\]

Where \( a_{ij}^{(k)} \) are the augmentation coefficients for category \( i, j \) and \( k \). are first augmentation coefficients for category 1, 2 and 3. And \( a_{ij}^{(k)} \) are second augmentation coefficients for category 1, 2 and 3. \( a_{ij}^{(k)} \) are third augmentation coefficients for category 1, 2 and 3. \( a_{ij}^{(k)} \) are fourth augmentation coefficients for category 1, 2, and 3. \( a_{ij}^{(k)} \) are fifth augmentation coefficients for category 1, 2, and 3. \( a_{ij}^{(k)} \) are sixth augmentation coefficients for category 1, 2, and 3.

\[
\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{28} - \left[ (b_{28}^{(5)} - (b_{26}^{(5)}) (G_{31}, t) - (b_{24}^{(4,4)}) (G_{23}, t) - (b_{22}^{(6,6)}) (G_{35}, t) - (b_{13}^{(1,1,1,1,1)}) (G_{14}, t) - (b_{12}^{(2,2,2,2,2)}) (G_{17}, t) - (b_{11}^{(3,3,3,3,3)}) (G_{21}, t) \right] T_{28}
\]

\[
\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{29} - \left[ (b_{29}^{(5)} - (b_{27}^{(5)}) (G_{32}, t) - (b_{25}^{(4,4)}) (G_{23}, t) - (b_{23}^{(6,6)}) (G_{35}, t) - (b_{14}^{(1,1,1,1,1)}) (G_{14}, t) - (b_{13}^{(2,2,2,2,2)}) (G_{17}, t) - (b_{12}^{(3,3,3,3,3)}) (G_{21}, t) \right] T_{29}
\]

\[
\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{30} - \left[ (b_{30}^{(5)} - (b_{28}^{(5)}) (G_{32}, t) - (b_{26}^{(4,4)}) (G_{23}, t) - (b_{24}^{(6,6)}) (G_{35}, t) - (b_{15}^{(1,1,1,1,1)}) (G_{14}, t) - (b_{14}^{(2,2,2,2,2)}) (G_{17}, t) - (b_{13}^{(3,3,3,3,3)}) (G_{21}, t) \right] T_{30}
\]

Where \( b_{ij}^{(k)} \) are the detrition coefficients for category 1, 2 and 3. are first detrition coefficients for category 1, 2 and 3. are second detrition coefficients for category 1, 2 and 3. are third detrition coefficients for category 1, 2 and 3.
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\[ \frac{dG_{32}}{dt} = (a_{32}^{(6)}) G_{33} \]

\[ = \left[ (a_{32}^{(6)})T_{33} + (a_{28}^{(5,5,5)})T_{29} + (a_{24}^{(4,4,4)})T_{25} \right] G_{32} \]

\[ + (a_{36}^{(7,7,7,7)})T_{37} \]

\[ \frac{dG_{33}}{dt} = (a_{33}^{(6)}) G_{32} \]

\[ = \left[ (a_{33}^{(6)})T_{33} + (a_{29}^{(5,5,5)})T_{29} + (a_{25}^{(4,4,4)})T_{25} \right] G_{33} \]

\[ + (a_{37}^{(7,7,7,7)})T_{37} \]

\[ \frac{dG_{34}}{dt} = (a_{34}^{(6)}) G_{33} \]

\[ = \left[ (a_{34}^{(6)})T_{33} + (a_{28}^{(5,5,5)})T_{29} + (a_{24}^{(4,4,4)})T_{25} \right] G_{34} \]

\[ + (a_{36}^{(7,7,7,7)})T_{37} \]

\[ \text{are first augmentation coefficients for category 1, 2 and 3} \]

\[ \text{are second augmentation coefficients for category 1, 2 and 3} \]

\[ \text{are third augmentation coefficients for category 1, 2 and 3} \]

\[ \text{are fourth augmentation coefficients} \]

\[ \text{are fifth augmentation coefficients} \]

\[ \text{sixth augmentation coefficients} \]

\[ \text{Secretary AUGMENTATION COEFFICIENTS} \]

\[ \frac{dT_{32}}{dt} = (b_{32}^{(6)}) T_{33} - \left[ (b_{28}^{(5,5,5)})T_{35} - (b_{24}^{(4,4,4)})T_{29} \right] G_{35} \]

\[ - (b_{24}^{(4,4,4)})T_{25} \]

\[ - (b_{36}^{(7,7,7,7)})T_{37} \]

\[ \text{are fourth detrition coefficients for category 1, 2, and 3} \]

\[ \text{are fifth detrition coefficients for category 1, 2, and 3} \]

\[ \text{are sixth detrition coefficients for category 1, 2, and 3} \]
\[ \frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[ \begin{array}{c} -(b_{14}^{(6)})^{(6)}(G_{35}, t) - (b_{29}^{(5,5,5)})(G_{31}, t) - (b_{29}^{(4,4,4)})(G_{27}, t) \\ -(b_{13}^{(1,1,1,1,1)})(G_{t}) - (b_{12}^{(2,2,2,2,2)})(G_{19}, t) - (b_{28}^{(3,3,3,3,3)})(G_{23}, t) \\ -(b_{17}^{(7,7,7,7,7)})(G_{39}, t) \end{array} \right] T_{33} \]

\[ \frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[ \begin{array}{c} -(b_{14}^{(6)})^{(6)}(G_{35}, t) - (b_{29}^{(5,5,5)})(G_{31}, t) - (b_{29}^{(4,4,4)})(G_{27}, t) \\ -(b_{13}^{(1,1,1,1,1)})(G_{t}) - (b_{12}^{(2,2,2,2,2)})(G_{19}, t) - (b_{28}^{(3,3,3,3,3)})(G_{23}, t) \\ -(b_{17}^{(7,7,7,7,7)})(G_{39}, t) \end{array} \right] T_{34} \]

\(- (b_{14})^{(6)}(G_{35}, t)\), \(- (b_{29})^{(5,5,5)}(G_{31}, t)\), \(- (b_{17})^{(7,7,7,7,7)}(G_{39}, t)\) are first deterioration coefficients for category 1, 2, and 3.

\(- (b_{14})^{(5,5,5)}(G_{31}, t)\), \(- (b_{17})^{(7,7,7,7,7)}(G_{39}, t)\) are second deterioration coefficients for category 1, 2, and 3.

\(- (b_{14})^{(4,4,4)}(G_{27}, t)\), \(- (b_{17})^{(2,2,2,2,2)}(G_{19}, t)\) are third deterioration coefficients for category 1, 2, and 3.

\(- (b_{14})^{(3,3,3,3,3)}(G_{23}, t)\) are fourth deterioration coefficients for category 1, 2, and 3.

\(- (b_{14})^{(2,2,2,2,2)}(G_{19}, t)\) are fifth deterioration coefficients for category 1, 2, and 3.

\(- (b_{14})^{(3,3,3,3,3)}(G_{23}, t)\) are sixth deterioration coefficients for category 1, 2, and 3.

\[- (b_{14})^{(7,7,7,7,7)}(G_{39}, t)\] are SEVENTH DETERIORATION COEFFICIENTS.

**SEVENTH MODULE CONCATENATION:**

\[ \frac{dg_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[ (a_{36})^{(7)}(T_{37}, t) + (a_{16})^{(7)}(T_{17}, t) + (a_{20})^{(7)}(T_{21}, t) + (a_{24})^{(7)}(T_{23}, t)G_{36} + (a_{28})^{(7)}(T_{29}, t) + (a_{32})^{(7)}(T_{33}, t) + (a_{13})^{(7)}(T_{14}, t) \right] G_{36} \]

\[ \frac{dg_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[ (a_{37})^{(7)}(T_{37}, t) + (a_{14})^{(7)}(T_{14}, t) + (a_{21})^{(7)}(T_{21}, t) + (a_{17})^{(7)}(T_{17}, t) + (a_{25})^{(7)}(T_{25}, t) + (a_{33})^{(7)}(T_{33}, t) + (a_{20})^{(7)}(T_{29}, t) \right] G_{37} \]

\[ \frac{dg_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[ (a_{38})^{(7)}(T_{37}, t) + (a_{15})^{(7)}(T_{14}, t) + (a_{22})^{(7)}(T_{21}, t) + (a_{18})^{(7)}(T_{17}, t) \right] G_{38} \]
Where we suppose

(A) \((a_i^0)^{(1)}, (a_j^0)^{(1)}, (a_k^0)^{(1)}, (b_i^0)^{(1)}, (b_j^0)^{(1)}, (b_k^0)^{(1)} > 0, \)
i, j = 13, 14, 15

(B) The functions \((a_i^0)^{(1)}, (b_k^0)^{(1)}\) are positive continuous increasing and bounded.

\text{Definition of } (p_i)^{(1)}, (r_i)^{(1)}:

\((a_i^0)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\bar{A}_{13})^{(1)}\)

\((b_i^0)^{(1)}(G, t) \leq (r_i)^{(1)} \leq (\bar{B}_{13})^{(1)}\)

(C) \(\lim_{T_2 \to \infty} (a_i^0)^{(1)}(T_{14}, t) = (p_i)^{(1)}\)

\(\lim_{G \to \infty} (b_i^0)^{(1)}(G, t) = (r_i)^{(1)}\)

\text{Definition of } (\bar{A}_{13})^{(1)}, (\bar{B}_{13})^{(1)}:

Where \((\bar{A}_{13})^{(1)}, (\bar{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}\) are positive constants

and \(i = 13, 14, 15\)

They satisfy Lipschitz condition:

\(|(a_i^0)^{(1)}(T_{14}, t) - (a_i^0)^{(1)}(T_{14}, t)| \leq (\bar{K}_{13})^{(1)}|T_{14} - T_1^{14}|e^{-(\bar{A}_{13})^{(1)}t}\)

\(|(b_i^0)^{(1)}(G, t) - (b_i^0)^{(1)}(G, t)| \leq (\bar{K}_{13})^{(1)}|G - G'|e^{-(\bar{A}_{13})^{(1)}t}\)

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i^0)^{(1)}(T_{14}, t)\) and \((a_i^0)^{(1)}(T_{14}, t)\).

\(\text{.}(T_{14}, t)\) and \((T_{14}, t)\) are points belonging to the interval \([\bar{K}_{13})^{(1)}, (\bar{M}_{13})^{(1)}\] . It is to be noted that \((a_i^0)^{(1)}(T_{14}, t)\)
is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{13})^{(1)} = 1\) then the function \((a_{1}''(1)(T_{14}, t))\), the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

**Definition of** \((\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}\):

(D) \((\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}\), are positive constants

\[
\frac{(a_{1})^{(1)}}{(\hat{M}_{13})^{(1)}} \cdot \frac{(b_{1})^{(1)}}{(\hat{M}_{13})^{(1)}} < 1
\]

**Definition of** \((\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}\):

(E) There exists two constants \((\hat{P}_{13})^{(1)}\) and \((\hat{Q}_{13})^{(1)}\) which together with \((\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}\) and \((\hat{B}_{13})^{(1)}\) and the constants \((a_{1})^{(1)}, (a_{1}')^{(1)}, (b_{1})^{(1)}, (b_{1}')^{(1)}, (p_{1})^{(1)}, (r_{1})^{(1)}, i = 13, 14, 15,\) satisfy the inequalities

\[
\frac{1}{(\hat{M}_{13})^{(1)}}[(a_{1})^{(1)} + (a_{1}')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1
\]

\[
\frac{1}{(\hat{M}_{13})^{(1)}}[(b_{1})^{(1)} + (b_{1}')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1
\]
\[
\frac{dT_{38}}{dt} = \left( b_{38}^{(7)}(T_{37}, t) - \left( b_{38}^{(7)}(T_{37}, t) - \left( b_{38}^{(7)}(G_{19}, t) - \left( b_{18}^{(7)}(G_{19}, t) - \left( b_{26}^{(7)}(G_{14}, t) \right) \right) \right) \right) \right) T_{38}
\]

\[+(a_{36}^{(7)}(T_{37}, t) = \text{First augmentation factor}\]

\[(1)(a_i^{(2)}, a_j^{(2)}, a_{11}^{(2)}, b_i^{(2)}, b_j^{(2)}, b_{11}^{(2)}) > 0, \quad i, j = 16, 17, 18\]

\[(F) \quad (2) \text{The functions (} a_i^{(2)}, b_i^{(2)} \text{ are positive continuous increasing and bounded.}\]

**Definition of (p_i)^{(2)}, (r_j)^{(2)}:**

\[a_i^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{i6})^{(2)}\]

\[b_i^{(2)}(G_{19}, t) \leq (r_j)^{(2)} \leq (\hat{B}_{i6})^{(2)}\]

\[(G) \quad (3) \lim_{T_2 \to 0} a_i^{(2)}(T_{17}, t) = (p_i)^{(2)}\]

\[\lim_{G \to 0} b_i^{(2)}(G_{19}, t) = (r_j)^{(2)}\]

**Definition of (\hat{A}_{i6})^{(2)}, (\hat{B}_{i6})^{(2)}:**

Where \[(\hat{A}_{i6})^{(2)}, (\hat{B}_{i6})^{(2)}, (p_i)^{(2)}, (r_j)^{(2)}\] are positive constants and \[i = 16, 17, 18\]

They satisfy Lipschitz condition:

\[|(a_i^{(2)}(T_{17}, t) - (a_i^{(2)}(T_{17}, t)| \leq (\hat{\kappa}_{16})^{(2)}|T_{17} - T_{17}|e^{-(\hat{A}_{i6})^{(2)}t}\]

\[|(b_i^{(2)}(G_{19}, t) - (b_i^{(2)}(G_{19}, t)| < (\hat{\kappa}_{16})^{(2)}||G_{19} - (G_{19})'||e^{-(\hat{A}_{i6})^{(2)}t}\]

With the Lipschitz condition, we place a restriction on the behavior of functions (a_i^{(2)}(T_{17}, t) and (a_j^{(2)}(T_{17}, t). (T_{17}, t) \text{ And (T_{17}, t) are points belonging to the interval } [(\hat{\kappa}_{16})^{(2)}, (\hat{\kappa}_{16})^{(2)}]. \text{ It is to be noted that (a_i^{(2)}(T_{17}, t) is uniformly continuous. In the eventuality of the fact, that if (M_{16})^{(2)} = 1 \text{ then the function (a_i^{(2)}(T_{17}, t) , the SECOND augmentation coefficient would be absolutely continuous.}\]

**Definition of (\hat{M}_{16})^{(2)}, (\hat{\kappa}_{16})^{(2)}:**

\[\hat{M}_{16}^{(2)}, (\hat{\kappa}_{16})^{(2)}, \text{ are positive constants}\]

\[\frac{(a_i)^{(2)}}{(\hat{A}_{i6})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{A}_{i6})^{(2)}} < 1\]

**Definition of (\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}:**

There exists two constants (\hat{P}_{13})^{(2)} and (\hat{Q}_{13})^{(2)} which together with (\hat{M}_{16})^{(2)}, (\hat{\kappa}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)} and the constants (a_i)^{(2)}, (a_j)^{(2)}, (b_i)^{(2)}, (b_j)^{(2)}, (p_i)^{(2)}, (r_j)^{(2)} i = 16, 17, 18,
satisfy the inequalities
\[
\frac{1}{(\mathcal{M}_{16})^2} \left[ (a_i)^{(2)} + (a'_i)^{(2)} + (\mathcal{A}_{16})^{(2)} + (\mathcal{P}_{16})^{(2)} (\mathcal{K}_{16})^{(2)} \right] < 1
\]
\[
\frac{1}{(\mathcal{K}_{16})^2} \left[ (b_j)^{(2)} + (b'_j)^{(2)} + (\mathcal{B}_{16})^{(2)} + (\mathcal{Q}_{16})^{(2)} (\mathcal{K}_{16})^{(2)} \right] < 1
\]
Where we suppose

\( (5) \quad (a_i)^{(3)}, (a'_i)^{(3)}, (a''_i)^{(3)}, (b_j)^{(3)}, (b'_j)^{(3)}, (b''_j)^{(3)} > 0, \quad i, j = 20, 21, 22 \)

The functions \((a''_i)^{(3)}, (b''_j)^{(3)}\) are positive continuous increasing and bounded.

**Definition of** \((p_i)^{(3)}, (r_j)^{(3)}\):

\[
(a''_i)^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\mathcal{A}_{20})^{(3)}
\]
\[
(b''_j)^{(3)}(G_{23}, t) \leq (r_j)^{(3)} \leq (\mathcal{B}_{20})^{(3)}
\]
\[
\lim_{t_2 \to \infty} (a''_i)^{(3)}(T_{21}, t) = (p_i)^{(3)}
\]
\[
\lim_{G \to \infty} (b''_j)^{(3)}(G_{23}, t) = (r_j)^{(3)}
\]

**Definition of** \((\mathcal{A}_{20})^{(3)}, (\mathcal{B}_{20})^{(3)}\):

Where \([ (\mathcal{A}_{20})^{(3)}, (\mathcal{B}_{20})^{(3)}, (p_i)^{(3)}, (r_j)^{(3)} ] \) are positive constants and \( i = 20, 21, 22 \)

They satisfy Lipschitz condition:

\[
| (a''_i)^{(3)}(T_{21}', t) - (a''_i)^{(3)}(T_{21}, t) | \leq (\mathcal{A}_{20})^{(3)} |T_{21} - T_{21}'| e^{-((\mathcal{A}_{20})^{(3)}) t}
\]
\[
| (b''_j)^{(3)}(G_{23}', t) - (b''_j)^{(3)}(G_{23}, t) | \leq (\mathcal{B}_{20})^{(3)} |G_{23} - G_{23}'| e^{-((\mathcal{B}_{20})^{(3)}) t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a''_i)^{(3)}(T_{21}, t)\)

and \((a''_i)^{(3)}(T_{21}', t)\). And \((T_{21}, t)\) are points belonging to the interval \([ (\mathcal{A}_{20})^{(3)}, (\mathcal{B}_{20})^{(3)} ] \). It is to be noted that \((a''_i)^{(3)}(T_{21}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\mathcal{M}_{20})^{(3)} = 1\) then the function \((a''_i)^{(3)}(T_{21}, t)\), the THIRD augmentation coefficient, would be absolutely continuous.

**Definition of** \((\mathcal{M}_{20})^{(3)}, (\mathcal{K}_{20})^{(3)}\):

\( (6) \quad (\mathcal{M}_{20})^{(3)}, (\mathcal{K}_{20})^{(3)}, \) are positive constants

\[
\frac{(a_i)^{(3)}}{(\mathcal{M}_{20})^{(3)}} < \frac{(a'_i)^{(3)}}{(\mathcal{K}_{20})^{(3)}} < 1
\]

There exists two constants \((\mathcal{P}_{20})^{(3)}\) and \((\mathcal{Q}_{20})^{(3)}\) which together with \((\mathcal{M}_{20})^{(3)}, (\mathcal{K}_{20})^{(3)}, (\mathcal{A}_{20})^{(3)}\), \((\mathcal{B}_{20})^{(3)}\) and the constants \((a_i)^{(3)}, (a'_i)^{(3)}, (b_j)^{(3)}, (b'_j)^{(3)}, (p_i)^{(3)}, (r_j)^{(3)}, i = 20, 21, 22\), satisfy the inequalities

\[
\frac{1}{(\mathcal{M}_{20})^{(3)}} \left[ (a_i)^{(3)} + (a'_i)^{(3)} + (\mathcal{A}_{20})^{(3)} + (\mathcal{P}_{20})^{(3)} (\mathcal{K}_{20})^{(3)} \right] < 1
\]
\[
\frac{1}{(\mathcal{M}_{20})^{(3)}} \left[ (b_j)^{(3)} + (b'_j)^{(3)} + (\mathcal{B}_{20})^{(3)} + (\mathcal{Q}_{20})^{(3)} (\mathcal{K}_{20})^{(3)} \right] < 1
\]
Where we suppose

\( (a_i^{(4)}, a_j^{(4)}, a_{ii}^{(4)}, a_{jj}^{(4)}, b_i^{(4)}, b_j^{(4)}, b_{ii}^{(4)}, b_{jj}^{(4)}) > 0, \quad i, j = 24, 25, 26 \)  

(L) \hspace{1cm} (7) \hspace{1cm} The functions \((a_i^{(4)}), (b_i^{(4)})\) are positive continuous increasing and bounded.

**Definition of** \((p_i^{(4)}), (r_i^{(4)})\):

\[
(a_i^{(4)})(T_{25}, t) \leq (p_i^{(4)}) \leq (\hat{A}_{24})^{(4)}
\]
\[
(b_i^{(4)})(G_{27}, t) \leq (r_i^{(4)}) \leq (\hat{B}_{24})^{(4)}
\]

(M) \hspace{1cm} (8) \hspace{1cm} \lim_{T_{25} \to \infty}(a_i^{(4)})(T_{25}, t) = (p_i^{(4)})
\lim_{G_{27} \to \infty}(b_i^{(4)})(G_{27}, t) = (r_i^{(4)})

**Definition of** \((\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}\):

Where \((\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i^{(4)}), (r_i^{(4)})\) are positive constants and \(i = 24, 25, 26\)

They satisfy Lipschitz condition:

\[
|a_i^{(4)}(T_{25}, t) - a_i^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}^*|e^{-((\hat{A}_{24})^{(4)})t}
\]
\[
|b_i^{(4)}(G_{27}, t) - b_i^{(4)}(G_{27}, t)| \leq (\hat{k}_{24})^{(4)}|G_{27} - G_{27}^*|e^{-((\hat{B}_{24})^{(4)})t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i^{(4)})(T_{25}, t)\) and \((b_i^{(4)})(G_{27}, t)\). And \((T_{25}, t)\) and \((G_{27}, t)\) are points belonging to the interval \([ (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)}]\). It is to be noted that \((a_i^{(4)})(T_{25}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{A}_{24})^{(4)} = 4\) then the function \((a_i^{(4)})(T_{25}, t)\), the FOURTH augmentation coefficient WOULD be absolutely continuous.

**Definition of** \((\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}\):

\[
(\hat{M}_{24})^{174}_{(4), (\hat{k}_{24})^{(4)}}, \quad \text{are positive constants}
\]

\[
\frac{(a_i^{(4)})}{(\hat{M}_{24})^{(4)}}, \frac{(b_i^{(4)})}{(\hat{M}_{24})^{(4)}} \leq 1
\]

**Definition of** \((\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}\):

(P) \hspace{1cm} (9) \hspace{1cm} There exists two constants \((\hat{P}_{24})^{(4)}\) and \((\hat{Q}_{24})^{(4)}\) which together with \((\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)}\) and \((\hat{B}_{24})^{(4)}\) and the constants \((a_i^{(4)}), (a_j^{(4)}), (b_i^{(4)}), (b_j^{(4)}), (p_i^{(4)}), (r_i^{(4)}), i = 24, 25, 26\), satisfy the inequalities

\[
1 \frac{1}{(\hat{M}_{24})^{(4)}}[(a_i^{(4)}) + (a_j^{(4)}) + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1
\]
\[
1 \frac{1}{(\hat{M}_{24})^{(4)}}[(b_i^{(4)}) + (b_j^{(4)}) + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1
\]
Where we suppose

\[(a_j^{(5)}, (a_j^{'(5)}, (a_j^{''(5)}, (b_j^{(5)}, (b_j^{'(5)}, (b_j^{''(5)}) > 0, \quad i, j = 28, 29, 30\]

(10) The functions \((a_j^{''(5)}, (b_j^{''(5)})\) are positive continuous increasing and bounded.

**Definition of** \((p_i^{(5)}, (r_i^{(5)})\):

\[
(a_i^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (A_{29})^{(5)}
\]

\[
(b_i^{(5)}((G_{31}, t) \leq (r_i)^{(5)} \leq (B_{28})^{(5)}
\]

(11) \(\lim_{T_2 \rightarrow 00}(a_i^{(5)}(T_{29}, t) = (p_i)^{(5)}\)

\[\lim_{G \rightarrow 00}(b_i^{(5)}((G_{31}, t) = (r_i)^{(5)}\]

**Definition of** \((A_{29})^{(5)}, (B_{28})^{(5)}\):

Where \((A_{29})^{(5)}, (B_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}\) are positive constants and \(i = 28, 29, 30\)

They satisfy Lipshitz condition:

\[|(a_i^{(5)}(T_{29}, t) - (a_i^{(5)}(T_{29}, t)| \leq (K_{28})^{(5)}|T_{29} - T_{29}'|e^{-(M_{28})^{(5)}t}
\]

\[|(b_i^{(5)}((G_{31}, t) - (b_i^{(5)}((G_{31}, t)| \leq (K_{28})^{(5)}|((G_{31}) - (G_{31}))|e^{-(M_{28})^{(5)}t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i^{''(5)}(T_{29}, t)\)

and \((a_i^{''(5)}(T_{29}, t) . (T_{29}, t)\) and \((T_{29}, t)\) are points belonging to the interval \(\{(K_{28})^{(5)}, (M_{28})^{(5)}\}\). It is to be noted that \((a_i^{''(5)}(T_{29}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((M_{28})^{(5)} = S\) then the function \((a_i^{''(5)}(T_{29}, t)\), the FIFTH augmentation coefficient attributable would be absolutely continuous.

**Definition of** \((M_{28})^{(5)}, (K_{28})^{(5)}\):

\((M_{28})^{(5)}, (K_{28})^{(5)}, (a_{(5)}, (b_{(5)})\), \quad \frac{M_{28}}{(M_{28})^{(5)}}, \frac{K_{28}}{(M_{28})^{(5)}} < 1\]

**Definition of** \((P_{28})^{(5)}, (Q_{28})^{(5)}\):

There exists two constants \((P_{28})^{(5)}\) and \((Q_{28})^{(5)}\) which together with \((M_{28})^{(5)}, (K_{28})^{(5)}, (A_{29})^{(5)}\) and \((B_{28})^{(5)}\) and the constants \((a_{(5)}, (a_{(5)}), (b_{(5)}), (b_{(5)}), (p_{(5)}), (r_{(5)}), i = 28, 29, 30, \ satisfy the inequalities\)

\[\frac{1}{(M_{28})^{(5)}} \left[ (a_j^{(5)} + (a_j^{(5)} + (A_{29})^{(5}} + (P_{28})^{(5}} (K_{28})^{(5}} \right] < 1\]

\[\frac{1}{(M_{28})^{(5)}} \left[ (b_j^{(5)} + (b_j^{(5)} + (B_{28})^{(5}} + (Q_{28})^{(5}} (K_{28})^{(5}} \right] < 1\]

Where we suppose

\[(a_j)^{(6)}, (a_j)^{(6)}, (a_j^{''(6)}, (b_j)^{(6)}, (b_j)^{(6)}, (b_j^{''(6)}) > 0, \quad i, j = 32, 33, 34\]

(12) The functions \((a_j^{''(6)}, (b_j^{''(6)})\) are positive continuous increasing and bounded.
Definition of \((p_i)^{(6)}\), \((r_i)^{(6)}\):

\[
(a_i^{(6)})(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}
\]

\[
(b_i^{(6)})(G_{33}, t) \leq (r_i)^{(6)} \leq (\hat{B}_{32})^{(6)}
\]

(13) \[
\lim_{T_{33} \to \infty} (a_i^{(6)})(T_{33}, t) = (p_i)^{(6)}
\]

\[
\lim_{G_{33} \to \infty} (b_i^{(6)})(G_{33}, t) = (r_i)^{(6)}
\]

Definition of \((\hat{A}_{32})^{(6)}\), \((\hat{B}_{32})^{(6)}\):

Where \((\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}\) are positive constants and \(i = 32, 33, 34\)

They satisfy Lipschitz condition:

\[
|((a_i)^{(6)})(T_{33}, t) - (a_i^{(6)})(T_{33}, t)| \leq (\hat{K}_{32})^{(6)}|T_{33} - T_{33}^{*}|e^{-((\hat{B}_{32})^{(6)})t}
\]

\[
|((b_i)^{(6)})(G_{33}, t) - (b_i^{(6)})(G_{33}, t)| \leq (\hat{K}_{32})^{(6)}|G_{33} - G_{33}^{*}|e^{-((\hat{B}_{32})^{(6)})t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i^{(6)})(T_{33}, t)\) and \((b_i^{(6)})(T_{33}, t)\). \((T_{33}, t)\) and \((T_{33}, t)\) are points belonging to the interval \([\hat{K}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\]. It is to be noted that \((a_i^{(6)})(T_{33}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{32})^{(6)} = 6\) then the function \((a_i^{(6)})(T_{33}, t)\), the SIXTH augmentation coefficient, would be absolutely continuous.

Definition of \((\hat{M}_{32})^{(6)}, (\hat{K}_{32})^{(6)}\):

\[
(\hat{M}_{32})^{(6)}, (\hat{K}_{32})^{(6)},\text{ are positive constants}
\]

\[
\frac{(a_i^{(6)})}{(M_{32})^{(6)}} - \frac{(b_i^{(6)})}{(M_{32})^{(6)}} < 1
\]

Definition of \((\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}\):

There exist two constants \((\hat{P}_{32})^{(6)}\) and \((\hat{Q}_{32})^{(6)}\) which together with \((\hat{M}_{32})^{(6)}, (\hat{K}_{32})^{(6)}, (\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}\) and the constants \((a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}\), \(i = 32, 33, 34\), satisfy the inequalities

\[
\frac{1}{(M_{32})^{(6)}}[ (a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{K}_{32})^{(6)}] < 1
\]

\[
\frac{1}{(M_{32})^{(6)}}[ (b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} + (\hat{K}_{32})^{(6)}] < 1
\]

Where we suppose
The functions \((a_i)_{(7)}\) and \((b_i)_{(7)}\) are positive continuous, increasing and bounded.

