Control Rods Drop Failure On Reactors Stability And Safety

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ABSTRACT

This paper examined the control rod drop-failure in nuclear power plants. Safety margin test was conducted on some typical water-cooled reactor design (WCRD) models at an accident situation, secondly safety margin test was carried out on the thermal efficiency and thermal power output of the reactor when power supply failed and thirdly, safety margin test was perform on the reactor in relation to the high temperature within reactor core and the fuel temperature. The results of the statistical analysis on these types of nuclear reactor models reveals that the typical water-cooled reactor design (WCRD) models promises most stability under thermal efficiency of 45% and above. Meanwhile, at anything below 45% thermal efficiency the fuel element seems to be unstable in the reactor as the regression plot could not fit optimal. At this point the fuel temperature seems at maximum, the reactor agrees to be stable as the regression plot was at the best fit, that is the least squares method finds its optimum when the sum, S, of squared residuals became minimal. Safety margin prediction of 4.42% was validated for a typical WCRD model as an advantage over the current 5.1% challenging problem for plant engineers to predict the safety margin limit.

Keywords: water-cooled reactor design models, control rods drop failure, high fuel temperature, thermal efficiency and thermal power, reactor stability and safety.

Introduction

There have been past and recent event examples of control rod trip-failure especially in typical operating water-cooled power reactors [1]. If this occurs, the atom reactivity increase dramatically and leads to an increase in power, fuel enthalpy and fuel temperature. The fuel and reactor can be damaged. These failures have caused some major fatal accidents, these accidents has received international attention and, although there are still gaps in knowledge relating to details of some phenomena involved in the accident, the causes and the failure have been clearly identified and measures implemented to avoid a repetition of these events. As is often the case in major disasters, the causes relate to two areas – poor design of the reactor and its shut-down facilities, coupled with the lack of a safety culture which led to violation of standard operating procedures. Therefore, the purpose of this paper is to assess the control rod trip - failure in nuclear power plants. Therefore, regression analysis approach was adopted to test the stability margin of the reactor when control rod fails. The literature introduce control rod design failures, operational factor of hardware, procedures, and the human causes of trip- failures in control rod in the past and present times.

Researches have shown that system failure cases in nuclear reactor operation results from a variety of factors, including inadequate design, inadequate materials testing, and poor procedures and training [2]. In the studied cases of control rod failure, the design fault, engineering deficiencies and human errors have been the major causes of trip failure[3]. After several incidents from problems with control rods that could not be inserted in the core and loss of burnable absorbers, the reactor was shut down due to maintenance, it was stated that the reactor could go critical just by removing the central control rod to 75% [4]. System innovative technologies under consideration need safety hazards analyses process before testing or experimentation in other to avoid sudden failure that can lead to severe disaster in the economy. Malfunctioning of control rods of the nuclear plant could lead to overheating of the reactor core and this could subsequently result to dangerous accident. In this work comparism of different test on water-cooled reactor design (WCRD) models with respect to control rod failure during operation or accident was carried out by testing for thermal efficiency and thermal power using regression analysis technique before conclusion.

Thus, when a reactor is suddenly shut down by the insertion of the control rods, all of the prompt energy sources are removed because almost all fissions stop. β - decay and γ - decay, however, do not stop. Reactor thermal output does not drop to zero immediately after shut down; instead it drops to approximately 7 percent of the pre-shutdown power
and continues to decrease at a slower and slower rate as the fission fragments $\beta$- decay and $\gamma$- decay to stable daughter products.

2. Control Rod

Definition 1: “A control rod” is one of a number of rods or tubes containing a neutron absorber, such as boron, that can be inserted into or retracted from the core of a nuclear reactor in order to control its rate of reaction. That is a control rod is a rod used in nuclear reactors to control the rate of fission of uranium and plutonium.

They are made of chemical elements capable of absorbing many neutrons without fissioning themselves, such as boron, silver, indium and cadmium. Because these elements have different capture cross sections for neutrons of varying energies, the compositions of the control rods must be designed for the neutron spectrum of the reactor it is supposed to control. Light water reactors (BWR, PWR) and heavy water reactors (HWR) operate with “thermal” neutrons, whereas breeder reactors operate with "fast" neutrons.

A Reactor Control Rod defines an active fuel column within a multiblock reactor. Beneath a control rod is a column of yttrium fuel rods, which contain the fuel and waste in that fuel column. The control rod provides a UI to monitor the fuel column, showing you the column's overall heat and relative fuel/waste mixture. A control rod also provides a radiation-moderating "Control Rod", which can be extended into, or retracted from, a fuel column.

The further a control rod is extended into a fuel column, the less radioactivity will be generated by the fuel column, resulting in lower heat and power production, but also lower fuel consumption. A fully-extended control rod effectively shuts off a single fuel column.

Definition 2: SCRAM is an operation that shuts down a nuclear reactor. In a reactor, a SCRAM is achieved by a large insertion of negative reactivity by insertion of the control rods.

In some PWR reactors, special control rods are used to enable the core to sustain a low level of power efficiently. The Figure 1 presents PWR control rod assembly.

![Figure 1: PWR Control Rod Assembly, above Fuel Element](image)

Definition 3: The fuel grids consist of an egg-crate arrangement of interlocked straps that maintain lateral spacing between the rods. The grid straps have spring fingers and dimples that grip and support the fuel rods. The intermediate mixing vane grids also have coolant mixing vanes. In addition, there are four intermediate flow mixing (IFM) grids. The IFM grid straps contain support dimples and coolant mixing vanes only. The top and bottom grids do not contain mixing vanes.

Definition 4: Fuel rods are the containers for the uranium used in nuclear power plants or Fuel rod is a protective metal tube containing pellets of fuel for a nuclear reactor. That is fuel rod is a long tube, often made of a zirconium
alloy and containing uranium-oxide pellets, that is stacked in bundles of about 200 to provide the fuel in certain types of nuclear reactor. Fuel rods are assembled into bundles called fuel assemblies, which are loaded individually into the reactor core.

**Definition 5:** Fuel element is an arrangement of a number of fuel rods into which the nuclear fuel is inserted in the reactor. That is fuel element consisting of nuclear fuel and other materials for use in a reactor. A fuel element of a pressurized water reactor (PWR) contains about 530 kg, that of a boiling water reactor (BWR) about 190 kg of uranium. The pressurized water reactor of the Emsland nuclear power plant uses 193 fuel elements and the Krümmel boiling water reactor 840. A fuel element failure is a rupture in a nuclear reactor's fuel cladding that allows the nuclear fuel or fission products, either in the form of dissolved radioisotopes or hot particles, to enter the reactor coolant or storage water.

Control rods are devices that isolate the fuel elements and absorb neutrons. When a control rod is raised, exposing more of the fuel element to thermal neutrons, the rate of reaction increases; when it is lowered, it isolates the fuel element, and the reaction slows or stops. If control rods are not exercised correctly, an exponential unsteady state can occur by either increasing (also known as a supercritical state) or decreasing (also known as a subcritical state) the rate of nuclear reaction.

**Definition 6:** The nuclear fuel is a material that can be 'burned' by nuclear fission or fusion to derive nuclear energy.

**Definition 7:** Fuel temperature coefficient of reactivity is the change in reactivity of the nuclear fuel per degree change in the fuel temperature. The coefficient quantifies the amount of neutrons that the nuclear fuel (uranium-238) absorbs from the fission process as the fuel temperature increases. It is a measure of the stability of the reactor operations. This coefficient is also known as the Doppler coefficient [5].

When the nuclear fuel increases in temperature, the rapid motion of the atoms in the fuel causes an effect known as Doppler broadening. When thermal motion causes a particle to move towards the observer, the emitted radiation will be shifted to a higher frequency. Likewise, when the emitter moves away, the frequency will be lowered. For non-relativistic thermal velocities, the Doppler shift in frequency will be:

\[
f = f_0 \left(1 + \frac{v}{c}\right)\]

where \(f\) is the observed frequency, \(f_0\) is the rest frequency, \(v\) is the velocity of the emitter towards the observer, and \(c\) is the speed of light.

Since there is a distribution of speeds both toward and away from the observer in any volume element of the radiating body, the net effect will be to broaden the observed line.

If \(P_v(v)dv\) is the fraction of particles with velocity component \(v\) to \(v + dv\) along a line of sight, then the corresponding distribution of the frequencies is:

\[
P_f(f)df = P_v(v_f)\frac{dv}{df}df\]

where

\[v_f = c \left(\frac{f}{f_0} - 1\right)\]

is the velocity towards the observer corresponding to the shift of the rest frequency \(f_0\) to \(f\).

Therefore,
We can also express the broadening in terms of the wavelength $\lambda$. Recalling that in the non-relativistic limit

$$\frac{\lambda - \lambda_0}{\lambda_0} \approx -\frac{f - f_0}{f_0},$$

we obtain

$$P_\lambda(\lambda) d\lambda = \frac{c}{\lambda_0} P_v \left( c \left( 1 - \frac{\lambda}{\lambda_0} \right) \right) d\lambda.$$  

In the case of the thermal Doppler broadening, the velocity distribution is given by the Maxwell distribution

$$P_v(v) dv = \sqrt{\frac{m}{2\pi kT}} \exp\left( -\frac{mv^2}{2kT} \right) dv,$$

where,

- $m$ is the mass of the emitting particle,
- $T$ is the temperature and
- $k$ is the Boltzmann constant.

Then,

$$P_f(f) df = \left( \frac{c}{f_0} \right) \sqrt{\frac{m}{2\pi kT}} \exp\left( -\frac{m \left[ c \left( \frac{f}{f_0} - 1 \right) \right]^2}{2kT} \right) df,$$

which we immediately recognize as a Gaussian profile with the standard deviation

$$\sigma_f = \sqrt{\frac{kT}{mc^2} f_0}$$

and full width at half maximum (FWHM)

$$\Delta f_{\text{FWHM}} = \sqrt{\frac{8kT \ln 2}{mc^2} f_0}.$$  

The fuel sees a wider range of relative neutron speeds. Uranium-238, which forms the bulk of the uranium in the reactor, is much more likely to absorb fast or epithermal neutrons at higher temperatures. This reduces the number of neutrons available to cause fission, and reduces the power of the reactor. Doppler broadening therefore creates a negative feedback because as fuel temperature increases, reactor power decreases. All reactors have reactivity feedback mechanisms, except some gas reactor such as pebble-bed reactor which is designed so that this effect is very strong and does not depend on any kind of machinery or moving parts.