**Definition of** \((p_i)_{(7)}\), \((r_i)_{(7)}\):

\[(a_i)_{(7)}(T_{37}, t) \leq (p_i)_{(7)} \leq (\hat{A}_{36})_{(7)}\]

\[(b_i)_{(7)}(G, t) \leq (r_i)_{(7)} \leq (b_i)_{(7)} \leq (\hat{B}_{36})_{(7)}\]

**Definition of** \((\hat{A}_{36})_{(7)}, (\hat{B}_{36})_{(7)}\):

Where \([\hat{A}_{36}]_{(7)}, [\hat{B}_{36}]_{(7)}, [p_i]_{(7)}, [r_i]_{(7)}\) are positive constants and \(i = 36, 37, 38\)

They satisfy the Lipschitz condition:

\[|(a_i)_{(7)}(T_{37}, t) - (a_i)_{(7)}(T_{37}, t)| \leq (\hat{k}_{36})_{(7)}|T_{37} - T_{37}^*|e^{-\theta_{36}}_{(7)}t\]

\[|(b_i)_{(7)}((G_{39}), t) - (b_i)_{(7)}(G_{39}, T_{39}))| < (\hat{k}_{36})_{(7)}||G_{39} - (G_{39})||e^{-\theta_{36}}_{(7)}t\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i)_{(7)}(T_{37}, t)\) and \((b_i)_{(7)}(T_{37}, t)\). \((T_{37}, t)\) and \((T_{37}, t)\) are points belonging to the interval \([\hat{k}_{36}]_{(7)}, (\hat{M}_{36})_{(7)}\). It is to be noted that \((a_i)_{(7)}(T_{37}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\theta_{36})_{(7)} = 7\) then the function \((a_i)_{(7)}(T_{37}, t)\) the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

**Definition of** \((\theta_{36})_{(7)}, (\hat{M}_{36})_{(7)}\):
(Y) \( (\tilde{M}_{36})^{(7)}, (\tilde{k}_{36})^{(7)} \) are positive constants

\[
\left(\frac{a_{ij}^{(7)}}{M_{36}^{(7)}}, \frac{b_{ij}^{(7)}}{M_{36}^{(7)}}\right) < 1
\]

**Definition of** \((\tilde{p}_{36})^{(7)}, (\tilde{q}_{36})^{(7)}\):  

(Z) There exists two constants \((\tilde{p}_{36})^{(7)}\) and \((\tilde{q}_{36})^{(7)}\) which together with \((\tilde{M}_{36})^{(7)}, (\tilde{k}_{36})^{(7)}, (\tilde{A}_{36})^{(7)}\) and \((\tilde{B}_{36})^{(7)}\) and the constants \((a_{ij})^{(7)}, (a_{ij}^{(7)}), (b_{ij})^{(7)}, (b_{ij}^{(7)}), (p_{ij})^{(7)}, (r_{ij})^{(7)}\), \(i = 36, 37, 38\), satisfy the inequalities

\[
\frac{1}{(\tilde{M}_{36})^{(7)}} [(a_{ij})^{(7)} + (a_{ij})^{(7)} + (\tilde{A}_{36})^{(7)} + (\tilde{p}_{36})^{(7)} (\tilde{k}_{36})^{(7)}] < 1
\]

\[
\frac{1}{(\tilde{M}_{36})^{(7)}} [(b_{ij})^{(7)} + (b_{ij})^{(7)} + (\tilde{B}_{36})^{(7)} + (\tilde{q}_{36})^{(7)} (\tilde{k}_{36})^{(7)}] < 1
\]

**Definition of** \(G_{i}(0), T_{i}(0)\):

\(G_{i}(t) \leq (\tilde{P}_{28})^{(5)} e^{(\tilde{M}_{28})^{(5)} t}, \quad G_{i}(0) = G_{i}^{0} > 0\)

\(T_{i}(t) \leq (\tilde{Q}_{28})^{(5)} e^{(\tilde{M}_{28})^{(5)} t}, \quad T_{i}(0) = T_{i}^{0} > 0\)

**Definition of** \(G_{i}(0), T_{i}(0)\):

\(G_{i}(t) \leq (\tilde{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}, \quad G_{i}(0) = G_{i}^{0} > 0\)

\(T_{i}(t) \leq (\tilde{Q}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}, \quad T_{i}(0) = T_{i}^{0} > 0\)

========================================================================
\[ G_j(t) \leq (\mathcal{P}_{36})^{(t)}e^{(\mathcal{M}_{36})^{(t)}}t \quad , \quad G_j(0) = G_j^0 > 0 \]
\[ T_i(t) \leq (\mathcal{Q}_{36})^{(t)}e^{(\mathcal{M}_{36})^{(t)}}t \quad , \quad T_i(0) = T_i^0 > 0 \]

**Proof:** Consider operator \( \mathcal{A}^{(1)} \) defined on the space of sextuples of continuous functions \( G_i, T_i : \mathbb{R}_+ \to \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G_i^0 , \quad T_i(0) = T_i^0 \quad , \quad G_i \leq (\mathcal{P}_{13})^{(1)} , T_i \leq (\mathcal{Q}_{13})^{(1)}, \]
\[ 0 \leq G_i(t) - G_i^0 \leq (\mathcal{P}_{13})^{(1)}e^{(\mathcal{M}_{13})^{(t)}}, \]
\[ 0 \leq T_i(t) - T_i^0 \leq (\mathcal{Q}_{13})^{(1)}e^{(\mathcal{M}_{13})^{(t)}}, \]

By

\[ \bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)}T_{14}(s_{13}) - \left( (a_{13})^{(1)} + (a_{13})^{(1)} \right)(T_{14}(s_{13}), s_{13}) \right] dS_{13} \]
\[ \bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)}G_{13}(s_{13}) - \left( (a_{14})^{(1)} + (a_{14})^{(1)} \right)(G(s_{13}), s_{13}) \right] dS_{13} \]
\[ \bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)}G_{14}(s_{13}) - \left( (a_{15})^{(1)} + (a_{15})^{(1)} \right)(G(s_{13}), s_{13}) \right] dS_{13} \]
\[ \bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)}G_{14}(s_{13}) - \left( (b_{13})^{(1)} + (b_{13})^{(1)} \right)(G(s_{13}), s_{13}) \right] dS_{13} \]
\[ \bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)}T_{13}(s_{13}) - \left( (b_{14})^{(1)} + (b_{14})^{(1)} \right)(T(s_{13}), s_{13}) \right] dS_{13} \]
\[ \bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)}T_{14}(s_{13}) - \left( (b_{15})^{(1)} + (b_{15})^{(1)} \right)(T(s_{13}), s_{13}) \right] dS_{13} \]

Where \( s_{13} \) is the integrand that is integrated over an interval \((0, t)\)

if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

**Definition of** \( G_i(0) , T_i(0) : \)

\[ G_i(t) \leq (\mathcal{P}_{36})^{(t)}e^{(\mathcal{M}_{36})^{(t)}}, \quad G_i(0) = G_i^0 > 0 \]
\[ T_i(t) \leq (\mathcal{Q}_{36})^{(t)}e^{(\mathcal{M}_{36})^{(t)}}, \quad T_i(0) = T_i^0 > 0 \]

Consider operator \( \mathcal{A}^{(7)} \) defined on the space of sextuples of continuous functions \( G_i, T_i : \mathbb{R}_+ \to \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G_i^0 , \quad T_i(0) = T_i^0 \quad , \quad G_i \leq (\mathcal{P}_{36})^{(7)}, T_i \leq (\mathcal{Q}_{36})^{(7)}, \]
\[ 0 \leq G_i(t) - G_i^0 \leq (\mathcal{P}_{36})^{(7)}e^{(\mathcal{M}_{36})^{(t)}}, \]
\[0 \leq T_i(t) - T_i^0 \leq (\tilde{Q}_{3i})^{(7)} e^{(\tilde{Q}_{3i})^{(7)} t}\]

By

\[\tilde{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + (a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}\]

\[\tilde{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}\]

\[\tilde{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}\]

\[\tilde{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}\]

\[\tilde{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}\]

\[\tilde{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}\]

Where \(s_{(36)}\) is the integrand that is integrated over an interval \((0, t)\)
Consider operator $A^{(2)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+^0 \to \mathbb{R}_+^0$ which satisfy

\begin{align*}
G_i(0) &= G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\tilde{P}_{16})^{(2)}, \quad T_i^0 \leq (\tilde{Q}_{16})^{(2)}, \\
0 &\leq G_i(t) - G_i^0 \leq (\tilde{P}_{16})^{(2)}e^{(\beta_{16})^{(2)}t}, \\
0 &\leq T_i(t) - T_i^0 \leq (\tilde{Q}_{16})^{(2)}e^{(\beta_{16})^{(2)}t}.
\end{align*}

By

\begin{align*}
\tilde{g}_{16}(t) &= \tilde{G}_{16}^0 + \int_0^t \left[ (a_{16})^{(2)}G_{17}(s_{16}) - \left( (a'_{16})^{(2)} + (a''_{16})^{(2)} \right)(T_{17}(s_{16}), s_{16}) \right] G_{16}(s_{16}) ds_{16} \\
\tilde{g}_{17}(t) &= \tilde{G}_{17}^0 + \int_0^t \left[ (a_{17})^{(2)}G_{16}(s_{16}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} \right)(T_{17}(s_{16}), s_{16}) \right] G_{17}(s_{16}) ds_{16} \\
\tilde{g}_{18}(t) &= \tilde{G}_{18}^0 + \int_0^t \left[ (a_{18})^{(2)}G_{17}(s_{16}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} \right)(T_{17}(s_{16}), s_{16}) \right] G_{18}(s_{16}) ds_{16} \\
\tilde{t}_{16}(t) &= \tilde{T}_{16}^0 + \int_0^t \left[ (b_{16})^{(2)}T_{17}(s_{16}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)} \right)(G(s_{16}), s_{16}) \right] T_{16}(s_{16}) ds_{16} \\
\tilde{t}_{17}(t) &= \tilde{T}_{17}^0 + \int_0^t \left[ (b_{17})^{(2)}T_{16}(s_{16}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} \right)(G(s_{16}), s_{16}) \right] T_{17}(s_{16}) ds_{16} \\
\tilde{t}_{18}(t) &= \tilde{T}_{18}^0 + \int_0^t \left[ (b_{18})^{(2)}T_{17}(s_{16}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} \right)(G(s_{16}), s_{16}) \right] T_{18}(s_{16}) ds_{16}
\end{align*}

Where $s_{16}$ is the integrand that is integrated over an interval $(0,t)$

Consider operator $A^{(3)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+^0 \to \mathbb{R}_+^0$ which satisfy

\begin{align*}
G_i(0) &= G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\tilde{P}_{20})^{(3)}, \quad T_i^0 \leq (\tilde{Q}_{20})^{(3)}, \\
0 &\leq G_i(t) - G_i^0 \leq (\tilde{P}_{20})^{(3)}e^{(\beta_{20})^{(3)}t}, \\
0 &\leq T_i(t) - T_i^0 \leq (\tilde{Q}_{20})^{(3)}e^{(\beta_{20})^{(3)}t}.
\end{align*}

By

\begin{align*}
\tilde{g}_{20}(t) &= \tilde{G}_{20}^0 + \int_0^t \left[ (a_{20})^{(3)}G_{21}(s_{20}) - \left( (a'_{20})^{(3)} + (a''_{20})^{(3)} \right)(T_{21}(s_{20}), s_{20}) \right] G_{20}(s_{20}) ds_{20} \\
\tilde{g}_{21}(t) &= \tilde{G}_{21}^0 + \int_0^t \left[ (a_{21})^{(3)}G_{20}(s_{20}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} \right)(T_{21}(s_{20}), s_{20}) \right] G_{21}(s_{20}) ds_{20} \\
\tilde{g}_{22}(t) &= \tilde{G}_{22}^0 + \int_0^t \left[ (a_{22})^{(3)}G_{21}(s_{20}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} \right)(T_{21}(s_{20}), s_{20}) \right] G_{22}(s_{20}) ds_{20} \\
\tilde{t}_{20}(t) &= \tilde{T}_{20}^0 + \int_0^t \left[ (b_{20})^{(3)}T_{21}(s_{20}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} \right)(G(s_{20}), s_{20}) \right] T_{20}(s_{20}) ds_{20}.
\end{align*}
\[ \bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[ (b_{21})^{(3)}(s_{20}) - \left( (b_{21})^{(3)} - (b_{21})^{(3)}(G(s_{20}), s_{20}) \right) T_{21}(s_{20}) \right] ds_{20} \]

\[ \bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{20}) - \left( (b_{22})^{(3)} - (b_{22})^{(3)}(G(s_{20}), s_{20}) \right) T_{22}(s_{20}) \right] ds_{20} \]

Where \( s_{20} \) is the integrand that is integrated over an interval \((0, t)\).

Consider operator \( \mathcal{A}^{(4)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\bar{P}_{24})^{(4)}, \quad T_i^0 \leq (\bar{Q}_{24})^{(4)}, \]

\[ 0 \leq G_i(t) - G_i^0 \leq (\bar{P}_{24})^{(4)}e^{(\bar{G}_{24})^{(4)}t} \]

\[ 0 \leq T_i(t) - T_i^0 \leq (\bar{Q}_{24})^{(4)}e^{(\bar{G}_{24})^{(4)}t} \]

By

\[ \bar{g}_{24}(t) = G_{24}^0 + \int_0^t \left[ (a_{24})^{(4)}G_{25}(s_{24}) - \left( (a_{24})^{(4)} + (a_{24})^{(4)}(T_{25}(s_{24}), s_{24}) \right) G_{25}(s_{24}) \right] ds_{24} \]

\[ \bar{g}_{25}(t) = G_{25}^0 + \int_0^t \left[ (a_{25})^{(4)}G_{24}(s_{24}) - \left( (a_{25})^{(4)} + (a_{25})^{(4)}(T_{25}(s_{24}), s_{24}) \right) G_{24}(s_{24}) \right] ds_{24} \]

\[ \bar{g}_{26}(t) = G_{26}^0 + \int_0^t \left[ (a_{26})^{(4)}G_{25}(s_{24}) - \left( (a_{26})^{(4)} + (a_{26})^{(4)}(T_{25}(s_{24}), s_{24}) \right) G_{25}(s_{24}) \right] ds_{24} \]

\[ \bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[ (b_{24})^{(4)}T_{25}(s_{24}) - \left( (b_{24})^{(4)} - (b_{24})^{(4)}(G(s_{24}), s_{24}) \right) T_{25}(s_{24}) \right] ds_{24} \]

\[ \bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[ (b_{25})^{(4)}T_{24}(s_{24}) - \left( (b_{25})^{(4)} - (b_{25})^{(4)}(G(s_{24}), s_{24}) \right) T_{24}(s_{24}) \right] ds_{24} \]

\[ \bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[ (b_{26})^{(4)}T_{25}(s_{24}) - \left( (b_{26})^{(4)} - (b_{26})^{(4)}(G(s_{24}), s_{24}) \right) T_{25}(s_{24}) \right] ds_{24} \]

Where \( s_{24} \) is the integrand that is integrated over an interval \((0, t)\).

Consider operator \( \mathcal{A}^{(5)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\bar{P}_{28})^{(5)}, \quad T_i^0 \leq (\bar{Q}_{28})^{(5)}, \]

\[ 0 \leq G_i(t) - G_i^0 \leq (\bar{P}_{28})^{(5)}e^{(\bar{G}_{28})^{(5)}t} \]

\[ 0 \leq T_i(t) - T_i^0 \leq (\bar{Q}_{28})^{(5)}e^{(\bar{G}_{28})^{(5)}t} \]

By

\[ \bar{g}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)}G_{29}(s_{28}) - \left( (a_{28})^{(5)} + (a_{28})^{(5)}(T_{29}(s_{28}), s_{28}) \right) G_{29}(s_{28}) \right] ds_{28} \]

\[ \bar{g}_{29}(t) = G_{29}^0 + \int_0^t \left[ (a_{29})^{(5)}G_{28}(s_{28}) - \left( (a_{29})^{(5)} + (a_{29})^{(5)}(T_{29}(s_{28}), s_{28}) \right) G_{28}(s_{28}) \right] ds_{28} \]

\[ \bar{g}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30})^{(5)}G_{29}(s_{28}) - \left( (a_{30})^{(5)} + (a_{30})^{(5)}(T_{29}(s_{28}), s_{28}) \right) G_{29}(s_{28}) \right] ds_{28} \]
\[
\tilde{T}_{28}(t) = T_{28}^0 + \int_0^t \left[ (b_{28}^{(s)}(s_{28})) T_{28}(s_{28}) - \left( (b_{28}^{(s)}(s_{28})) - (b_{28}^{(s)}(s_{28})) (G(s_{28}), s_{28}) \right) T_{28}(s_{28}) \right] ds_{28}
\]

\[
\tilde{T}_{29}(t) = T_{29}^0 + \int_0^t \left[ (b_{29}^{(s)}(s_{29})) T_{29}(s_{29}) - \left( (b_{29}^{(s)}(s_{29})) - (b_{29}^{(s)}(s_{29})) (G(s_{29}), s_{29}) \right) T_{29}(s_{29}) \right] ds_{29}
\]

\[
\tilde{T}_{30}(t) = T_{30}^0 + \int_0^t \left[ (b_{30}^{(s)}(s_{30})) T_{30}(s_{30}) - \left( (b_{30}^{(s)}(s_{30})) - (b_{30}^{(s)}(s_{30})) (G(s_{30}), s_{30}) \right) T_{30}(s_{30}) \right] ds_{30}
\]

Where \( s_{28}, s_{29}, s_{30} \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( \mathcal{A}^{(6)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+ \) which satisfy

\[
G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i \leq (\tilde{p}_{32})^{(6)}, \quad T_i \leq (\tilde{q}_{32})^{(6)},
\]

\[
0 \leq G_i(t) - G_i^0 \leq (\tilde{p}_{32})^{(6)} e^{(\mathcal{A}_{32})^{(6)} t}
\]

\[
0 \leq T_i(t) - T_i^0 \leq (\tilde{q}_{32})^{(6)} e^{(\mathcal{A}_{32})^{(6)} t}
\]

By

\[
\tilde{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32}^{(s)}(s_{32})) G_{32}(s_{32}) - \left( (a_{32}^{(s)}(s_{32})) + (a_{32}^{(s)}(s_{32})) (G(s_{32}), s_{32}) \right) G_{32}(s_{32}) \right] ds_{32}
\]

\[
\tilde{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33}^{(s)}(s_{33})) G_{33}(s_{33}) - \left( (a_{33}^{(s)}(s_{33})) + (a_{33}^{(s)}(s_{33})) (G(s_{33}), s_{33}) \right) G_{33}(s_{33}) \right] ds_{33}
\]

\[
\tilde{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34}^{(s)}(s_{34})) G_{34}(s_{34}) - \left( (a_{34}^{(s)}(s_{34})) + (a_{34}^{(s)}(s_{34})) (G(s_{34}), s_{34}) \right) G_{34}(s_{34}) \right] ds_{34}
\]

\[
\tilde{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32}^{(s)}(s_{32})) T_{32}(s_{32}) - \left( (b_{32}^{(s)}(s_{32})) - (b_{32}^{(s)}(s_{32})) (G(s_{32}), s_{32}) \right) T_{32}(s_{32}) \right] ds_{32}
\]

\[
\tilde{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33}^{(s)}(s_{33})) T_{33}(s_{33}) - \left( (b_{33}^{(s)}(s_{33})) - (b_{33}^{(s)}(s_{33})) (G(s_{33}), s_{33}) \right) T_{33}(s_{33}) \right] ds_{33}
\]

\[
\tilde{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34}^{(s)}(s_{34})) T_{34}(s_{34}) - \left( (b_{34}^{(s)}(s_{34})) - (b_{34}^{(s)}(s_{34})) (G(s_{34}), s_{34}) \right) T_{34}(s_{34}) \right] ds_{34}
\]

Where \( s_{32}, s_{33}, s_{34} \) is the integrand that is integrated over an interval \((0, t)\)

: if the conditions IN THE FOREGOING are fulfilled, there exists a solution satisfying the conditions

\[
G_i(t) \leq (\tilde{p}_{32})^{(7)} e^{(\mathcal{A}_{36})^{(7)} t}, \quad G_i(0) = G_i^0 > 0
\]

\[
T_i(t) \leq (\tilde{q}_{32})^{(7)} e^{(\mathcal{A}_{36})^{(7)} t}, \quad T_i(0) = T_i^0 > 0
\]

**Proof:**

Consider operator \( \mathcal{A}^{(7)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+ \)
which satisfy

\[ G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\tilde{p}_{36})^{(7)}(T_i^0), \quad T_i^0 \leq (\tilde{q}_{36})^{(7)}, \]

\[ 0 \leq G_i(t) - G_i^0 \leq (\tilde{p}_{36})^{(7)}e^{(\tilde{p}_{36})^{(7)}t} \]

\[ 0 \leq T_i(t) - T_i^0 \leq (\tilde{q}_{36})^{(7)}e^{(\tilde{q}_{36})^{(7)}t} \]

By

\[ \tilde{g}_{36}(t) = G_{36}^0 + \int_0^t [(a_{36})^{(7)}G_{37}(s_{36}) - \left( (a_{36})^{(7)} + a_{36}^{*7} (T_{37}(s_{36}), s_{36}) \right) G_{36}(s_{36})] ds_{36} \]

\[ \tilde{g}_{37}(t) = G_{37}^0 + \int_0^t [(a_{37})^{(7)}G_{38}(s_{36}) - \left( (a_{37})^{(7)} + (a_{37})^{*7} (T_{38}(s_{36}), s_{36}) \right) G_{37}(s_{36})] ds_{36} \]

\[ \tilde{g}_{38}(t) = G_{38}^0 + \int_0^t [(a_{38})^{(7)}G_{39}(s_{36}) - \left( (a_{38})^{(7)} + a_{38}^{*7} (T_{39}(s_{36}), s_{36}) \right) G_{38}(s_{36})] ds_{36} \]

\[ T_{36}(t) = T_{36}^0 + \int_0^t [(b_{36})^{(7)}T_{36}(s_{36}) - \left( (b_{36})^{(7)} - (b_{36})^{*7} (G(s_{36}), s_{36}) \right) T_{36}(s_{36})] ds_{36} \]

\[ T_{37}(t) = T_{37}^0 + \int_0^t [(b_{37})^{(7)}T_{37}(s_{36}) - \left( (b_{37})^{(7)} - (b_{37})^{*7} (G(s_{36}), s_{36}) \right) T_{37}(s_{36})] ds_{36} \]

\[ T_{38}(t) = T_{38}^0 + \int_0^t [(b_{38})^{(7)}T_{38}(s_{36}) - \left( (b_{38})^{(7)} - (b_{38})^{*7} (G(s_{36}), s_{36}) \right) T_{38}(s_{36})] ds_{36} \]
Where $S_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

(a) The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself.

Indeed it is obvious that

\[
G_{24}(t) \leq G_{24}^{0} + \int_{0}^{t} \left[ (a_{24})^{(4)} \left( G_{25}^{0} + \left( \hat{P}_{24} \right)^{(4)}e^{\left( \hat{\theta}_{24} \right)^{(4)}x_{(24)}} \right) \right] dS_{(24)} = \\
\left( 1 + (a_{24})^{(4)}t \right)G_{25}^{0} + \frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{\theta}_{24})^{(4)}} \left( e^{(\hat{\theta}_{24})^{(4)}t} - 1 \right)
\]

From which it follows that

\[
(G_{24}(t) - G_{24}^{0}) e^{-\left( \hat{\theta}_{24} \right)^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(\hat{\theta}_{24})^{(4)}} \left( (\hat{P}_{24})^{(4)} + G_{25}^{0} \right) e^{\left( \hat{P}_{24} \right)^{(4)}x_{(24)}^{2}} + (\hat{P}_{24})^{(4)}
\]

$(G_{24}^{0})$ is as defined in the statement of theorem 1.

(b) The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself.

Indeed it is obvious that

\[
G_{28}(t) \leq G_{28}^{0} + \int_{0}^{t} \left[ (a_{28})^{(5)} \left( G_{29}^{0} + \left( \hat{P}_{28} \right)^{(5)}e^{\left( \hat{\theta}_{28} \right)^{(5)}x_{(28)}} \right) \right] dS_{(28)} = \\
\left( 1 + (a_{28})^{(5)}t \right)G_{29}^{0} + \frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{\theta}_{28})^{(5)}} \left( e^{(\hat{\theta}_{28})^{(5)}t} - 1 \right)
\]

From which it follows that

\[
(G_{28}(t) - G_{28}^{0}) e^{-\left( \hat{\theta}_{28} \right)^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{\theta}_{28})^{(5)}} \left( (\hat{P}_{28})^{(5)} + G_{29}^{0} \right) e^{\left( \hat{P}_{28} \right)^{(5)}x_{(28)}^{2}} + (\hat{P}_{28})^{(5)}
\]

$(G_{28}^{0})$ is as defined in the statement of theorem 1.

(c) The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself.

Indeed it is obvious that

\[
G_{32}(t) \leq G_{32}^{0} + \int_{0}^{t} \left[ (a_{32})^{(6)} \left( G_{33}^{0} + \left( \hat{P}_{32} \right)^{(6)}e^{\left( \hat{\theta}_{32} \right)^{(6)}x_{(32)}} \right) \right] dS_{(32)} = \\
\left( 1 + (a_{32})^{(6)}t \right)G_{33}^{0} + \frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{\theta}_{32})^{(6)}} \left( e^{(\hat{\theta}_{32})^{(6)}t} - 1 \right)
\]

From which it follows that

\[
(G_{32}(t) - G_{32}^{0}) e^{-\left( \hat{\theta}_{32} \right)^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{\theta}_{32})^{(6)}} \left( (\hat{P}_{32})^{(6)} + G_{33}^{0} \right) e^{\left( \hat{P}_{32} \right)^{(6)}x_{(32)}^{2}} + (\hat{P}_{32})^{(6)}
\]

$(G_{32}^{0})$ is as defined in the statement of theorem 1.
Analogous inequalities hold also for \( G_{25}, G_{26}, T_{24}, T_{25}, T_{26} \)

(d) The operator \( \mathcal{A}^{(7)} \) maps the space of functions satisfying 37,35,36 into itself. Indeed it is obvious that

\[
G_{36}(t) \leq G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} \left( G_{36}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \right) \right] ds_{36} = \\
(1 + (a_{36})^{(7)} t)G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left( e^{(\hat{M}_{36})^{(7)} t} - 1 \right)
\]

From which it follows that

\[
(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \leq \left( (\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{\left( -\frac{(\hat{P}_{36})^{(7)} t}{G_{37}^0} \right) + (\hat{P}_{36})^{(7)}}
\]

\( (G_i^0) \) is as defined in the statement of theorem 7

It is now sufficient to take \( \frac{(a_{i1})^{(1)}}{(\hat{M}_{13})^{(1)}} \), \( \frac{(b_{i1})^{(1)}}{(\hat{M}_{13})^{(1)}} < 1 \) and to choose \( (\hat{P}_{13})^{(1)} \) and \( (\hat{Q}_{13})^{(1)} \) large to have

\[
\frac{(a_{i1})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[ (\hat{P}_{13})^{(1)} + (\hat{P}_{13})^{(1)} + G_{37}^0 \right] e^{\left( -\frac{(\hat{P}_{13})^{(1)} + G_{37}^0}{G_{37}^0} \right) + (\hat{P}_{13})^{(1)}} \leq (\hat{P}_{13})^{(1)}
\]

\[
\frac{(b_{i1})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[ (\hat{Q}_{13})^{(1)} + T_{37}^0 \right] e^{\left( -\frac{(\hat{Q}_{13})^{(1)} + T_{37}^0}{T_{37}^0} \right) + (\hat{Q}_{13})^{(1)}} \leq (\hat{Q}_{13})^{(1)}
\]

In order that the operator \( \mathcal{A}^{(1)} \) transforms the space of sextuples of functions \( G_i, T_i \) satisfying GLOBAL EQUATIONS into itself

The operator \( \mathcal{A}^{(1)} \) is a contraction with respect to the metric

\[
d \left( (G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)}) \right) = \\
\sup_{i \in R^m} \max_{t \in R^*} \left| G_i^{(1)}(t) - G_i^{(2)}(t) e^{-(\hat{M}_{13})^{(1)} t} \right| \max_{i \in R^m} \left| T_i^{(1)}(t) - T_i^{(2)}(t) e^{-(\hat{M}_{13})^{(1)} t} \right|
\]

Indeed if we denote

**Definition of \( \bar{G}, \bar{T} \):**

\[
(\bar{G}, \bar{T}) = \mathcal{A}^{(1)}(G, T)
\]

It results
\[ |G_{13}^{(1)} - \bar{G}_{i}^{(2)}| \leq \int_{0}^{t} (a_{13})^{(1)} |G_{14}^{(1)} - G_{14}^{(2)}| e^{-\langle R_{13} \rangle^{(1)}(s_{13})} e^{\langle R_{13} \rangle^{(1)}(s_{13})} ds_{13} + \]

\[ \int_{0}^{t} (a_{13}'')^{(1)} G_{13}^{(1)} - G_{13}^{(2)}| e^{-\langle R_{13} \rangle^{(1)}(s_{13})} e^{\langle R_{13} \rangle^{(1)}(s_{13})} + \]

\[ (a_{13}'')^{(1)} (T_{14}^{(1)}, s_{13}) |G_{13}^{(1)} - G_{13}^{(2)}| e^{-\langle R_{13} \rangle^{(1)}(s_{13})} e^{\langle R_{13} \rangle^{(1)}(s_{13})} + \]

\[ G_{13}^{(2)} [(a_{13}'')^{(1)} (T_{14}^{(1)}, s_{13}) - (a_{13}'')^{(1)} (T_{14}^{(2)}, s_{13})] e^{-\langle R_{13} \rangle^{(1)}(s_{13})} e^{\langle R_{13} \rangle^{(1)}(s_{13})} ds_{13} \]

Where \( s_{13} \) represents integrand that is integrated over the interval \([0,t]\).