3. **Nuclear Fuel Performance in Reactors**

Nuclear fuel operates in a harsh environment in which high temperature, chemical corrosion, radiation damage and physical stresses may attack the integrity of a fuel assembly. The life of a fuel assembly in the reactor core is therefore regulated to a burn-up level at which the risk of its failure is still low. Fuel ‘failure’ refers to a situation when the cladding has been breached and radioactive material leaks from the fuel ceramic (pellet) into the reactor.
cooler water. The radioactive materials with most tendencies to leak through a cladding breach into the reactor coolant are fission-product gases and volatile elements, notably; krypton, xenon, iodine and cesium.

Fuel leaks do not present a significant risk to plant safety, though they have a big impact on reactor operations and (potentially) on plant economics. For this reason, primary coolant water is monitored continuously for these species so that any leak is quickly detected. The permissible level of released radioactivity is strictly regulated against specifications which take into account the continuing safe operation of the fuel. Depending on its severity a leak will require different levels of operator intervention:
1. Very minor leak: no change to operations – the faulty fuel assembly with leaking rod(s) is removed at next refueling, inspected, and possibly re-loaded.
2. Small leak: allowable transient for the reactor are restricted. This might prevent the reactors from being able to operate in a “load-follow” mode and require careful monitoring of reactor physics. The faulty fuel assembly with leaking rod is generally removed and evaluated at the next scheduled refueling.
3. Significant leak: the reactor is shut down and the faulty assembly located and removed.

A leaking fuel rod can sometimes be repaired but it is more usual that a replacement assembly is needed (this having a matching level of remaining enrichment). Replacement fuel is one cost component associated with failed fuel. There is also the cost penalty and/or replacement power from having to operate at reduced power or having an unscheduled shutdown. There may also be higher operation and maintenance costs associated with mitigating increased radiation levels in the plant.

Fuel management is a balance between the economic imperative to burn fuel for longer and the need to keep within failure-risk limits. Improving fuel reliability extends these limits, and therefore is a critical factor in providing margin to improve fuel burn-up.

The nuclear industry has made significant performance improvements reducing fuel failure rates by about 60% in the 20 years to 2006 to an average of some 14 leaks per million rods loaded [IAEA 2010]. The reliability drive continues. Industry-wide programs led by EPRI and the US Institute of Nuclear Power Operations (INPO) have produced guidelines to help eliminate fuel failures (there was an ambitious goal to achieve zero fuel failures by 2010). Fuel engineering continues to improve, eg, with more sophisticated debris filters in assembly structures. Utilities themselves impose more rigorous practices to exclude foreign material entering primary coolant water. Global Nuclear Fuels (GNF) in 2013 had 2 million fuel rods in operation and claimed to have no leakers among them. (In the early 1970s hydriding and pellet-clad interaction caused a lot of leaks. The 1980s saw an order of magnitude improvement).

At the same time there has been a gradual global trend toward higher fuel burnup*, however, there is a limit on how far fuel burnup can be stretched given the strict criticality safety limitation imposed on fuel fabrication facilities such that a maximum uranium enrichment level of 5% can be handled.

* **Higher burnup does not necessarily mean better energy economics.** Utilities must carefully balance the benefits of greater cycle length against higher front-end fuel costs (uranium, enrichment). Refueling outage costs may also be higher, depending on length, frequency and the core re-load fraction. An equally important trend in nuclear fuel engineering is to be able to increase the power rating for fuels, ie, how much energy can be extracted per length of fuel rod. Currently this is limited by the material properties of the zirconium cladding.

4. **Materials used for Control Rod**

Chemical elements with a sufficiently high capture cross section for neutrons include silver, indium and cadmium. Other elements that can be used include boron, cobalt, hafnium, dysprosium, gadolinium, samarium, erbium, and europium, or their alloys and compounds, e.g. high-boron steel, silver-indium-cadmium alloy, boron carbide, zirconium diboride, titanium diboride, hafnium diboride, gadolinium titanate, and dysprosium titanate. The choice of materials is influenced by the energy of neutrons in the reactor, their resistance to neutron-induced swelling, and the required mechanical and lifetime properties. The rods may have the form of stainless steel tubes filled with neutron absorbing pellets or powder. The swelling of the material in the neutron flux can cause deformation of the rod, leading to its premature replacement. The burn up of the absorbing isotopes is another limiting lifetime factor.

5. **Materials Selection of Control Rod**

The material used for the control rods varies depending on reactor design. Generally, the material selected should have a good absorption cross section for neutrons and have a long lifetime as an absorber (not burn out rapidly). The ability of a control rod to absorb neutrons can be adjusted during manufacture. A control rod that is referred to as a "black" absorber absorbs essentially all incident neutrons. A "grey" absorber absorbs only a part of them. While it takes more grey rods than black rods for a given reactivity effect, the grey rods are often preferred because they cause smaller depressions in the neutron flux and power in the vicinity of the
rod. This leads to a flatter neutron flux profile and more even power distribution in the core. If grey rods are desired, the amount of material with a high absorption cross section that is loaded in the rod is limited.

Material with a very high absorption cross section may not be desired for use in a control rod, because it will burn out rapidly due to its high absorption cross section. The same amount of reactivity worth can be achieved by manufacturing the control rod from material with a slightly lower cross section and by loading more of the material. This also results in a rod that does not burn out as rapidly. Another factor in control rod material selection is that materials that resonantly absorb neutrons are often preferred to those that merely have high thermal neutron absorption cross sections. Resonance neutron absorbers absorb neutrons in the epithermal energy range. The path length traveled by the epithermal neutrons in a reactor is greater than the path length traveled by thermal neutrons.

Therefore, a resonance absorber absorbs neutrons that have their last collision farther (on the average) from the control rod than a thermal absorber. This has the effect of making the area of influence around a resonance absorber larger than around a thermal absorber and is useful in maintaining a flatter flux profile.

6. Types of Control Rods
There are several ways to classify the types of control rods. One classification method is by the purpose of the control rods. Three purposes of control rods are listed as follows:
(i) Shim rods - used for coarse control and/or to remove reactivity in relatively large amounts.
(ii) Regulating rods - used for fine adjustments and to maintain desired power or temperature.
(iii) Safety rods - provide a means for very fast shutdown in the event of an unsafe condition. Addition of a large amount of negative reactivity by rapidly inserting the safety rods is referred to as a "scram" or "trip."

7. Operation Principle
Control rods are usually combined into control rod assemblies — typically 20 rods for a commercial Pressurized Water Reactor (PWR) assembly — and inserted into guide tubes within a fuel element. A control rod is removed from or inserted into the central core of a nuclear reactor in order to control the neutron flux — to increase or decrease the number of neutrons which will split further uranium atoms. This in turn affects the thermal power of the reactor, the amount of steam produced, and hence the electricity generated.

Control rods often stand vertically within the core (figure 1). In pressurised water reactors (PWR), they are inserted from above, the control rod drive mechanisms being mounted on the reactor pressure vessel head. But in boiling water reactor (BWR) the control rods is inserted from underneath the core, this is due to the necessity of a steam dryer above the core. The control rods are partially removed from the core to allow a chain reaction to occur. The number of control rods inserted and the distance by which they are inserted can be varied to control the reactivity of the reactor.

8. Safety Measure for Control Rods
In most reactor designs, as a safety measure, control rods are attached to the lifting machinery by electromagnets, rather than direct mechanical linkage. This means that automatically in the event of power failure, or if manually invoked due to failure of the lifting machinery, the control rods will fall, under gravity, fully into the pile to stop the reaction. A notable exception to this fail-safe mode of operation is the BWR which requires the hydraulic insertion of control rods in the event of an emergency shut-down, using water from a special tank that is under high nitrogen pressure. Quickly shutting down a reactor in this way is called scramming the reactor.

The subject of control rods and control assemblies has been treated only as a part of more extensive safety studies of International Atomic Energy Agency (IAEA) programmes in the past[4]. In the Chernobyl reactor 4, control rod was characteristics cause of instability in the reactor (a rapid uncontrollable power surge) during low power operation (now known to correspond to a power level of less than about 700 MW), due to a phenomenon known as a positive void coefficient.

In a water cooled reactor, steam may accumulate to form pockets, known as voids. If excess steam is produced, creating more voids than normal, operation of the reactor is disturbed because:
1) water is a more efficient coolant than steam
2) water acts a moderator and neutron absorber
A reactor is said to have a positive void coefficient if excess steam voids lead to increased power generation. Positive void coefficients can lead to rapid power increases because power increases lead to increased steam generation. Most reactor’s have a negative void coefficient because water is used as both moderator and coolant, and steam generation also reduces the moderation (fail safe).
9. Criticality Accident Prevention
Mismanagement or control rod failure was often the cause or aggravating factor for nuclear accidents[6]. Homogeneous neutron absorbers have often been used to manage criticality accidents which involve aqueous solutions of fissile metals. In several such accidents, either borax (sodium borate) or a cadmium compound has been added to the system. The cadmium can be added as a metal to nitric acid solutions of fissile material; the corrosion of the cadmium in the acid will then generate cadmium nitrate in situ.

In carbon dioxide-cooled reactors such as the AGR, if the solid control rods were to fail to arrest the nuclear reaction, nitrogen gas can be injected into the primary coolant cycle. This is because nitrogen has a larger absorption cross-section for neutrons than carbon or oxygen; hence, the core would then become less reactive.

10. Operation of Nuclear Control Rods
Nuclear reactors work by using the heat generated by nuclear fission to produce steam that powers a turbine to produce electricity. Fission is when the nucleus of an atom (in most cases Uranium-235) splits in two, creating heat and expelling free neutrons. When these neutrons collide with other U-235 atoms, its causes more fission; this creates a nuclear chain reaction. If left unchecked the chain reaction will grow exponentially and result in a nuclear meltdown.