From the hypotheses it follows

\[ \left| G^{(1)} - G^{(2)} \right| e^{-\langle R_{13} \rangle^{(1)} t} \leq \]

\[ \frac{1}{\langle R_{13} \rangle^{(1)}} \left( (a_{13})^{(1)} + (a_{13})^{(1)} + (A_{13})^{(1)} + (P_{13})^{(1)}(\bar{k}_{13})^{(1)} \right) d \left( (G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}) \right) \]

And analogous inequalities for \( G_{i} \) and \( T_{i} \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{13}'')^{(1)}\) and \((b_{13}')^{(1)}\) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate a condition necessary to prove the uniqueness of the solution bounded by \((\bar{P}_{13})^{1}(\langle R_{13} \rangle^{(1)}e^{\langle R_{13} \rangle^{(1)}t})\) and \((\bar{Q}_{13})^{1}(\langle R_{13} \rangle^{(1)}e^{\langle R_{13} \rangle^{(1)}t})\) respectively of \( \mathbb{R}^{+} \).

If instead of proving the existence of the solution on \( \mathbb{R}^{+} \), we have to prove it only on a compact then it suffices to consider that \((a_{13}'')^{(1)}\) and \((b_{13}')^{(1)}\), \( i = 13,14,15 \) depend only on \( T_{14} \) and respectively on \( G(\text{and not on } t) \) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_{i} (t) = 0 \) and \( T_{i} (t) = 0 \)

From 19 to 24 it results

\[ G_{i} (t) \geq G_{i}^{0} e^{-\langle R_{13} \rangle^{(1)} t} \geq 0 \]

\[ T_{i} (t) \geq T_{i}^{0} e^{-\langle b_{13} \rangle^{(1)} t} > 0 \] for \( t > 0 \)

**Definition of** \( (\bar{M}_{13})^{(1)} \) and \( (\bar{M}_{13})^{(1)} \)

**Remark 3:** If \( G_{13} \) is bounded, the same property have also \( G_{14} \) and \( G_{15} \). Indeed if

\[ G_{13} \leq (\bar{M}_{13})^{(1)}(t) \]

it follows \( \frac{dG_{14}}{dt} \leq (\bar{M}_{13})^{(1)}(\bar{k}_{14})^{(1)} G_{14} \) and by integrating

\[ G_{14} \leq (\bar{M}_{13})^{(1)}_{2} = G_{14}^{0} + 2(a_{14})^{(1)}(\bar{M}_{13})^{(1)}_{1}/(a_{14})^{(1)} \]

In the same way, one can obtain

\[ G_{15} \leq (\bar{M}_{13})^{(1)}_{3} = G_{15}^{0} + 2(a_{15})^{(1)}(\bar{M}_{13})^{(1)}_{2}/(a_{15})^{(1)} \]

If \( G_{14} \) or \( G_{15} \) is bounded, the same property follows for \( G_{13} \), \( G_{15} \) and \( G_{13} \), \( G_{14} \) respectively.

**Remark 4:** If \( G_{13} \) is bounded, from below, the same property holds for \( G_{14} \) and \( G_{15} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{14} \) is bounded from below.

**Remark 5:** If \( T_{13} \) is bounded from below and \( \lim_{t \to \infty} (b_{14}')^{(1)} (G_{i} (t), t) = (b_{14}')^{(1)} \) then \( T_{14} \to \infty \).

**Definition of** \((m)^{(1)}\) and \( \varepsilon_{1} \):
Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_{14}')^{(1)} (G(t), t) < \epsilon_1, T_{13} (t) > (m)^{(1)}$$

Then $\frac{d\tau_{14}}{dt} \geq (a_{14})^{(1)} (m)^{(1)} - \epsilon_1 T_{14}$ which leads to

$$T_{14} \geq \left(\frac{(a_{14})^{(1)} (m)^{(1)}}{\epsilon_1}\right) (1 - e^{-\epsilon_1 t}) + T_{15}^0 e^{-\epsilon_1 t}$$

If we take $t$ such that $e^{-\epsilon_1 t} = \frac{1}{2}$ it results

$$T_{14} \geq \left(\frac{(a_{14})^{(1)} (m)^{(1)}}{2 \epsilon_1}\right)$$

$t = \log \frac{2}{\epsilon_1}$ By taking now $\epsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded. The same property holds for $T_{15}$ if $\lim_{t \rightarrow \infty} (b_{15}')^{(1)} (G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

$$\left(\frac{\hat{P}_{16}}{\hat{M}_{16}}\right)^{(2)} \text{ and } \left(\frac{\hat{Q}_{16}}{\hat{M}_{16}}\right)^{(2)}$$

large to have

$$\left(\frac{a_{16}}{\hat{M}_{16}}\right)^{(2)} \left[ \left(\hat{P}_{16}\right)^{(2)} + \left(\hat{Q}_{16}\right)^{(2)} + G_1^0 \right] e^{-\left(\frac{\left(\hat{P}_{16}\right)^{(2)} + G_1^0}{\sigma_j^2}\right)} \leq \left(\hat{P}_{16}\right)^{(2)}$$

$$\left(\frac{b_{16}}{\hat{M}_{16}}\right)^{(2)} \left[ \left(\hat{Q}_{16}\right)^{(2)} + T_j^0 \right] e^{-\left(\frac{\left(\hat{Q}_{16}\right)^{(2)} + T_j^0}{\sigma_j^2}\right)} + \left(\hat{Q}_{16}\right)^{(2)} \leq \left(\hat{Q}_{16}\right)^{(2)}$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric

$$d \left( (G_{19})^{(1)}, (T_{19})^{(1)} \right), (G_{19})^{(2)}, (T_{19})^{(2)}\right) =$$

$$\sup_{t \in \mathbb{R}^+} \left| G_i^{(1)} (t) - G_i^{(2)} (t) \right| e^{-\left(\frac{\left(\hat{M}_{16}\right)^{(1)}}{\sigma_j^2}\right)} \sup_{t \in \mathbb{R}^+} \left| T_i^{(1)} (t) - T_i^{(2)} (t) \right| e^{-\left(\frac{\left(\hat{M}_{16}\right)^{(2)}}{\sigma_j^2}\right)}$$

Indeed if we denote

**Definition of**

$\tilde{G}_{19}, \tilde{T}_{19} : \left( \tilde{G}_{19}, \tilde{T}_{19} \right) = \mathcal{A}^{(2)} (G_{19}, T_{19})$

It results

$$\left| \tilde{G}_{i}^{(1)} - \tilde{a}_{i}^{(2)} \right| \leq \int_0^t (a_{16})^{(2)} \left[ G_{17}^{(1)} - G_{17}^{(2)} \right] e^{-\left(\frac{\left(\hat{M}_{16}\right)^{(1)}}{\sigma_j^2}\right) S_{(16)}} \sigma_j dS_{(16)} +$$

$$\int_0^t (a_{16}')^{(2)} \left[ G_{16}^{(1)} - G_{16}^{(2)} \right] e^{-\left(\frac{\left(\hat{M}_{16}\right)^{(1)}}{\sigma_j^2}\right) S_{(16)}} \sigma_j dS_{(16)} +$$

$$(a_{16}')^{(2)} \left[ T_{17}^{(1)}, S_{(16)} \right] G_{16}^{(1)} - G_{16}^{(2)} \left[ T_{17}^{(2)}, S_{(16)} \right] e^{-\left(\frac{\left(\hat{M}_{16}\right)^{(1)}}{\sigma_j^2}\right) S_{(16)}} \sigma_j dS_{(16)} +$$

$$G_{16}^{(2)} \left[ (a_{16}')^{(2)} \left[ T_{17}^{(1)}, S_{(16)} \right] - (a_{16}')^{(2)} \left[ T_{17}^{(2)}, S_{(16)} \right] \right] e^{-\left(\frac{\left(\hat{M}_{16}\right)^{(1)}}{\sigma_j^2}\right) S_{(16)}} \sigma_j dS_{(16)}$$

Where $S_{(16)}$ represents integrand that is integrated over the interval $[0, t]$
From the hypotheses it follows

\[
\left| (G_{19})^{(1)} - (G_{19})^{(2)} \right| e^{-(M_{16})^{(2)}t} \leq \frac{1}{(M_{16})^{(2)}} \left( (a_{16}^{(2)}) + (A_{16}^{(2)}) + (P_{16}^{(2)}(M_{16}^{(2)})) d \left( \left( (G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)} \right) \right) \right)
\]

And analogous inequalities for \( G_t \) and \( T_t \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{16}^{(2)}) \) and \((b_{16}^{(2)}) \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((P_{16}^{(2)})c^{(M_{16}^{(2)})t}\) and \((Q_{16}^{(2)})c^{(M_{16}^{(2)})t}\) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \((a_{17}^{(2)}) \) and \((b_{17}^{(2)}) \), \( i = 16, 17, 18 \) depend only on \( T_{17} \) and respectively on \( G_{19} \)(and not on \( t \) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_t (t) = 0 \) and \( T_t (t) = 0 \)

From 19 to 24 it results

\[
G_t (t) \geq G_t^0 e^{\left[ -\int_{t}^{\infty} (a_t^{(2)} - \int_{0}^{t}(T_t; \psi(t))) dt \right]} \geq 0
\]

\[
T_t (t) \geq T_t^0 e^{\left[ -\int_{t}^{\infty} (b_t^{(2)}) dt \right]} > 0 \quad \text{for} \ t > 0
\]

**Definition of** \((\overline{M}_{16}^{(2)})_1, (\overline{M}_{16}^{(2)})_2 \) and \((\overline{M}_{16}^{(2)})_3\) :

**Remark 3:** if \( G_{16} \) is bounded, the same property have also \( G_{17} \) and \( G_{18} \). indeed if \( G_{16} \leq (\overline{M}_{16}^{(2)})_2 \) it follows \( \frac{dG_{12}}{dt} \leq (\overline{M}_{16}^{(2)})_2 - (a_{17}^{(2)}) G_{17} \) and by integrating

\[
G_{17} \leq (\overline{M}_{16}^{(2)})_2 = G_{16}^0 + 2(a_{17}^{(2)}) (\overline{M}_{16}^{(2)})_1 / (a_{17}^{(2)})
\]

In the same way, one can obtain

\[
G_{18} \leq (\overline{M}_{16}^{(2)})_3 = G_{16}^0 + 2(a_{18}^{(2)}) (\overline{M}_{16}^{(2)})_2 / (a_{18}^{(2)})
\]

If \( G_{17} \) or \( G_{18} \) is bounded, the same property follows for \( G_{16} \), \( G_{18} \) and \( G_{16}, G_{17} \) respectively.

**Remark 4:** If \( G_{16} \) is bounded, from below, the same property holds for \( G_{17} \) and \( G_{18} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{17} \) is bounded from below.

**Remark 5:** If \( T_{16} \) is bounded from below and \( \lim_{t \to \infty} ((b_{17}^{(2)})(G_{19})(t), t) = (b_{17}^{(2)}) \) then \( T_{17} \to \infty \).

**Definition of** \((m)^{(2)} \) and \( \varepsilon_2 \):

Indeed let \( t_2 \) be so that for \( t > t_2 \)

\[
(b_{17}^{(2)}) - (b_{17}^{(2)})(G_{19})(t), t < \varepsilon_2, T_{16} (t) > (m)^{(2)}
\]

Then \( \frac{dT_{17}}{dt} \geq (a_{17}^{(2)})(m)^{(2)} - \varepsilon_2 T_{17} \) which leads to

\[
T_{17} \geq \frac{(a_{17}^{(2)})(m)^{(2)}}{\varepsilon_2}(1 - e^{-\varepsilon_2 t}) + T_{17,0} e^{-\varepsilon_2 t}
\]

If we take \( t \) such that \( e^{-\varepsilon_2 t} = \frac{1}{2} \) it results
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T_{17} \geq \left( a_{12}^{(2)}(m)^{(2)} \right), \quad t = \log \frac{2}{\varepsilon_{2}} \text{ By taking now } \varepsilon_{2} \text{ sufficiently small one sees that } T_{17} \text{ is unbounded. The same property holds for } T_{18} \text{ if } \lim_{t \to \infty}(b_{18}^{(2)}(G_{19})(t), t) = (b_{18}^{(2)})

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take \((a_{20}^{(3)})\), \((b_{20}^{(3)})\) large to have

\[(a_{20}^{(3)}) \left( (P_{20})^{(3)} + (G_{t}^{0} + (Q_{20})^{(3)} \right) e^{-\left( \frac{(P_{20})^{(3)} + Q_{20}}{t_{0}} \right)} \leq (P_{20}^{(3)}) \]

In order that the operator \(A^{(3)}\) transforms the space of sextuples of functions \(G_{t}, T_{t}\) into itself

The operator \(A^{(3)}\) is a contraction with respect to the metric

\[d \left( (G_{23}^{(1)}, T_{23}^{(1)}), (G_{23}^{(2)}, T_{23}^{(2)}) \right) = \sup_{t \in \mathbb{R}_{+}} \left| G_{t}^{1}(t) - G_{t}^{2}(t) \right| e^{-\left( (M_{20})^{(3)} t \right)} \max_{t \in \mathbb{R}_{+}} \left| T_{t}^{1}(t) - T_{t}^{2}(t) \right| e^{-\left( (M_{20})^{(3)} t \right)} \]

Indeed if we denote

**Definition of \(G_{23}, T_{23}^{(3)} : (G_{23}, T_{23}) = A^{(3)}(G_{23}, T_{23})\)**

It results

\[\left| a_{20}^{(3)} - G_{t}^{(2)} \right| \leq \int_{0}^{t} (a_{20}^{(3)}) \left| G_{21}^{(1)} - G_{21}^{(2)} \right| e^{-\left( (M_{20})^{(3)} \xi_{20} \right)} e^{\left( (M_{20})^{(3)} \xi_{20} \right)} \left| a_{21}^{(3)} \right| d \xi_{20} + \int_{0}^{t} (a_{20}^{(3)}) \left| G_{20}^{(1)} - G_{20}^{(2)} \right| e^{-\left( (M_{20})^{(3)} \xi_{20} \right)} e^{\left( (M_{20})^{(3)} \xi_{20} \right)} \left| a_{20}^{(3)} \right| d \xi_{20} + \]

\[\left( a_{20}^{(3)}(T_{21}^{(1)}, s_{20}^{(1)}) \right) e^{-\left( (M_{20})^{(3)} \xi_{20} \right)} e^{\left( (M_{20})^{(3)} \xi_{20} \right)} + \left( a_{20}^{(3)}(T_{21}^{(2)}, s_{20}^{(2)}) \right) e^{-\left( (M_{20})^{(3)} \xi_{20} \right)} e^{\left( (M_{20})^{(3)} \xi_{20} \right)} \]

Where \(s_{20}\) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[\left| G^{(1)} - G^{(2)} \right| e^{-\left( (M_{20})^{(3)} t \right)} \leq \int_{(M_{20})^{(3)}}^{1} \left( (a_{20}^{(3)} + (a_{20}^{(3)} + (a_{20}^{(3)})) \right) d \left( (G_{23}^{(1)}, T_{23}^{(1)}), (G_{23}^{(2)}, T_{23}^{(2)}) \right) \]

And analogous inequalities for \(G_{t}\) and \(T_{t}\). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{20}^{(3)})\) and \((b_{20}^{(3)})\) depending also on \(t\) can be considered as

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not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\tilde{P}_{20}(3)e^{(\tilde{M}_{20})^{(3)} t_1})$ and $(\tilde{Q}_{20}(3)e^{(\tilde{M}_{20})^{(3)} t_1})$ respectively of $\mathbb{R}_+$. 

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i''(3))$, $(b_i''(3))$, $i=20,21,22$ depend only on $T_{21}$ and respectively on $(G_{23})$, and not on $t$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ and $T_i(t) = 0$.

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\frac{1}{d_{2i}}((a_i''(3)) - (a_i''(3)(T_{21}(s_{(20)}), x_{(20)})))ds_{(20)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-b_i''(3)t)} > 0 \text{ for } t > 0$$

**Definition of** $(\tilde{M}_{20}(3))$, $(\tilde{M}_{20}(3))$, and $(\tilde{M}_{20}(3))$.

**Remark 3:** If $G_{20}$ is bounded, the same property have also $G_{21}$ and $G_{22}$. Indeed if $G_{20} < (\tilde{M}_{20}(3))$ it follows $\frac{dG_{21}}{dt} \leq ((\tilde{M}_{20}(3))_1 - (a_{21}'(3))G_{21}$ and by integrating

$$G_{21} \leq ((\tilde{M}_{20}(3))_2 = G_{21}^0 + 2(a_{21}'(3)(((\tilde{M}_{20}(3))_1)/(a_{21}'(3))$$

In the same way, one can obtain

$$G_{22} \leq ((\tilde{M}_{20}(3))_3 = G_{22}^0 + 2(a_{22}'(3)(((\tilde{M}_{20}(3))_1)/(a_{22}'(3))$$

If $G_{21}$ or $G_{22}$ is bounded, the same property follows for $G_{20}$, $G_{22}$, and $G_{20}$, $G_{21}$ respectively.

**Remark 4:** If $G_{20}$ is bounded, from below, the same property holds for $G_{21}$ and $G_{22}$. The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below.

**Remark 5:** If $T_{20}$ is bounded from below and $\lim_{t \to \infty}((b_i''(3))(G_{23}(t), t)) = (b_{23}'(3))$ then $T_{21} \to \infty$.

**Definition of** $(m)(3)$ and $\varepsilon_3$:

Indeed let $\varepsilon_3$ be so that for $t > t_3$

$$(b_{21}'(3))(3) < (b_{21}'(3))(G_{23}(t), t) < \varepsilon_3, T_{20}(t) > (m)(3)$$

Then $\frac{dT_{21}}{dt} \geq (a_{21}'(3))(m)(3) - \varepsilon_3 T_{21}$ which leads to

$$T_{21} \geq\left(\frac{(a_{21}'(3))(m)(3)}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$$. If we take $t$ such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$$T_{21} \geq\left(\frac{(a_{21}'(3))(m)(3)}{2}\right). t = \log\frac{2}{\varepsilon_3}$$. By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded. The same property holds for $T_{22}$ if $\lim_{t \to \infty}(b_{22}'(3))(G_{23}(t), t) = (b_{22}'(3))$.

We now state a more precise theorem about the behaviors at infinity of the solutions.

It is now sufficient to take $\frac{(a_i''(3))}{(\tilde{M}_{21}(3))} < 1$ and to choose $\varepsilon_3$.
\[(\bar{P}_{24})^{(4)}\] and \[(\bar{Q}_{24})^{(4)}\] large to have

\[
\frac{(a)\bar{G}^{(4)}}{(M_{24})^{(4)}} \left[ (\bar{P}_{24})^{(4)} + ((\bar{P}_{24})^{(4)} + G_{ij}^{(4)} e^{-\frac{(\bar{P}_{24})^{(4)} + G_{ij}^{(4)}}{G_{ij}^{(4)}}} \right) \leq (\bar{P}_{24})^{(4)}
\]

\[
\frac{(b)\bar{G}^{(4)}}{(M_{24})^{(4)}} \left[ (\bar{Q}_{24})^{(4)} + T_{ij}^{(4)} e^{-\frac{(\bar{Q}_{24})^{(4)} + T_{ij}^{(4)}}{T_{ij}^{(4)}}} + (\bar{Q}_{24})^{(4)} \right) \leq (\bar{Q}_{24})^{(4)}
\]

In order that the operator \(\mathcal{A}^{(4)}\) transforms the space of sextuples of functions \(G_i, T_i\) satisfying IN to itself

The operator \(\mathcal{A}^{(4)}\) is a contraction with respect to the metric

\[
d \left( (G_{27})^{(1)}, (T_{27})^{(1)} \right), (G_{27})^{(2)}, (T_{27})^{(2)} \right) =
\]

\[
\sup_{t} \max_{\in \mathbb{R}^+} \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-\left((\bar{G}_{24})^{(4)}(t)\right)} + \max_{\in \mathbb{R}^+} \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-\left((\bar{T}_{24})^{(4)}(t)\right)}
\]

Indeed if we denote

**Definition of** \((\bar{G}_{27}), (\bar{T}_{27}) : (\bar{G}_{27}), (\bar{T}_{27}) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))\)

It results

\[
\left| G_{24}^{(1)} - G_i^{(2)} \right| \leq \int_{0}^{t} (a_{4})^{(4)} \left( G_{24}^{(1)} - G_{24}^{(2)} \right) e^{-\left((\bar{G}_{24})^{(4)}(s)\right)} e^{-\left((\bar{G}_{24})^{(4)}(s)\right)} ds_{(24)} +
\]

\[
\left( a_{4}^{(4)} \left( T_{24}^{(1)}, s(24) \right) \right) \left| G_{24}^{(1)} - G_{24}^{(2)} \right| e^{-\left((\bar{G}_{24})^{(4)}(s)\right)} e^{-\left((\bar{G}_{24})^{(4)}(s)\right)} +
\]

\[
G_{24}^{(2)} \left( a_{4}^{(4)} \left( T_{24}^{(2)}, s(24) \right) \right) \left| G_{24}^{(2)} \right| e^{-\left((\bar{G}_{24})^{(4)}(s)\right)} e^{-\left((\bar{G}_{24})^{(4)}(s)\right)} ds_{(24)}
\]

Where \(s_{(24)}\) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[
\left| (G_{27})^{(1)} - (G_{27})^{(2)} \right| e^{-\left((\bar{G}_{24})^{(4)}(t)\right)} \leq
\]

\[
\frac{1}{(M_{24})^{(4)}} \left( (a_{4}^{(4)}) + (a_{4}^{(4)}) + (\bar{A}_{24})^{(4)} +
\right)
\]

\[
(\bar{P}_{24})^{(4)}(\bar{Q}_{24})^{(4)} d \left( (G_{27})^{(1)}, (T_{27})^{(1)}; (G_{27})^{(2)}, (T_{27})^{(2)} \right)
\]

And analogous inequalities for \(G_i\) and \(T_i\). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{4}^{(4)})\) and \((b_{4}^{(4)})\) depending also on \(t\) can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\bar{P}_{24})^{(4)} e^{-((\bar{G}_{24})^{(4)}(t))}\) and \((\bar{Q}_{24})^{(4)} e^{-((\bar{G}_{24})^{(4)}(t))}\) respectively of \(\mathbb{R}^+\).
If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \((a'_{i})^{(4)}\) and \((b'_{i})^{(4)}\), \(i = 24, 25, 26\) depend only on \(T_{25}\) and respectively on \((G_{27})(\text{and not on } t)\) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_i(t) = 0 \) and \( T_i(t) = 0 \)

From GLOBAL EQUATIONS it results

\[
G_i(t) \geq G^0_i e^{-\int_0^t \left[ f_i(a''_i, a'_i(t), t, z(t)) dt \right]} \geq 0
\]

\[
T_i(t) \geq T^0_i e^{-\int_0^t \left[ f_i(a''_i, a'_i(t), t, z(t)) dt \right]} > 0 \quad \text{for } t > 0
\]

**Definition of** \( (\overline{M}_{24})^{(4)} \), \( (\overline{M}_{24})^{(4)}_2 \) and \( (\overline{M}_{24})^{(4)}_3 \):

**Remark 3:** If \( G_{24} \) is bounded, the same property have also \( G_{25} \) and \( G_{26} \). Indeed if \( G_{24} < (\overline{M}_{24})^{(4)} \) it follows \( \frac{dG_{25}}{dt} \leq (\overline{M}_{24})^{(4)}_2 - (a_{25})^{(4)}G_{25} \) and by integrating

\[
G_{25} \leq (\overline{M}_{24})^{(4)}_2 G_{25}^0 + 2(a_{25})^{(4)}(\overline{M}_{24})^{(4)}_2 / (a_{25})^{(4)}
\]

In the same way, one can obtain

\[
G_{26} \leq (\overline{M}_{24})^{(4)}_3 G_{26}^0 + 2(a_{26})^{(4)}(\overline{M}_{24})^{(4)}_3 / (a_{26})^{(4)}
\]

If \( G_{25} \) or \( G_{26} \) is bounded, the same property follows for \( G_{24}, G_{26} \) and \( G_{24}, G_{25} \) respectively.

**Remark 4:** If \( G_{24} \) is bounded, from below, the same property holds for \( G_{25} \) and \( G_{26} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{25} \) is bounded from below.

**Remark 5:** If \( T_{24} \) is bounded from below and \( \lim_{t \to \infty} ((b''_{25})^{(4)} ((G_{27})(t), t)) = (b''_{25})^{(4)} \) then \( T_{25} \to \infty \).

**Definition of** \( (m)^{(4)} \) and \( \varepsilon_{4} \):

Indeed let \( t_{4} \) be so that for \( t > t_{4} \),

\[
(b_{25})^{(4)} - (b''_{25})^{(4)}((G_{27})(t), t) < \varepsilon_{4}, T_{24}(t) > (m)^{(4)}
\]

Then \( \frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_{4} T_{25} \) which leads to

\[
T_{25} \geq \left( (a_{25})^{(4)}(m)^{(4)} / \varepsilon_{4} \right) (1 - e^{-\varepsilon_{4}t}) + T_{25}^{0} e^{-\varepsilon_{4}t}
\]

If we take \( t \) such that

\[
e^{-\varepsilon_{4}t} = \frac{1}{2}
\]

it results

\[
T_{25} \geq \left( (a_{25})^{(4)}(m)^{(4)} / \varepsilon_{4} \right) \cdot \left( \log \frac{4}{\varepsilon_{4}} \right)
\]

By taking now \( \varepsilon_{4} \) sufficiently small one sees that \( T_{25} \) is unbounded. The same property holds for \( T_{26} \) if \( \lim_{t \to \infty} ((b''_{26})^{(4)} ((G_{27})(t), t)) = (b''_{26})^{(4)} \)

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for \( G_{29}, G_{30}, T_{29}, T_{29}, T_{30} \)

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It is now sufficient to take $\frac{(a_l)^{(5)}}{\langle M_{28} \rangle^{(5)}}$, $\frac{(b_l)^{(5)}}{\langle M_{28} \rangle^{(5)}} < 1$ and to choose

$$(P_{28})^{(5)}$$ and $$(Q_{28})^{(5)}$$ large to have

$$\frac{(a_l)^{(5)}}{\langle M_{28} \rangle^{(5)}} \left[ (P_{28})^{(5)} + \left( (P_{28})^{(5)} + Q_l^{(5)} \right) e^{-\left( \frac{(Q_{28})^{(5)}}{(T_l)^{(5)}} \right)} \right] \leq (P_{28})^{(5)}$$

$$\frac{(b_l)^{(5)}}{\langle M_{28} \rangle^{(5)}} \left[ (Q_{28})^{(5)} + T_l^{(5)} e^{-\left( \frac{(Q_{28})^{(5)}}{(T_l)^{(5)}} \right)} \right] \leq (Q_{28})^{(5)}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i$, $T_i$ into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric

$$d \left( (G_{31})^{(1)}, (T_{31})^{(1)}, (G_{31})^{(2)}, (T_{31})^{(2)} \right) = \sup_{t \in \mathbb{R}^+} \max_{i} \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-\left( \langle M_{28} \rangle^{(5)} \right) t} \max_{i} \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-\left( \langle M_{28} \rangle^{(5)} \right) t}$$

Indeed if we denote

**Definition of** $(G_{31}), (T_{31})$ : $\left( (G_{31}), (T_{31}) \right) = \mathcal{A}^{(5)}(\left( (G_{31}), (T_{31}) \right))$

It results

$$\left| G_{28}^{(1)} - G_{28}^{(2)} \right| \leq \int_0^t \left| \left( a_{28}^{(5)} \right) \left( \int S_{28}^{(1)} \right) \right| e^{-\left( \langle M_{28} \rangle^{(5)} \right) x(28) e^{-\left( \langle M_{28} \rangle^{(5)} \right) x(28)}} dS_{(28)} +$$

$$\int_0^t \left| \left( a_{28}^{(5)} \right) \left( \int S_{28}^{(1)} \right) \right| e^{-\left( \langle M_{28} \rangle^{(5)} \right) x(28) e^{-\left( \langle M_{28} \rangle^{(5)} \right) x(28)}} dS_{(28)} +$$

$$\left( a_{28}^{(5)} \right) \left( \int S_{28}^{(1)} \right) \left| G_{28}^{(1)} - G_{28}^{(2)} \right| e^{-\left( \langle M_{28} \rangle^{(5)} \right) x(28) e^{-\left( \langle M_{28} \rangle^{(5)} \right) x(28)}} +$$

$$\left( a_{28}^{(5)} \right) \left( \int S_{28}^{(1)} \right) \left| G_{28}^{(1)} - G_{28}^{(2)} \right| e^{-\left( \langle M_{28} \rangle^{(5)} \right) x(28) e^{-\left( \langle M_{28} \rangle^{(5)} \right) x(28)}} +$$

Where $S_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left| (G_{31})^{(1)} - (G_{31})^{(2)} \right| e^{-\left( \langle M_{28} \rangle^{(5)} \right) t} \leq$$

$$\frac{1}{\langle M_{28} \rangle^{(5)}} \left[ (a_{28}^{(5)} + (a_{28}^{(5)}) + (a_{28}^{(5)}) + (a_{28}^{(5)}) \right] d \left( (G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)} \right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis $(35,35,36)$ the result follows
Remark 1: The fact that we supposed \((a''_{28})^{(5)}\) and \((b'_{29})^{(5)}\) depending also on \(t\) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}t}\) and \((\hat{Q}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}t}\) respectively of \(\mathbb{R}_+\).