A nuclear reactor needs to maintain enough of reaction to generate heat, but not allow the core to become super critical and melt down. To do this control rod, which are made of a neutron absorbing material, are placed into the core and are literally raised and lowered to tweak the reaction – if you need to generate more heat, you raise the rods out of the core to let more neutrons split more atoms.

To curb the reaction, you lower the rods into the core to absorb more of the neutrons before they have a chance to come in contact with the uranium. In emergency cases (like recently in Japan), the rods are automatically shoved into the core using gravity, hydraulics or a mechanical spring, causing the chain reaction to stop. This is called “SCRAMing” the reactor. The table 1 presents required numerical parameters reactor AP1000 design control document.

Table 1: Reactor Design Comparison Table

<table>
<thead>
<tr>
<th>Reactor Design Comparison Table</th>
<th>AP1000</th>
<th>AP600</th>
<th>Typical XL Plant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rod Cluster Control Assemblies</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neutron Absorber</td>
<td></td>
<td></td>
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<tr>
<td>RCCA</td>
<td>24 Ag-In-Cd Rodlets 20 304 SS rodlets</td>
<td>24 Ag-In-Cd Rodlets 20 304 SS Rodlets 4 Ag-In-Cd rodlets</td>
<td>24 Hafnium or Ag-In-Cd</td>
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<tr>
<td>GRCA</td>
<td>4 Ag-In-Cd Rodlets</td>
<td>4 Ag-In-Cd Rodlets</td>
<td></td>
</tr>
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<td>Type 304 SS, cold-worked</td>
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<td>0.0185</td>
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<tr>
<td>Number of clusters</td>
<td>53 RCCAs 16 GRCAs</td>
<td>45 RCCAs 16 GRCAs</td>
<td>57 RCCAs 0 GRCAs</td>
</tr>
</tbody>
</table>

11. Mathematical Definition of Reliability
The life of a system or a device under reliability study follows a sequence that results in an observable time to failure. A new device is put into service, it functions acceptably for a period of time and then it fails to function satisfactorily. The observed time to failure is a value of the random variable T, which represents the lifetime of the device. T takes its values in an interval of the real numbers, R, most often in the closed interval [0,∞). Since the lifetime of a device is represented by a random variable T, there is a probability distribution function (cdf) of T,

\[
FT(t) = P(T \leq t), \quad 0 < t \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdOTS

FT(t) = \int_{0}^{t} f_T(t') \, dt' \tag{11}

FT(t) is usually called the unreliability at time t. It represents the probability of failure in the interval [0,t]. The probability of failure in the interval (t1,t2) equals F(t2) − F(t1).

Definition 5: The reliability function is:

\[
RT(t) = P(T > t) = 1 − FT(t) \tag{12}
\]

Thus, reliability is the probability of no failures in the interval [0,t] or equivalently, the probability of failure after time t. Sometimes T will take on only a countable number of values in R. This case, called the discrete case, occurs when T is a number of cycles, for example, or when the failure time can occur at only discrete points.

Most of the time, however, T will be a continuous random variable and its distribution FT(t) will be a continuous distribution having a density fT(t).
Reliability with Continuous Random Variables:
Assume T is a continuous random variable, taking values in open interval \((0,\infty)\) and with density function \(f_T(t)\). The reliability function \(RT(t)\) is:

\[
RT(t) = \int_t^\infty f_T(x)dx = 1 - \int_0^t f_T(x)dx = 1 - FT(t) \quad \ldots \quad (13)
\]

where, \(FT(t) \geq 0\) and \(\int_0^\infty f_T(x)dx = 1\)

12. Failure and Accident Analysis
Some reports on the failures- trip of control rods and system failure analysis include “Advances in Control Assembly Materials for Water Reactors”[6], “Regulatory Guide”[7], “The unsteady state operation of chemical reactors”[8] “Derivation of correlation coefficient formula for determination of Doppler angle using time domain correlation ultrasonic flowmeter”[9], “Doppler coefficient of reactivity — benchmark calculations for different enrichments of \(\text{uo}_2\)”[10] and “Investigating Progress in Arab Electricity Markets”[11].

These accidents may perhaps be a result of design concept process of some of these reactors (which could involve novel technologies) that have inherent risk of failure in operation and were not well studied/understood. In avoiding such accidents the industry has been very successful. As in over 14,500 cumulative reactor-years of commercial operation in 32 countries, there have been only three major accidents to nuclear power plants – Fukushima, Chernobyl and Three Mile Island. As in other industries, the design and operation of nuclear power plants aims to reduce the likelihood of accidents, and avoid major human consequences when they occur.

However, recent study of the reactor fuel under accident conditions, reveal that after subjecting the fuel to extreme temperatures — far greater temperatures than it would experience during normal operation or postulated accident conditions — TRISO fuel is even more robust than expected. Specifically, the research revealed that at 1,800 degrees Celsius (more than 200 degrees Celsius greater than postulated accident conditions) most fission products remained inside the fuel particles, which each boast their own primary containment system.

13. Methodology
A brief discussion of some past and recent accident of nuclear power plant due to control rod trip – failures was investigated. Literature review of risk of the control rod trip failures and comparative analyses of incidents caused by trip failures was also carry out. The design parameters of control rod were used to test the correlation between reactor safety margin and fuel temperature were highlighted. Therefore, the safety factor \(\bar{Y}\), of the reactor can be calculated or determined using the linear regression empirical formula.

In this work, Ordinary Least Square (OLS) methodology, which is largely used in nuclear industry for modeling safety is employed. Some related previous works on the application of regression analysis technique include: “Statistical Analysis of Reactor Pressure Vessel Fluence Calculation Benchmark Data Using Multiple Regression Techniques”[12], “Simplified modeling of a PWR reactor pressure vessel lower head failure in the case of a severe accident”[13].


14. Objective of the Research
In this work comparism of different test on water-cooled reactor design (WCRD) models with respect to drop failure or malfunction of control rod during operation or accident was carried out by testing for thermal efficiency and thermal power using regression analysis technique before reaching conclusion. The research aimed at demonstrating sufficient safety margins, for nuclear power plants. One objective of this research is to evaluate power system reliability analysis improvements with distributed generators while satisfying equipment handling constraints.

In this research, a computer algorithm involving pointers and linked list is developed to analyze the power system reliability. This algorithm needs to converge rapidly as it is to be used for systems containing thousands of components. So an efficient “object-oriented” computer software design and implementation is investigated. This algorithm is also used to explore the placement of control rod and how the different placements affect system
reliability, which has not been done in previous research. This exploration makes possible the comparison of alternative system designs to discover systems yielding desired reliability material properties. In this paper, variation of system reliability with the varying loads is also investigated. Other publications of distribution system reliability analysis associated with time varying loads have not been found.

15. Motivation of the Research

The purpose of this work is to assist countries wishing to include nuclear energy for the generation of electricity, like Nigeria, to secure a reactor that is better and safe. Also, the studies intended to provide guidance in developing practical catalytic materials for power generation reactor and to help researchers make appropriate recommendation for Nigeria nuclear energy proposition as one of the solutions to Nigeria energy crisis. Moreover, this is to help Nigeria meet its international obligations to use nuclear technology for peaceful means. Finally, the achievement is to make worldwide contribution to knowledge.

16. Research Design/Approach

The design of control rods plays significant role in the safety of the reactor as in the case of emergency it allows the safe shut-down of the nuclear power plant and prevent reactor meltdown during accident. Hence, in this work, a statistical analysis of a design input parameters of some typical reactor water-cooled reactor was investigated for safety under a failed control rod dropping. Specifically, the studies concentrated on technical factors that limit the functionality of control rods, such as the mechanical interaction, malfunctioning, failure and the reactor thermal efficiency and thermal power. More also, the study examined the temperature of the fuel behaviour under reactor accident conditions. The Table 2 presents data input for safety margin against thermal power and thermal efficiency of some typical water-cooled reactor design model.

Table 2: Data input for thermal power and thermal efficiency of some typical water-cooled reactor design model.
Source : [3]

<table>
<thead>
<tr>
<th>Nos. of trial (j)</th>
<th>Thermal Power (MW)</th>
<th>Thermal Power (MWe)</th>
<th>Thermal Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>100</td>
<td>30.00</td>
</tr>
<tr>
<td>2</td>
<td>210</td>
<td>105</td>
<td>31.00</td>
</tr>
<tr>
<td>3</td>
<td>215</td>
<td>107</td>
<td>32.50</td>
</tr>
<tr>
<td>4</td>
<td>218</td>
<td>110</td>
<td>33.30</td>
</tr>
<tr>
<td>5</td>
<td>225</td>
<td>112</td>
<td>34.80</td>
</tr>
<tr>
<td>6</td>
<td>233</td>
<td>115</td>
<td>35.00</td>
</tr>
<tr>
<td>7</td>
<td>240</td>
<td>117</td>
<td>36.70</td>
</tr>
<tr>
<td>8</td>
<td>247</td>
<td>119</td>
<td>41.00</td>
</tr>
<tr>
<td>9</td>
<td>250</td>
<td>120</td>
<td>45.00</td>
</tr>
<tr>
<td>10</td>
<td>253</td>
<td>123</td>
<td>47.60</td>
</tr>
<tr>
<td>11</td>
<td>260</td>
<td>129</td>
<td>49.80</td>
</tr>
<tr>
<td>12</td>
<td>263</td>
<td>130</td>
<td>50.00</td>
</tr>
</tbody>
</table>

Table 3 highlights input data for safety margin with respect to fuel size in a typical water-cooled reactor.

Table 3: Input data for fuel size and heat generated in a typical water-cooled reactor.
Source : [3]

<table>
<thead>
<tr>
<th>Nos. of trial (j)</th>
<th>Fuel size in Mass (g)</th>
<th>Heat Generated °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.8</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>3.5</td>
<td>270</td>
</tr>
<tr>
<td>3</td>
<td>4.2</td>
<td>300</td>
</tr>
<tr>
<td>4</td>
<td>5.0</td>
<td>440</td>
</tr>
<tr>
<td>5</td>
<td>5.7</td>
<td>480</td>
</tr>
<tr>
<td>6</td>
<td>6.0</td>
<td>520</td>
</tr>
<tr>
<td>7</td>
<td>7.4</td>
<td>600</td>
</tr>
<tr>
<td>8</td>
<td>8.3</td>
<td>760</td>
</tr>
<tr>
<td>9</td>
<td>9.0</td>
<td>900</td>
</tr>
<tr>
<td>10</td>
<td>10.6</td>
<td>1050</td>
</tr>
<tr>
<td>11</td>
<td>11.0</td>
<td>1100</td>
</tr>
<tr>
<td>12</td>
<td>12.0</td>
<td>1200</td>
</tr>
</tbody>
</table>
RESULTS AND ANALYSES