If instead of proving the existence of the solution on \(\mathbb{R}_+\), we have to prove it only on a compact then it suffices to consider that \((a''_{28})^{(5)}\) and \((b'_{29})^{(5)}\), \(i = 28,29,30\) depend only on \(T_29\) and respectively on \((G_{31})\) (not on \(t\)) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any \(t\) where \(G_1(t) = 0\) and \(T_1(t) = 0\)

From GLOBAL EQUATIONS it results

\[
G_i(t) \geq G_i^0 e \left[ - \int_i^t (a'_i)^{(5)} - (a''_i)^{(5)}(T_29(x(T_29)),x(T_29))]\left| ds \right. \right] \geq 0 \\
T_i(t) \geq T_i^0 e^{-b'_i(t)} > 0 \quad \text{for } t > 0
\]

Definition of \(((\hat{M}_{28})^{(5)})_1\), \(((\hat{M}_{28})^{(5)})_2\) and \(((\hat{M}_{28})^{(5)})_3\):

Remark 3: If \(G_{28}\) is bounded, the same property have also \(G_{28} + G_{30}\) \cdot indeed if

\[
G_{28} < ((\hat{M}_{28})^{(5)}) \text{ it follows } \frac{dG_{29}}{dt} \leq \left((\hat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)}G_{29} \text{ and by integrating}
\]

\[
G_{29} \leq \left((\hat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\hat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}
\]

In the same way, one can obtain

\[
G_{30} \leq \left((\hat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\hat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}
\]

If \(G_{29}\) or \(G_{30}\) is bounded, the same property follows for \(G_{28}, G_{30} + G_{28}, G_{29}\) respectively.

Remark 4: If \(G_{28}\) is bounded, from below, the same property holds for \(G_{29}\) and \(G_{30}\). The proof is analogous with the preceding one. An analogous property is true if \(G_{29}\) is bounded from below.

Remark 5: If \(T_{29}\) is bounded from below and \(\lim_{t \to \infty}(b''_{29}) ((G_{31})(t),t) = (b'_{29})^{(5)}\) then \(T_{29} \to \infty\).

Definition of \((m)^{(5)}\) and \(\varepsilon_5\):

Indeed let \(t_5\) be so that for \(t > t_5\)

\[
(b_{29})^{(5)} - (b''_{29})((G_{31})(t),t) < \varepsilon_5, T_{29}(t) > (m)^{(5)}
\]

Then \(\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}\) which leads to

\[
T_{29} \geq \left(\frac{(a_{29})^{(5)(m)^{(5)})}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t} \quad \text{if we take } t \quad \text{such that } e^{-\varepsilon_5 t} = \frac{1}{2} \quad \text{it results}
\]

\[
T_{29} \geq \left(\frac{(a_{29})^{(5)(m)^{(5)})}{2}\right), \quad t = \log \frac{2}{\varepsilon_5} \quad \text{By taking now } \varepsilon_5 \quad \text{sufficiently small one sees that } T_{29} \quad \text{is}
\]
unbounded. The same property holds for $T_{36}$ if $\lim_{t \to \infty} (b_{36}^{(5)}) ((G_{31})(t), t) = (b_{36}^{(5)})$

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$.

It is now sufficient to take $\frac{(a_{i})^{(6)}}{(M_{32})^{(6)}} \frac{(b_{j})^{(6)}}{(M_{32})^{(6)}} < 1$ and to choose

$$\left( \bar{P}_{32} \right)^{(6)} \text{ and } \left( \bar{Q}_{32} \right)^{(6)} \text{ large to have}$$

$$\frac{(a_{i})^{(6)}}{(M_{32})^{(6)}}, \left( \bar{P}_{32} \right)^{(6)} + \left( \bar{P}_{32} \right)^{(6)} + G_{j}^{(6)} e^{-\left( \frac{\left( \bar{P}_{32} \right)^{(6)} e^{\alpha_{j}^{(6)}}}{\alpha_{j}^{(6)}} \right)} \leq \left( \bar{P}_{32} \right)^{(6)}$$

$$\frac{(b_{j})^{(6)}}{(M_{32})^{(6)}}, \left( \bar{Q}_{32} \right)^{(6)} + T_{j}^{(6)} e^{-\left( \frac{\left( \bar{Q}_{32} \right)^{(6)} e^{\alpha_{j}^{(6)}}}{\alpha_{j}^{(6)}} \right)} + \left( \bar{Q}_{32} \right)^{(6)} \leq \left( \bar{Q}_{32} \right)^{(6)}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_{1}, T_{i}$ into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric

$$d \left( (G_{33})^{(1)}, (T_{35})^{(1)} ), (G_{33})^{(2)}, (T_{35})^{(2)} \right) =$$

$$\sup_{t \in \mathbb{R}^{+}} \max_{i} \left( G_{i}^{(1)}(t) - a_{i}^{(1)}(t) \right) e^{-\left( \frac{\left( \bar{G}_{32} \right)^{(6)} e^{\alpha_{j}^{(6)}}}{\alpha_{j}^{(6)}} \right)} \max_{t \in \mathbb{R}^{+}} \left( T_{i}^{(1)}(t) - T_{i}^{(2)}(t) \right) e^{-\left( \frac{\left( \bar{T}_{32} \right)^{(6)} e^{\alpha_{j}^{(6)}}}{\alpha_{j}^{(6)}} \right)}$$

Indeed if we denote

**Definition of** $(G_{35}, T_{35}) : \left( (G_{35}), (T_{35}) \right) = (G_{35}, T_{35})$

It results

$$\left| \bar{G}_{32} - \bar{G}_{32}^{(2)} \right| \leq \int_{0}^{t} \left( a_{32}^{(1)} \right)^{(6)} \left| G_{33} - G_{33}^{(2)} \right| e^{-\left( \frac{\left( \bar{G}_{32} \right)^{(6)} e^{\alpha_{j}^{(6)}}}{\alpha_{j}^{(6)}} \right)} dS_{(32)} +$$

$$\int_{0}^{t} \left( a_{32}^{(1)} \right)^{(6)} \left( G_{33}^{(1)} - G_{33}^{(2)} \right) e^{-\left( \frac{\left( \bar{G}_{32} \right)^{(6)} e^{\alpha_{j}^{(6)}}}{\alpha_{j}^{(6)}} \right)} dS_{(32)} +$$

$$\left( a_{32}^{(2)} \right)^{(6)} \left( T_{33}^{(1)}, S_{(32)} \right) \left| G_{32}^{(1)} - G_{32}^{(2)} \right| e^{-\left( \frac{\left( \bar{G}_{32} \right)^{(6)} e^{\alpha_{j}^{(6)}}}{\alpha_{j}^{(6)}} \right)} dS_{(32)} +$$

$$\left( a_{32}^{(2)} \right)^{(6)} \left( T_{33}^{(2)}, S_{(32)} \right) - \left( a_{32}^{(2)} \right)^{(6)} \left( T_{33}^{(2)}, S_{(32)} \right) \left| e^{-\left( \frac{\left( \bar{G}_{32} \right)^{(6)} e^{\alpha_{j}^{(6)}}}{\alpha_{j}^{(6)}} \right)} dS_{(32)} \right.$$}

Where $S_{(32)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

1. $\left( a_{i}^{(1)} \right)^{(1)}, \left( a_{i}^{(2)} \right)^{(1)}, \left( b_{i}^{(1)} \right)^{(1)}, \left( b_{i}^{(2)} \right)^{(1)}, \left( a_{i}^{(3)} \right)^{(1)}, \left( a_{i}^{(4)} \right)^{(1)}, \left( a_{i}^{(5)} \right)^{(1)}, \left( a_{i}^{(6)} \right)^{(1)}, \left( b_{i}^{(3)} \right)^{(1)}, \left( b_{i}^{(4)} \right)^{(1)}, \left( b_{i}^{(5)} \right)^{(1)}, \left( b_{i}^{(6)} \right)^{(1)} > 0,$

2. The functions $\left( a_{i}^{(2)} \right)^{(1)}, \left( b_{i}^{(4)} \right)^{(1)}$ are positive continuous increasing and bounded.
Definition of \((p_i)^{(1)}\), \((r_i)^{(1)}\):
\[
(a_i')^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (A_{13})^{(1)}
\]
\[
(b_i')^{(1)}(G, t) \leq (r_i)^{(1)} \leq (B_{13})^{(1)}
\]

\(3\) \(\lim_{T_{14} \to 0}(a_i'')^{(1)}(T_{14}, t) = (p_i)^{(1)}\)
\(\lim_{G \to 0}(b_i'')^{(1)}(G, t) = (r_i)^{(1)}\)

Definition of \((\tilde{A}_{13})^{(1)}, (\tilde{B}_{13})^{(1)}\):
Where \((\tilde{A}_{13})^{(1)}, (\tilde{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}\) are positive constants
and \(l = 13, 14, 15\).

They satisfy Lipschitz condition:
\[
|\{(a_i'')^{(1)}(T_{14}, t) - (a_i'')^{(1)}(T_{14}, t)\} \leq (\tilde{A}_{13})^{(1)}|T_{14} - T_{14}'|e^{-|l_{13}|t}
\]
\[
|\{(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, T)\} < (\tilde{B}_{13})^{(1)}||G - G'||e^{-|l_{13}|t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i'')^{(1)}(T_{14}, t)\) and \((a_i'')^{(1)}(T_{14}, t)\) .\((T_{14}, t)\) and \((T_{14}, t)\) are points belonging to the interval \([\tilde{A}_{13})^{(1)}, (\tilde{B}_{13})^{(1)}]\). It is to be noted that \((a_i'')^{(1)}(T_{14}, t)\) is uniformly continuous. In the eventual fact, that if \((\tilde{M}_{13})^{(1)} = 1\) then the function \((a_i'')^{(1)}(T_{14}, t)\), the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

Definition of \((\tilde{M}_{13})^{(1)}, (\tilde{K}_{13})^{(1)}\):

\(AA\) \((\tilde{M}_{13})^{(1)}, (\tilde{K}_{13})^{(1)}, \) are positive constants
\[
\frac{(a_i')^{(1)} + (b_i')^{(1)}}{(\tilde{M}_{13})^{(1)}} < 1
\]

Definition of \((\tilde{P}_{13})^{(1)}, (\tilde{Q}_{13})^{(1)}\):

\(BB\) There exist two constants \((\tilde{P}_{13})^{(1)}\) and \((\tilde{Q}_{13})^{(1)}\) which together with \((\tilde{M}_{13})^{(1)}, (\tilde{K}_{13})^{(1)}, (A_{13})^{(1)}\) and \((B_{13})^{(1)}\) and the constants \((a_i')^{(1)}, (a_i')^{(1)}, (b_i')^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}\), \(l = 13, 14, 15\), satisfy the inequalities
\[
\frac{1}{(\tilde{M}_{13})^{(1)}}[\{(a_i')^{(1)} + (a_i')^{(1)} + (A_{13})^{(1)} + (\tilde{P}_{13})^{(1)}(\tilde{K}_{13})^{(1)}\] < 1
\]
\[
\frac{1}{(\tilde{M}_{13})^{(1)}}[\{(b_i')^{(1)} + (b_i')^{(1)} + (B_{13})^{(1)} + (\tilde{Q}_{13})^{(1)}(\tilde{K}_{13})^{(1)}\] < 1
\]

Analogous inequalities hold also for \(G_{37}, G_{38}, T_{36}, T_{37}, T_{38}\)

It is now sufficient to take \(\frac{(a_i')^{(7)}}{(M_{36})^{(7)}}, \frac{(b_i')^{(7)}}{(M_{36})^{(7)}} < 7\) and to choose
In order that the operator \( \mathcal{A} \) transforms the space of sextuples of functions \( G_i, T_i \) satisfying \( 37,35,36 \) into itself

The operator \( \mathcal{A} \) is a contraction with respect to the metric

\[
d \left( (G_{39})^{(1)}, (T_{39})^{(1)} \right) = \sup \left( \max_{i} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-\langle G_{36} \rangle^{(7)} t}, \max_{i} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-\langle G_{36} \rangle^{(7)} t} \right)
\]

Indeed if we denote

**Definition of** \( (G_{39}, (T_{39})) \):

\[
(G_{39}, (T_{39})) = \mathcal{A} \left( (G_{39}, (T_{39})) \right)
\]

It results

\[
\left| \tilde{G}_i^{(1)} - \tilde{G}_i^{(2)} \right| \leq \int_{0}^{t} \left( a_{36}^{(1)} \rangle^{(7)} \right) \left| G_{36}^{(1)} - G_{36}^{(2)} \right| e^{-\langle G_{36} \rangle^{(7)} \langle \xi_{36} \rangle_{36} e^{(G_{36})^{(7)} \langle \xi_{36} \rangle_{36} dS_{36}} + \\
\int_{0}^{t} \left( a_{36}^{(2)} \rangle^{(7)} \right) G_{36}^{(1)} e^{-\langle G_{36} \rangle^{(7)} \langle \xi_{36} \rangle_{36} e^{(G_{36})^{(7)} \langle \xi_{36} \rangle_{36} dS_{36}} + \\
\left| a_{36}^{(1)} \rangle^{(7)} (T_{37}^{(1)}, S_{36}) \right| G_{36}^{(1)} - G_{36}^{(2)} \right| e^{-\langle G_{36} \rangle^{(7)} \langle \xi_{36} \rangle_{36} e^{(G_{36})^{(7)} \langle \xi_{36} \rangle_{36} dS_{36}} + \\
G_{26}^{(2)} \left( a_{36}^{(7)} (T_{37}^{(1)}, S_{36}) - \left| a_{36}^{(7)} (T_{37}^{(2)}, S_{36}) \right| e^{-\langle G_{36} \rangle^{(7)} \langle \xi_{36} \rangle_{36} e^{(G_{36})^{(7)} \langle \xi_{36} \rangle_{36}} dS_{36}
\]

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Where \( s_{(36)} \) represents integrand that is integrated over the interval \([0, t]\).

From the hypotheses it follows

\[
\left| (G_{39})^{(1)} - (G_{39})^{(2)} \right| e^{-\langle \theta_{36} \rangle t} \leq \frac{1}{\langle \bar{a}_{36} \rangle} (a_{36}^{(7)} + (a_{36}^{(7)} + (\bar{A}_{36})^{(7)} + (\bar{P}_{36})^{(7)} k_{36}^{(7)}) d \left( ((G_{39})^{(1)}, (T_{39})^{(1)}; (G_{39})^{(2)}, (T_{39})^{(2)}) \right)
\]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis (37,35,36) the result follows

**Remark 1:** The fact that we supposed \((a_{36}^{(7)} \quad b_{36}^{(7)})\) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((P_{36})^{(7)} e^{(\bar{M}_{36}) t} and (Q_{36})^{(7)} e^{(M_{36}) t}\) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \((u_i^{(7)} \quad h_i^{(7)}), i = 36,37,38\) depend only on \( T_{37} \) and respectively on \((G_{39})(and \ not \ on \ t)\) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_i (t) = 0 \ and \ T_i (t) = 0 \)

From 79 to 36 it results

\[
G_i (t) \geq G_i^0 e^{-\int_0^t \left[ (a_i^{(5)}) - (a_i^{(5)} (T_{37} (x_{(36)}), x_{(36)})) \right] dx_{(36)}} \geq 0
\]
\[ T_i(t) \geq T_i^0 e^{-(b_i^0)^{(7)} t)} > 0 \text{ for } t > 0 \]

**Definition of** \((\overline{M}_{36})^{(7)}\)_1, \((\overline{M}_{36})^{(7)}\)_2, and \((\overline{M}_{36})^{(7)}\)_3 :

**Remark 3:** If \(G_{36}\) is bounded, the same property have also \(G_{37}\) and \(G_{38}\). Indeed if \(G_{36} < (\overline{M}_{36})^{(7)}\) it follows \(\frac{dG_{37}}{dt} \leq ((\overline{M}_{36})^{(7)} - (a_{37})^{(7)}G_{37}\) and by integrating

\[ G_{37} \leq (\overline{M}_{36})^{(7)} \frac{G_{37}^0}{2} + 2(a_{37})^{(7)}((\overline{M}_{36})^{(7)}_1/a_{37})^{(7)} \]

In the same way, one can obtain

\[ G_{38} \leq (\overline{M}_{36})^{(7)} \frac{G_{38}^0}{2} + 2(a_{38})^{(7)}((\overline{M}_{36})^{(7)}_2/a_{38})^{(7)} \]

If \(G_{37}\) or \(G_{38}\) is bounded, the same property follows for \(G_{36}, G_{38}\) and \(G_{36}, G_{37}\) respectively.

**Remark 7:** If \(G_{36}\) is bounded, from below, the same property holds for \(G_{37}\) and \(G_{38}\). The proof is analogous with the preceding one. An analogous property is true if \(G_{37}\) is bounded from below.

**Remark 5:** If \(T_{36}\) is bounded from below and \(\lim_{t \to \infty}((b_i^1)^{(7)}((G_{39}(t), t)) = (b_{37}^1)^{(7)}\) then

\[ T_{37} \to \infty. \]

**Definition of** \((m)^{(7)}\) and \(\varepsilon_7\):

Indeed let \(t_7\) be so that for \(t > t_7\)

\[ (b_{37}^1)^{(7)} - (b_i^1)^{(7)}((G_{39}(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)} \]
Then \( \frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon T_{37} \) which leads to

\[
T_{37} \geq \left( \frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon} \right) (1 - e^{-\varepsilon t}) + T_{37}^0 e^{-\varepsilon t} 
\]

If we take \( t \) such that \( e^{-\varepsilon t} = \frac{1}{2} \) it results

\[
T_{37} \geq \frac{(a_{37})^{(7)}(m)^{(7)}}{2}. \quad t = \log \frac{2}{\varepsilon} \]

By taking now \( \varepsilon \) sufficiently small one sees that \( T_{37} \) is unbounded. The same property holds for \( T_{38} \) if \( \lim_{t \to \infty} (b_{38})^{(7)}(G_{99})_t(t), t = (b_{38})^{(7)} \)

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 72

In order that the operator \( A^{(7)} \) transforms the space of sextuples of functions \( G_i, T_i \) satisfying

GLOBAL EQUATIONS AND ITS CONCOMITANT CONDITIONALITIES into itself

The operator \( A^{(7)} \) is a contraction with respect to the metric

\[
d \left( \left( \left( G_{39} \right)^{(1)}, \left( T_{39} \right)^{(1)} \right), \left( \left( G_{39} \right)^{(2)}, \left( T_{39} \right)^{(2)} \right) \right) = \\
\sup \left\{ \max \left\{ \left| G^{(1)}_i(t) - G^{(2)}_i(t) \right| e^{-\left( h_{36} \right)^{(7)} t}, \max \left\{ \left| T^{(1)}_i(t) - T^{(2)}_i(t) \right| e^{-\left( h_{36} \right)^{(7)} t} \right\} \right\} \\
\right.
\]

Indeed if we denote

\[ (G_{39}, T_{39}) \]

**Definition of \( (G_{39}), (T_{39}) \):**

\[ ( (G_{39}), (T_{39}) ) = A^{(7)}((G_{39}), (T_{39})) \]

It results

\[
\left| \tilde{G}^{(1)}_{36} - \tilde{G}^{(2)}_{36} \right| \leq \int_0^t (a_{36})^{(7)} \left| G^{(1)}_{37} - G^{(2)}_{37} \right| e^{-\left( h_{36} \right)^{(7)} \chi_{36})} e^{\left( h_{36} \right)^{(7)} \chi_{36})} dS_{36} \]

\[
\int_0^t (a_{36})^{(7)} \left| G^{(1)}_{36} - G^{(2)}_{36} \right| e^{-\left( h_{36} \right)^{(7)} \chi_{36})} e^{\left( h_{36} \right)^{(7)} \chi_{36})} + \\
(a_{36})^{(7)} \left( T^{(1)}_{37}, \chi_{36} \right) \left| G^{(1)}_{36} - G^{(2)}_{36} \right| e^{-\left( h_{36} \right)^{(7)} \chi_{36})} e^{\left( h_{36} \right)^{(7)} \chi_{36})} + \\
(a_{36})^{(7)} \left( T^{(1)}_{37}, \chi_{36} \right) \left| G^{(1)}_{36} - G^{(2)}_{36} \right| e^{-\left( h_{36} \right)^{(7)} \chi_{36})} e^{\left( h_{36} \right)^{(7)} \chi_{36})} + \\
\]

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\[ G_{36}^{(2)}(a_{36}^{(7)}(T_{37}^{(1)}, S_{(36)}), a_{36}^{(7)}(T_{37}^{(2)}, S_{(36)}) ] e^{-(\bar{M}_{36})^{(7)}x_{(36)}k_{(36)}e^{(\bar{M}_{36})^{(7)}x_{(36)}}} dS_{(36)} \]

Where \( s_{(36)} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[ |(G_{39})^{(1)} - (G_{39})^{(2)}| e^{-(\bar{M}_{36})^{(7)}t} \leq \]

\[ \frac{1}{(\bar{M}_{36})^{(7)} (a_{36}^{(7)} + a_{36}^{(7)}) + (A_{36})^{(7)} + (P_{36})^{(7)} (\bar{M}_{36})^{(7)} ) d \left( \left( (G_{39})^{(1)}(T_{39})^{(1)}; (G_{39})^{(2)}, (T_{39})^{(2)} \right) \right) \]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{36}^{(7)})^{(7)} \) and \((b_{36}^{(7)})^{(7)} \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( (P_{36})^{(7)} e^{(\bar{M}_{36})^{(7)}t} \) and \( (Q_{36})^{(7)} e^{(\bar{M}_{36})^{(7)}t} \) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \((a_{36}^{(7)})^{(7)} \) and \((b_{36}^{(7)})^{(7)} \), \( i = 36, 37, 38 \) depend only on \( T_{37} \) and respectively on \( (G_{39}) \) and not on \( t \) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( G_i(t) = 0 \) and \( T_i(t) = 0 \)

From CONCATENATED GLOBAL EQUATIONS it results

\[ G_i(t) \geq G_i \left[ \int_{0}^{1} \left( (a_{36}^{(7)})^{(7)}(T_{37}^{(1)}, S_{(36)})dS_{(36)} \right) \right] \geq 0 \]

\[ T_i(t) \geq T_i \left[ \int_{0}^{1} \left( (b_{36}^{(7)})^{(7)} \right) \right] > 0 \] for \( t > 0 \)

**Definition of** \( (\bar{M}_{36})^{(7)} \) and \( (\bar{M}_{36})^{(7)} \) : 387

**Remark 3:** if \( G_{36} \) is bounded, the same property have also \( G_{37} \) and \( G_{38} \), indeed if

\[ G_{36} < (\bar{M}_{36})^{(7)} \text{ it follows } \frac{dG_{37}}{dt} \leq (\bar{M}_{36})^{(7)} - (a_{37}^{(7)})^{(7)} G_{37} \] and by integrating

\[ G_{37} \leq (\bar{M}_{36})^{(7)} + 2(a_{37}^{(7)}(\bar{M}_{36})^{(7)})/(a_{37}^{(7)})^{(7)} \]

In the same way, one can obtain
If $G_{38}$ or $G_{39}$ is bounded, the same property follows for $G_{36}$, $G_{38}$ and $G_{36}$, $G_{37}$ respectively.

**Remark 7:** If $G_{36}$ is bounded, from below, the same property holds for $G_{37}$ and $G_{38}$. The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below.

**Remark 5:** If $G_{36}$ is bounded from below and

\[ (a_{16})^{(2)}(m)^{(2)} - (a_{16})^{(2)}(m)^{(2)} + (a_{17})^{(2)}(m)^{(2)} - (a_{17})^{(2)}(m)^{(2)} \leq -(\tau_2)^{(2)} \]

then $T_{37}$ is unbounded. The same property holds for $T_{38}$ if $\lim_{t \to \infty} (b_{38}^{''})^{(1)}((G_{39})(t), t) = (b_{38}^{''})^{(1)}$ then $T_{37} \to \infty$.

**Definition of** $(m)^{(2)}$ and $\varepsilon_T$:

Indeed let $t_T$ be so that for $t > t_T$

\[ (b_{37})^{(2)}(m)^{(2)} - (a_{16})^{(2)}(m)^{(2)} < \varepsilon_T, T_{36}(t) > (m)^{(2)} \]

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(2)}(m)^{(2)} - \varepsilon_T T_{37}$ which leads to

\[ T_{37} \geq \left( \frac{(a_{37})^{(2)}(m)^{(2)}}{2} \right) \left( 1 - e^{-\varepsilon_T t} \right) + T_0^{(2)} e^{-\varepsilon_T t} \] If we take $t$ such that $e^{-\varepsilon_T t} = \frac{1}{2}$ it results

\[ T_{37} \geq \left( \frac{(a_{37})^{(2)}(m)^{(2)}}{2} \right), t = \log \frac{2}{\varepsilon_T} \] By taking now $\varepsilon_T$ sufficiently small one sees that $T_{37}$ is unbounded. The same property holds for $T_{38}$ if $\lim_{t \to \infty} (b_{38}^{''})^{(1)}((G_{39})(t), t) = (b_{38}^{''})^{(1)}$.

We now state a more precise theorem about the behaviors at infinity of the solutions

\[ -(a_2)^{(2)} - (a_2)^{(2)} - (a_2)^{(2)}(T_1, t) + (a_2)^{(2)}(T_1, t) \leq -(\sigma_2)^{(2)} \]

\[ -(\tau_2)^{(2)} - (a_2)^{(2)} + (b_1)^{(2)} - (b_1)^{(2)}(G_{19}, t) - (b_1)^{(2)}(G_{19}, t) \leq -(\tau_2)^{(2)} \]

**Definition of** $(v_1)^{(2)}$, $(v_2)^{(2)}$, $(u_1)^{(2)}$, $(u_2)^{(2)}$:

By $(v_1)^{(2)} > 0$, $(v_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots

\[ (a_17)^{(2)}(v)^{(2)} + (a_17)^{(2)}(u)^{(2)} - (a_16)^{(2)} = 0 \]

and $(b_14)^{(2)}(u)^{(2)} + (\tau_2)^{(2)}(u)^{(2)} - (b_16)^{(2)} = 0$ and

**Definition of** $(\bar{v}_1)^{(2)}$, $(\bar{v}_2)^{(2)}$, $(\bar{u}_1)^{(2)}$, $(\bar{u}_2)^{(2)}$:

By $(\bar{v}_1)^{(2)} > 0$, $(\bar{v}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the roots of the equations $(a_17)^{(2)}(v)^{(2)} + (a_17)^{(2)}(u)^{(2)} - (a_16)^{(2)} = 0$

and $(b_14)^{(2)}(u)^{(2)} + (\tau_2)^{(2)}(u)^{(2)} - (b_16)^{(2)} = 0$

**Definition of** $(m_1)^{(2)}$, $(m_2)^{(2)}$, $(\mu_1)^{(2)}$, $(\mu_2)^{(2)}$:

(b) If we define $(m_1)^{(2)}$, $(m_2)^{(2)}$, $(\mu_1)^{(2)}$, $(\mu_2)^{(2)}$ by

\[ m_1 \leq m_2 \leq \mu_1 \leq \mu_2 \]

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\((m_2)^{(2)} = (v_0)^{(2)} , (m_1)^{(2)} = (v_1)^{(2)} \), \(\text{if } (v_0)^{(2)} < (v_1)^{(2)}\)

\((m_2)^{(2)} = (v_1)^{(2)} , (m_1)^{(2)} = (\bar{v}_1)^{(2)} \), \(\text{if } (v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)}\),

\[
(V_0)^{(2)} = \frac{\theta_0}{u_{16}}
\]

\((m_2)^{(2)} = (v_1)^{(2)} , (m_1)^{(2)} = (v_0)^{(2)} \), \(\text{if } (v_1)^{(2)} < (v_0)^{(2)}\)

and analogously

\((\mu_2)^{(2)} = (u_0)^{(2)} , (\mu_1)^{(2)} = (u_1)^{(2)} \), \(\text{if } (u_0)^{(2)} < (u_1)^{(2)}\)

\((\mu_2)^{(2)} = (u_1)^{(2)} , (\mu_1)^{(2)} = (\bar{u}_1)^{(2)} \), \(\text{if } (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}\),

and \((u_0)^{(2)} = \frac{\theta_0}{T_{16}}\)

\((\mu_2)^{(2)} = (u_1)^{(2)} , (\mu_1)^{(2)} = (u_0)^{(2)} \), \(\text{if } (u_1)^{(2)} < (u_0)^{(2)}\)

Then the solution satisfies the inequalities

\[G_{16}^0 e^{(S_1)^{(2)} - (P_{16})^{(2)} t} \leq G_{16}^0 e^{(S_2)^{(2)} t}\]

\((p_j)^{(2)}\) is defined

\[
-\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (P_{16})^{(2)} t)} \leq G_{17}^0 (t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_2)^{(2)} t}
\]

\[
\frac{G_{16}^0 e^{((S_1)^{(2)} - (P_{16})^{(2)} t) - e^{-(S_2)^{(2)} t} + G_{16}^0 e^{-(S_2)^{(2)} t} \leq G_{16}^0 (t)} \leq G_{18}^0 (t) \leq \frac{G_{16}^0 e^{-(S_1)^{(2)} t} - e^{-(S_1)^{(2)} t} + G_{16}^0 e^{-(S_2)^{(2)} t}}{
\frac{(m_1)^{(2)}}{(m_1)^{(2)} + (a_{16})^{(2)}} \frac{(m_1)^{(2)}}{(m_1)^{(2)} + (a_{16})^{(2)}}}
\]

\[
T_{16}^0 e^{(R_2)^{(2)} t} \leq \leq T_{16}^0 e^{((R_2)^{(2)} + (r_{16})^{(2)} t)}
\]

\[
-\frac{1}{(m_2)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \leq T_{16}^0 (t) \leq \frac{1}{(m_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)} t) t}
\]

\[
\frac{(b_{16})^{(2)}}{(m_2)^{(2)} + (b_{16})^{(2)}} \frac{(b_{16})^{(2)}}{(m_2)^{(2)} + (b_{16})^{(2)}} \frac{(b_{16})^{(2)}}{(m_2)^{(2)} + (b_{16})^{(2)}} \frac{(b_{16})^{(2)}}{(m_2)^{(2)} + (b_{16})^{(2)}} \frac{(b_{16})^{(2)}}{(m_2)^{(2)} + (b_{16})^{(2)}} \frac{(b_{16})^{(2)}}{(m_2)^{(2)} + (b_{16})^{(2)}} \frac{(b_{16})^{(2)}}{(m_2)^{(2)} + (b_{16})^{(2)}} \frac{(b_{16})^{(2)}}{(m_2)^{(2)} + (b_{16})^{(2)}} \frac{(b_{16})^{(2)}}{(m_2)^{(2)} + (b_{16})^{(2)}} \frac{(b_{16})^{(2)}}{(m_2)^{(2)} + (b_{16})^{(2)}} \frac{(b_{16})^{(2)}}{(m_2)^{(2)} + (b_{16})^{(2)}}
\]

**Definition of** \((S_1)^{(2)} , (S_2)^{(2)} , (R_1)^{(2)} , (R_2)^{(2)}\):

Where \((S_2)^{(2)} = (a_{16})^{(2)} (m_2)^{(2)} - (a_{16})^{(2)}\)

\[(S_2)^{(2)} = (a_{16})^{(2)} - (P_{16})^{(2)}\]

\[(R_1)^{(2)} = (b_{16})^{(2)} (\mu_2)^{(1)} - (b_{16})^{(2)}\]

\[(R_2)^{(2)} = (b_{16})^{(2)} - (r_{16})^{(2)}\]

**Behavior of the solutions**
If we denote and define

**Definition of** \((\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}\):

(a) \((\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}\) four constants satisfying

\[-(\sigma_2)^{(3)} \leq -(a_{20}^{'})^{(3)} + (a_{21}^{'})^{(3)} - (a_{20}^{'})^{(3)}(T_{21},t) + (a_{21}^{'})^{(3)}(T_{21},t) \leq -(\sigma_1)^{(3)}\]

\[-(\tau_2)^{(3)} \leq -(b_{20}^{'})^{(3)} + (b_{21}^{'})^{(3)} - (b_{20}^{'})^{(3)}(G, t) - (b_{21}^{'})^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)}\]

**Definition of** \((v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}\):

(b) By \((v_1)^{(3)} > 0, (v_2)^{(3)} < 0\) and respectively \((u_1)^{(3)} > 0, (u_2)^{(3)} < 0\) the roots of the equations

\((a_{21}^{'})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20}^{'})^{(3)} = 0\)

and \((b_{21}^{'})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20}^{'})^{(3)} = 0\) and

By \((\tilde{v}_1)^{(3)} > 0, (\tilde{v}_2)^{(3)} < 0\) and respectively \((\tilde{u}_1)^{(3)} > 0, (\tilde{u}_2)^{(3)} < 0\) the roots of the equations

\((a_{21}^{'})^{(3)}(\tilde{v}^{(3)})^2 + (\sigma_2)^{(3)}\tilde{v}^{(3)} - (a_{20}^{'})^{(3)} = 0\)

and \((b_{21}^{'})^{(3)}(\tilde{u}^{(3)})^2 + (\tau_2)^{(3)}\tilde{u}^{(3)} - (b_{20}^{'})^{(3)} = 0\)

**Definition of** \((m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}\):

(c) If we define \((m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}\) by

\((m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}, \text{ if } (v_0)^{(3)} < (v_1)^{(3)}\)

\((m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\tilde{v}_1)^{(3)}, \text{ if } (v_1)^{(3)} < (v_0)^{(3)} < (\tilde{v}_1)^{(3)}\)

and \((v_0)^{(3)} = \frac{\sigma_2}{\sigma_1}\)

\((m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}, \text{ if } (v_1)^{(3)} < (v_0)^{(3)}\)

and analogously

\((\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \text{ if } (u_0)^{(3)} < (u_1)^{(3)}\)

\((\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\tilde{u}_1)^{(3)}, \text{ if } (u_1)^{(3)} < (u_0)^{(3)} < (\tilde{u}_1)^{(3)}\)

and \((u_0)^{(3)} = \frac{\sigma_2}{\sigma_1}\)

\((\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \text{ if } (u_1)^{(3)} < (u_0)^{(3)}\)

Then the solution satisfies the inequalities

\[G_{20}^0 e^{((S_1)^{(3)}-(P_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{((S_1)^{(3)})t}\]

\((p_j)^{(3)}\) is defined

\[
\frac{1}{(m_2)^{(3)}} G_{20}^0 e^{((S_1)^{(3)}-P_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{((S_1)^{(3)})t}\]

\[
\left(\frac{(a_{22})^{'}}{(m_1)^{(3)}((S_1)^{(3)}-P_{20})^{(3)}-S_2}\right) \left[e^{((S_1)^{(3)}-P_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + c_{22}^0 e^{-(S_2)^{(3)}t} \leq G_{22}(t) \leq \]

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\[
\frac{(a_{22}^{(3)}G_{20}^{(3)})}{(m_{2}^{(3)}d_{1}^{(3)}-a_{22}^{(3)}b_{20}^{(3)})}\left[e^{(a_{11}^{(3)})t} - e^{-(a_{22}^{(3)})t}\right] + G_{22}^{(3)}e^{-(a_{22}^{(3)})t}
\]

\[T_{20}^{0}e^{(R_{1}^{(3)})^{2}}t \leq T_{20}^{0}e^{(R_{1}^{(3)})^{2}}t \leq T_{20}^{0}e^{((R_{1}^{(3)})+(r_{20}^{(3)})^{2})t}\]

\[\frac{1}{(\mu_{1}^{(3)}T_{20}^{0}e^{(R_{1}^{(3)})^{2}}t) \leq T_{20}^{0}t} \leq \frac{1}{(\mu_{2}^{(3)}T_{20}^{0}e^{((R_{1}^{(3)})+(r_{20}^{(3)})^{2})t}}\]

\[\frac{(b_{22}^{(3)})^{2}}{(\mu_{1}^{(3)}(R_{2}^{(3)})^{2}-(b_{22}^{(3)})^{2})^{2}} \left[e^{(R_{1}^{(3)})^{2}}t - e^{-(b_{22}^{(3)})^{2}}t\right] + T_{22}^{0}e^{-(b_{22}^{(3)})^{2}}t \leq T_{22}^{0}t \leq T_{22}^{0}e^{(R_{1}^{(3)})^{2}}t + T_{22}^{0}e^{-(R_{2}^{(3)})^{2}}t\]

**Definition of** \((S_{1}^{(3)}, S_{2}^{(3)}, R_{1}^{(3)}, R_{2}^{(3)})\):

Where \((S_{1}^{(3)}) = (a_{20}^{(3)})(m_{2}^{(3)}) - (a_{20}^{(3)})^{2}\)
\[(S_{2}^{(3)}) = (a_{22}^{(3)}) - (b_{22}^{(3)})\]
\[(R_{1}^{(3)}) = (b_{20}^{(3)})(\mu_{2}^{(3)}) - (b_{20}^{(3)})^{2}\]
\[(R_{2}^{(3)}) = (b_{22}^{(3)}) - (r_{22}^{(3)})\]

\[\text{If we denote and define} \]

**Definition of** \((\sigma_{1}^{(4)}, \sigma_{2}^{(4)}, \tau_{1}^{(4)}, \tau_{2}^{(4)})\):

(d) \((\sigma_{1}^{(4)}, \sigma_{2}^{(4)}, \tau_{1}^{(4)}, \tau_{2}^{(4)})\) four constants satisfying

\[-(\sigma_{2}^{(4)}) \leq -(a_{24}^{(4)}) + (b_{24}^{(4)}) - (a_{24}^{(4)})(T_{25}, t) + (a_{25}^{(4)})(T_{25}, t) \leq -(\sigma_{1}^{(4)})\]
\[-(\tau_{2}^{(4)}) \leq -(b_{24}^{(4)}) + (b_{24}^{(4)}) - (b_{24}^{(4)})(G_{27}, t) - (b_{25}^{(4)})(G_{27}, t) \leq -(\tau_{1}^{(4)})\]

**Definition of** \((\nu_{i}^{(4)}, \nu_{2}^{(4)}, \nu_{3}^{(4)}, \nu_{4}^{(4)}, \nu_{5}^{(4)}, u^{(4)})\):

(e) By \((\nu_{1}^{(4)}) > 0, \nu_{2}^{(4)} < 0\) and respectively \((u_{1}^{(4)}) > 0, (u_{2}^{(4)}) < 0\) the roots of the equations \((a_{25}^{(4)})^{2}(\nu^{(4)})^{2} + (\sigma_{1}^{(4)})^{2}(\nu^{(4)}) - (a_{24}^{(4)})^{2}\) and \((b_{25}^{(4)})^{2}(u^{(4)})^{2} + (\tau_{1}^{(4)})^{2}(u^{(4)}) - (b_{24}^{(4)})^{2}\) are

**Definition of** \((\bar{\nu}_{i}^{(4)}, \bar{\nu}_{2}^{(4)}, \bar{\nu}_{3}^{(4)}, \bar{\nu}_{4}^{(4)}, \bar{\nu}_{5}^{(4)}, \bar{\nu}_{6}^{(4)})\):

By \((\bar{\nu}_{1}^{(4)}) > 0, (\bar{\nu}_{2}^{(4)}) < 0\) and respectively \((\bar{\nu}_{1}^{(4)}) > 0, (\bar{\nu}_{2}^{(4)}) < 0\) the roots of the equations \((a_{25}^{(4)})^{2}(\bar{\nu}^{(4)})^{2} + (\sigma_{1}^{(4)})^{2}(\bar{\nu}^{(4)}) - (a_{24}^{(4)})^{2}\) and \((b_{25}^{(4)})^{2}(\bar{u}^{(4)})^{2} + (\tau_{1}^{(4)})^{2}(\bar{u}^{(4)}) - (b_{24}^{(4)})^{2}\)

**Definition of** \((m_{1}^{(4)}, m_{2}^{(4)}, \mu_{1}^{(4)}, \mu_{2}^{(4)}, v_{0}^{(4)})\):

(f) If we define \((m_{1}^{(4)}, m_{2}^{(4)}, \mu_{1}^{(4)}, \mu_{2}^{(4)})\) by

\[m_{2}^{(4)} = \nu_{0}^{(4)}, m_{1}^{(4)} = \nu_{1}^{(4)}, \text{if} \quad \nu_{0}^{(4)} < \nu_{1}^{(4)}\]
\[(m_1)^{(4)} = (v_4)^{(4)}, (m_2)^{(4)} = (\bar{v}_4)^{(4)}, \text{ if } (v_4)^{(4)} < (\bar{v}_4)^{(4)} \]
and
\[(v_0)^{(4)} = \frac{\bar{v}_2}{\bar{g}_2} \]
\[(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}, \text{ if } (\bar{v}_4)^{(4)} < (v_0)^{(4)} \]
and analogously
\[(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}, \text{ if } (u_0)^{(4)} < (u_1)^{(4)} \]
\[(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \text{ if } (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)} , \]
and
\[(u_0)^{(4)} = \frac{T_2}{T_0} \]
\[(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \text{ if } (\bar{u}_1)^{(4)} < (u_0)^{(4)} \]
where \((u_1)^{(4)}\), \((\bar{u}_1)^{(4)}\) are defined respectively.

Then the solution satisfies the inequalities
\[G_2 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{24} e^{(S_1)^{(4)}t} \leq G_{24} e^{((S_2)^{(4)} - (p_{24})^{(4)})t} \]
where \((p_j)^{(4)}\) is defined

\[
\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{24}^0 e^{(S_1)^{(4)}t} \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{((S_2)^{(4)} - (p_{24})^{(4)})t} \]
\[
\frac{(a_{24})^{(4)} e_{24}^{(S_1)^{(4)} - (p_{24})^{(4)}}}{(m_1)^{(4)}} \left[ e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t} \right] + G_{26}^0 e^{-((S_2)^{(4)} - (a_{26})^{(4)})t} \leq G_{26}^0 e^{-((S_2)^{(4)} - (a_{26})^{(4)})t} \]
\[
T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \leq T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \leq T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \]
\[
\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \leq T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \]
\[
\frac{(b_{24})^{(4)} e_{24}^{(R_1)^{(4)} + (r_{24})^{(4)}}}{(\mu_1)^{(4)}} \left[ e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-((R_2)^{(4)} - (b_{26})^{(4)})t} \right] + T_{26}^0 e^{-((R_2)^{(4)} - (b_{26})^{(4)})t} \leq T_{26}^0 e^{-((R_2)^{(4)} - (b_{26})^{(4)})t} \]
\[
\frac{(a_{24})^{(4)} e_{24}^{(R_1)^{(4)} + (r_{24})^{(4)}}}{(\mu_2)^{(4)}} \left[ e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-((R_2)^{(4)} - (a_{26})^{(4)})t} \right] + T_{26}^0 e^{-((R_2)^{(4)} - (a_{26})^{(4)})t} \leq T_{26}^0 e^{-((R_2)^{(4)} - (a_{26})^{(4)})t} \]

**Definition of** \((S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}):\)

Where \((S_1)^{(4)} = (a_{24})^{(4)} (m_2)^{(4)} - (a_{24})^{(4)} \)
\[(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)} \]
\[(R_1)^{(4)} = (b_{24})^{(4)} (\mu_2)^{(4)} - (b_{24})^{(4)} \]
\[(R_2)^{(4)} = (b_{26})^{(4)} - (r_{26})^{(4)} \]
Behavior of the solutions

If we denote and define

**Definition of \((\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}\):**

\[(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}\] four constants satisfying

\[-(\sigma_2)^{(5)} \leq -(a_2^{(5)}T_{29})^2 + (a_2^{(5)}T_{29}) \leq -(\sigma_1)^{(5)}\]

\[-(\tau_2)^{(5)} \leq -(b_2^{(5)}T_{29}) - (b_2^{(5)}T_{29})^2 \leq -(\tau_1)^{(5)}\]

**Definition of \((\nu_1)^{(5)}, (\nu_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu)^{(5)}\) and \((\mu)^{(5)}\):**

\[0 < (\nu_1)^{(5)} < (\nu_2)^{(5)} < 0\text{ and respectively } (\mu_1)^{(5)} < (\mu_2)^{(5)} < 0\] the roots of the equations

\[a_2^{(5)}(\nu)^{(5)} + (\sigma_1)^{(5)}\nu - (a_2^{(5)}) = 0\]

and

\[b_2^{(5)}(\mu)^{(5)} + (\tau_1)^{(5)}\mu - (b_2^{(5)}) = 0\]

**Definition of \((\bar{\nu}_1)^{(5)}, (\bar{\nu}_2)^{(5)}, (\bar{\mu}_1)^{(5)}, (\bar{\mu}_2)^{(5)}\):**

\[0 < (\bar{\nu}_1)^{(5)} < (\bar{\nu}_2)^{(5)} < 0\text{ and respectively } (\bar{\mu}_1)^{(5)} < (\bar{\mu}_2)^{(5)} < 0\] the roots of the equations

\[a_2^{(5)}(\bar{\nu}) - (a_2^{(5)}) = 0\]

and

\[(\bar{\nu}) - (b_2^{(5)}) = 0\]

**Definition of \((\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_1)^{(5)}, (\nu_2)^{(5)}\):**

If we define \((\mu_1)^{(5)}, (\mu_2)^{(5)}\) by

\[(\mu_2)^{(5)} = (\nu_0)^{(5)}, (\mu_1)^{(5)} = (\nu_1)^{(5)}, \text{ if } (\nu_0)^{(5)} < (\nu_1)^{(5)}\]

\[(\mu_2)^{(5)} = (\nu_1)^{(5)}, (\mu_1)^{(5)} = (\nu_0)^{(5)}, \text{ if } (\nu_1)^{(5)} < (\nu_0)^{(5)}\]

and analogously

\[(\mu_2)^{(5)} = (\nu_0)^{(5)}, (\mu_1)^{(5)} = (\nu_1)^{(5)}, \text{ if } (\nu_0)^{(5)} < (\nu_1)^{(5)}\]

\[(\mu_2)^{(5)} = (\nu_1)^{(5)}, (\mu_1)^{(5)} = (\nu_0)^{(5)}, \text{ if } (\nu_1)^{(5)} < (\nu_0)^{(5)}\]

where \((\nu_1)^{(5)}, (\nu_0)^{(5)}\) are defined respectively

Then the solution satisfies the inequalities

\[G_28^0((s_1)^{(5)} - (p_{28})^{(5)})t \leq G_28^0((s_1)^{(5)}t) \leq G_28^0((s_2)^{(5)}t)\]

where \((p_{28})^{(5)}\) is defined

\[\frac{1}{(m_2)^{(5)}} G_28^0((s_1)^{(5)} - (p_{28})^{(5)})t \leq G_28^0((s_1)^{(5)}t) \leq \frac{1}{(m_2)^{(5)}} G_28^0((s_2)^{(5)}t)\]
\[
\frac{(a_{20})^{(5)}G_{20}^{0}}{(m_{1})^{(5)}(S_{1})^{(5)}(p_{20})^{(5)}(S_{1})^{(5)}} \left[ e^{((S_{1})^{(5)}-(p_{20})^{(5)}t)}e^{-((S_{2})^{(5)}t)} \right] + G_{30}^{0}e^{-((S_{2})^{(5)}t)} \leq G_{30}(t) \leq
\]

\[
\frac{(a_{30})^{(5)}G_{30}^{0}}{(m_{2})^{(5)}(S_{1})^{(5)}(a_{30})^{(5)}} \left[ e^{((S_{1})^{(5)}t)} - e^{-(a_{30})^{(5)}t} \right] + G_{30}^{0}e^{-(a_{30})^{(5)}t}
\]

\[
\frac{T_{20}^{0}e^{-((R_{1})^{(5)}t)}}{(μ_{1})^{(5)}} \leq T_{20}^{0} \leq \frac{T_{30}^{0}e^{-((R_{1})^{(5)}+(R_{2})^{(5)}t)}}{(μ_{2})^{(5)}}
\]

\[
\frac{1}{(μ_{1})^{(5)}}T_{20}^{0}e^{-((R_{1})^{(5)}t)} \leq T_{20}(t) \leq \frac{1}{(μ_{2})^{(5)}}T_{30}^{0}e^{-((R_{1})^{(5)}+(R_{2})^{(5)}t)}
\]

Where \((S_{1})^{(5)} = (a_{20})^{(5)}(m_{2})^{(5)} - (a'_{20})^{(5)}\)

\((S_{2})^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}\)

\((R_{1})^{(5)} = (b_{20})^{(5)}(μ_{2})^{(5)} - (b'_{20})^{(5)}\)

\((R_{2})^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}\)

**Behavior of the solutions**

If we denote and define

**Definition of** \((\sigma_{1})^{(6)}, (\sigma_{2})^{(6)}, (τ_{1})^{(6)}, (τ_{2})^{(6)}:\)

\(- (\sigma_{1})^{(6)} \leq - (a'_{32})^{(6)} + (a'_{33})^{(6)} - (a'_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq - (\sigma_{1})^{(6)}\)

\(- (τ_{2})^{(6)} \leq - (b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}(G_{33}, t) - (b''_{33})^{(6)}(G_{33}, t) \leq - (τ_{1})^{(6)}\)

**Definition of** \((v_{1})^{(6)}, (v_{2})^{(6)}, (u_{1})^{(6)}, (u_{2})^{(6)}, v^{(6)}, u^{(6)}:\)

By \((v_{1})^{(6)} > 0, (v_{2})^{(6)} < 0 \) and respectively \((u_{1})^{(6)} > 0, (u_{2})^{(6)} < 0 \) the roots of the equations \((a_{32})^{(6)}(v^{(6)})^{2} + (\sigma_{1})^{(6)}(v^{(6)} - (a_{32})^{(6)} = 0\)

and \((b_{33})^{(6)}(u^{(6)})^{2} + (τ_{1})^{(6)}(u^{(6)}) - (b_{32})^{(6)} = 0\)

**Definition of** \((\tilde{v}_{1})^{(6)}, (\tilde{v}_{2})^{(6)}, (\tilde{u}_{1})^{(6)}, (\tilde{u}_{2})^{(6)}:\)

By \((\tilde{v}_{1})^{(6)} > 0, (\tilde{v}_{2})^{(6)} < 0 \) and respectively \((\tilde{u}_{1})^{(6)} > 0, (\tilde{u}_{2})^{(6)} < 0 \) the roots of the equations \((\tau_{33})^{(6)}(v^{(6)})^{2} + (\sigma_{2})^{(6)}(v^{(6)} - (a_{32})^{(6)} = 0\)

and \((b_{33})^{(6)}(u^{(6)})^{2} + (τ_{2})^{(6)}(u^{(6)}) - (b_{32})^{(6)} = 0\)

**Definition of** \((m_{1})^{(6)}, (m_{2})^{(6)}, (μ_{1})^{(6)}, (μ_{2})^{(6)}, (v_{0})^{(6)}:\)

If we define \((m_{1})^{(6)}, (m_{2})^{(6)}, (μ_{1})^{(6)}, (μ_{2})^{(6)}\) by

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\[ (m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}, \text{ if } (v_0)^{(6)} < (v_1)^{(6)} \]

\[ (m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_0)^{(6)}, \text{ if } (v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)} \]

and

\[ (v_0)^{(6)} = \frac{c_{12}^6}{c_{14}^6} \]

\[ (m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \text{ if } (\bar{v}_1)^{(6)} < (v_0)^{(6}) \]

and analogously

\[ (\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \text{ if } (u_0)^{(6)} < (u_1)^{(6}) \]

\[ (\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \text{ if } (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)} \]

and

\[ (u_0)^{(6)} = \frac{T_0}{T_3^6} \]

\[ (\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (\bar{u}_1)^{(6)} < (u_0)^{(6}) \]

where \((u_0)^{(6)}, (\bar{u}_1)^{(6)}\) are defined respectively.

Then the solution satisfies the inequalities

\[ a_{32}^6 e^{((S_1)^{(6)} - (p_{32})^{(6)}) t} \leq G_{32}^0 e^{(S_1)^{(6)} t} \]

where \((p_j)^{(6)}\) is defined

\[ \frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)}) t} \leq G_{33}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)}) t} \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \]

\[ \left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}(S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)}}\right) \left[e^{((S_1)^{(6)} - (p_{32})^{(6)}) t} - e^{((S_2)^{(6)} - (p_{32})^{(6)}) t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}^0 \]

\[ T_{32}^0 e^{((R_1)^{(6)} - (r_{32})^{(6)}) t} \]

\[ T_{32}^0 e^{((R_1)^{(6)} - (r_{32})^{(6)}) t} \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)}) t} \]

\[ \frac{1}{(\mu_1)^{(6)} T_{32}^0 (R_1)^{(6)} t} \leq T_{32}^0 (R_1)^{(6)} t \leq \frac{1}{(\mu_2)^{(6)} T_{32}^0 (R_1)^{(6)} + (r_{32})^{(6)} t} \]

\[ \frac{(b_{34})^{(6)} e^{(R_1)^{(6)} t} - e^{-(b_{34})^{(6)} t}}{(\mu_1)^{(6)} (R_1)^{(6)} + (b_{34})^{(6)} t} + T_{34}^0 e^{-(b_{34})^{(6)} t} \leq T_{34}^0 \leq \frac{(a_{34})^{(6)} e^{(R_1)^{(6)} + (r_{32})^{(6)} t} - e^{-(R_2)^{(6)} t}}{(\mu_2)^{(6)} (R_1)^{(6)} + (r_{32})^{(6)} t + (R_2)^{(6)} t} + T_{34}^0 e^{-(R_2)^{(6)} t} \]

Definition of \((S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}\):-

Where \((S_1)^{(6)} = (a_{32})^{(6)} (m_2)^{(6)} - (a_{32}')^{(6)}\)

\[ (S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)} \]

\[ (R_1)^{(6)} = (b_{32})^{(6)} (\mu_2)^{(6)} - (b_{32}')^{(6)} \]

\[ (R_2)^{(6)} = (b_{34})^{(6)} - (r_{34})^{(6)} \]
If we denote and define

**Definition of** \((\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}\): 

\[(m) \ (\sigma_2)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)} \text{ four constants satisfying} \]
\[-(\sigma_2)^{(7)} \leq -(a_{36})^{(7)} + (a_{37})^{(7)} - (a_{36}''^{(7)})(T_{37}, t) + (a_{37}''^{(7)})(T_{37}, t) \leq -(\sigma_1)^{(7)} \]
\[-(\tau_2)^{(7)} \leq -(b_{36}''^{(7)} + (b_{36}''^{(7)})(G_{39}, t) - (b_{37}''^{(7)})(G_{39}, t) \leq -(\tau_2)^{(7)} \]

**Definition of** \((\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, (v)^{(7)}, (u)^{(7)}\): 

\[(n) \ By \ (\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0 \text{ and respectively } (u_1)^{(7)} > 0, (u_2)^{(7)} < 0 \text{ the roots of} \]
\[\text{the equations} \ (a_{37})^{(7)}(v)^{(7)} + (\sigma_1)^{(7)}v - (a_{36})^{(7)} = 0 \]
\[\text{and} \ (b_{37})^{(7)}(u)^{(7)} + (\tau_2)^{(7)}u - (b_{36})^{(7)} = 0 \]

**Definition of** \((\tilde{\nu})^{(7)}, (\tilde{\nu}_2)^{(7)}, (\tilde{u}_1)^{(7)}, (\tilde{u}_2)^{(7)}\): 

By \((\tilde{\nu}_1)^{(7)} > 0, (\tilde{\nu}_2)^{(7)} < 0 \text{ and respectively } (\tilde{u}_1)^{(7)} > 0, (\tilde{u}_2)^{(7)} < 0 \text{ the roots of} \]
\[\text{the equations} \ (a_{37})^{(7)}(\nu)^{(7)} + (\sigma_2)^{(7)}\nu - (a_{36})^{(7)} = 0 \]
\[\text{and} \ (b_{37})^{(7)}(u)^{(7)} + (\tau_2)^{(7)}u - (b_{36})^{(7)} = 0 \]

**Definition of** \((m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}\):

\[(o) \ If \ we \ define \ (m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)} \text{ by} \]
\[(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}, \ if \ (v_0)^{(7)} < (v_1)^{(7)} \]
\[(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\tilde{\nu}_1)^{(7)}, \ if \ (v_1)^{(7)} < (v_0)^{(7)} < (\tilde{\nu}_1)^{(7)} \]
\[\text{and} \quad (v_0)^{(7)} = \frac{\sigma_{36}}{\sigma_{37}} \]

\[(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ if \ (\nu_0)^{(7)} < (\nu_1)^{(7)} \]

\[(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\tilde{\nu}_2)^{(7)}, \ if \ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\tilde{\nu}_2)^{(7)} \]

and analogously

\[(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ if \ (u_0)^{(7)} < (u_1)^{(7)} \]
\[(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\tilde{u}_1)^{(7)}, \ if \ (u_1)^{(7)} < (u_0)^{(7)} < (\tilde{u}_1)^{(7)} \]
\[\text{and} \quad (u_0)^{(7)} = \frac{\tau_{36}}{\tau_{37}} \]
\[(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \text{if } (\bar{u}_2)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}\]

are defined respectively.