1. Water-Cooled Reactor Design Model (WCRDM)

The result of the application of the linear regression analysis of the data in Tables 2 and 3 of some typical water-cooled reactor design model is presented as follows:

(i) Empirical Expression for Safety Factor, Ŷ

In the assessment of control rod drop failure on reactor stability and safety, the data obtained in Tables 2 and 3 which represents parameters for some typical water-cooled reactor design model was used in order to obtain the best fit for the model. The new conceptual fuel design for reactor operation could optimize the performance of this type of water-cooled reactor design model. The linear regression model equation to be solved is given by:

\[ Ŷ = B_0 + B_1X_j + e_j \]  

where,

- \( B_0 \) is an intercept,
- \( B_1 \) is the slope,
- \( X_j \) is the rate of increase in fuel volume
- \( e_j \) = error or residual, \( j = 1,2,3,...,k \) and \( k \) is the last term.

Empirical Expression for Safety Factor, Ŷ

The model empirical expression is the equation of the straight line relating heat in the reactor and the volume of fuel in the reactor as a measure of safety factor estimated as:

\[ Ŷ = (-49.6924) + (0.7664)(X_j) + e_j \]  

where,

- \( Ŷ \) = Dependent Variable, Intercept = -49.6924,
- Slope = 0.7664,
- \( X \) = Independent Variable,
- \( e \) = error or residual,
- \( j = 1,2,3,...,12 \) and
- 12 is the last term of trial.

The Figure 2 shows the linear regression plot section on thermal efficiency and thermal power

(ii) Linear Regression Plot on the relationship between thermal efficiency and thermal power
Figure 2: Thermal efficiency and Thermal power

(iii) **F-test Result**

Table 4: Summary of F-test Statistical Data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable</td>
<td>Y</td>
</tr>
<tr>
<td>Independent Variable</td>
<td>X</td>
</tr>
<tr>
<td>Intercept($B_0$)</td>
<td>-49.6924</td>
</tr>
<tr>
<td>Slope($B_1$)</td>
<td>0.7664</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.9135</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.9558</td>
</tr>
<tr>
<td>Mean Square Error (MSE)</td>
<td>$5.275179 \times 10^{-2}$</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>0.0591</td>
</tr>
<tr>
<td>Square Root of MSE</td>
<td>2.296776</td>
</tr>
</tbody>
</table>

Table 5: Descriptive Statistics Section

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dependent</th>
<th>Independent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Thermal efficiency</td>
<td>Thermal power</td>
</tr>
<tr>
<td>Count</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Mean</td>
<td>38.8917</td>
<td>115.5833</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>7.4476</td>
<td>9.2879</td>
</tr>
<tr>
<td>Minimum</td>
<td>30.0000</td>
<td>100.0000</td>
</tr>
<tr>
<td>Maximum</td>
<td>50.0000</td>
<td>130.0000</td>
</tr>
</tbody>
</table>

The Table 6 is the regression estimation section results that show the least-squares estimates of the intercept and slope followed by the corresponding standard errors, confidence intervals, and hypothesis tests. These results are based on several assumptions that are validated before they are used.
In Table 7 the analysis of variance shows that the F-Ratio testing whether the slope is zero, the degrees of freedom, and the mean square error. The mean square error, which estimates the variance of the residuals, was used extensively in the calculation of hypothesis tests and confidence intervals.

Table 7: Analysis of Variance Section

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Squares</th>
<th>F-Ratio</th>
<th>Prob Level</th>
<th>Power(5%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>1</td>
<td>18150.74</td>
<td>18150.74</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>1</td>
<td>557.3774</td>
<td>557.3774</td>
<td>105.6604</td>
<td>0.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>Error</td>
<td>10</td>
<td>52.75179</td>
<td>5.275179 X10^{-2}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adj. Total</td>
<td>11</td>
<td>610.1292</td>
<td>55.46629</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>12</td>
<td>18760.87</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

S = Square Root(5.275179 X10^{-2}) = 2.296776

In Table 8 Anderson Darling method confirms the rejection of H0 at 20% level of significance but all of the above methods agreed that H0 Should not be rejected at 5% level of significance. Hence the normality assumption is satisfied as one of the assumptions of the Linear Regression Analysis is that the variance of the error variable $\delta^2$ has to be constant.

Table 8: Tests of Assumptions Section

<table>
<thead>
<tr>
<th>Assumption/Test Residuals follow Normal Distribution?</th>
<th>Test Value</th>
<th>Prob Level</th>
<th>Is the Assumption Reasonable at the 20% or 0.2000 Level of Significance?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shapiro Wilk</td>
<td>0.8901</td>
<td>0.169812</td>
<td>No</td>
</tr>
<tr>
<td>Anderson Darling</td>
<td>0.5842</td>
<td>0.128324</td>
<td>No</td>
</tr>
<tr>
<td>D'Agostino Skewness</td>
<td>1.0600</td>
<td>0.289166</td>
<td>Yes</td>
</tr>
<tr>
<td>D'Agostino Kurtosis</td>
<td>-0.5545</td>
<td>0.579233</td>
<td>Yes</td>
</tr>
<tr>
<td>D'Agostino Omnibus</td>
<td>1.4310</td>
<td>0.488954</td>
<td>Yes</td>
</tr>
<tr>
<td>Constant Residual Variance?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modified Levene Test</td>
<td>0.3515</td>
<td>0.569628</td>
<td>Yes</td>
</tr>
<tr>
<td>Relationship is a Straight Line?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lack of Linear Fit F(0, 0) Test</td>
<td>0.0000</td>
<td>0.000000</td>
<td>No</td>
</tr>
</tbody>
</table>

Notes:
A 'Yes' means there is not enough evidence to make this assumption seem unreasonable.
A 'No' means that the assumption is not reasonable
(iv) Residual Plots Section

The plot section is used as further check on the validity of the model to satisfy all the assumptions of the linear regression analysis.

Amir D. Aczel (2002, P528) have stated that the normality assumption can be checked by the use of plot of errors against the predicted values of the dependent variable against each of the independent variable and against time (the order of selection of the data points) and on a probability scale.

The diagnostic plot for linear regression analysis is a scatter plot of the prediction errors or residuals against predicted values and is used to decide whether there is any problem in the data at hand Siegel F (2002, p.578).

The Figure 2 is for the plot of errors against the order to selection of the data points (e = 1,2,…,12). Although the order of selection was not used as a variable in the mode, the plot reveal whether order of selection of the data points should have been included as one of the variables in our regression model. This plot shows no particular pattern in the error as the period increases or decreases and the residuals appear to be randomly distributed about their mean zero, indicating independence. The residuals are randomly distributed with no pattern and with equal variance as volume of fuel increases.

![Figure 2: Residuals of Heat (°C) versus Fuel (g) ](image)

**Note:**
1. Residual = original value for heat (Y) minors predicted value for heat, Ŷ
2. Count = the design number (design 1, 2, 3, …, 12 )

Figure 3 shows the histogram of residuals of error (e₁) and this is nearly skewed to the right but the software used indicated that the plot is normal.
While Figure 4 is the result on plot graph of experimental errors. The residuals are perfectly normally distributed as most of the error terms align themselves along the diagonal straight line with some error terms outside the arc above and below the diagonal line. This further indicates that the estimated model is valid.

2. Summary/Conclusion
In summary this paper examined the possibilities to derive and implement a method for safety assessment based on regression analysis techniques. Regression analysis approach was applied to test the stability margin of the reactor when control rod fails. That is the research conducted safety margin test on some typical water-cooled reactor design (WCRD) models at an accident situation and at same time loss of emergency power supply occurred, secondly safety margin test was carried out on the thermal efficiency and thermal power output of the reactor when power supply failed and thirdly, safety margin test was perform on the reactor in relation to the high temperature effect within...
reactor core and the fuel temperature. The results of the statistical analysis on these types of nuclear reactor models reveals that the typical water-cooled reactor design (WCRD) models promises most stability under thermal efficiency of 45% and above.

The research implication is that the WCRD models could be significantly most stable at thermal efficiency of 45% and above. Secondly, the safety margin prediction of up to 4.42% has been validated for reactor design models on water-cooled reactor. The research effort served as an advantage over the current 5.1% challenging problem for plant engineers to predict the safety margin limit. According to Xianxun Yuan (2007, p49) in “Stochastic Modeling of Deterioration in Nuclear Power Plants Components” a challenging problem of plant engineers is to predict the end of life of a system safety margin up to 5.1% validation.

The current design limits for various reactors safety in a nuclear power plant, defined by the relative increase and decrease in the parametric range at a chosen operating point from its original value, varies from station to station. However, the finding in the work would suggest that the design of the plant should ensure that operating reactor core are made up of large graphite core in order to minimize core melting in an extreme high temperature condition which can damaged the reactor. It is suggested that the WCRD models “should allow for thermal efficiency of 45% and above in their construction and possibly provision for extra in-built control rods in the design features to ensure safe operation of nuclear reactor”.

If control rods technology solution must be addressed properly then the following areas of applicable EPS technology needs to be well study these include power system reliability analysis improvements with distributed generators while satisfying equipment power handling constraints. An efficient “object-oriented” computer software design and implementation needs employ for investigation. Dynamic and seismic analysis; safety and reliability; and verification and qualification of analysis with relevant software.

The design of the plant should ensure thermal efficiency of 45% during operation for safety purpose. The discoveries shall provide a novel approach and method for multi-objective decision-making based on seven dissimilar objectives attributes: materials selection, evolving technology, effectiveness, efficiency, cost, safety and failure. The implication of this research effort to Nigeria’s nuclear power project drive. It is therefore recommended that for countries wishing to include nuclear energy for the generation of electricity, like Nigeria, the design input parameters of the selected nuclear reactor should undergo test and analysis using this method for optimization and choice.

Acknowledgments
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