Then the solution satisfies the inequalities

\[G_{36}^0 e^{((s_1)^{(7)}-(p_{36})^{(7)})t} \leq G_{36}(t) \leq G_{36}^0 e^{(s_1)^{(7)}t}\]

where \((p_2)^{(7)}\) is defined

\[\frac{1}{(m_2)^{(7)}} G_{36}^0 e^{((s_1)^{(7)}-(p_{36})^{(7)})t} \leq G_{36}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(s_1)^{(7)}t}\]

\[\frac{(a_{36})^{(7)0} e_{36}}{(m_2)^{(7)}} e^{((s_1)^{(7)}-(p_{36})^{(7)})t} = e^{((s_1)^{(7)}-(p_{36})^{(7)})t} - e^{-(s_2)^{(7)}t} + G_{38}^0 e^{-(s_2)^{(7)}t} \leq G_{38}(t) \leq G_{38}^0 e^{-(s_2)^{(7)}t}\]

\[T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)}+(p_{36})^{(7)})t}\]

\[\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)}+(p_{36})^{(7)})t}\]

Definition of \((S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}\):

Where \((S_1)^{(7)} = (a_{36})^{(7)0} (m_2)^{(7)} - (a_{36})^{(7)}\)

\[(S_2)^{(7)} = (a_{36})^{(7)} - (p_{36})^{(7)}\]
\[(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b_3^{(7)})^{(7)}\]
\[(R_2)^{(7)} = (b_3^{(7)})^{(7)} - (r_{36})^{(7)}\]

From GLOBAL EQUATIONS we obtain

\[
\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left( (a_{37}^{(7)})^{(7)} - (a_{37}^{(7)})^{(7)} + (a_{36}^{(7)})^{(7)}(T_{37}, t) \right) - \\
(a_{37}^{(7)}(T_{37}, t)v^{(7)} - (a_{37})^{(7)}v^{(7)}
\]

**Definition of** \(\nu^{(7)} :\)

\[\nu^{(7)} = \frac{g_3}{g_2}\]

It follows

\[-\left((a_{37})^{(7)}(\nu^{(7)})^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq \]
\[-\left((a_{37})^{(7)}(\nu^{(7)})^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)\]

From which one obtains

**Definition of** \((\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)} :\)

(a) For \(0 < \frac{(\nu_0)^{(7)}}{(\nu_1)^{(7)}} = \frac{\sigma_2^0}{\sigma_2^1} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}\)

\[\nu^{(7)}(t) \geq \frac{(\nu_3)^{(7)}(C)^{(7)}(\nu_2)^{(7)}}{1 + (C)^{(7)}} e^{-(a_{37})^{(7)}(\nu_1^{(7)} - (\nu_0)^{(7)})t} = \]
\[\frac{(\nu_3)^{(7)}(C)^{(7)}(\nu_2)^{(7)}}{1 + (C)^{(7)}} e^{-(a_{37})^{(7)}(\nu_1^{(7)} - (\nu_0)^{(7)})t}
\]

\[\frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}\]

it follows \((\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}\)

In the same manner, we get
\[ v^{(7)}(t) \leq \frac{(v_3^{(7)})^2 + (c)^{7}\left|\frac{\partial}{\partial t}\left[\frac{v_1^{(7)} - v_2^{(7)}}{1 + (c)^{7}}\right]\right|}{1 + (c)^{7}} \leq \frac{(v_3^{(7)})^2}{(v_0^{(7)})^2} \]

From which we deduce \((v_0^{(7)}) \leq v^{(7)}(t) \leq (\bar{v}_1^{(7)})\)

(b) If \(0 < (v_1^{(7)}) < (v_0^{(7)}) = \frac{\delta_0}{g_{27}} < (\bar{v}_1^{(7)})\) we find like in the previous case,

\[ (v_2^{(7)}) \leq \frac{(v_3^{(7)})^2 + (c)^{7}\left|\frac{\partial}{\partial t}\left[\frac{v_1^{(7)} - v_2^{(7)}}{1 + (c)^{7}}\right]\right|}{1 + (c)^{7}} \leq v^{(7)}(t) \leq \frac{(v_3^{(7)})^2}{(v_0^{(7)})^2} \]

(c) If \(0 < (v_1^{(7)}) \leq (\bar{v}_2^{(7)}) \leq (v_0^{(7)}) = \frac{\delta_0}{g_{27}}\) we obtain

\[ (v_4^{(7)}) \leq v^{(7)}(t) \leq \frac{(v_3^{(7)})^2 + (c)^{7}\left|\frac{\partial}{\partial t}\left[\frac{v_1^{(7)} - v_2^{(7)}}{1 + (c)^{7}}\right]\right|}{1 + (c)^{7}} \leq (v_0^{(7)}) \]

And so with the notation of the first part of condition (c), we have

**Definition of** \(v^{(7)}(t):\)

\[ (m_2^{(7)}) \leq v^{(7)}(t) \leq (m_1^{(7)}), \quad v^{(7)}(t) = \frac{g_{36}(t)}{g_{27}(t)} \]

In a completely analogous way, we obtain

**Definition of** \(u^{(7)}(t):\)

\[ (\mu_2^{(7)}) \leq u^{(7)}(t) \leq (\mu_1^{(7)}), \quad u^{(7)}(t) = \frac{f_{36}(t)}{f_{27}(t)} \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.
Particular case:

If \( a^{n}_{36}(t) = a^{n}_{37}(t) \), then \( \sigma(t) = \sigma_2(t) \) and in this case \( v_1(t) = \bar{v}_1(t) \) if in addition \( v_0(t) = \bar{v}_0(t) \) then \( v(t) = v_0(t) \) and as a consequence \( G_{36}(t) = v_0(t)G_{37}(t) \) this also defines \( v_0(t) \) for the special case.

Analogously if \( b^{n}_{36}(t) = b^{n}_{37}(t) \), then \( \tau_1(t) = \tau_2(t) \) and then:

\[
(u_1(t) = \bar{u}_1) \text{ if in addition } (u_0(t) = u_1(t) \text{ then } T_{36}(t) = u_0(t)T_{37}(t) \text{ This is an important consequence of the relation between } (v_1(t) \text{ and } \bar{v}_1(t), \text{ and definition of } (u_0(t)).}
\]

We can prove the following

If \( (a^{n}_{36}(t) \text{ and } (b^{n}_{36}(t) \text{ are independent on } t, \text{ and the conditions}

\[
a^{n}_{36}(t)a^{n}_{37}(t) - (a^{n}_{36}(t)a^{n}_{37}(t) < 0
\]

\[
(a^{n}_{36}(t)a^{n}_{37}(t) - (a^{n}_{36}(t)a^{n}_{37}(t) + (a^{n}_{36}(t)p^{n}_{36}(t) + (a^{n}_{37}(t)p^{n}_{36}(t) + (p^{n}_{36}(t)p^{n}_{37}(t) > 0
\]

\[
(b^{n}_{36}(t)b^{n}_{37}(t) - (b^{n}_{36}(t)b^{n}_{37}(t) > 0,
\]

\[
(b^{n}_{36}(t)b^{n}_{37}(t) - (b^{n}_{36}(t)b^{n}_{37}(t) - (b^{n}_{37}(t)r^{n}_{37}(t) - (b^{n}_{37}(t)r^{n}_{37}(t) + (r^{n}_{36}(t)r^{n}_{37}(t) < 0
\]

with \( (p^{n}_{36}(t), r^{n}_{37}(t) \text{ as defined are satisfied, then the system WITH THE SATISFACTION OF THE FOLLOWING PROPERTIES HAS A SOLUTION AS DERIVED BELOW.}

Particular case:

If \( a^{n}_{29}(t) = a^{n}_{27}(t) \), then \( \sigma(t) = \sigma_2(t) \) and in this case \( v_1(t) = \bar{v}_1(t) \) if in addition \( v_0(t) = \bar{v}_0(t) \) then \( v(t) = v_0(t)G_{36}(t) = v_0(t)G_{37}(t) \)

Analogously if \( b^{n}_{29}(t) = b^{n}_{27}(t) \), then \( \tau_1(t) = \tau_2(t) \) and then:

\[
(u_1(t) = \bar{u}_1) \text{ if in addition } (u_0(t) = u_1(t) \text{ then } T_{16}(t) = u_0(t)T_{17}(t) \text{ This is an important consequence of the relation between } (v_1(t) \text{ and } \bar{v}_1(t) \text{ and definition of } (u_0(t)).}
\]

From GLOBAL EQUATIONS we obtain

\[
\frac{dv^{(3)}}{dt} = (a^{(3)}_{20}) - \left( (a^{(3)}_{20}) - (a^{(3)}_{21}) + (a^{(3)}_{20})(T_{21}, t) \right) - (a^{(3)}_{21})(T_{23}, t)v^{(3)} - (a^{(3)}_{21})v^{(3)}
\]
Definition of $v^{(3)}$:

$$v^{(3)} = \frac{g_{20}}{g_{21}}$$

It follows

$$-\left((a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}(v^{(3)} - (a_{20})^{(3)})\right) \leq \frac{dt}{dt} \leq -\left((a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}(v^{(3)} - (a_{20})^{(3)})\right)$$

From which we deduce

From which one obtains

(a) For $0 < (v_0)^{(3)} = \frac{g_{20}}{g_{21}} < (v_1)^{(3)} < (\tilde{v}_1)^{(3)}$

$$v^{(3)}(t) \geq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_0)^{(3)}]t}}{1 + (C)^{(3)} e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_0)^{(3)}]t}}$$

$$\left(C\right)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}$$

it follows $(v_0)^{(3)} \leq v^{(3)}(t) \leq (v_1)^{(3)}$

In the same manner, we get

$$v^{(3)}(t) \leq \frac{(v_1)^{(3)} + (\tilde{C})^{(3)}(v_2)^{(3)} e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_2)^{(3)}]t}}{1 + (\tilde{C})^{(3)} e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_2)^{(3)}]t}}$$

$$\left(\tilde{C}\right)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}$$

Definition of $(\tilde{v}_1)^{(3)}$:

From which we deduce $(v_0)^{(3)} \leq v^{(3)}(t) \leq (\tilde{v}_1)^{(3)}$

(b) If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{g_{20}}{g_{21}} < (\tilde{v}_1)^{(3)}$ we find like in the previous case,

$$(v_1)^{(3)} \leq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_2)^{(3)}]t}}{1 + (C)^{(3)} e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_2)^{(3)}]t}} \leq v^{(3)}(t) \leq \frac{(v_1)^{(3)} + (\tilde{C})^{(3)}(v_2)^{(3)} e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_2)^{(3)}]t}}{1 + (\tilde{C})^{(3)} e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_2)^{(3)}]t}}$$

(c) If $0 < (v_1)^{(3)} \leq (\tilde{v}_1)^{(3)} \leq (v_0)^{(3)} = \frac{g_{20}}{g_{21}}$, we obtain

$$(v_1)^{(3)} \leq v^{(3)}(t) \leq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_2)^{(3)}]t}}{1 + (C)^{(3)} e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_2)^{(3)}]t}} \leq (v_0)^{(3)}$$

And so with the notation of the first part of condition (c), we have

Definition of $v^{(3)}(t)$:

$$(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}$$

$$v^{(3)}(t) = \frac{g_{20}(t)}{g_{21}(t)}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:
\((\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}\).

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case:**

If \((a_2^{(3)}) = (a_1^{(3)})\), then \((\sigma^2)^{(3)} = (\sigma_1)^{(3)}\) and in this case \((v_1)^{(3)} = (\bar{v}_1)^{(3)}\) if in addition \((v_0)^{(3)} = (v_1)^{(3)}\) then \(v^{(3)}(t) = (v_0)^{(3)}\) and as a consequence \(G_20(t) = (v_0)^{(3)}\).

Analogously if \((b_2^{(3)}) = (b_1^{(3)})\), then \((\tau^2)^{(3)} = (\tau_1)^{(3)}\) and then

\((u_1)^{(3)} = (\tilde{u}_1)^{(3)}\) if in addition \((u_0)^{(3)} = (u_1)^{(3)}\) then \(T_20(t) = (u_0)^{(3)}\).

This is an important consequence of the relation between \((v_1)^{(3)}\) and \((\bar{v}_1)^{(3)}\).

\[\frac{dv}{dt} = (a_2v)^{(4)} - \left((a_{20})^{(4)} - (a_{21})^{(4)} + (a_{20})^{(4)}(T_{20}) - (a_{21})^{(4)}(T_{21})\right) - (a_{20})^{(4)}(T_{20})v^{(4)} - (a_{21})^{(4)}v^{(4)}\]

**Definition of \(v^{(4)}\):**

\[v^{(4)} = \frac{G_{2x}}{G_{2x}}\]

It follows

\[-(a_{2x})^{(4)}v^{(4)} + (a_{2x})^{(4)}v^{(4)} = \frac{dv}{dt} \leq -\left((a_{2x})^{(4)}v^{(4)} + (a_{4x})^{(4)}v^{(4)}\right)\]

From which one obtains

**Definition of \((\bar{v}_1)^{(4)}\), \((v_0)^{(4)}\):**

\[(d)\] For \(0 < (v_0)^{(4)} = \frac{G_{2x}}{G_{2x}} < (v_1)^{(4)} < (\bar{v}_1)^{(4)}\)

\[v^{(4)}(t) \geq \frac{(v_1)^{(4) + (4)}(v_2)^{(4)}e^{-\left((a_{2x})^{(4)}(v_{1}^{(4)} - (v_0)^{(4)})\right)}}{4 + (C)^{(4)}e^{-\left((a_{2x})^{(4)}(v_{1}^{(4)} - (v_0)^{(4)})\right)}}\]

\[C^{(4)} = \frac{(v_1)^{(4) - (v_0)^{(4)}}}{(v_0)^{(4) - (v_2)^{(4)}}}\]

it follows \((v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)}\)

In the same manner, we get

\[v^{(4)}(t) \leq \frac{(v_2)^{(4) + (4)}(v_1)^{(4)}e^{-\left((a_{2x})^{(4)}(v_{2}^{(4)} - (v_0)^{(4)})\right)}}{4 + (\bar{C})^{(4)}e^{-\left((a_{2x})^{(4)}(v_{2}^{(4)} - (v_0)^{(4)})\right)}}\]

\[\bar{C}^{(4)} = \frac{(v_2)^{(4) - (v_0)^{(4)}}}{(v_0)^{(4) - (v_2)^{(4)}}}\]

From which we deduce \((v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)}\)

\[(e)\] If \(0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{G_{2x}}{G_{2x}} < (\bar{v}_1)^{(4)}\) we find like in the previous case,

\[v^{(4)}(t) \leq \frac{\left((v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)}e^{-\left((a_{2x})^{(4)}(v_{1}^{(4)} - (v_0)^{(4)})\right)}\right)}{1 + (C)^{(4)}e^{-\left((a_{2x})^{(4)}(v_{1}^{(4)} - (v_0)^{(4)})\right)}} \leq v^{(4)}(t) \leq \]

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\[
\frac{(\bar{v}_2)^{(4)} + (\bar{c})^{(4)}(\bar{v}_2)^{(4)} e^{-(\theta_25)^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t}}{1 + (\bar{c})^{(4)} e^{-(\theta_25)^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t}} \leq (\bar{v}_1)^{(4)}
\]

(f) If \(0 < (v_1)^{(4)} \leq (\bar{v}_1)^{(4)} \leq \frac{(v_0)^{(4)}}{g_{24}(t)}\), we obtain

\[
(v_1)^{(4)} \leq v(t) \leq \frac{(v_1)^{(4)} + (\bar{c})^{(4)}(\bar{v}_2)^{(4)} e^{-(\theta_25)^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t}}{1 + (\bar{c})^{(4)} e^{-(\theta_25)^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t}} \leq (v_0)^{(4)}
\]

And so with the notation of the first part of condition (c), we have

**Definition of** \(v(t)\) :

\[
(m_2)^{(4)} \leq v(t) \leq (m_1)^{(4)}, \quad v(t) = \frac{g_{24}(t)}{g_{25}(t)}
\]

In a completely analogous way, we obtain

**Definition of** \(u(t)\) :

\[
(\mu_2)^{(4)} \leq u(t) \leq (\mu_1)^{(4)}, \quad u(t) = \frac{r_{24}(t)}{r_{25}(t)}
\]

Now, using this result and replacing it in **GLOBAL EQUATIONS** we get easily the result stated in the theorem.

**Particular case:**

If \((a_{28}^\nu)^{(4)} = (a_{29}^\nu)^{(4)}, then \((\sigma_1)^{(4)} = (\sigma_2)^{(4)}\) and in this case \((v_1)^{(4)} = (\bar{v}_1)^{(4)}\) if in addition \((v_0)^{(4)} = (v_1)^{(4)}\) then \(v(t) = (v_0)^{(4)}\) and as a consequence \(g_{24}(t) = (v_0)^{(4)}g_{25}(t)\) **this also defines** \((v_0)^{(4)}\) for the special case.

Analogously if \((b_{28}^\nu)^{(4)} = (b_{29}^\nu)^{(4)}, then \((\tau_1)^{(4)} = (\tau_2)^{(4)}\) and then \((u_1)^{(4)} = (\bar{u}_1)^{(4)}\) if in addition \((u_0)^{(4)} = (u_1)^{(4)}\) then \(r_{24}(t) = (u_0)^{(4)}r_{25}(t)\) This is an important consequence of the relation between \((v_1)^{(4)}\) and \((\bar{v}_1)^{(4)}\), **and definition of** \((u_0)^{(4)}\).

**From** **GLOBAL EQUATIONS** we obtain

\[
\frac{d\nu}{dt} = (a_{28})^{(5)} - \left((a_{29})^{(5)} - (a_{28})^{(5)} + (a_{28})^{(5)}(T_{29}, t) - (a_{29})^{(5)}(T_{29}, t)\right)\nu^{(5)} - (a_{28})^{(5)}\nu^{(5)}
\]

**Definition of** \(\nu^{(5)}\) :

\[
\nu^{(5)} = \frac{g_{28}}{g_{29}}
\]

It follows

\[
-\left((a_{29})^{(5)}\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} \leq \frac{d\nu}{dt} \leq -\left((a_{29})^{(5)}\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}
\]

From which one obtains

**Definition of** \((\bar{v}_1)^{(5)}, (v_0)^{(5)}\) :

(g) For \(0 < (v_0)^{(5)} = \frac{g_{28}}{g_{29}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}\)
we obtain

\[ v^{(5)}(t) \geq \frac{(v_1^{(5)} + C^{(5)}(v_2^{(5)} - \frac{v_0^{(5)} - v_1^{(5)} + C^{(5)}(v_2^{(5)} - \frac{v_0^{(5)} - v_1^{(5)} - v_2^{(5)}}{v_0^{(5)} - v_2^{(5)}})}{v_0^{(5)} - v_2^{(5)}}) e^{-a_2^{(5)}(v_2^{(5)} - v_1^{(5)} - v_2^{(5)}) t}}{v_0^{(5)} - v_2^{(5)}} e^{-a_2^{(5)}(v_2^{(5)} - v_1^{(5)} - v_2^{(5)}) t}} \]

it follows \( v_0^{(5)} \leq v^{(5)}(t) \leq v_2^{(5)} \)

In the same manner, we get

\[ v^{(5)}(t) \leq \frac{(v_2^{(5)} + C^{(5)}(v_1^{(5)} - \frac{v_0^{(5)} - v_1^{(5)} + C^{(5)}(v_2^{(5)} - \frac{v_0^{(5)} - v_1^{(5)} - v_2^{(5)}}{v_0^{(5)} - v_2^{(5)}})}{v_0^{(5)} - v_2^{(5)}}) e^{-a_2^{(5)}(v_2^{(5)} - v_1^{(5)} - v_2^{(5)}) t}}{v_0^{(5)} - v_2^{(5)}} e^{-a_2^{(5)}(v_2^{(5)} - v_1^{(5)} - v_2^{(5)}) t}} \]

From which we deduce \( v_0^{(5)} \leq v^{(5)}(t) \leq v_2^{(5)} \)

(h) If 0 < \( v_1^{(5)} \) < \( v_0^{(5)} = \frac{\sigma_0}{\sigma_2} \) < \( \bar{v}_1^{(5)} \) we find like in the previous case,

\[ v_0^{(5)} = \frac{\sigma_0}{\sigma_2} \]

we obtain

\[ v^{(5)}(t) \leq \frac{(v_2^{(5)} + C^{(5)}(v_1^{(5)} - \frac{v_0^{(5)} - v_1^{(5)} + C^{(5)}(v_2^{(5)} - \frac{v_0^{(5)} - v_1^{(5)} - v_2^{(5)}}{v_0^{(5)} - v_2^{(5)}})}{v_0^{(5)} - v_2^{(5)}}) e^{-a_2^{(5)}(v_2^{(5)} - v_1^{(5)} - v_2^{(5)}) t}}{v_0^{(5)} - v_2^{(5)}} e^{-a_2^{(5)}(v_2^{(5)} - v_1^{(5)} - v_2^{(5)}) t}} \]

(i) If 0 < \( v_1^{(5)} \) ≤ \( \bar{v}_1^{(5)} \) ≤ \( v_0^{(5)} = \frac{\sigma_0}{\sigma_2} \), we obtain

\[ v^{(5)}(t) \leq \frac{(v_2^{(5)} + C^{(5)}(v_1^{(5)} - \frac{v_0^{(5)} - v_1^{(5)} + C^{(5)}(v_2^{(5)} - \frac{v_0^{(5)} - v_1^{(5)} - v_2^{(5)}}{v_0^{(5)} - v_2^{(5)}})}{v_0^{(5)} - v_2^{(5)}}) e^{-a_2^{(5)}(v_2^{(5)} - v_1^{(5)} - v_2^{(5)}) t}}{v_0^{(5)} - v_2^{(5)}} e^{-a_2^{(5)}(v_2^{(5)} - v_1^{(5)} - v_2^{(5)}) t}} \]

And so with the notation of the first part of condition (c), we have

**Definition of** \( v^{(5)}(t) \) :

\[ m_1^{(5)} \leq v^{(5)}(t) \leq m_1^{(5)} \]

In a completely analogous way, we obtain

**Definition of** \( u^{(5)}(t) \) :

\[ m_2^{(5)} \leq u^{(5)}(t) \leq m_2^{(5)} \]

**Particular case** :

If \( a^{(5)} = a^{(5)} \), then \( v_1^{(5)} = v_2^{(5)} \) and in this case \( v_0^{(5)} = v_1^{(5)} \) and as a consequence \( u^{(5)}(t) = v^{(5)}(t) \) this also defines \( v^{(5)}(t) \) for the special case.

Similarly, if \( b^{(5)} = b^{(5)} \), then \( v_1^{(5)} = v_2^{(5)} \) and then \( u^{(5)}(t) = v^{(5)}(t) \) if in addition \( u^{(5)}(t) = u^{(5)}(t) \). This is an important consequence of the relation between \( v_1^{(5)} \) and \( v_2^{(5)} \) and definition of \( u^{(5)}(t) \).
\[ \frac{dv^{(6)}}{dt} = (a_{32}^{(6)}) - \left( (a_{32}^{(6)}) - (a_{13}^{(6)}) + (a_{32}^{(6)})T_{33}^{(6)}(t) \right) - (a_{33}^{(6)})(T_{33}^{(6)}(t)v^{(6)} - (a_{33}^{(6)})v^{(6)}) \]

**Definition of** \( v^{(6)} \):

\[ v^{(6)} = \frac{\sigma_{32}}{\sigma_{33}} \]

It follows

\[ -\left( (a_{33}^{(6)})v^{(6)} \right)^2 + (\sigma_{2}^{(6)})v^{(6)} - (a_{32}^{(6)}) \leq \frac{dv^{(6)}}{dt} \leq -\left( (a_{33}^{(6)})v^{(6)} \right)^2 + (\sigma_{1}^{(6)})v^{(6)} - (a_{32}^{(6)}) \]

From which one obtains

**Definition of** \( (\bar{v}_1)^{(6)} \), \( (v_0)^{(6)} \):

(i) For \( 0 < (v_0)^{(6)} = \frac{\sigma_{32}}{\sigma_{33}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)} \)

\[ v^{(6)}(t) \geq \frac{(v_1)^{(6)}+(v_2)^{(6)}(v_3)(1-e^{[-(a_{33}^{(6)})(v_1)^{(6)}-v_0)^{(6)}]t})}{1+(v_2)^{(6)}(v_1)^{(6)}(v_3)} \]

\[ (C)^{(6)} = \frac{(v_1)^{(6)}-(v_0)^{(6)}}{(v_0)^{(6)}-(v_2)^{(6)}} \]

it follows \( (v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)} \)

In the same manner , we get

\[ v^{(6)}(t) \leq \frac{(v_2)^{(6)}+(v_3)^{(6)}(v_4)(1-e^{[-(a_{33}^{(6)})(v_2)^{(6)}-v_0)^{(6)}]t})}{1+(v_2)^{(6)}(v_1)^{(6)}(v_3)} \]

\[ (C)^{(6)} = \frac{(v_2)^{(6)}-(v_0)^{(6)}}{(v_0)^{(6)}-(v_3)^{(6)}} \]

From which we deduce \( (v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)} \)

(k) If \( 0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{\sigma_{32}}{\sigma_{33}} < (\bar{v}_1)^{(6)} \) we find like in the previous case,

\[ (v_1)^{(6)} \leq \frac{(v_1)^{(6)}+(v_2)^{(6)}(v_3)(1-e^{[-(a_{33}^{(6)})(v_2)^{(6)}-v_0)^{(6)}]t})}{1+(v_2)^{(6)}(v_1)^{(6)}(v_3)} \]

\[ (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \]

(l) If \( 0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq (v_0)^{(6)} = \frac{\sigma_{32}}{\sigma_{33}} \) we obtain

\[ (v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(v_1)^{(6)}+(v_2)^{(6)}(v_3)(1-e^{[-(a_{33}^{(6)})(v_2)^{(6)}-v_0)^{(6)}]t})}{1+(v_2)^{(6)}(v_1)^{(6)}(v_3)} \]

And so with the notation of the first part of condition (c) , we have

**Definition of** \( v^{(6)}(t) \):

\[ (m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)}, \quad v^{(6)}(t) = \frac{\sigma_{32}(t)}{\sigma_{33}(t)} \]

In a completely analogous way, we obtain
**Definition of** \( u^{(6)}(t) := \)
\[
(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}
\]

Now, using this result and replacing it in **GLOBAL EQUATIONS** we get easily the result stated in the theorem.

**Particular case:**

If \( (a^{(6)}_{22})' = (a^{(6)}_{23})' \), then \( (\sigma_1)^{(6)} = (\sigma_2)^{(6)} \) and in this case \( (\nu_1)^{(6)} = (\nu_2)^{(6)} = (\bar{\nu}_1)^{(6)} \) if in addition \( (\nu_0)^{(6)} = (\nu_1)^{(6)} \) then \( \nu^{(6)}(t) = (\nu_0)^{(6)} \) and as a consequence \( \tilde{g}_{32}(t) = (\nu_0)^{(6)} \tilde{g}_{33}(t) \) this also defines \( (\nu_0)^{(6)} \) for the special case.

Analogously if \( (b^{(6)}_{22})' = (b^{(6)}_{23})' \), then \( (\tau_1)^{(6)} = (\tau_2)^{(6)} \) and then \( (u_1)^{(6)} = (\bar{u}_1)^{(6)} \) if in addition \( (u_0)^{(6)} = (u_1)^{(6)} \) then \( T_{32}(t) = (u_0)^{(6)} \) \( T_{33}(t) \) This is an important consequence of the relation between \( (\nu_1)^{(6)} \) and \( (\bar{\nu}_1)^{(6)} \), and **definition of** \( (u_0)^{(6)} \).

**Behavior of the solutions**

If we denote and define

**Definition of** \( (\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)} : \)
\[
(p) \quad (\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)} \quad \text{four constants satisfying}
\]
\[
-(\sigma_2)^{(7)} \leq -(a^{(7)}_{36}) + (a^{(7)}_{37}) - (a^{(7)}_{36})(T_{37}, t) + (a^{(7)}_{37})(T_{37}, t) \leq -(\sigma_1)^{(7)}
\]
\[
-(\tau_2)^{(7)} \leq -(b^{(7)}_{36}) + (b^{(7)}_{37}) - (b^{(7)}_{36})(G_{39}, t) - (b^{(7)}_{37})(G_{39}, t) \leq -(\tau_1)^{(7)}
\]

**Definition of** \( (\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, (v)^{(7)}, (u)^{(7)} : \)
\[
(q) \quad \text{By} \quad (\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0 \quad \text{and respectively} \quad (u_1)^{(7)} > 0, (u_2)^{(7)} < 0 \quad \text{the roots of} \quad \text{the}
\]
\[
(a^{(7)}_{37})(v)^{(7)} + (a^{(7)}_{36})(v)^{(7)} = 0
\]
\[
(b^{(7)}_{37})(u)^{(7)} + (b^{(7)}_{36})(u)^{(7}) = 0
\]

**Definition of** \( (\bar{\nu}_1)^{(7)}, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7}) : \)
\[
(\bar{\nu}_1)^{(7)}, (\bar{\nu}_2)^{(7)} < 0 \quad \text{and respectively} \quad (\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0 \quad \text{the}
\]
\[
(a^{(7)}_{37})(v)^{(7)} + (a^{(7)}_{36})(v)^{(7}) = 0
\]
and \((b_{37}^{(r)}(u^{(r)})^2 + (\tau_2^{(r)})u^{(r)} - (b_{36}^{(r)}) = 0\)

Definition of \((m_1)^{(r)}\), \((m_2)^{(r)}\), \((\mu_1)^{(r)}\), \((\mu_2)^{(r)}\), \((v_0)^{(r)}\): -

(r) If we define \((m_1)^{(r)}\), \((m_2)^{(r)}\), \((\mu_1)^{(r)}\), \((\mu_2)^{(r)}\) by

\((m_2)^{(r)} = (v_0)^{(r)}, (m_1)^{(r)} = (v_1)^{(r)}, \text{ if } (v_0)^{(r)} < (v_1)^{(r)}\)

\((m_2)^{(r)} = (v_1)^{(r)}, (m_1)^{(r)} = (\bar{v}_1)^{(r)}, \text{ if } (v_0)^{(r)} < (\bar{v}_1)^{(r)},\)

and \((v_0)^{(r)} = \frac{a_{36}^{(r)}}{a_{37}^{(r)}}\)

\((m_2)^{(r)} = (v_1)^{(r)}, (m_1)^{(r)} = (v_0)^{(r)}, \text{ if } (\bar{v}_1)^{(r)} < (v_0)^{(r)}\)

and analogously

\((\mu_2)^{(r)} = (u_0)^{(r)}, (\mu_1)^{(r)} = (u_1)^{(r)}, \text{ if } (u_0)^{(r)} < (u_1)^{(r)}\)

\((\mu_2)^{(r)} = (u_1)^{(r)}, (\mu_1)^{(r)} = (\bar{u}_1)^{(r)}, \text{ if } (u_0)^{(r)} < (\bar{u}_1)^{(r)}\)

and \((u_0)^{(r)} = \frac{\bar{a}_{36}^{(r)}}{\bar{a}_{37}^{(r)}}\)

\((\mu_2)^{(r)} = (u_1)^{(r)}, (\mu_1)^{(r)} = (u_0)^{(r)}, \text{ if } (\bar{u}_1)^{(r)} < (u_0)^{(r)}\)

are defined by 59 and 67 respectively

Then the solution of GLOBAL EQUATIONS satisfies the inequalities

\[ a_{36}^{(r)}e^{((S_1)^{(r)} - (p_{36})^{(r)})t} \leq G_{36}(t) \leq a_{36}^{(r)}e^{((S_1)^{(r)})t} \]

where \((p_{I})^{(r)}\) is defined
\[ \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{\left((s_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(s_1)^{(7)}t} \]

\[ \left( \frac{(a_{36})^{(7)} e_0^0}{(m_3)^{(7)}} \right) \left[ e^{(s_1)^{(7)} - (p_{36})^{(7)}t} - e^{-(s_2)^{(7)}t} \right] + G_{38}^0 e^{-(a_{36})^{(7)}t} \leq G_{38}(t) \leq \]

\[ \frac{(a_{36})^{(7)} e_0^0}{(m_2)^{(7)}} \left[ e^{(s_1)^{(7)}t} - e^{-(a_{36})^{(7)}t} \right] + G_{38}^0 e^{-(a_{36})^{(7)}t} \]

\[ T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \]

\[ \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \]

\[ \left( \frac{(b_{38})^{(7)} e_0^0}{(\mu_1)^{(7)}} \right) \left[ e^{(R_2)^{(7)}t} - e^{-(b_{38})^{(7)}t} \right] + T_{38}^0 e^{-(b_{38})^{(7)}t} \leq T_{38}(t) \leq \]

\[ \left( \frac{(a_{36})^{(7)} e_0^0}{(\mu_1)^{(7)}} \right) \left[ e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)}t} \right] + T_{38}^0 e^{-(R_2)^{(7)}t} \]

**Definition of** \((S_1)^{(7)}\), \((S_2)^{(7)}\), \((R_1)^{(7)}\), \((R_2)^{(7)}\):-

Where \((S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a_{36})^{(7)}\)

\((S_2)^{(7)} = (a_{36})^{(7)} - (p_{36})^{(7)}\)

\((R_1)^{(7)} = (b_{38})^{(7)}(\mu_2)^{(7)} - (b_{38})^{(7)}\)

\((R_2)^{(7)} = (b_{38})^{(7)} - (r_{38})^{(7)}\)

From CONCATENATED GLOBAL EQUATIONS we obtain

\[ \frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left( (a_{36})^{(7)} - (a_{36})^{(7)} + (a_{36})^{(7)}(T_{37}, t) \right) - \]

\[ (a_{36})^{(7)}(T_{37}, t)v^{(7)} - (a_{36})^{(7)}v^{(7)} \]

**Definition of** \(v^{(7)}\):-

\[ v^{(7)} = \frac{e_0^0}{G_{37}} \]

It follows
\[
- \left( (a_{37})^{(7)}(v^{(7)})^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} \right) \leq \frac{dv^{(7)}}{dt} \leq - \left( (a_{37})^{(7)}(v^{(7)})^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} \right)
\]

From which one obtains

**Definition of** \((\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}\) :-

- (m) For \(0 < (\nu_0)^{(7)} = \frac{\partial \rho}{\partial \nu} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}\)

\[
v^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (\bar{\nu}_1)^{(7)}(\bar{\nu}_2)^{(7)} e^{-[(a_{37})^{(7)}(v_1^{(7)} - v_0^{(7)})]t}}{1 + (\bar{\nu}_1)^{(7)} e^{-[(a_{37})^{(7)}(v_1^{(7)} - v_0^{(7)})]t}}, \quad \left( \bar{\nu}_1 \right)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}
\]

it follows \((\nu_0)^{(7)} \leq v^{(7)}(t) \leq (\nu_1)^{(7)}\)

In the same manner, we get

\[
v^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{\nu}_1)^{(7)}(\bar{\nu}_2)^{(7)} e^{-[(a_{37})^{(7)}(v_1^{(7)} - v_2^{(7)})]t}}{1 + (\bar{\nu}_1)^{(7)} e^{-[(a_{37})^{(7)}(v_1^{(7)} - v_2^{(7)})]t}}, \quad \left( \bar{\nu}_1 \right)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}
\]

From which we deduce \((\nu_0)^{(7)} \leq v^{(7)}(t) \leq (\nu_1)^{(7)}\)

- (n) If \(0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{\partial \rho}{\partial \nu} < (\bar{\nu}_1)^{(7)}\) we find like in the previous case,

\[
(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)} + (\bar{\nu}_1)^{(7)}(\bar{\nu}_2)^{(7)} e^{-[(a_{37})^{(7)}(v_1^{(7)} - v_2^{(7)})]t}}{1 + (\bar{\nu}_1)^{(7)} e^{-[(a_{37})^{(7)}(v_1^{(7)} - v_2^{(7)})]t}} \leq v^{(7)}(t) \leq (\nu_1)^{(7)}
\]

\[
(\bar{\nu}_1)^{(7)} \leq \frac{(\nu_1)^{(7)} + (\bar{\nu}_1)^{(7)}(\bar{\nu}_2)^{(7)} e^{-[(a_{37})^{(7)}(v_1^{(7)} - v_2^{(7)})]t}}{1 + (\bar{\nu}_1)^{(7)} e^{-[(a_{37})^{(7)}(v_1^{(7)} - v_2^{(7)})]t}} \leq (\bar{\nu}_1)^{(7)}
\]

- (o) If \(0 < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)} \leq (\nu_0)^{(7)} = \frac{\partial \rho}{\partial \nu}\), we obtain

\[
(\nu_1)^{(7)} \leq v^{(7)}(t) \leq \frac{(\nu_1)^{(7)} + (\bar{\nu}_1)^{(7)}(\bar{\nu}_2)^{(7)} e^{-[(a_{37})^{(7)}(v_1^{(7)} - v_2^{(7)})]t}}{1 + (\bar{\nu}_1)^{(7)} e^{-[(a_{37})^{(7)}(v_1^{(7)} - v_2^{(7)})]t}} \leq (\nu_0)^{(7)}
\]

And so with the notation of the first part of condition (c), we have

**Definition of** \(v^{(7)}(t) : -

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(m_2)^{(7)} \leq \psi^{(7)}(t) \leq (m_1)^{(7)}, \quad \psi^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}$

In a completely analogous way, we obtain

**Definition of** \(u^{(7)}(t)\) :-

\((\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}\)

Now, using this result and replacing it in CONCATENATED GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If \((a_{36}^{\ast})^{(7)} = (a_{37}^{\ast})^{(7)}, \text{ then } (\sigma_1)^{(7)} = (\sigma_2)^{(7)} \text{ and in this case } (v_1)^{(7)} = (\bar{\nu}_1)^{(7)} \text{ if in addition } (v_0)^{(7)} = (v_1)^{(7)} \text{ then } \psi^{(7)}(t) = (v_0)^{(7)} \text{ and as a consequence } G_{36}(t) = (v_0)^{(7)} G_{37}(t) \text{ this also defines } (v_0)^{(7)} \text{ for the special case}.\)

Analogously if \((b_{36}^{\ast})^{(7)} = (b_{37}^{\ast})^{(7)}, \text{ then } (\tau_1)^{(7)} = (\bar{\tau}_2)^{(7)} \text{ and then } (u_2)^{(7)} = (\bar{\nu}_1)^{(7)} \text{ if in addition } (u_0)^{(7)} = (u_1)^{(7)} \text{ then } T_{36}(t) = (u_0)^{(7)} T_{37}(t) \text{ This is an important consequence of the relation between } (v_1)^{(7)} \text{ and } (\bar{\nu}_1)^{(7)}, \text{ and definition of } (u_0)^{(7)}.\)

\((b_1^{(1)})_1 T_{13} - [(b_1^{(1)}) - (b_1^{(1)}) (G)]T_{14} = 0 \quad 544\)
\((b_1^{(1)})_1 T_{14} - [(b_1^{(1)}) - (b_1^{(1)}) (G)]T_{15} = 0 \quad 545\)

has a unique positive solution, which is an equilibrium solution for the system

\((a_{16})^{(2)} G_{16} - [(a_1^{(2)}) + (a_1^{(2)}) (T_{17})] G_{16} = 0 \quad 546\)
\((a_{17})^{(2)} G_{16} - [(a_1^{(2)}) + (a_1^{(2)}) (T_{17})] G_{16} = 0 \quad 547\)
\((a_{18})^{(2)} G_{16} - [(a_1^{(2)}) + (a_1^{(2)}) (T_{17})] G_{16} = 0 \quad 548\)
\((b_{16})^{(2)} T_{17} - [(b_1^{(2)}) - (b_1^{(2)}) (G_{19})] T_{18} = 0 \quad 549\)
\((b_{17})^{(2)} T_{18} - [(b_1^{(2)}) - (b_1^{(2)}) (G_{19})] T_{18} = 0 \quad 550\)
\((b_{18})^{(2)} T_{18} - [(b_1^{(2)}) - (b_1^{(2)}) (G_{19})] T_{18} = 0 \quad 551\)

has a unique positive solution, which is an equilibrium solution for

\((a_{20})^{(3)} G_{21} - [(a_2^{(3)}) + (a_2^{(3)}) (T_{21})] G_{20} = 0 \quad 552\)
\((a_{20})^{(3)} G_{22} - [(a_2^{(3)}) + (a_2^{(3)}) (T_{22})] G_{21} = 0 \quad 553\)
\((a_{20})^{(3)} G_{22} - [(a_2^{(3)}) + (a_2^{(3)}) (T_{22})] G_{21} = 0 \quad 554\)
\[(a_{21})^3 G_{20} - [(a'_{21})^3 + (a''_{21})^3(T_{21})]G_{21} = 0\]
\[(a_{22})^3 G_{21} - [(a'_{22})^3 + (a''_{22})^3(T_{21})]G_{22} = 0\]
\[(b_{20})^3 T_{21} - [(b'_{20})^3 - (b''_{20})^3(G_{23})]T_{20} = 0\]
\[(b_{21})^3 T_{20} - [(b'_{21})^3 - (b''_{21})^3(G_{23})]T_{21} = 0\]
\[(b_{22})^3 T_{21} - [(b'_{22})^3 - (b''_{22})^3(G_{23})]T_{22} = 0\]

has a unique positive solution, which is an equilibrium solution for the system.

\[(a_{24})^4 G_{25} - [(a'_{24})^4 + (a''_{24})^4(T_{25})]G_{24} = 0\]
\[(a_{25})^4 G_{24} - [(a'_{25})^4 + (a''_{25})^4(T_{25})]G_{25} = 0\]
\[(a_{26})^4 G_{25} - [(a'_{26})^4 + (a''_{26})^4(T_{25})]G_{26} = 0\]
\[(b_{24})^4 T_{25} - [(b'_{24})^4 - (b''_{24})^4(G_{27})]T_{24} = 0\]
\[(b_{25})^4 T_{24} - [(b'_{25})^4 - (b''_{25})^4(G_{27})]T_{25} = 0\]
\[(b_{26})^4 T_{25} - [(b'_{26})^4 - (b''_{26})^4(G_{27})]T_{26} = 0\]

has a unique positive solution, which is an equilibrium solution for the system.

\[(a_{28})^5 G_{29} - [(a'_{28})^5 + (a''_{28})^5(T_{29})]G_{28} = 0\]
\[(a_{29})^5 G_{28} - [(a'_{29})^5 + (a''_{29})^5(T_{29})]G_{29} = 0\]
\[(a_{30})^5 G_{29} - [(a'_{30})^5 + (a''_{30})^5(T_{29})]G_{30} = 0\]
\[(b_{28})^5 T_{29} - [(b'_{28})^5 - (b''_{28})^5(G_{31})]T_{28} = 0\]
\[(b_{29})^5 T_{28} - [(b'_{29})^5 - (b''_{29})^5(G_{31})]T_{29} = 0\]
\[(b_{30})^5 T_{29} - [(b'_{30})^5 - (b''_{30})^5(G_{31})]T_{30} = 0\]

has a unique positive solution, which is an equilibrium solution for the system.
(b_{33})^{(6)}T_{32} - [(b_{33})^{(6)} - (b''_{33})^{(6)}(G'_{35})]T_{33} = 0 \hspace{5cm} 580

(b_{34})^{(6)}T_{33} - [(b_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \hspace{5cm} 584

has a unique positive solution, which is an equilibrium solution for the system \hspace{5cm} 582

(a_{36})^{(7)}G_{37} - [(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})]G_{36} = 0 \hspace{5cm} 583

(a_{37})^{(7)}G_{36} - [(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})]G_{37} = 0 \hspace{5cm} 584

(a_{38})^{(7)}G_{37} - [(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})]G_{38} = 0 \hspace{5cm} 585

(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \hspace{5cm} 586

(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})]T_{37} = 0 \hspace{5cm} 587

(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})]T_{38} = 0 \hspace{5cm} 588

has a unique positive solution, which is an equilibrium solution for the system \hspace{5cm} 589

(a) Indeed the first two equations have a nontrivial solution \(G_{36}, G_{37}\) if \hspace{5cm} 560

\[
F(T_{39}) = (a'_{36})^{(7)}(T_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{37})^{(7)}(a'_{37})^{(7)}(T_{37})^{(7)} + (a''_{37})^{(7)}(a''_{37})^{(7)}(T_{37})^{(7)} = 0
\]

Definition and uniqueness of \(T_{37}^{*}\) \hspace{5cm} 561

After hypothesis \(f(0) < 0, f(\infty) > 0\) and the functions \((a'_{ii})^{(7)}(T_{37})\) being increasing, it follows
that there exists a unique \(T_{37}^{*}\) for which \(f(T_{37}^{*}) = 0\). With this value, we obtain from the three first equations

\[
G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})^{(7)}]} \hspace{1cm} G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})^{(7)}]}\]

(e) By the same argument, the equations (SOLUTIONAL) admit solutions \(G_{36}, G_{37}\) if

\[
\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -
\]

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\[(b_{36})^{(7)}(b_{37})^{(7)}(G_{39}) + (b_{37}^{(7)}(b_{36})^{(7)}(G_{39}) + (b_{36}^{(7)}(G_{39})(b_{37}^{(7)}(G_{39}) = 0\]

Where in \((G_{39})G_{36}, G_{37}, G_{38}\), \(G_{46}, G_{39}\) must be replaced by their values from 96. It is easy to see that \(\varphi\) is a decreasing function in \(G_{39}\), taking into account the hypothesis \(\varphi(0) > 0, \varphi(\infty) < 0\) it follows that there exists a unique \(G_{37}^*\) such that \(\varphi(G^*) = 0\)

Finally we obtain the unique solution of the system

\[
G_{36}^* = \frac{(a_{36})^{(7)}G_{37}^*}{([a_{36}^{(7)}(G_{39}), \quad G_{39}^* = \frac{(a_{39})^{(7)}G_{37}^*}{([a_{39}^{(7)}(G_{39})},
\]

\[
T_{37}^* = \frac{(b_{36})^{(7)}G_{37}^*}{([b_{36}^{(7)}(G_{39})], \quad T_{38}^* = \frac{(b_{38})^{(7)}G_{37}^*}{([b_{38}^{(7)}(G_{39})]}
\]

**Definition and uniqueness of** \(T_{21}\) :

After hypothesis \(f(0) < 0, f(\infty) > 0\) and the functions \((a_i^{(1)}(T_{21})\) being increasing, it follows that there exists a unique \(T_{21}^*\) for which \(f(T_{21}^*) = 0\). With this value, we obtain from the three first equations

\[
G_{20} = \frac{(a_{20})^{(3)}G_{21}}{([a_{20}^{(3)}(G_{21})]}, \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{([a_{22}^{(3)}(G_{21})]}
\]

**Definition and uniqueness of** \(T_{25}\) :

After hypothesis \(f(0) < 0, f(\infty) > 0\) and the functions \((a_i^{(4)}(T_{25})\) being increasing, it follows that there exists a unique \(T_{25}^*\) for which \(f(T_{25}^*) = 0\). With this value, we obtain from the three first equations

\[
G_{24} = \frac{(a_{24})^{(4)}G_{25}}{([a_{24}^{(4)}(G_{25})]}, \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{([a_{26}^{(4)}(G_{25})]}
\]

**Definition and uniqueness of** \(T_{29}\) :

After hypothesis \(f(0) < 0, f(\infty) > 0\) and the functions \((a_i^{(5)}(T_{29})\) being increasing, it follows that there exists a unique \(T_{29}^*\) for which \(f(T_{29}^*) = 0\). With this value, we obtain from the three first equations

\[
G_{28} = \frac{(a_{28})^{(5)}G_{29}}{([a_{28}^{(5)}(G_{29})]}, \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{([a_{30}^{(5)}(G_{29})]}
\]

**Definition and uniqueness of** \(T_{33}\) :

After hypothesis \(f(0) < 0, f(\infty) > 0\) and the functions \((a_i^{(6)}(T_{33})\) being increasing, it follows that there exists a unique \(T_{33}^*\) for which \(f(T_{33}^*) = 0\). With this value, we obtain from the three first equations
(f) By the same argument, the equations 92,93 admit solutions \( G_{13}, G_{14} \) if

\[
\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - \\
[(b'_{13})^{(1)}(b'_{14})^{(1)}(G) + (b'_{14})^{(1)}(b_{13})^{(1)}(G)] + (b''_{13})^{(1)}(G)(b_{14})^{(1)}(G) = 0
\]

Where in \( G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{14} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G'_{14} \) such that \( \varphi(G'_{14}) = 0 \)

(g) By the same argument, the equations 92,93 admit solutions \( G_{16}, G_{17} \) if

\[
\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - \\
[(b'_{16})^{(2)}(b'_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})] + (b''_{16})^{(2)}(G_{19})(b_{17})^{(2)}(G_{19}) = 0
\]

Where in \( G_{19}(G_{16}, G_{17}, G_{18}), G_{16}, G_{18} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{17} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G'_{14} \) such that \( \varphi(G'_{19}) = 0 \)

(a) By the same argument, the concatenated equations admit solutions \( G_{20}, G_{21} \) if

\[
\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - \\
[(b'_{20})^{(3)}(b'_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})] + (b''_{20})^{(3)}(G_{23})(b_{21})^{(3)}(G_{23}) = 0
\]

Where in \( G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{21} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G'_{21} \) such that \( \varphi(G_{23})' = 0 \)

(b) By the same argument, the equations of modules admit solutions \( G_{24}, G_{25} \) if

\[
\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - \\
[(b'_{24})^{(4)}(b'_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})] + (b''_{24})^{(4)}(G_{27})(b_{25})^{(4)}(G_{27}) = 0
\]

Where in \( G_{27}(G_{24}, G_{25}, G_{26}), G_{24}, G_{26} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{25} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G'_{25} \) such that \( \varphi(G_{27})' = 0 \)

(c) By the same argument, the equations (modules) admit solutions \( G_{28}, G_{29} \) if

\[
\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - \\
[(b'_{28})^{(5)}(b'_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})] + (b''_{28})^{(5)}(G_{31})(b_{29})^{(5)}(G_{31}) = 0
\]

Where in \( G_{31}(G_{28}, G_{29}, G_{30}), G_{28}, G_{30} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{29} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G'_{29} \) such that \( \varphi(G_{31})' = 0 \)
(d) By the same argument, the equations (modules) admit solutions $G_{32}, G_{33}$ if

\[ \varphi(G_{32}) = (b_{32})^{(6)}(b_{33})^{(6)} - (b_{32})(b_{33})^{(6)} - \]

\[ \left[(b_{32})^{(6)}(b_{33})^{(6)}G_{32} + (b_{32})^{(6)}(b_{33})^{(6)}G_{33}\right] + (b_{32})^{(6)}(b_{33})^{(6)}G_{32} = 0 \]

Where in $(G_{32})G_{32}, G_{33}, G_{34}, G_{32}, G_{34}$ must be replaced by their values. It is easy to see that $\varphi$ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0 , \varphi(\infty) < 0$ it follows that there exists a unique $G_{33}$ such that $\varphi(G'_{33}) = 0$

Finally we obtain the unique solution of 89 to 94

\[ G_{14}^* \text{ given by } \varphi(G_{14}^*) = 0 , \quad T_{14}^* \text{ given by } f(T_{14}^*) = 0 \] and

\[ G_{13}^* = \frac{(a_{13})^{(2)}G_{14}}{[a_{13}^{(2)}+a_{13}^{(2)}(T_{14})]} , \quad G_{15}^* = \frac{(a_{15})^{(2)}G_{14}}{[a_{15}^{(2)}+a_{15}^{(2)}(T_{14})]} \]

\[ T_{13}^* = \frac{(b_{13})^{(1)}T_{14}}{[b_{13}^{(1)}(b_{13}^{(1)}(G_{14}^*))]} , \quad T_{15}^* = \frac{(b_{13})^{(1)}T_{14}}{[b_{13}^{(1)}(b_{13}^{(1)}(G_{15}^*))]} \]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{17}^* \text{ given by } \varphi(G_{19}) = 0 , \quad T_{17}^* \text{ given by } f(T_{17}) = 0 \] and

\[ G_{16}^* = \frac{(a_{16})^{(2)}G_{17}}{[a_{16}^{(2)}+a_{16}^{(2)}(T_{17})]} , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}}{[a_{18}^{(2)}+a_{18}^{(2)}(T_{17})]} \]

\[ T_{16}^* = \frac{(b_{16})^{(2)}T_{17}}{[b_{16}^{(2)}(b_{16}^{(2)}(G_{17}^*))]} , \quad T_{18}^* = \frac{(b_{16})^{(2)}T_{17}}{[b_{16}^{(2)}(b_{16}^{(2)}(G_{18}^*))]} \]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{21}^* \text{ given by } \varphi(G_{23}) = 0 , \quad T_{21}^* \text{ given by } f(T_{21}) = 0 \] and

\[ G_{20}^* = \frac{(a_{20})^{(3)}G_{21}}{[a_{20}^{(3)}+a_{20}^{(3)}(T_{21})]} , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}}{[a_{22}^{(3)}+a_{22}^{(3)}(T_{21})]} \]

\[ T_{20}^* = \frac{(b_{20})^{(3)}T_{21}}{[b_{20}^{(3)}(b_{20}^{(3)}(G_{21}^*))]} , \quad T_{22}^* = \frac{(b_{20})^{(3)}T_{21}}{[b_{20}^{(3)}(b_{20}^{(3)}(G_{22}^*))]} \]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{25}^* \text{ given by } \varphi(G_{27}) = 0 , \quad T_{25}^* \text{ given by } f(T_{25}) = 0 \] and

\[ G_{24}^* = \frac{(a_{24})^{(4)}G_{25}}{[a_{24}^{(4)}+a_{24}^{(4)}(T_{25})]} , \quad G_{26}^* = \frac{(a_{26})^{(4)}G_{25}}{[a_{26}^{(4)}+a_{26}^{(4)}(T_{25})]} \]

\[ T_{24}^* = \frac{(b_{24})^{(4)}T_{25}}{[b_{24}^{(4)}(b_{24}^{(4)}(G_{25}^*))]} , \quad T_{26}^* = \frac{(b_{24})^{(4)}T_{25}}{[b_{24}^{(4)}(b_{24}^{(4)}(G_{26}^*))]} \]
Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{29}^* \] given by \( \varphi((G_{31})^*) = 0 \), \( T_{29}^* \) given by \( f(T_{29}^*) = 0 \) and

\[ G_{30}^* = \frac{(a_{28})^{(5)}(a_{29})^*}{[(a_{28})^{(5)} + (a_{29})^{(5)}(T_{29}^*)]} \]

\[ T_{28}^* = \frac{(b_{28})^{(5)}T_{28}}{[(b_{28})^{(5)} - (b_{29})^{(5)}(G_{31})^*]} \]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{33}^* \] given by \( \varphi((G_{33})^*) = 0 \), \( T_{33}^* \) given by \( f(T_{33}^*) = 0 \) and

\[ G_{34}^* = \frac{(a_{32})^{(6)}(a_{33})^*}{[(a_{32})^{(6)} + (a_{33})^{(6)}(T_{33}^*)]} \]

\[ T_{32}^* = \frac{(b_{32})^{(6)}T_{32}}{[(b_{32})^{(6)} - (b_{33})^{(6)}(G_{33})^*]} \]

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions \((a_{ij}^{(1)})^*\) and \((b_{ij}^{(1)})^*\) Belong to \(C^{(1)}(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** \( G_i, T_i \):

\[ G_i = G_i^* + G_i \]

\[ T_i = T_i^* + T_i \]

\[ \frac{\theta(a_{ij}^{(1)^*})}{\theta} (T_{14}) = (q_{14})^{(1)} \]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[ \frac{dG_{13}}{dt} = -((a_{13})^{(1)} + (p_{13})^{(1)}(G_{14} - (q_{13})^{(1)}}G_{13}^* T_{14} \]

\[ \frac{dG_{14}}{dt} = -((a_{14})^{(1)} + (p_{14})^{(1)} G_{14} + (a_{14})^{(1)}G_{14} - (q_{14})^{(1)}}G_{14}^* T_{14} \]

\[ \frac{dG_{15}}{dt} = -((a_{15})^{(1)} + (p_{15})^{(1)} G_{15} + (a_{15})^{(1)}G_{15} - (q_{15})^{(1)}}G_{15}^* T_{14} \]

\[ \frac{dT_{13}}{dt} = -((b_{13})^{(1)} - (r_{13})^{(1)}}T_{13} + (b_{13})^{(1)}T_{14} + \sum_{j=13}^{15} S_{(13,j)}T_{13}G_{j} \]

\[ \frac{dT_{14}}{dt} = -((b_{14})^{(1)} - (r_{14})^{(1)}}T_{14} + (b_{14})^{(1)}T_{13} + \sum_{j=14}^{15} S_{(14,j)}T_{14}G_{j} \]

\[ \frac{dT_{15}}{dt} = -((b_{15})^{(1)} - (r_{15})^{(1)}}T_{15} + (b_{15})^{(1)}T_{13} + \sum_{j=15}^{15} S_{(15,j)}T_{15}G_{j} \]

If the conditions of the previous theorem are satisfied and if the functions \((a_{ij}^{(2)})^*\) and \((b_{ij}^{(2)})^*\)
Belong to $C^2(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

**Definition of $G_i, T_i$:**

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial (a_{17}^{(2)}(2))}{\partial T_{17}} = (q_{17})^{(2)}, \quad \frac{\partial (b_{17}^{(2)}(2))}{\partial G_j} = s_{ij}$$

taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{dG_{16}}{dt} = -(a_{16}^{(2)} + (p_{16})^{(2)})G_{16} + (a_{17}^{(2)}G_{17} - (q_{16})^{(2)}G_{16}^* T_{17}$$

$$\frac{dG_{17}}{dt} = -(a_{17}^{(2)} + (p_{17})^{(2)})G_{17} + (a_{17}^{(2)}G_{16} - (q_{17})^{(2)}G_{17}^* T_{17}$$

$$\frac{dG_{18}}{dt} = -(a_{18}^{(2)} + (p_{18})^{(2)})G_{18} + (a_{17}^{(2)}G_{17} - (q_{18})^{(2)}G_{18}^* T_{17}$$

$$\frac{dT_{16}}{dt} = -(b_{16}^{(2)} - (r_{16})^{(2)})T_{16} + (b_{16}^{(2)}T_{16} + \sum_{j=16}^{18}(s_{16}(j)T_{16}^* G_j_j)$$

$$\frac{dT_{17}}{dt} = -(b_{17}^{(2)} - (r_{17})^{(2)})T_{17} + (b_{17}^{(2)}T_{16} + \sum_{j=16}^{18}(s_{17}(j)T_{17}^* G_j_j)$$

If the conditions of the previous theorem are satisfied and if the functions $(a_{17}^{(2)}(3)$ and $(b_{17}^{(2)}(3)$

Belong to $C^3(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

**Definition of $G_i, T_i$:**

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial (a_{21}^{(3)}(3))}{\partial T_{21}} = (q_{21})^{(3)}, \quad \frac{\partial (b_{21}^{(3)}(3))}{\partial G_j} = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{dG_{20}}{dt} = -(a_{20}^{(3)}(3) + (p_{20})^{(3)})G_{20} + (a_{20}^{(3)}G_{21} - (q_{20})^{(3)}G_{20}^* T_{21}$$

$$\frac{dG_{21}}{dt} = -(a_{21}^{(3)}(3) + (p_{21})^{(3)})G_{21} + (a_{21}^{(3)}G_{20} - (q_{21})^{(3)}G_{21}^* T_{21}$$

$$\frac{dG_{22}}{dt} = -(a_{22}^{(3)}(3) + (p_{22})^{(3)})G_{22} + (a_{22}^{(3)}G_{21} - (q_{22})^{(3)}G_{22}^* T_{21}$$

$$\frac{dT_{20}}{dt} = -(b_{20}^{(3)}(3) - (r_{20})^{(3)})T_{20} + (b_{20}^{(3)}T_{20} + \sum_{j=20}^{22}(s_{20}(j)T_{20}^* G_j)$$

$$\frac{dT_{21}}{dt} = -(b_{21}^{(3)}(3) - (r_{21})^{(3)})T_{21} + (b_{21}^{(3)}T_{21} + \sum_{j=20}^{22}(s_{21}(j)T_{21}^* G_j)$$
\[
\frac{dT_{22}}{dt} = -\left( (b_{22})^{(3)} - (r_{22})^{(3)} \right) T_{22} + (b_{22})^{(3)} T_{21} + \sum_{j=20}^{22} (s_{(22)(j)} T_{22} T_{2j})
\]

If the conditions of the previous theorem are satisfied and if the functions \((a_i^{(4)})^\prime\) and \((b_i^{(4)})^\prime\) Belong to \(C^{(4)}(\mathbb{R}_+)^\star\) then the above equilibrium point is asymptotically stable.

Denote

**Definition of** \(G_i, T_i :\)

\[G_i = G_i^\prime + G_i, \quad T_i = T_i^\prime + T_i\]

\[\frac{\partial (a_i^{(4)})^\prime}{\partial T_{28}} (T_{28}^\star) = (q_{28})^{(4)} , \quad \frac{\partial (b_i^{(4)})^\prime}{\partial T_{28}} (T_{28}^\star) = s_{ij}\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{24}}{dt} = -((a_{24})^{(4)} + (p_{24})^{(4)}) G_{24} + (a_{24})^{(4)} G_{25} - (q_{24})^{(4)} G_{24}^\star T_{25}
\]

\[
\frac{dG_{25}}{dt} = -((a_{25})^{(4)} + (p_{25})^{(4)}) G_{25} + (a_{25})^{(4)} G_{24} - (q_{25})^{(4)} G_{25}^\star T_{24}
\]

\[
\frac{dG_{26}}{dt} = -((a_{26})^{(4)} + (p_{26})^{(4)}) G_{26} + (a_{26})^{(4)} G_{25} - (q_{26})^{(4)} G_{26}^\star T_{25}
\]

\[
\frac{dT_{24}}{dt} = -((b_{24})^{(4)} - (r_{24})^{(4)}) T_{24} + (b_{24})^{(4)} T_{25} + \sum_{j=24}^{26} (s_{(24)(j)} T_{24} T_{2j})
\]

\[
\frac{dT_{25}}{dt} = -((b_{25})^{(4)} - (r_{25})^{(4)}) T_{25} + (b_{25})^{(4)} T_{24} + \sum_{j=24}^{26} (s_{(25)(j)} T_{25} T_{2j})
\]

\[
\frac{dT_{26}}{dt} = -((b_{26})^{(4)} - (r_{26})^{(4)}) T_{26} + (b_{26})^{(4)} T_{25} + \sum_{j=24}^{26} (s_{(26)(j)} T_{26} T_{2j})
\]

If the conditions of the previous theorem are satisfied and if the functions \((a_i^{(5)})^\prime\) and \((b_i^{(5)})^\prime\) Belong to \(C^{(5)}(\mathbb{R}_+)^\star\) then the above equilibrium point is asymptotically stable.

Denote

**Definition of** \(G_i, T_i :\)

\[G_i = G_i^\prime + G_i, \quad T_i = T_i^\prime + T_i\]

\[\frac{\partial (a_i^{(5)})^\prime}{\partial T_{29}} (T_{29}^\star) = (q_{29})^{(5)} , \quad \frac{\partial (b_i^{(5)})^\prime}{\partial T_{29}} (T_{29}^\star) = s_{ij}\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{28}}{dt} = -((a_{28})^{(5)} + (p_{28})^{(5)}) G_{28} + (a_{28})^{(5)} G_{29} - (q_{28})^{(5)} G_{28}^\star T_{29}
\]

\[
\frac{dG_{29}}{dt} = -((a_{29})^{(5)} + (p_{29})^{(5)}) G_{29} + (a_{29})^{(5)} G_{28} - (q_{29})^{(5)} G_{29}^\star T_{28}
\]

\[
\frac{dG_{30}}{dt} = -((a_{30})^{(5)} + (p_{30})^{(5)}) G_{30} + (a_{30})^{(5)} G_{29} - (q_{30})^{(5)} G_{30}^\star T_{29}
\]
If the conditions of the previous theorem are satisfied and if the functions \((a'_i)^{(6)}\) and \((b''_i)^{(6)}\) Belong to \(C^{(6)}(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

**Definition of** \(G_i, T_i :\)

\[
G_i = G^*_i + G_{i}, \quad T_i = T^*_i + T_i
\]

\[
\frac{\partial (a''(6))}{\partial T_3}(T_{33}) = (q_{33})^{(6)}, \quad \frac{\partial (b''(6))}{\partial G_j}((G_{3j})^*) = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{32}}{dt} = -((a'_{32})^{(6)} + (p_{32})^{(6)})G_{32} + (a_{32})^{(6)}G_{33} - (q_{32})^{(6)}G^*_3 T_{33}
\]

\[
\frac{dG_{33}}{dt} = -((a'_{33})^{(6)} + (p_{33})^{(6)})G_{33} + (a_{33})^{(6)}G_{32} - (q_{33})^{(6)}G^*_3 T_{33}
\]

\[
\frac{dG_{34}}{dt} = -((a'_{34})^{(6)} + (p_{34})^{(6)})G_{34} + (a_{34})^{(6)}G_{33} - (q_{34})^{(6)}G^*_3 T_{33}
\]

\[
\frac{dT_{32}}{dt} = -((b'_{32})^{(6)} - (r_{32})^{(6)})T_{32} + (b_{32})^{(6)}T_{33} + \sum_{j=32}^{4} (s_{(3j)(j)}T^*_j T_j)
\]

\[
\frac{dT_{33}}{dt} = -((b'_{33})^{(6)} - (r_{33})^{(6)})T_{33} + (b_{33})^{(6)}T_{32} + \sum_{j=32}^{4} (s_{(33)(j)}T^*_j T_j)
\]

\[
\frac{dT_{34}}{dt} = -((b'_{34})^{(6)} - (r_{34})^{(6)})T_{34} + (b_{34})^{(6)}T_{33} + \sum_{j=32}^{4} (s_{(34)(j)}T^*_j T_j)
\]

Obviously, these values represent an equilibrium solution of 79,20,36,22,23, If the conditions of the previous theorem are satisfied and if the functions \((a''(7))\) and \((b''(7))\) Belong to \(C^{(7)}(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** \(G_i, T_i :\)

\[
G_i = G^*_i + G_{i}, \quad T_i = T^*_i + T_i
\]
Then taking into account equations (SOLUTIONAL) and neglecting the terms of power 2, we obtain

\[
\frac{\partial (a_{37}^{(7)})}{\partial T_{37}^{(7)}} = (q_{37}^{(7)}) \quad , \quad \frac{\partial (b_{36}^{(7)})}{\partial G_j} = s_{ij} 
\]

\[
\frac{dG_{36}}{dt} = -((a_{36}^{(7)})^{T}_{36} + (p_{36}^{(7)})G_{36} + (a_{36}^{(7)})G_{37} - (q_{36}^{(7)})G_{36}^* T_{37} 
\]

\[
\frac{dG_{37}}{dt} = -((a_{37}^{(7)})^{T}_{37} + (p_{37}^{(7)})G_{37} + (a_{37}^{(7)})G_{36} - (q_{37}^{(7)})G_{37}^* T_{37} 
\]

\[
\frac{dG_{38}}{dt} = -((a_{38}^{(7)})^{T}_{38} + (p_{38}^{(7)})G_{38} + (a_{38}^{(7)})G_{37} - (q_{38}^{(7)})G_{38}^* T_{37} 
\]

\[
\frac{dT_{36}}{dt} = -((b_{36}^{(7)})^{T}_{36} - (r_{36}^{(7)})T_{36} + (b_{36}^{(7)})T_{37} + \sum_{j=36}^{T_{36}(j)} T_{36} G_{j} 
\]

\[
\frac{dT_{37}}{dt} = -((b_{37}^{(7)})^{T}_{37} - (r_{37}^{(7)})T_{37} + (b_{37}^{(7)})T_{36} + \sum_{j=36}^{T_{37}(j)} T_{37} G_{j} 
\]

\[
\frac{dT_{38}}{dt} = -((b_{38}^{(7)})^{T}_{38} - (r_{38}^{(7)})T_{38} + (b_{38}^{(7)})T_{37} + \sum_{j=36}^{T_{38}(j)} T_{38} G_{j} 
\]

2.

The characteristic equation of this system is

\[
(\lambda)^{(1)} + (b_{15}^{(1)})^{T}_{15} - (r_{15}^{(1)})^{T}_{15} (a_{15}^{(1)})^{T}_{15} + (p_{15}^{(1)})^{T}_{15} 
\]

\[
\left[ (\lambda)^{(1)} + (a_{13}^{(1)})^{T}_{13} + (p_{13}^{(1)})^{T}_{13} (q_{14}^{(1)})^{T}_{14} + (a_{14}^{(1)})^{T}_{14} (q_{13}^{(1)})^{T}_{13} \right] 
\]

\[
((\lambda)^{(1)} + (b_{13}^{(1)})^{T}_{13} - (r_{13}^{(1)})^{T}_{13}) s_{14(14)} T_{14} + (b_{14}^{(1)})^{T}_{14} s_{13(14)} T_{14} 
\]

\[
((\lambda)^{(1)} + (a_{13}^{(1)})^{T}_{13} + (p_{14}^{(1)})^{T}_{14} (q_{13}^{(1)})^{T}_{14} + (a_{14}^{(1)})^{T}_{14} (q_{13}^{(1)})^{T}_{13} 
\]

\[
((\lambda)^{(1)} + (b_{13}^{(1)})^{T}_{13} - (r_{13}^{(1)})^{T}_{13}) s_{14(13)} T_{14} + (b_{14}^{(1)})^{T}_{13} s_{13(13)} T_{13} 
\]

\[
((\lambda)^{(1)} + (a_{13}^{(1)})^{T}_{13} + (p_{14}^{(1)})^{T}_{14} (q_{13}^{(1)})^{T}_{14} + (a_{14}^{(1)})^{T}_{14} (q_{13}^{(1)})^{T}_{13} 
\]

\[
((\lambda)^{(1)} + (b_{13}^{(1)})^{T}_{13} - (r_{13}^{(1)})^{T}_{13}) s_{14(13)} T_{14} + (b_{14}^{(1)})^{T}_{13} s_{13(13)} T_{13} 
\]
\[
+ \left[ \left( \lambda^{(1)} \right)^2 + \left( a_{13}^{(1)} + a_{14}^{(1)} + p_{13}^{(1)} + p_{14}^{(1)} \right) \left( \lambda^{(1)} \right) (q_{15})^{(1)} G_{15} \right]
+ \left( \lambda^{(1)} + a_{13}^{(1)} + p_{13}^{(1)} \right) \left( a_{12}^{(1)} (q_{14})^{(1)} G_{14} + a_{14}^{(1)} (a_{15} G_{15}) (q_{13})^{(1)} G_{13} \right)
\]
\[
\left( \left( \lambda^{(1)} + b_{13}^{(1)} - r_{13}^{(1)} \right) s_{(14), (15)} T_{14}^{*} + b_{14}^{(1)} s_{(13), (15)} T_{13}^{*} \right) = 0
\]
\[
+ \left( \lambda^{(2)} + b_{19}^{(2)} - r_{18}^{(2)} \right) \left( \left( \lambda^{(2)} + a_{18}^{(2)} + p_{18}^{(2)} \right) \right)
\]
\[
\left[ \left( \left( \lambda^{(2)} + a_{16}^{(2)} + p_{16}^{(2)} \right) (q_{17})^{(2)} G_{17} + a_{17}^{(2)} (q_{16})^{(2)} G_{16} \right) \right]
\]
\[
\left( \left( \lambda^{(2)} + b_{16}^{(2)} - r_{16}^{(2)} \right) s_{(17), (16)} T_{17}^{*} + b_{17}^{(2)} s_{(16), (17)} T_{17}^{*} \right) = 0
\]
\[
+ \left( \left( \lambda^{(2)} + a_{17}^{(2)} + p_{17}^{(2)} \right) q_{16}^{(2)} G_{16} + a_{16}^{(2)} (q_{17})^{(2)} G_{17} \right)
\]
\[
\left( \left( \lambda^{(2)} + b_{16}^{(2)} - r_{16}^{(2)} \right) s_{(17), (16)} T_{17}^{*} + b_{17}^{(2)} s_{(16), (17)} T_{17}^{*} \right) = 0
\]
\[
+ \left( \left( \lambda^{(2)} + a_{16}^{(2)} + p_{16}^{(2)} \right) \left( a_{17}^{(2)} + p_{17}^{(2)} \right) \left( \lambda^{(2)} \right) \right)
\]
\[
\left( \left( \lambda^{(2)} + b_{16}^{(2)} + b_{17}^{(2)} - r_{16}^{(2)} + r_{17}^{(2)} \right) \left( \lambda^{(2)} \right) \right)
\]
\[
+ \left( \left( \lambda^{(2)} + a_{16}^{(2)} + p_{16}^{(2)} \right) \left( a_{17}^{(2)} + p_{17}^{(2)} \right) \left( \lambda^{(2)} \right) \right) q_{18}^{(2)} G_{18}
\]
\[
+ \left( \left( \lambda^{(2)} + a_{16}^{(2)} + p_{16}^{(2)} \right) \left( a_{17}^{(2)} + p_{17}^{(2)} \right) \left( \lambda^{(2)} \right) \right) q_{16}^{(2)} G_{16}
\]
\[
\left( \left( \lambda^{(2)} + b_{16}^{(2)} - r_{16}^{(2)} \right) s_{(17), (16)} T_{17}^{*} + b_{17}^{(2)} s_{(16), (17)} T_{17}^{*} \right) = 0
\]
\[
+ \left( \left( \lambda^{(3)} + b_{22}^{(3)} - r_{22}^{(3)} \right) \left( \left( \lambda^{(3)} + a_{22}^{(3)} + p_{22}^{(3)} \right) \right)
\]
\[
\left[ \left( \left( \lambda^{(3)} + a_{20}^{(3)} + p_{20}^{(3)} \right) \left( q_{21}^{(3)} G_{21} + a_{21}^{(3)} \left( q_{20}^{(3)} G_{20} \right) \right) \right]
\]
\[
\left( \left( \lambda^{(3)} + b_{20}^{(3)} - r_{20}^{(3)} \right) s_{(21), (21)} T_{21}^{*} + b_{21}^{(3)} s_{(20), (21)} T_{21}^{*} \right)
\]
\[
+ \left( \left( \lambda^{(3)} + a_{21}^{(3)} + p_{21}^{(3)} \right) q_{20}^{(3)} G_{20} + a_{20}^{(3)} (q_{21})^{(3)} G_{21} \right)
\]
\[
\left( (\lambda)^{(3)} + (b_20)^{(3)} - (r_{20})^{(3)} \right) s_{(21),(20)} T_{21}^* + (b_{21})^{(3)} s_{(20),(20)} T_{20}^*
\]

\[
\left( (\lambda)^{(3)} \right)^2 + \left( (a_{20}^{'})^{(3)} + (a_{21}^{'})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda)^{(3)}
\]

\[
\left( (\lambda)^{(3)} \right)^2 + \left( (b_{20}^{'})^{(3)} + (b_{21}^{'})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)} \right) (\lambda)^{(3)}
\]

\[
+ \left( (\lambda)^{(3)} \right)^2 + \left( (a_{20}^{'})^{(3)} + (a_{21}^{'})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda)^{(3)} (q_{22})^{(3)} G_{22}
\]

\[
+ (\lambda)^{(3)} + (a_{20}^{'})^{(3)} + (p_{20})^{(3)} \left( (a_{22}^{'})^{(3)} (q_{22})^{(3)} G_{21}^* + (a_{21}^{'})^{(3)} (a_{22}^{'})^{(3)} (q_{20})^{(3)} G_{20}^* \right)
\]

\[
\left( (\lambda)^{(3)} + (b_20^{'})^{(3)} - (r_{20})^{(3)} \right) s_{(21),(22)} T_{21}^* + (b_{21})^{(3)} s_{(20),(22)} T_{20}^* \right) = 0
\]

\[
(\lambda)^{(4)} + (b_{26}^{'})^{(4)} - (r_{26})^{(4)} \right) \left( (\lambda)^{(4)} + (a_{26}^{'})^{(4)} + (p_{26})^{(4)} \right)
\]

\[
\left[ \left( (\lambda)^{(4)} + (a_{24}^{'})^{(4)} + (p_{24})^{(4)} \right) (q_{25})^{(4)} G_{25}^* + (a_{25})^{(4)} (q_{24})^{(4)} G_{24}^* \right]
\]

\[
\left( (\lambda)^{(4)} + (b_{24}^{'})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(25)} T_{25}^* + (b_{25})^{(4)} s_{(24),(25)} T_{25}^*
\]

\[
+ \left( (\lambda)^{(4)} + (a_{25}^{'})^{(4)} + (p_{25})^{(4)} \right) (q_{24})^{(4)} G_{24}^* + (a_{24})^{(4)} (q_{25})^{(4)} G_{25}^* \right)
\]

\[
\left( (\lambda)^{(4)} + (b_{24}^{'})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(24)} T_{25}^* + (b_{25})^{(4)} s_{(24),(24)} T_{24}^*
\]

\[
\left( (\lambda)^{(4)} \right)^2 + \left( (a_{24}^{'})^{(4)} + (a_{25}^{'})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} \right) (\lambda)^{(4)}
\]

\[
\left( (\lambda)^{(4)} \right)^2 + \left( (b_{24}^{'})^{(4)} + (b_{25})^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)} \right) (\lambda)^{(4)}
\]

\[
+ \left( (\lambda)^{(4)} \right)^2 + \left( (a_{24})^{(4)} + (a_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} \right) (\lambda)^{(4)} (q_{26})^{(4)} G_{26}
\]

\[
+ (\lambda)^{(4)} + (a_{24})^{(4)} + (p_{24})^{(4)} \left( (a_{26})^{(4)} (q_{25})^{(4)} G_{25}^* + (a_{25})^{(4)} (a_{26})^{(4)} (q_{24})^{(4)} G_{24}^* \right)
\]

\[
\left( (\lambda)^{(4)} + (b_{24}^{'})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(20)} T_{25}^* + (b_{25})^{(4)} s_{(24),(20)} T_{24}^* \right) = 0
\]

\[
(\lambda)^{(5)} + (b_{30}^{'})^{(5)} - (r_{30})^{(5)} \right) \left( (\lambda)^{(5)} + (a_{30}^{'})^{(5)} + (p_{30})^{(5)} \right)
\]
\[
\left[ \left( (\lambda)^{(5)} + (a_{28}^{(5)}) + (p_{28}^{(5)}) \right) (q_{29}^{(5)}) G_{29}^{*} + (a_{29}^{(5)}) (q_{28}^{(5)}) G_{28}^{*} \right] \\
\left( (\lambda)^{(5)} + (b_{28}^{(5)}) - (r_{28}^{(5)}) \right) s_{(29),(29)} T_{29}^{*} + (b_{29}^{(5)}) s_{(28),(28)} T_{28}^{*} \\
+ \left( \left( (\lambda)^{(5)} + (a_{29}^{(5)}) + (p_{29}^{(5)}) \right) (q_{28}^{(5)}) G_{28}^{*} + (a_{28}^{(5)}) (q_{29}^{(5)}) G_{29}^{*} \right) \\
\left( \left( (\lambda)^{(5)} + (b_{29}^{(5)}) - (r_{29}^{(5)}) \right) s_{(29),(29)} T_{29}^{*} + (b_{28}^{(5)}) s_{(28),(28)} T_{28}^{*} \right) \\
\left( \left( (\lambda)^{(5)} + (a_{28}^{(5)}) + (a_{29}^{(5)}) + (p_{28}^{(5)}) + (p_{29}^{(5)}) \right) (\lambda)^{(5)} \right) \\
\left( \left( (\lambda)^{(5)} + (b_{28}^{(5)}) + (b_{29}^{(5)}) - (r_{28}^{(5)}) + (r_{29}^{(5)}) \right) (\lambda)^{(5)} \right) \\
+ \left( \left( (\lambda)^{(5)} + (a_{28}^{(5)}) + (p_{28}^{(5)}) \right) (a_{30}^{(5)})(q_{29}^{(5)}) G_{29}^{*} + (a_{29}^{(5)})(a_{30}^{(5)})(q_{28}^{(5)}) G_{28}^{*} \right) \\
\left( \left( (\lambda)^{(5)} + (b_{28}^{(5)}) - (r_{28}^{(5)}) \right) s_{(29),(30)} T_{29}^{*} + (b_{29}^{(5)}) s_{(28),(30)} T_{28}^{*} \right) = 0 \\
+ \\
\left( (\lambda)^{(6)} + (b_{34}^{(6)}) - (r_{34}^{(6)}) \right) \left( \left( (\lambda)^{(6)} + (a_{34}^{(6)}) + (p_{34}^{(6)}) \right) \right) \\
\left( \left( (\lambda)^{(6)} + (a_{32}^{(6)}) + (p_{32}^{(6)}) \right) (q_{33}^{(6)}) G_{33}^{*} + (a_{33}^{(6)})(q_{32}^{(6)}) G_{32}^{*} \right) \\
\left( \left( (\lambda)^{(6)} + (b_{32}^{(6)}) - (r_{32}^{(6)}) \right) s_{(33),(33)} T_{33}^{*} + (b_{33}^{(6)}) s_{(32),(33)} T_{33}^{*} \right) \\
+ \left( \left( (\lambda)^{(6)} + (a_{33}^{(6)}) + (p_{33}^{(6)}) \right) (q_{32}^{(6)}) G_{32}^{*} + (a_{32}^{(6)})(q_{33}^{(6)}) G_{33}^{*} \right) \\
\left( \left( (\lambda)^{(6)} + (b_{32}^{(6)}) - (r_{32}^{(6)}) \right) s_{(33),(32)} T_{33}^{*} + (b_{33}^{(6)}) s_{(32),(32)} T_{33}^{*} \right) \\
\left( \left( (\lambda)^{(6)} + (a_{32}^{(6)}) + (a_{33}^{(6)}) + (p_{32}^{(6)}) + (p_{33}^{(6)}) \right) (\lambda)^{(6)} \right) \\
\left( \left( (\lambda)^{(6)} + (b_{32}^{(6)}) + (b_{33}^{(6)}) - (r_{32}^{(6)}) + (r_{33}^{(6)}) \right) (\lambda)^{(6)} \right) \\
+ \left( \left( (\lambda)^{(6)} + (a_{32}^{(6)}) + (p_{32}^{(6)}) \right) (a_{34}^{(6)})(q_{33}^{(6)}) G_{33}^{*} + (a_{33}^{(6)})(a_{34}^{(6)})(q_{32}^{(6)}) G_{32}^{*} \right) \\
\right] = 226
\[
\left( (\lambda)^{(6)} + (b^{(6)}_{32}) - (r^{(6)}_{32}) \left[ s^{(33),(34)} T^{\ast}_{33} + (b^{(6)}_{33}) s^{(32),(34)} T^{\ast}_{32} \right] \right) = 0
\]

\[
\left( (\lambda)^{(7)} + (b^{(7)}_{38}) - (r^{(7)}_{38}) \left[ (\lambda)^{(7)} + (a^{(7)}_{38}) + (p^{(7)}_{38}) \right] \right)
\left[ \left[ (\lambda)^{(7)} + (a^{(7)}_{36}) + (p^{(7)}_{36}) \right] (q^{(7)}_{37}) G^{\ast}_{37} + (a^{(7)}_{37}) (q^{(7)}_{36}) G^{\ast}_{36} \right]
\left[ (\lambda)^{(7)} + (b^{(7)}_{36}) - (r^{(7)}_{36}) s^{(37),(36)} T^{\ast}_{37} + (b^{(7)}_{37}) s^{(36),(36)} T^{\ast}_{37} \right]
\left[ (\lambda)^{(7)} + (a^{(7)}_{37}) + (p^{(7)}_{37}) \right] (q^{(7)}_{36}) G^{\ast}_{37} + (a^{(7)}_{36}) (q^{(7)}_{37}) G^{\ast}_{37} \right)
\left[ (\lambda)^{(7)} + (b^{(7)}_{36}) - (r^{(7)}_{36}) s^{(37),(36)} T^{\ast}_{37} + (b^{(7)}_{37}) s^{(36),(36)} T^{\ast}_{37} \right]
\left[ (\lambda)^{(7)} + (b^{(7)}_{38}) - (r^{(7)}_{38}) s^{(37),(38)} T^{\ast}_{37} + (b^{(7)}_{37}) s^{(38),(38)} T^{\ast}_{37} \right] = 0
\]

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(13)^ Note that the relativistic mass, in contrast to the rest mass \( m_0 \), is not a relativistic invariant, and that the velocity is not a Minkowski four-vector, in contrast to the quantity \( \gamma \), where \( \gamma \) is the differential of the proper time. However, the energy-momentum four-vector is a genuine Minkowski four-vector, and the intrinsic origin of the square-root in the definition of the relativistic mass is the distinction between \( d\tau \) and \( dt \).


(20)^ [2] Cockcroft-Walton experiment

(21)^ Conversions used: 1956 International (Steam) Table (IT) values where one calorie
≡ 4.1868 J and one BTU ≡ 1055.05585262 J. Weapons designers’ conversion value of one gram TNT
≡ 1000 calories used.

(22)^ Assuming the dam is generating at its peak capacity of 6.809 MW.

(23)^ Assuming a 90/10 alloy of Pt/Ir by weight, a $C_p$ of 25.9 for Pt and 25.1 for Ir, a Pt-dominated
average $C_p$ of 25.8, 5.134 moles of metal, and 132 J.K$^{-1}$ for the prototype. A variation of
±1.5 picograms is of course, much smaller than the actual uncertainty in the mass of the international
prototype, which are ±2 micrograms.


(25)^ a,b Earth’s gravitational self-energy is $4.6 \times 10^{10}$ that of Earth’s total mass, or 2.7 trillion metric
tons. Citation: The Apache Point Observatory Lunar Laser-Ranging Operation (APOLLO), T. W.
Murphy, Jr. et al. University of Washington, Dept. of Physics (132 kB PDF, here.).

(26)^ There is usually more than one possible way to define a field energy, because any field can be
made to couple to gravity in many different ways. By general scaling arguments, the correct answer
at everyday distances, which are long compared to the quantum gravity scale, should be minimal
coupling, which means that no powers of the curvature tensor appear. Any non-minimal couplings,
along with other higher order terms, are presumably only determined by a theory of quantum gravity,
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